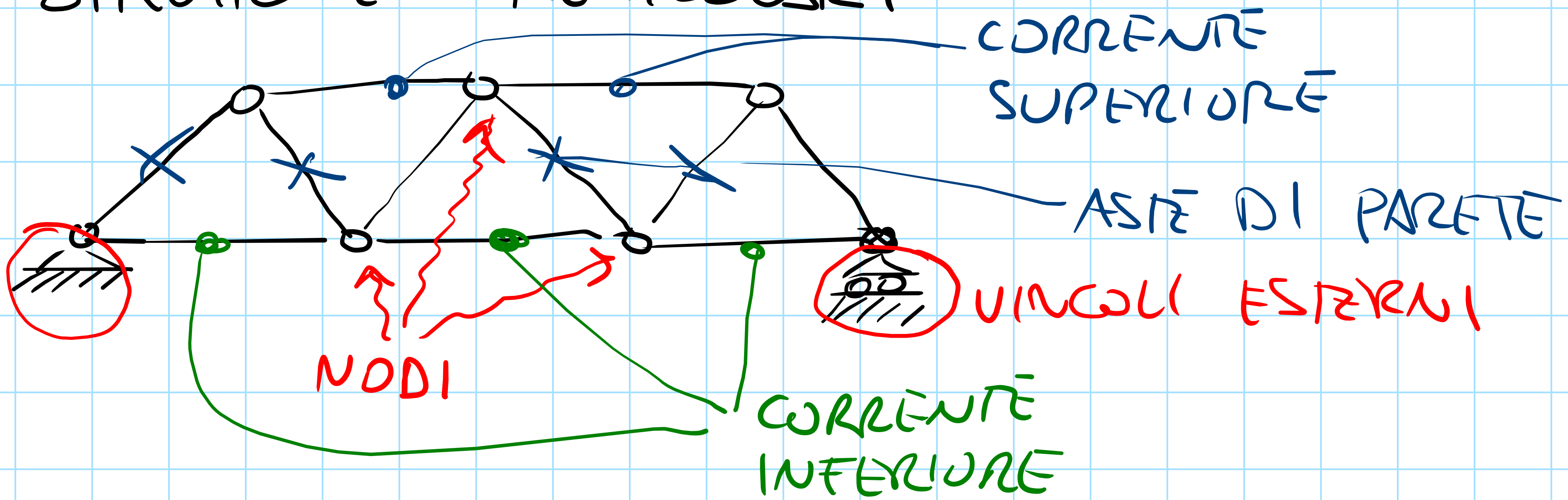
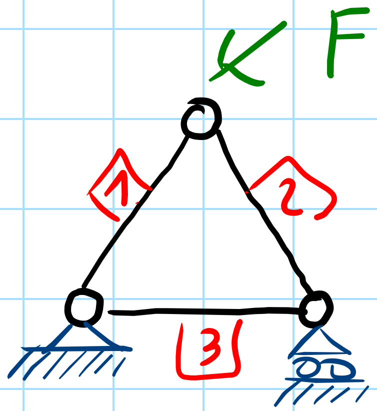
STRUTTURE RETICOLARI

14/5/26



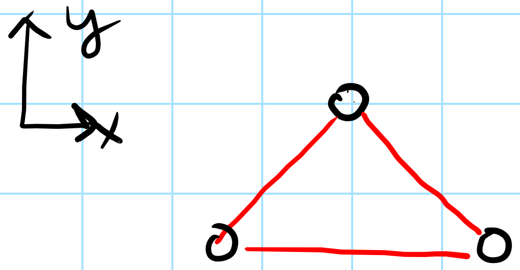


COL METODO TRADIZIONALE, LA VERIF DI ISOSTATICITÀ PREVEDE

3 CORPI $\longrightarrow$ $gdl = 3 \cdot 3 = 9$

$$\left.\begin{array}{l} \text{3 CERNIERE INTERNE} : \ m_{INT} = 3 \cdot 2 = 6 \\ \text{3 GRADI DI VINCOLO ESTERNO} \ \ m_{EST} = 3 \end{array}\right\} m_{TOT} = 9$$

PER LA VERIFICA DI ISOSTATICITÀ SI USA UN METODO ALTERNATIVO, PIÙ ADATTO ALLA "GEOMETRIA" DI QUESTE STRUTTURE.

