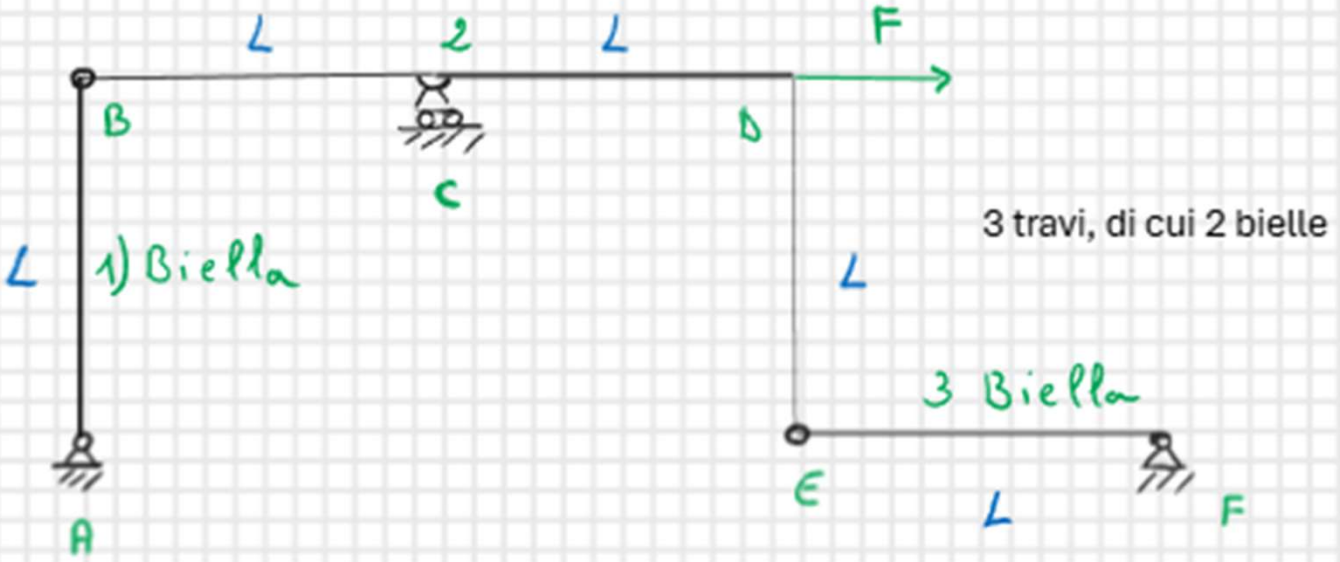
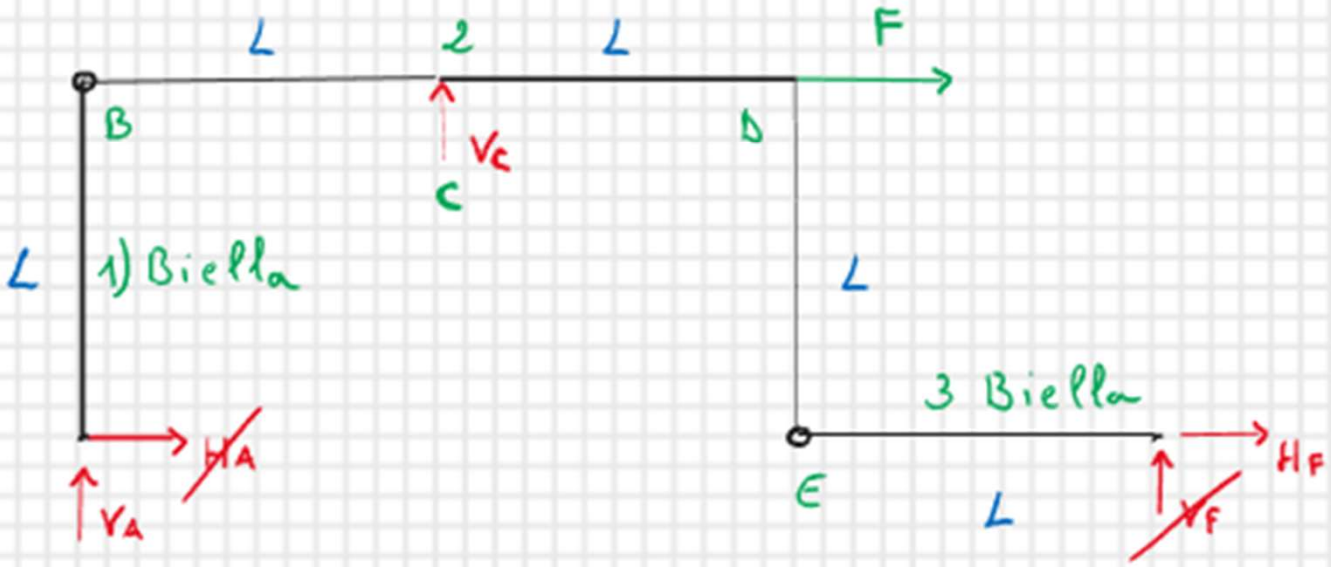
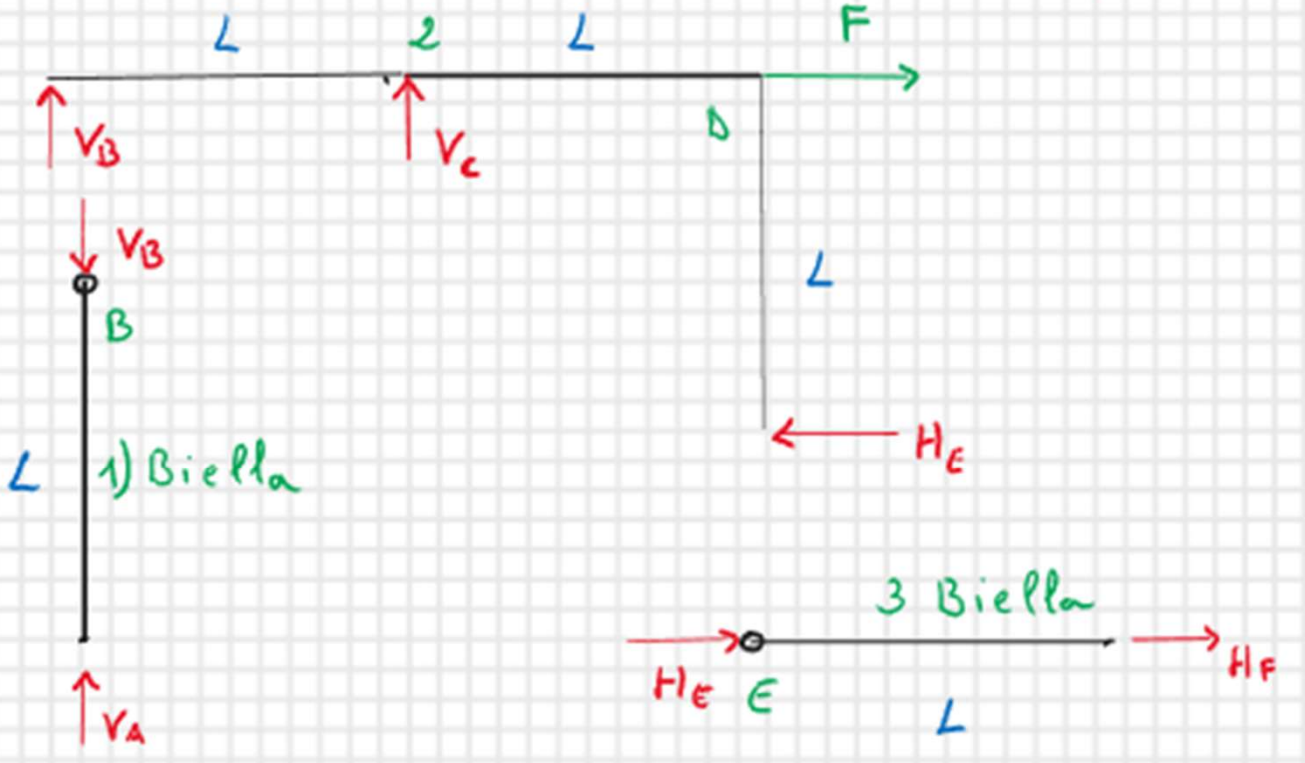
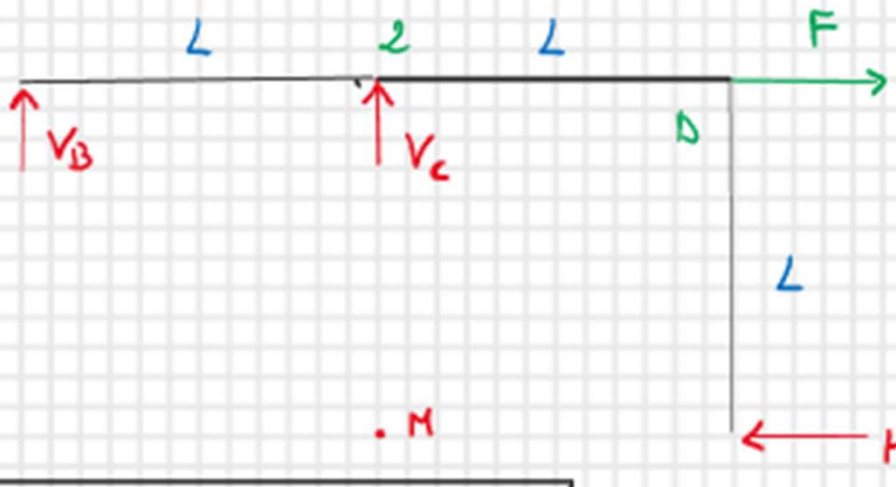
Esempio di struttura con biella



