Enrico Nobile
Dipartimento di Ingegneria e Architettura
Università degli Studi di Trieste

Computational Methods for Fluid Dynamics and Heat Transfer

Homework No. 1
AA 2024/2025

April 2025

Proposed problem

A *trapezoidal* fin, as shown in figure 1, is made with a uniform, isotropic material with a thermal conductivity $k = 40$ [W/(m K)]; its length is $L = 200$ mm, and the thickness at the base of the fin is $t = 20$ mm and the thickness at the end is $e = 10$ mm (see figure 1).
The fin is cooled only by convection, with a convective heat transfer coefficient $h = 400$ [W/(m^2 K)], and the surrounding fluid temperature $T_\infty = 25$ [°C]. The base of the fin is maintained at a temperature $T_b = 200$ [°C], while also the tip of the fin contribute, with the same heat transfer coefficient, to the overall heat flux.

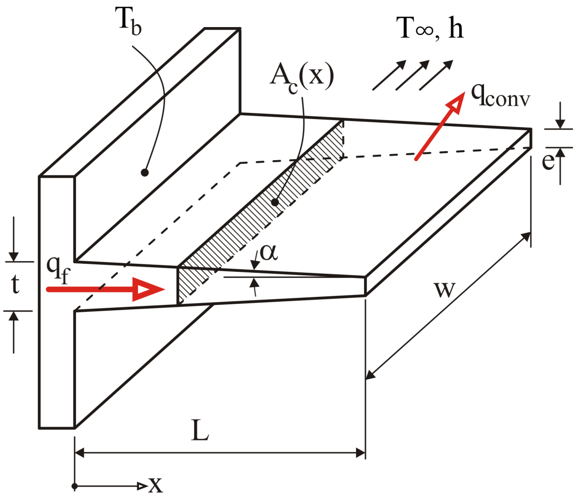

Figure 1: Trapezoidal fin.

Assuming a *1D* temperature distribution, i.e. $T \approx T(x)$, compute, with the Finite Volume method (FV), the heat flux per unit width q'_{num} [W/m], exchanged by the fin, using a number N of FVs equal to $N = 10$, 20, 40, and 80.
Plot in a *log-log* graph the behavior of the error vs *N*, verifying its quadratic trend. The error is defined as the difference between the numerical value of the computed heat flux q'_{num} and its analytical (exact) solution q'_f given in [1], which is obtained, in sequence, by the application of:

$$\tan\alpha = \frac{t-e}{2L} \tag{1}$$

$$K = \sqrt{\frac{h}{(k\sin\alpha)}} \tag{2}$$

$$\mu_a = 2K\left[\frac{e(1-\tan\alpha)}{2\tan\alpha}\right]^{1/2} \tag{3}$$

$$\mu_b = 2K\left[\left(L+\frac{e}{2}\right)+\frac{e(1-\tan\alpha)}{2\tan\alpha}\right]^{1/2} \tag{4}$$

$$q'_f = k\,\mu_b\,\tan\alpha\,(T_b - T_\infty)\,\frac{I_1(\mu_b)\,K_1(\mu_a) - I_1(\mu_a)\,K_1(\mu_b)}{I_0(\mu_b)\,K_1(\mu_a) + I_1(\mu_a)\,K_0(\mu_b)} \tag{5}$$

In (5) I_0 and I_1 represent the *modified Bessel functions of the first kind*, of *zero order* and of *order 1* respectively (in MATLAB: function `besseli`). K_0 and K_1 are instead the *modified Bessel functions of the second kind*, of *zero order* and of *order 1* respectively (in MATLAB: function `besselk`).

References

[1] A. D. Kraus, A. Aziz, J. Welty, EXTENDED SURFACE HEAT TRANSFER, J. Wiley & Sons, (2001).