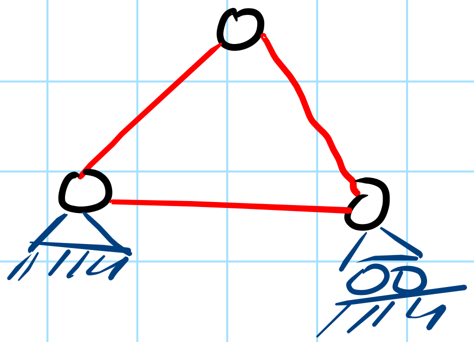


1) OGNI NODO LIBERO NEL PIANO HA 2 GRADI DI LIBERTA'
(LE COORD x, y DEL PUNTO)
(3 NODI : GDL TOTALI $2 \cdot 3 = 6$ GDL TOTALI)

2) OGNI BIELLA/PENDOLO INTERNA TOGLIE AL SISTEMA
1 GDL
(3 BIELLE : 3 GDL SOTTRATTI)

DOPO 1) e 2) IL SISTEMA HA $6 - 3 = \boxed{3}$ GDL RESIDUI

3) POSSO ORA "BLOCCARE" IL TRIANGOLO A TERRA CON
UNA CERNIERA ($m = \boxed{2}$) e UN CARRELLO ($m = \boxed{1}$) $3 = m_{EST}$



SCH. ISOSTATICO

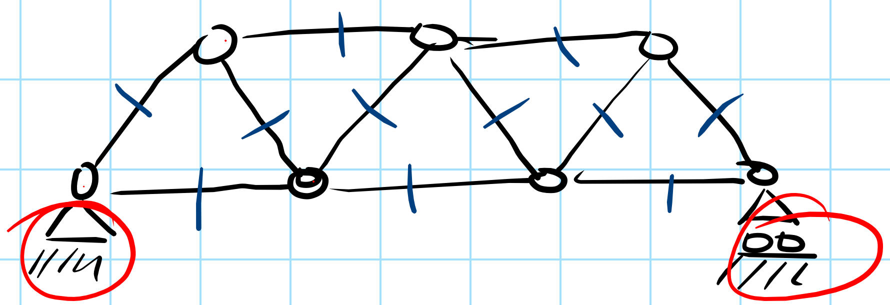
$m_{EST} = 3$

IN GENERALE, LA <u>CONDIZ. NECESSARIA</u> PER L'ISOSTATICITÀ È:

$$(*) \quad 2\, m_{NODI} = m_{ASTE} + \underbrace{m_{EST}}_{\text{MOLTEPLICITÀ ESTERNA (VALE 3)}}$$

(FORMULA DI MAXWELL)

ES: STR ISOSTATICA:

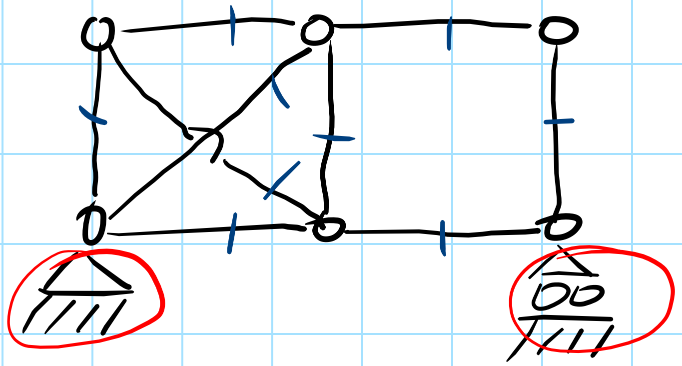


$$2\, m_{NODI} = 2 \cdot 7 = 14$$

$$\left.\begin{array}{l} m_{ASTE} = 11 \\ m_{EST} = 3 \end{array}\right\} 14$$

FORMULA (*) VERIFICATA

ES: STR NON ISOSTATICA (LABILE PER CATTIVA DISPOSIZ DELLE BIELLE INTERNE)