3 travi, di cui 2 bielle





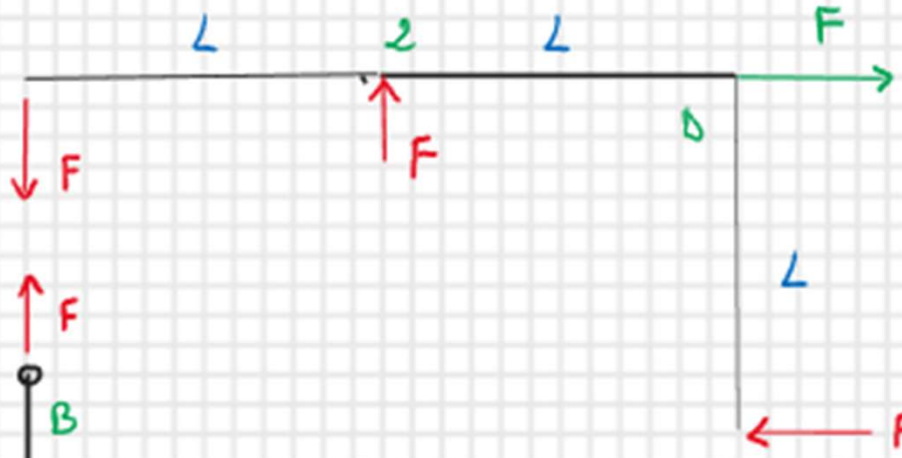


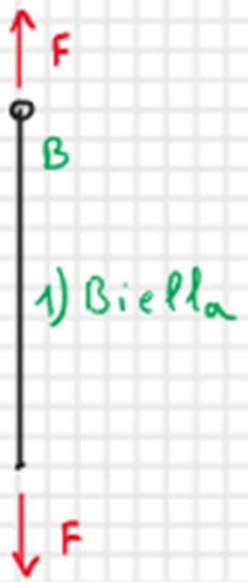
Risolvo la trave 2

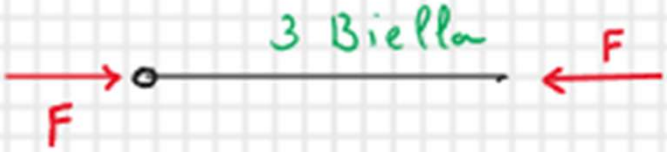
$H_E = F$

$V_B + V_C = 0$

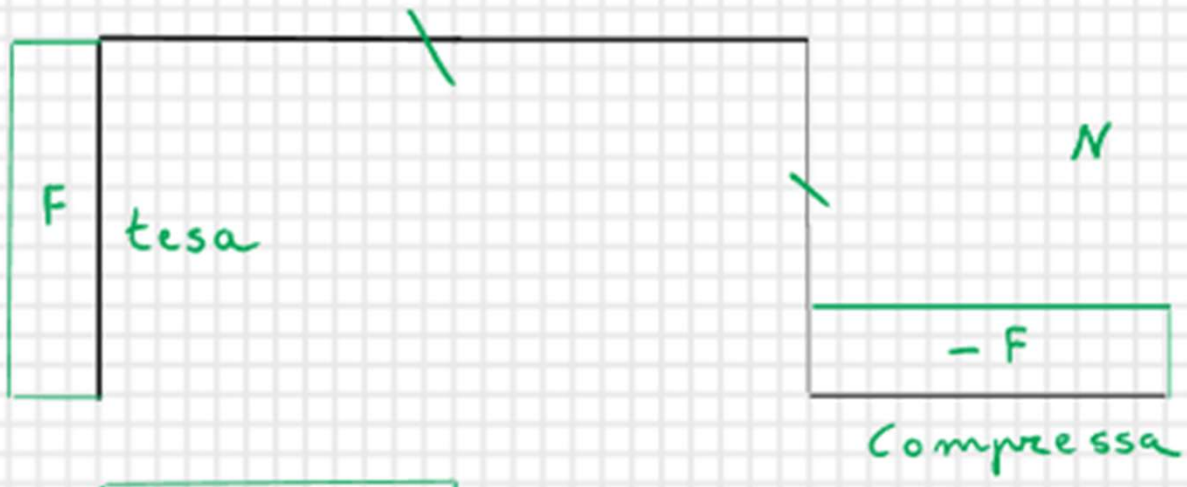
$V_B L + FL = 0$ in M

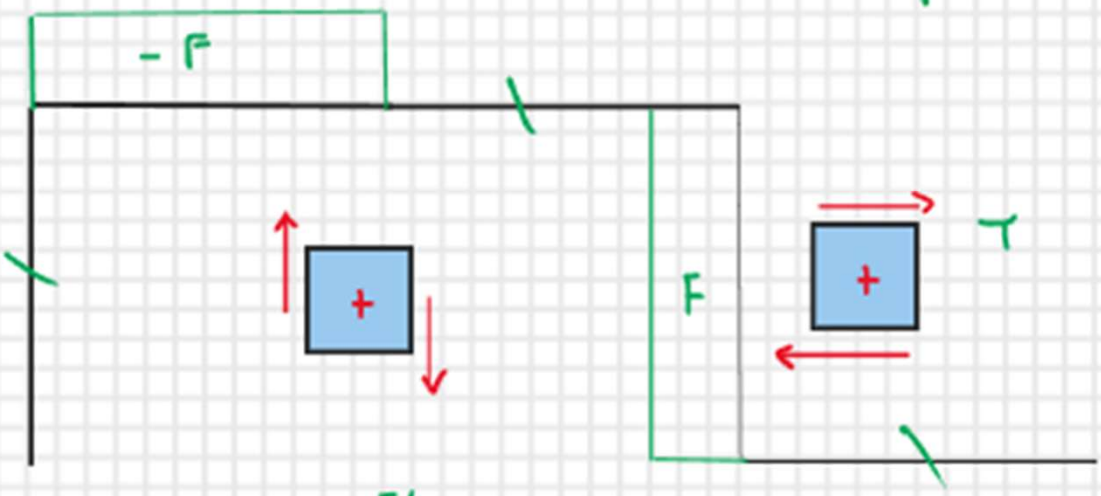


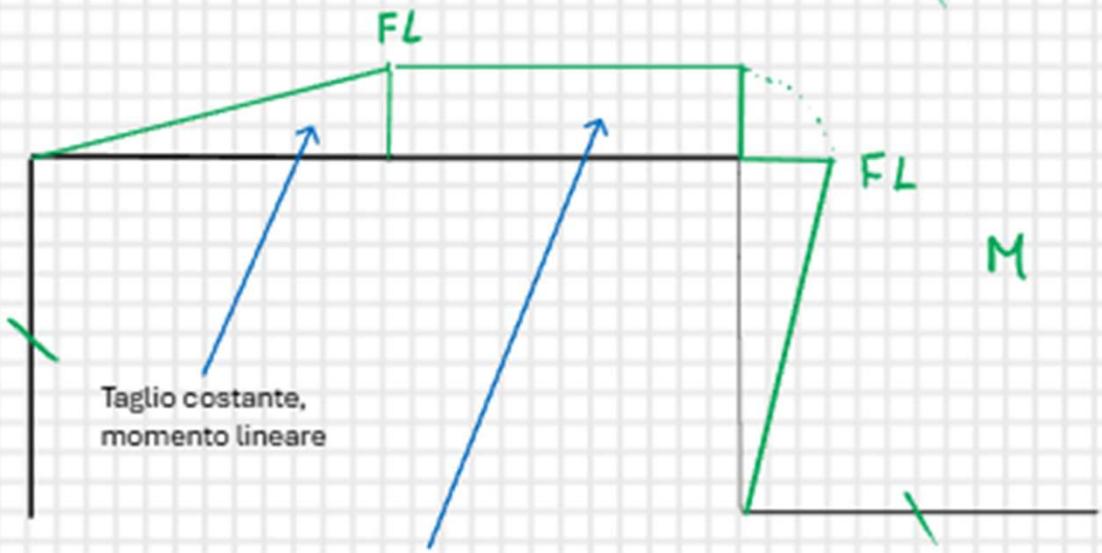