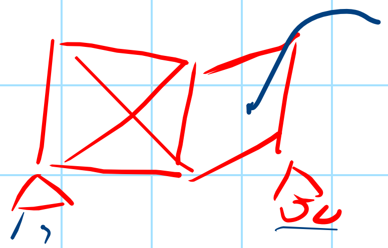


$$2\, m_{NODI} = 2 \cdot 6 = 12$$

$$\left.\begin{array}{l} m_{ASTE} = 9 \\ m_{EST} = 3 \end{array}\right\} 12$$

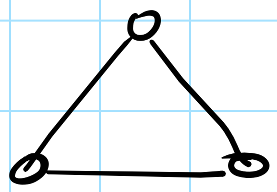
FORMULA (*) VERIFICATA

MA STR LABILE



MAGLIA LABILE

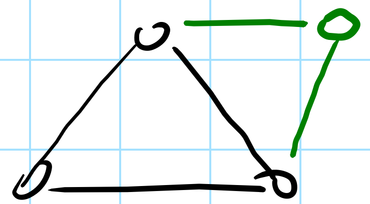
COSTRUZIONE DI UN "TRALICCIO" ISOSTATICO ATTRAVERSO LA GENERAZIONE TRIANGOLARE



TRIANGOLO ISOSTATICO.

NEL PIANO HA 3 GDL

(E' UN CORPO RIGIDO)

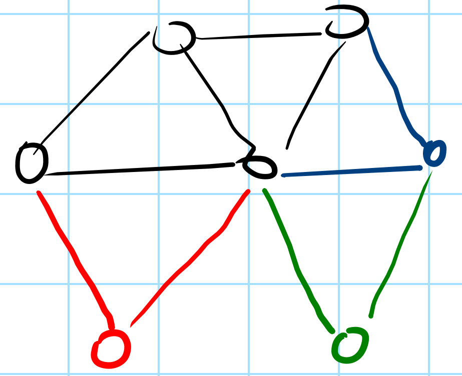


AGGIUNGO UN NODO E 2 BIELLE.

+NODO (+2 GDL)

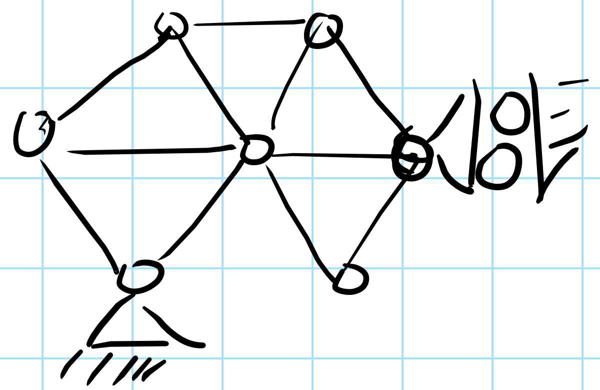
+2 BIELLE (−2 GDL)

QUESTO SCHEMA HA ANCORA 3 GDL NEL PIANO



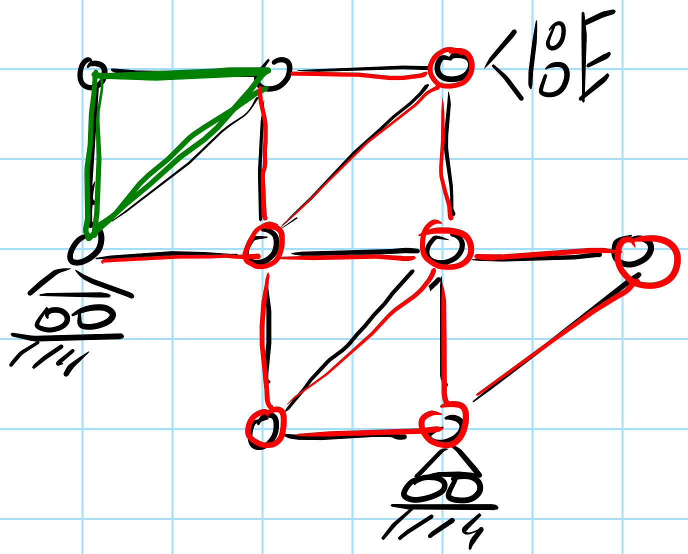
SCHEMA CON ANCORA 3 GDL NEL PIANO

VUOL DIRE CHE, SE LO BLOCCO A TERRA CON CERNIERA + CARRELLO OTTENGO UNA STR ISOSTATICA



ISOST.

ES



1) VERIFICO CHE $m_{EST} = 3$ (SÌ, HO 3 CARRELLI BEN DISPOSTI)

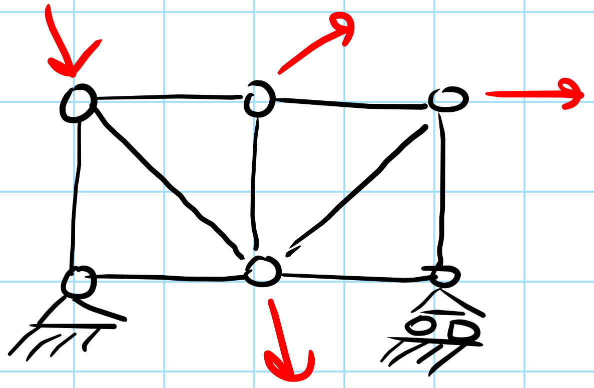
2) CERCO DI OTTENERE IL TRALICCIO CON LA GENERAZ. TRIANGOLARE

VERIFICA OK

1)+2): SONO SICURO CHE È ISOST !

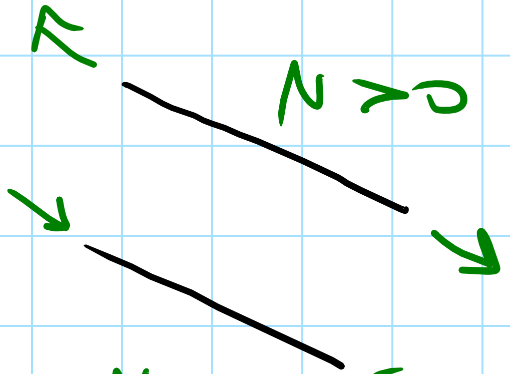
esercizio: verificare la form. di Maxwell

IPOTESI DI CALCOLO DEGLI "SFORZI" INTERNI DELLE ASTE



- NODI INCERNIERATI
- ASTE RETTILINEE
- FORZE APPLICATE AI NODI

L'UNICA AZIONE INTERNA o CDS PRESENTE È LA FORZA NORMALE (N). TAGLIO e MOMENTO FLETTENTE NULLI



N > 0 : ASTA TESA (<u>TIRANTE</u>)

N < 0 ASTA COMPRESSA (<u>PUNTONE</u>)

(N = 0 : ASTA SCARICA)