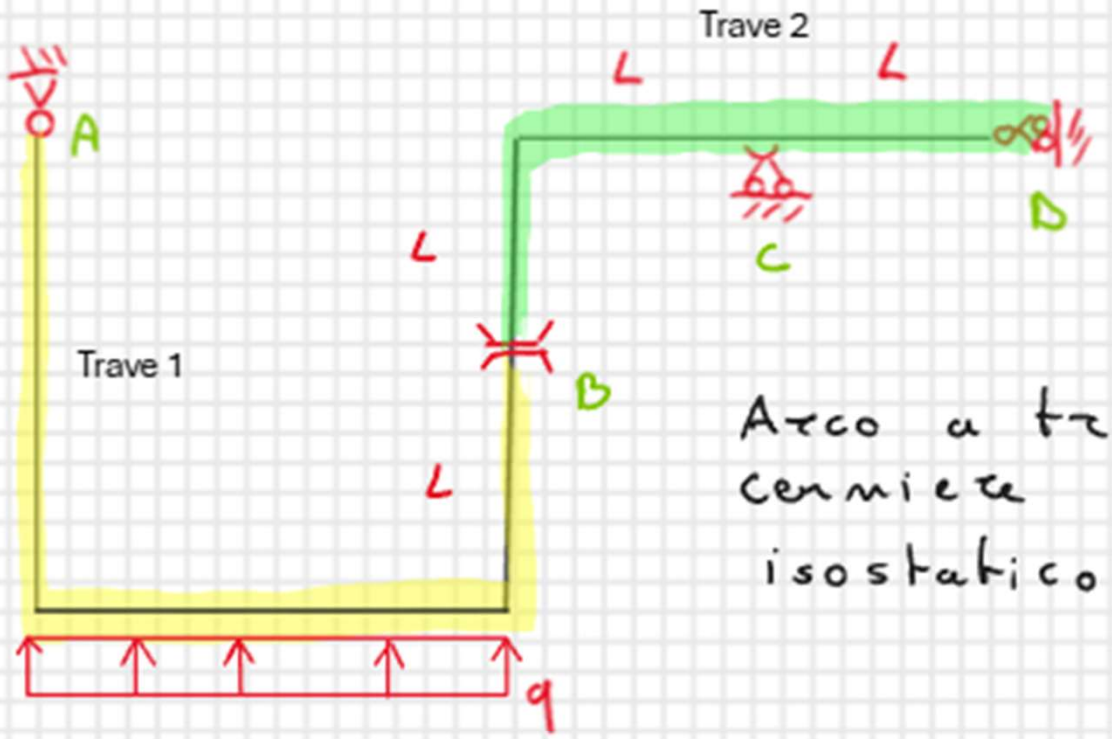
Diagrammi


F
tesa
N
- F
Compressa

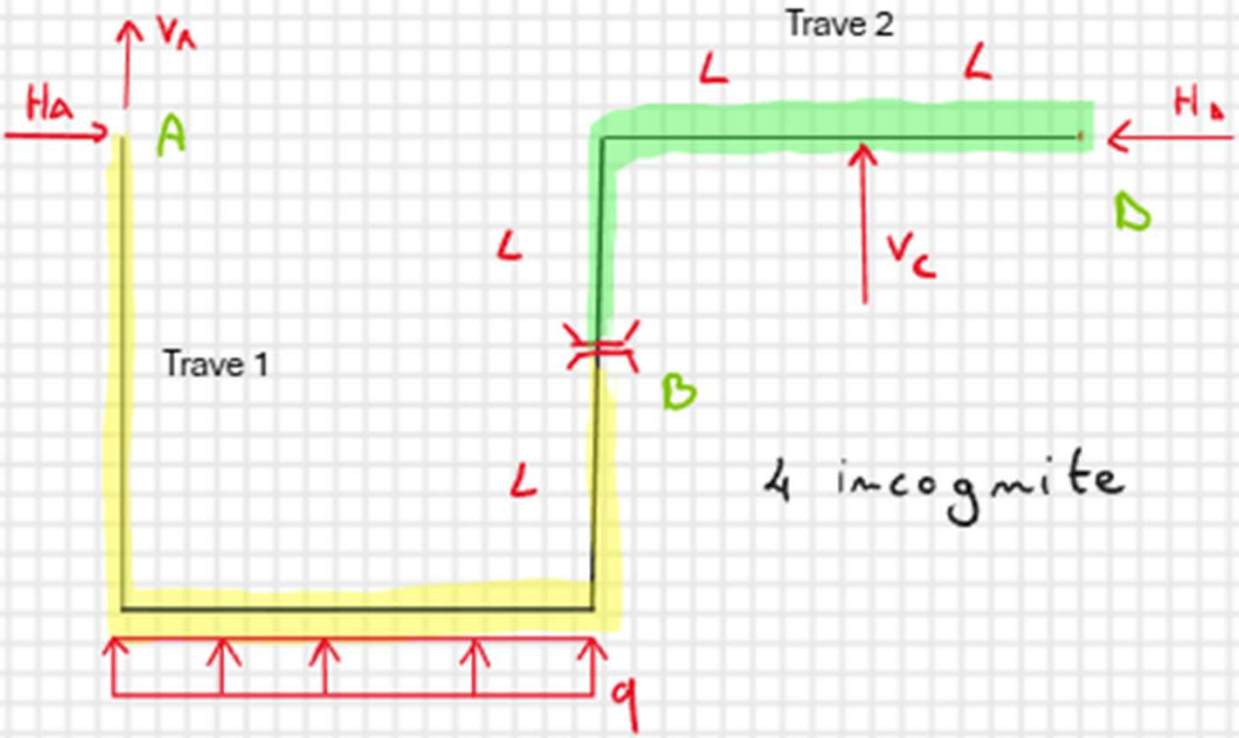

- F
+
F
+
T


FL
FL
M
Taglio costante,
momento lineare
Taglio nullo,
momento costante

Esempio



Arco a tre cerniere isostatico



4 incognite

Eq. c. statica

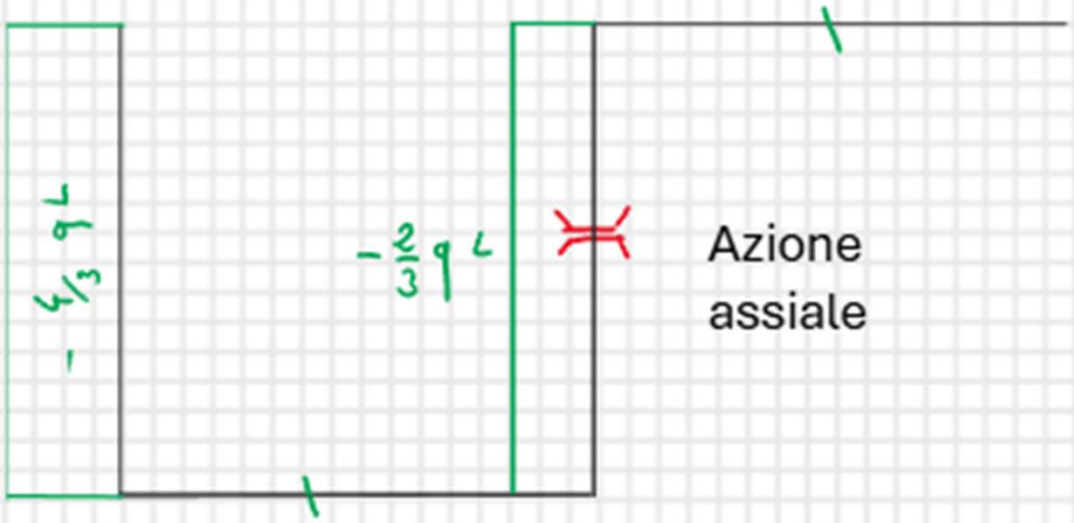
$H_A - H_D = 0$ 1)