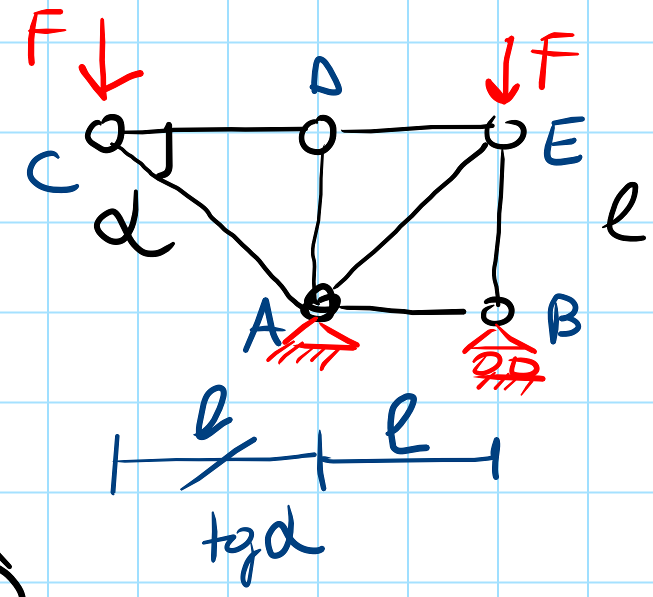
OBIETTIVO DEL CALCOLO DI UNA STR. RETICOLARE È:

- DETERMINARE LE REAZ. ESTERNE
- " GLI SFORZI (N) NELLE ASTE

METODO DEI NODI

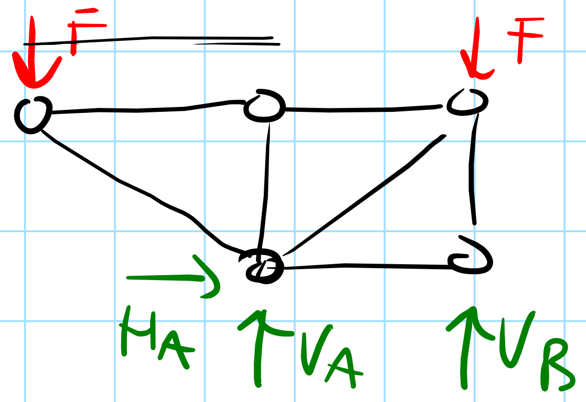
" DELLE SEZIONI (o RITTER)

ES: METODO DEI NODI



ISO

0) CALCOLO REAZ. ESTERNE



S.C.L.

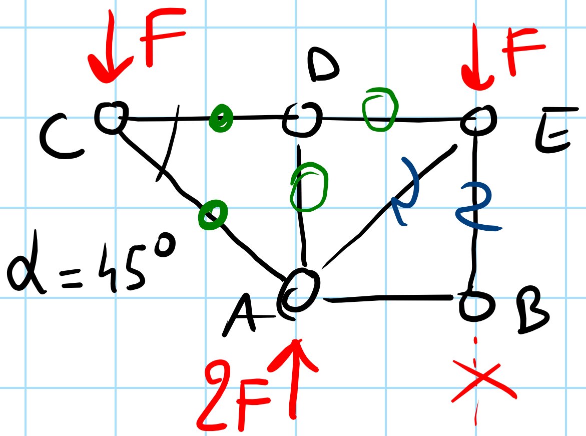
$$\xrightarrow{+} : \boxed{H_A = 0}$$

$$+\uparrow : +V_A + V_B - 2F = 0$$

$$+\circlearrowleft A : +F\frac{\cancel{\ell}}{tg\alpha} + V_B\cancel{\ell} - F\cancel{\ell} = 0$$

$$V_B = F - \frac{F}{tg\alpha} = F\left(1 - \frac{1}{tg\alpha}\right) \; ; \; V_A = 2F - V_B = 2F - F\left(1 - \frac{1}{tg\alpha}\right) = F\left(1 + \frac{1}{tg\alpha}\right)$$