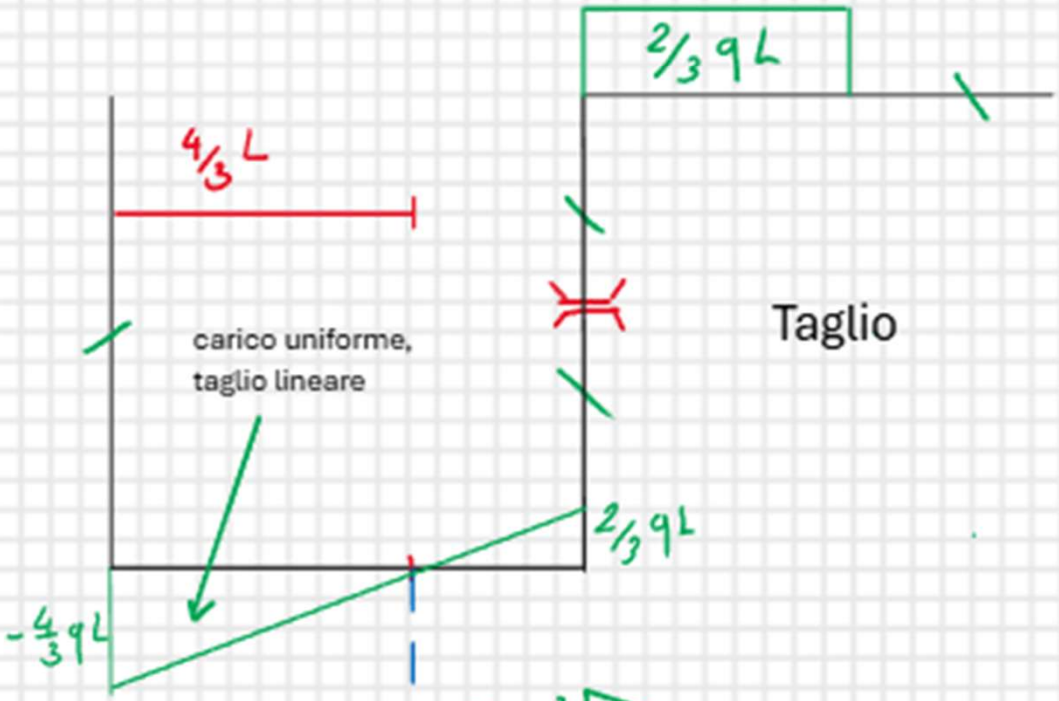
$V_A + V_C + 2qL = 0$ 2)

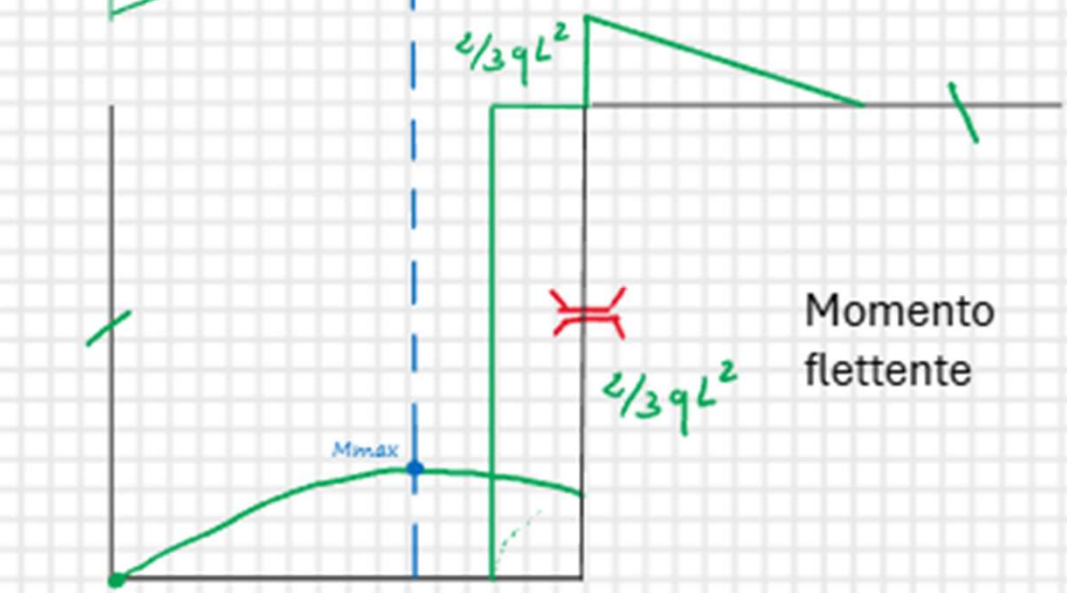
Aggiungo un'equazione ausiliaria

i_m A $2qL^2 + 3V_C L = 0$ 3)

Diagrammi

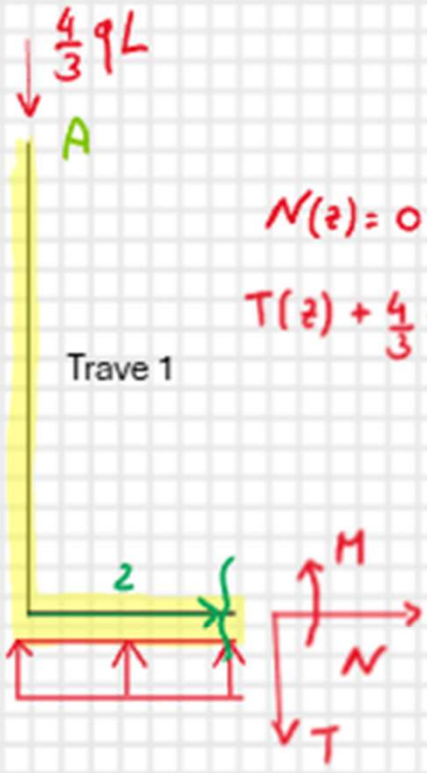






Carico uniforme, taglio lineare, momento parabolico

Potete sempre scrivere le equazioni di equilibrio relative al tratto di trave in esame



$N(z) = 0$

$T(z) + \frac{4}{3}qL - qz = 0 \longrightarrow T(z) = -\frac{4}{3}qL + qz$

$T(z=0) = -\frac{4}{3}qL$

$T(z=2L) = \frac{2}{3}qL$

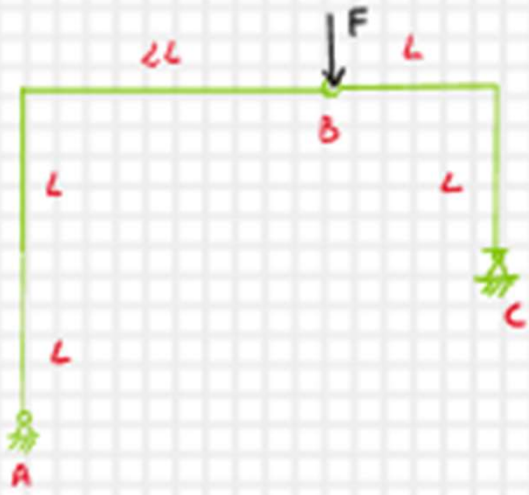
Taglio nullo

$-\frac{4}{3}qL + qz = 0 \longrightarrow z = \frac{4}{3}L$

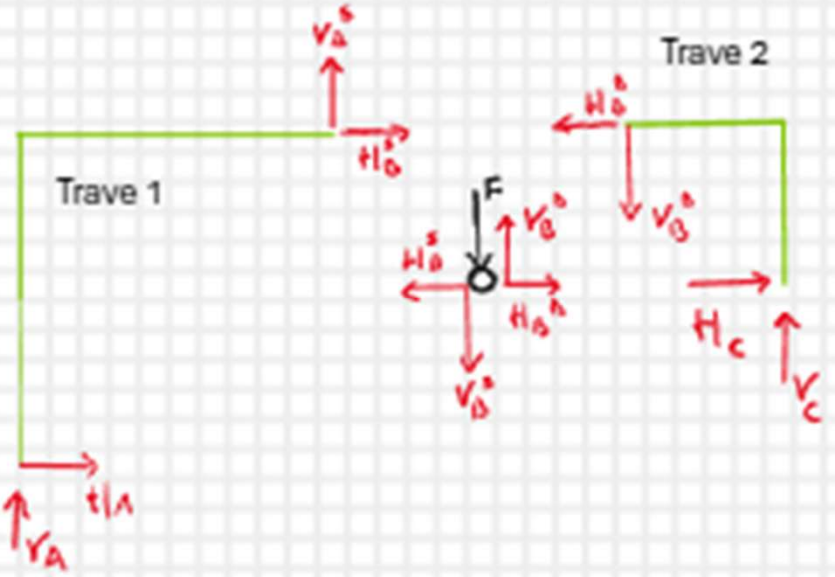
$M(z) + \frac{4}{3}qLz - q\frac{z^2}{2} = 0 \longrightarrow M(z) = -\frac{4}{3}qLz + q\frac{z^2}{2}$