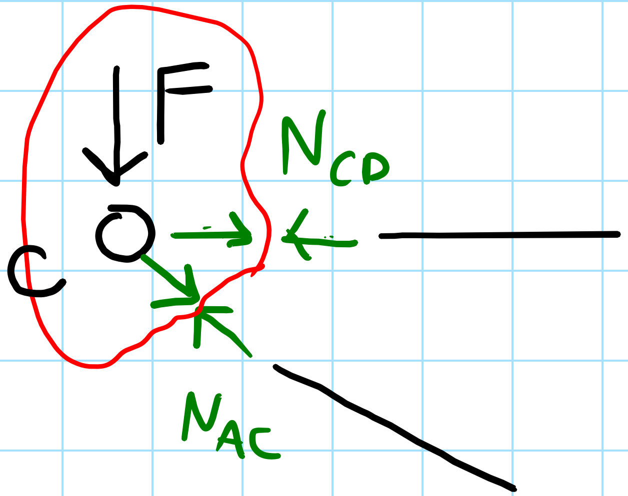
AL TERMINE DI 0) HO EQUILIBR. IL TRALICCIO. PASSO ORA AL CALCOLO DEGLI SFORZI DELLE ASTE CON IL METODO DEI NODI ($\alpha = 45°$, $tg\,\alpha = 1$)



1) METODO DEI NODI

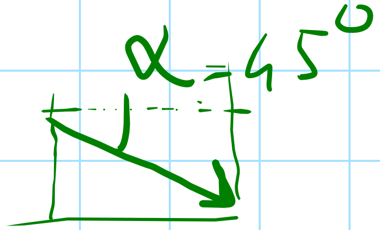
EQUILIBRIO NODO C

2 EQUAZIONI DI EQUIL.



INCOGNITE

N_{AC}, N_{CD}



$\overset{+}{\rightarrow}$: $+N_{CD} + N_{AC}\dfrac{1}{\sqrt{2}} = 0$

$+\uparrow$: $-F - N_{AC}\dfrac{1}{\sqrt{2}} = 0$

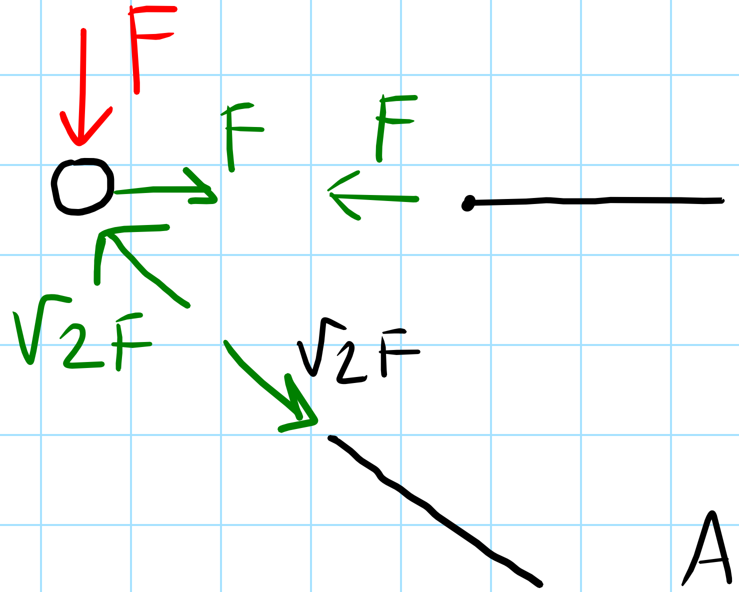
SIST DI 2 EQZ IN 2 INCOGNITE

↳ SI PUÒ RISOLVERE!