Carico applicato sulla cerniera

Isolo il nodo B nel quale è applicata la forza concentrata



S e D per indicare la sezione a sinistra e a destra del nodo



Abbiamo 8 incognite ($H_A, V_A, H_B^s, V_B^s, H_B^d, V_B^d, H_C, V_C$)

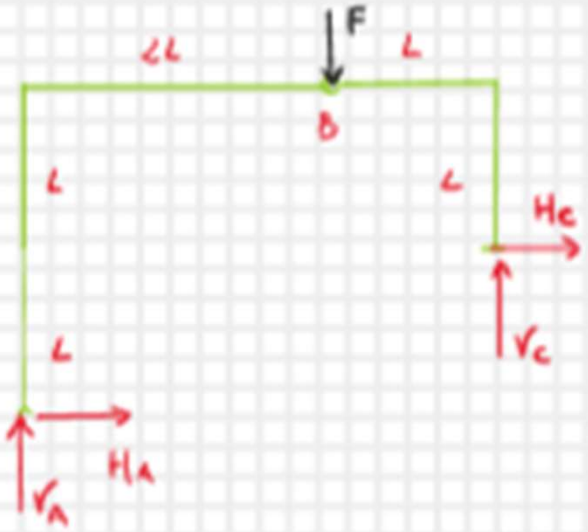
3 eq. cardinali per ciascuna trave = 6

Dobbiamo aggiungere 2 equazioni

Equazioni di equilibrio alla traslazione del nodo

Eq. orizzontale 1) $H_B^d - H_B^s = 0$

Eq. verticale 2) $V_B^s + F - V_B^d = 0$

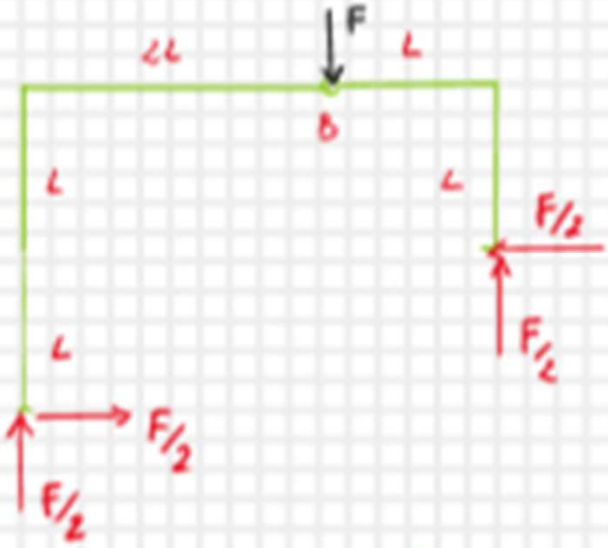


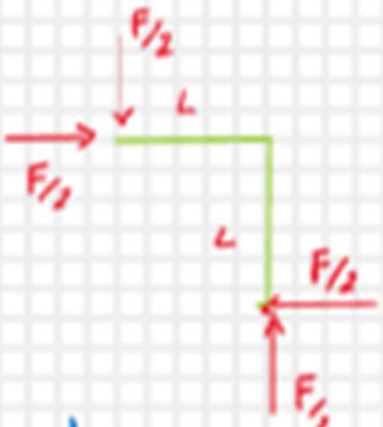
1) $H_A + H_C = 0$

2) $V_A + V_C = F$

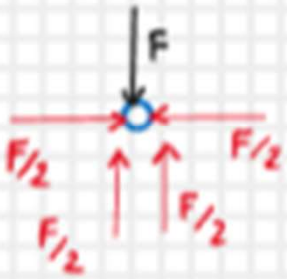
3) Im A $V_C \cdot 3L - 2FL - H_C L = 0$

4) $V_C L + H_C L = 0$ Ausiliaria





Verifica nel modo B



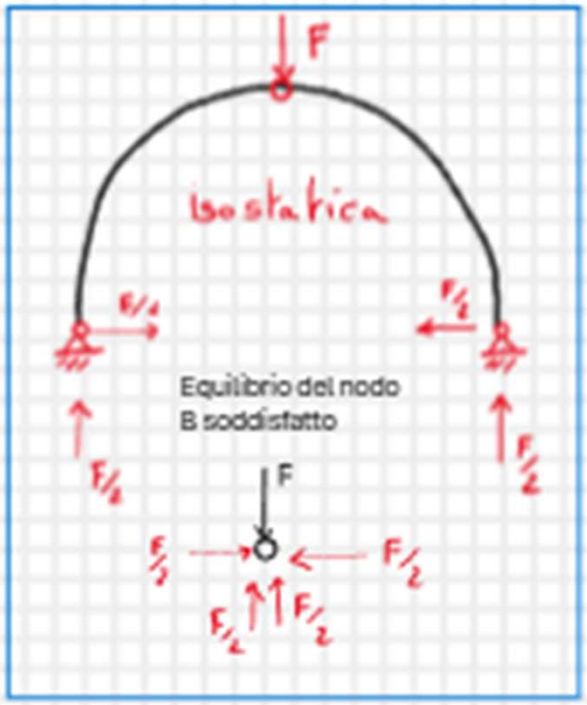
Eq. equilibrio modo

1) $F/2 - F/2 = 0$ O

2) $F - F/2 - F/2 = 0$ V

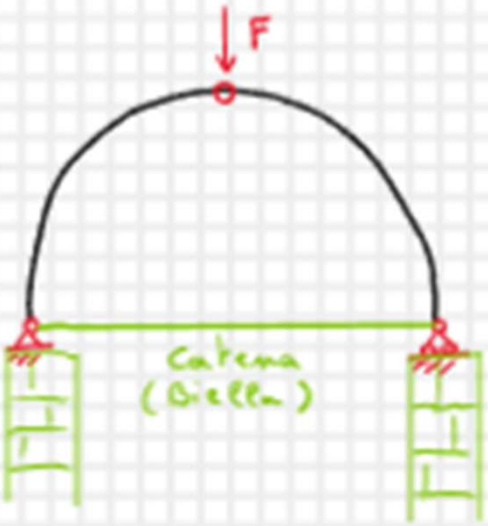
Eq. del modo verificato

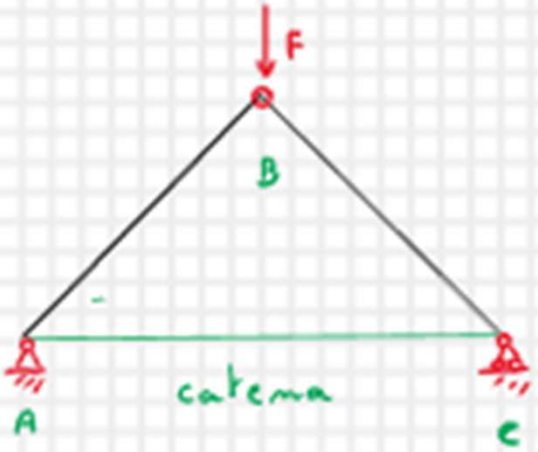
Esempio per arrivare alle strutture chiuse

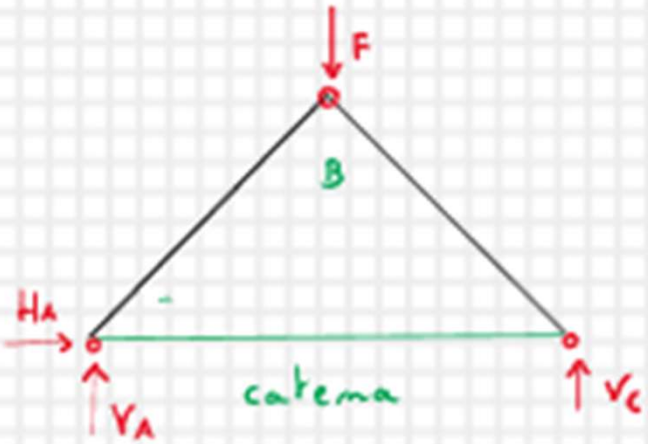


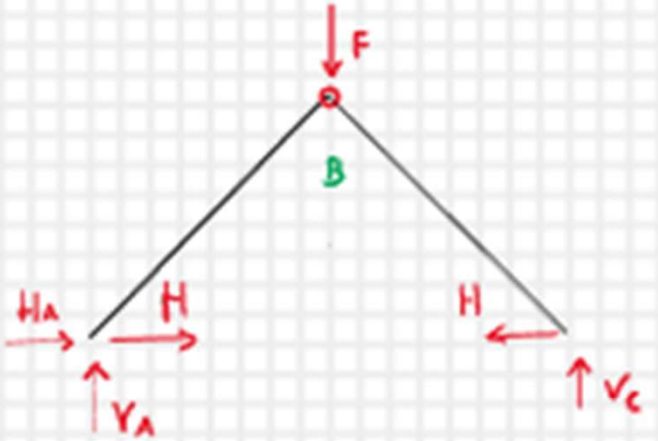
modifico

Modifico la cerniera con un carrello, introduco una biella per garantire l'isostaticità. (Genero una struttura chiusa)









Equazioni di equilibrio

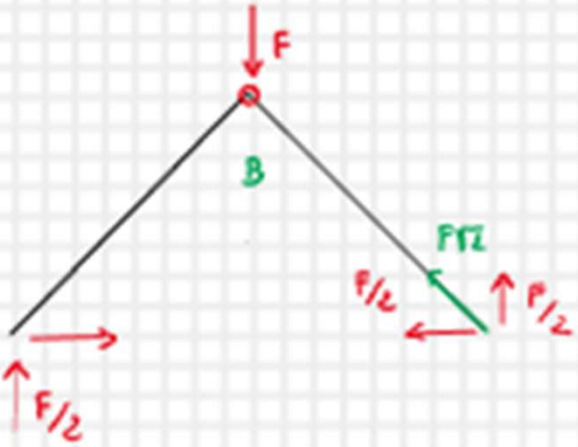
1) $H_A = 0$ $\quad H_A = 0$

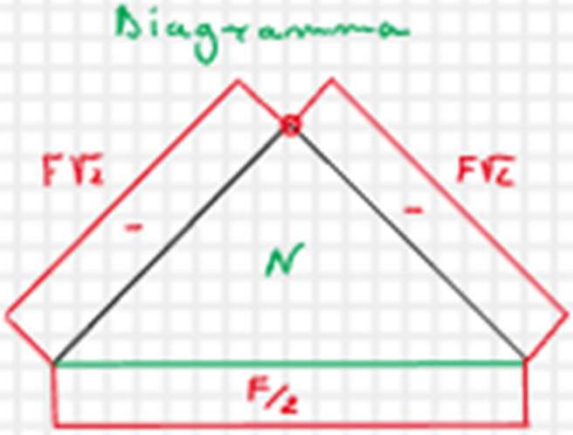
2) $V_A + V_C = F$ $\quad V_A = F/2$

3) $FL - V_C 2L = 0$ $\quad V_C = F/2$

Per ricavare la forza H nella biella utilizzo un'equazione ausiliaria

4) $V_C L - H L = 0$





3 Bielle, taglio e momento flettente nullo

Triangolo isostatico

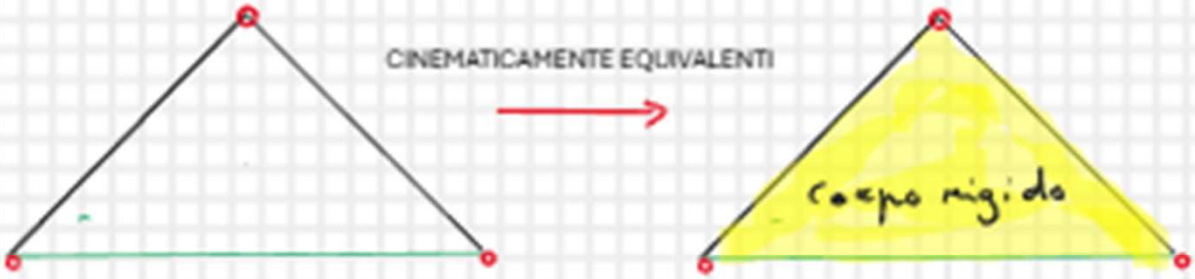
3 travi

3 vincoli relativi doppi

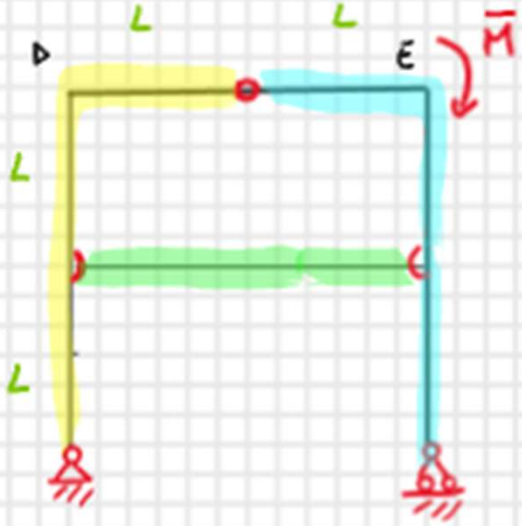
Le tre cerniere reali o "fittizie" non devono essere allineate

$gl = 3 \cdot 3 = 9$ $m_{tot} = 2 \times 3 = 6$

gl residui $9 - 6 = 3$ → Si comporta come un corpo rigido



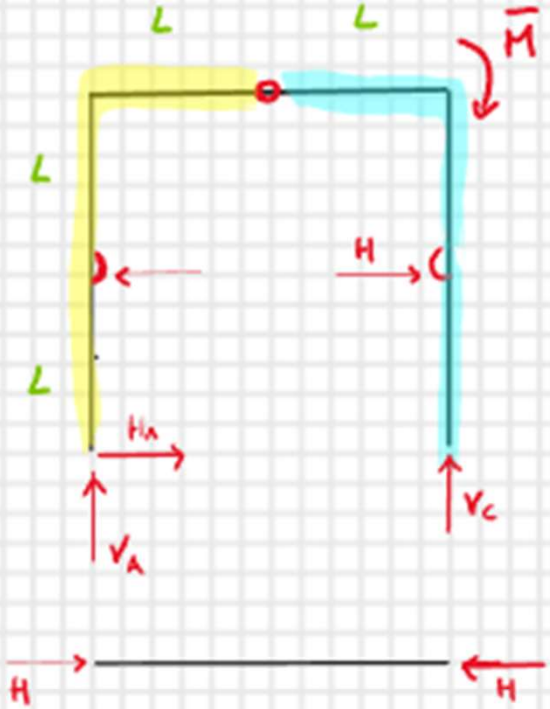
Strutture chiuse



Riconosciamo un triangolo isostatico: 3 travi con 3 vincoli doppi interni

CINEMATICAMENTE EQUIVALENTI

isostatica



Eq. di equilibrio

1) $H_A = 0$ $\quad H_A = 0$

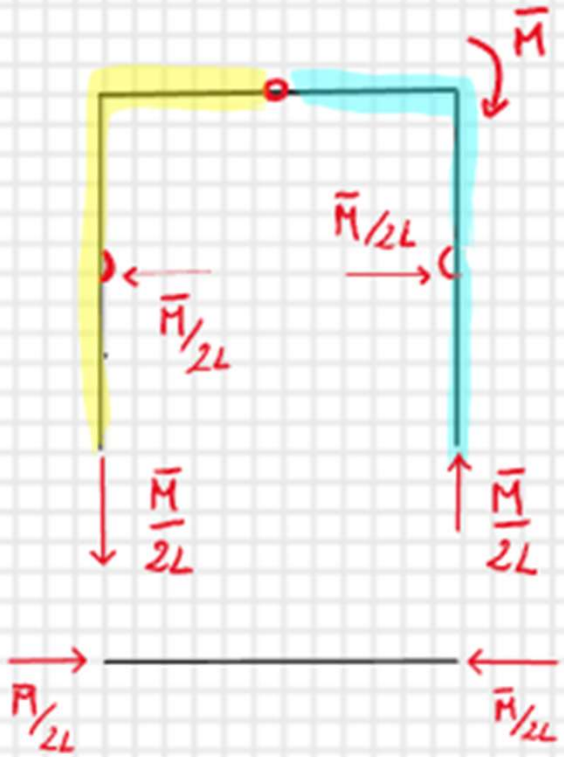
2) $V_A + V_C = 0$ $\quad V_A = -\frac{\bar{M}}{2L}$

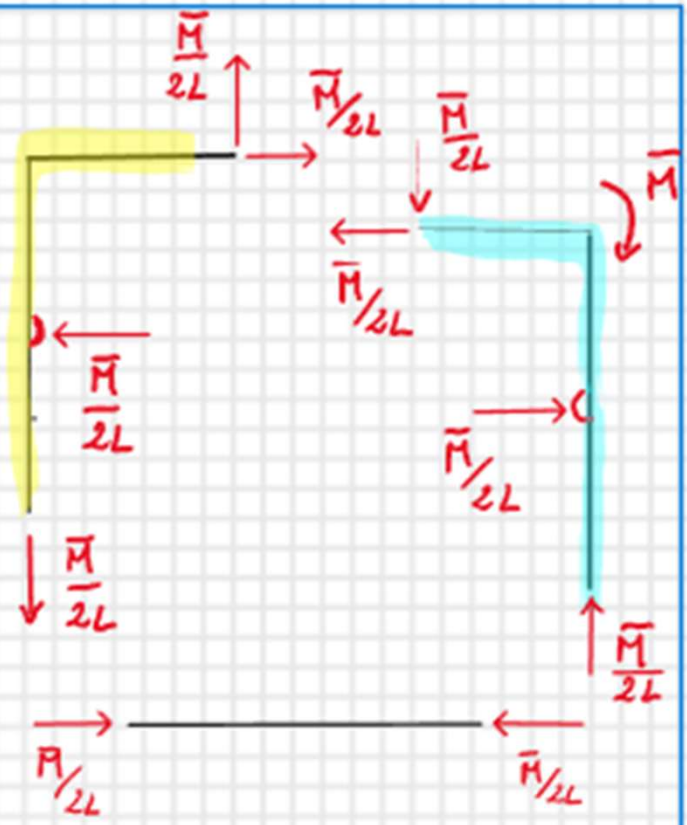
3) in A $\quad \bar{M} - 2V_C L = 0$ $\quad V_C = \frac{\bar{M}}{2L}$

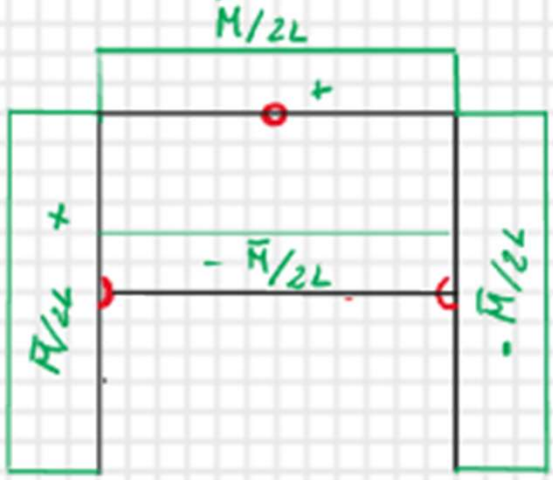
Calcolare H (eq. trave 1)

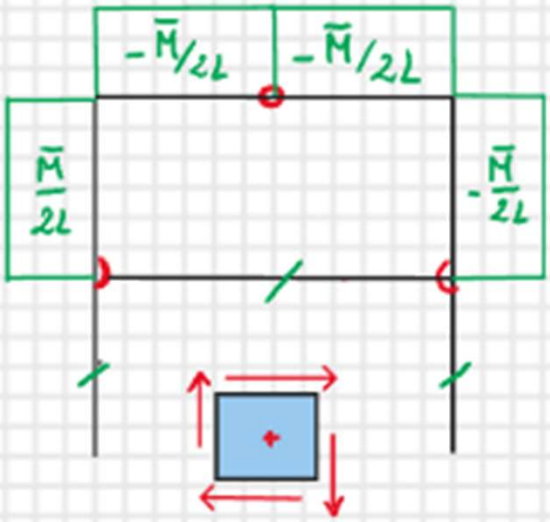
4) $HL + V_A L - 2H_A L = 0$

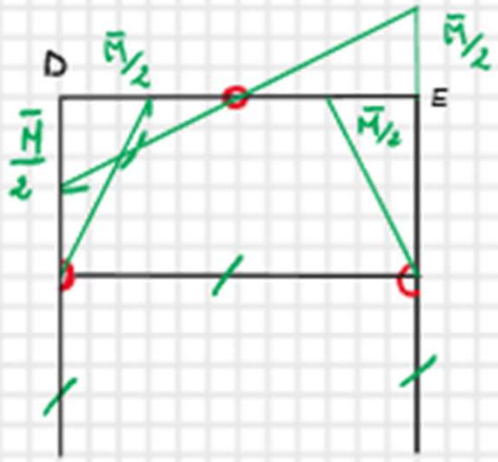
$H = \frac{\bar{M}}{2L}$











Momento lineari, taglio costante

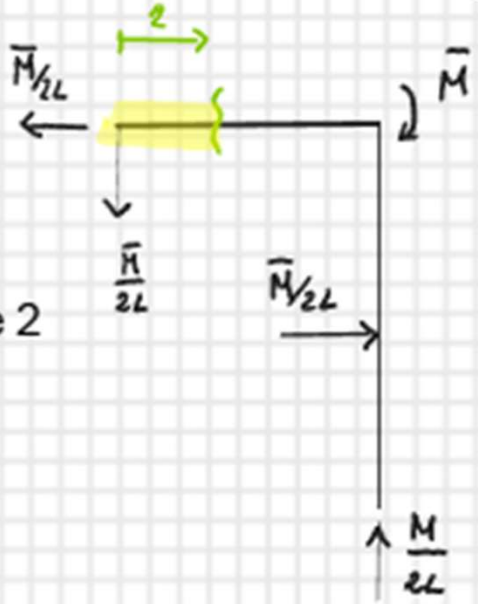
Verifica dei nodi

Nodo D

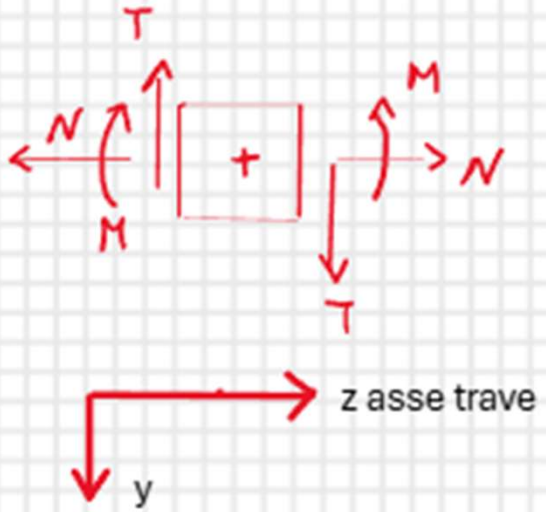
Nodo E

Eq. momenti soddisfatto

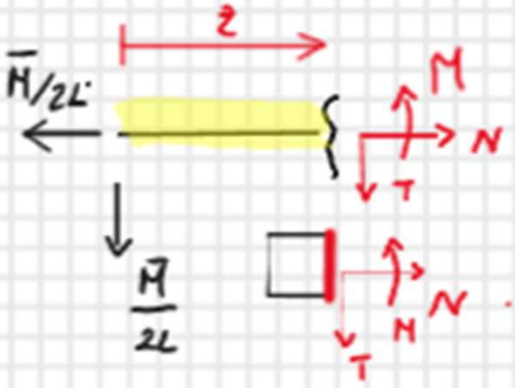
Osservazioni sul tracciamento dei diagrammi



Convenzione sul segno delle azioni interne



- Nelle lezioni precedenti imponevamo l'equilibrio della parte di trave individuata dalla sezione



$$-\frac{\bar{M}}{2L} + N(z) = 0 \qquad N(z) = \frac{\bar{M}}{2L}$$

$$-\frac{\bar{M}}{2L} - T(z) = 0 \longrightarrow T(z) = -\frac{\bar{M}}{2L}$$

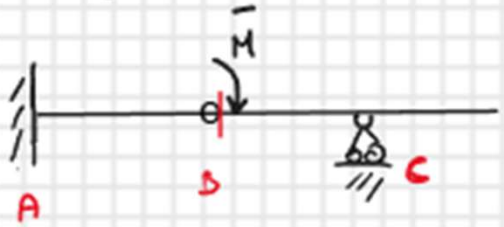
$$-M(z) - \frac{\bar{M}}{2L} z = 0 \qquad M(z) = -\frac{\bar{M}}{2L} z$$

Le azioni interne utilizzate sono quelle della faccia del concio elementare opposta al tratto di trave individuato dalla sezione (faccia rossa)

Il momento negativo delle formule lo disegnamo dal lato opposto alla freccia delle y rivolta in basso nella convenzione (o sul lato delle fibre tese)

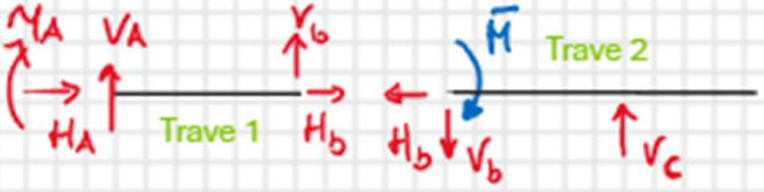
Materiale aggiuntivo, ulteriori esempi o curiosità

Esempi di strutture oltre a quelli dei temi d'esame



$\bar{M}$ a destra della cerniera

Risolvo prima la trave 1 (3 incognite)



$H_b = 0$

$V_b = V_C$

$-\bar{M} + V_C L = 0$

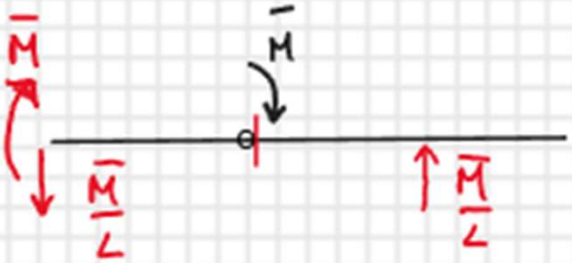
$H_b = 0$

$V_C = \bar{M}/L$

$V_b = \bar{M}/L$

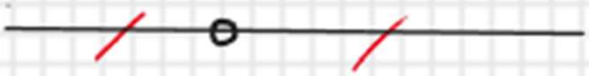
Risolta la struttura 2, calcolo la 1

$H_A = 0$ $\quad V_A = -\bar{M}/L$ $\quad M_A = \frac{\bar{M}}{L} L = \bar{M}$



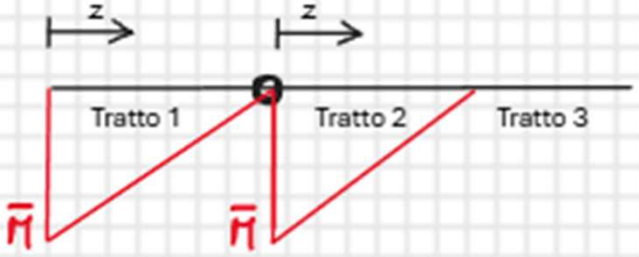
Schema di corpo libero

Azione assiale



Taglio

Momento flettente



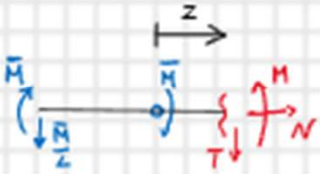
Tratto 3 $M(z) = 0$

Tratto 1

$M(z) = -\bar{M} + \frac{\bar{M}}{L} z = 0$

$M(z) = \bar{M} - \frac{\bar{M}}{L} z$

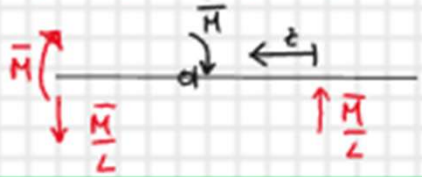
Tratto 2



$M(z) - \bar{M} - \bar{M} + \frac{\bar{M}}{L}(L+z) = 0$

$M(z) = \bar{M} - \frac{\bar{M}}{L} z$

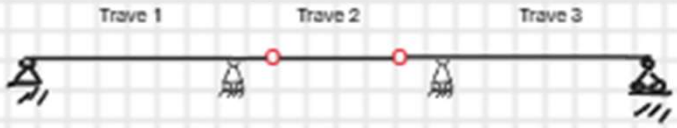
Alternativa tratto 2 con la z che parte da destra

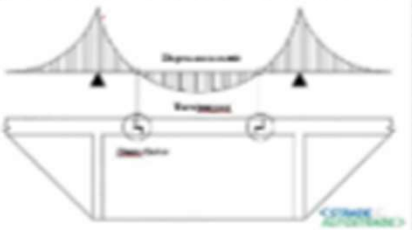


$M(z) = \frac{\bar{M}}{L} z$

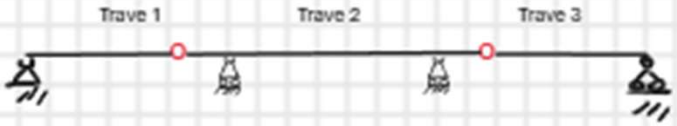
Travi Gerber (usate negli impalcati dei ponti)

schema n° 1

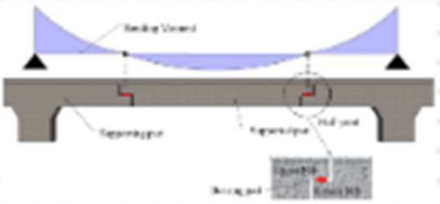




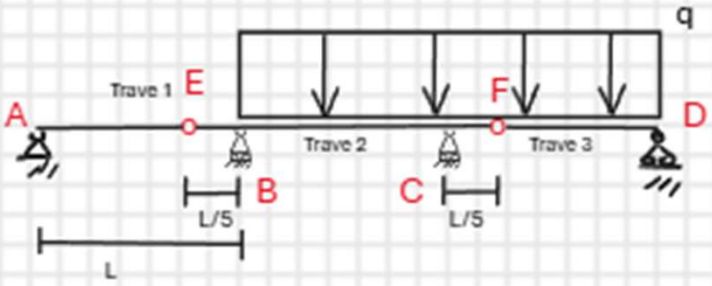
schema n° 2



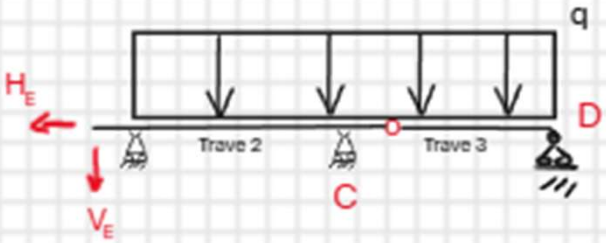
Isostatiche



Esempio



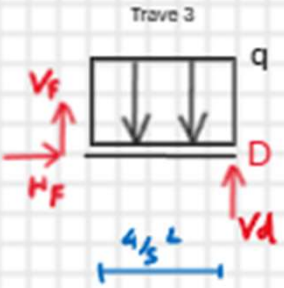
Risolvo la trave 1



Trave 1 scarica

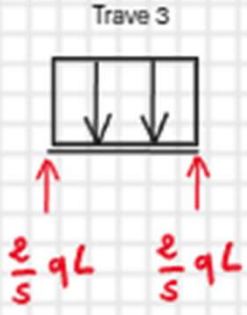
$H_E = 0$ $V_E = 0$

Risolvo la trave 3

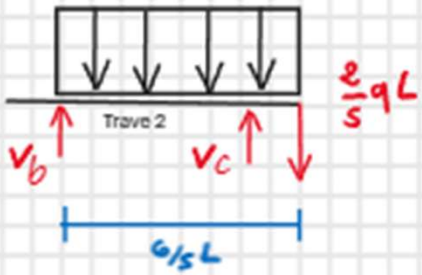


$H_F = 0$

$V_F = V_d = \frac{2}{5} qL$

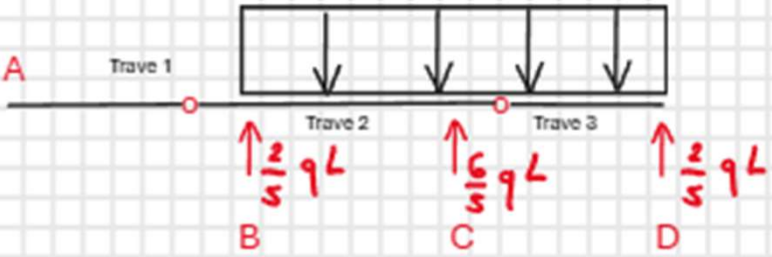


Risolvo la trave 2



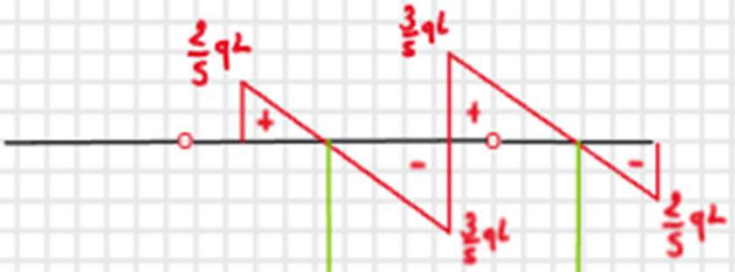
$V_b = \frac{2}{5}qL \qquad V_c = \frac{6}{5}qL$

Schema di corpo libero



Azione assiale N(z)=0

Taglio



Momento flettente