$\boxed{N_{AC} = -\sqrt{2}F}$ − : VERSO DA CAMBIARE

$\boxed{N_{CD} = +F}$ + : VERSO SCELTO PER N_{CD} È CORRETTO

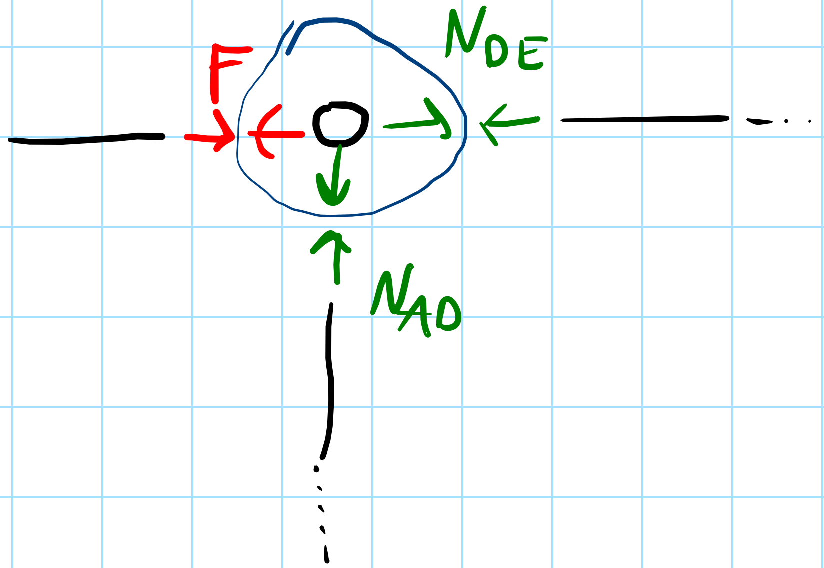
DALLA 1° EQ $\Rightarrow$ $N_{CD} = -N_{AC}\dfrac{1}{\sqrt{2}} = +F$



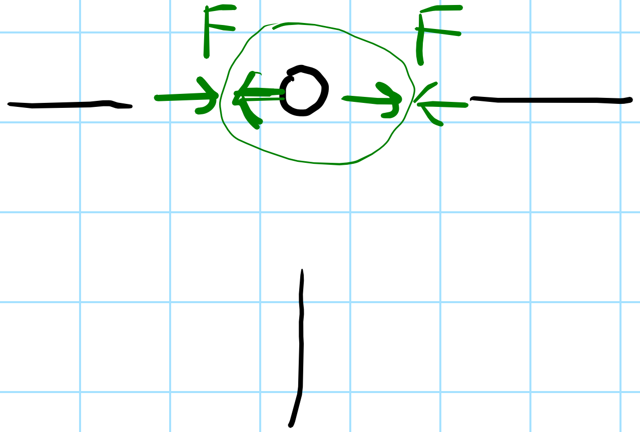
CD: TIRANTE

AC: PUNTONE

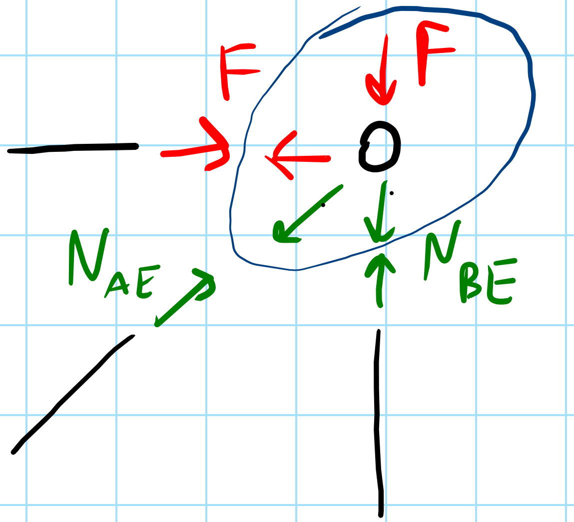
EQUILIBRIO NODO D



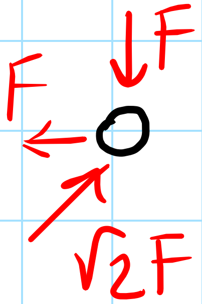
$$\left.\begin{array}{l} \overset{+}{\rightarrow}: \; N_{DE} - F = 0 \\ +\uparrow: \; -N_{AD} = 0 \end{array}\right\} \quad \begin{array}{l} N_{DE} = F \;, \quad DE: \text{TIRANTE} \\ N_{AD} = 0 \end{array}$$



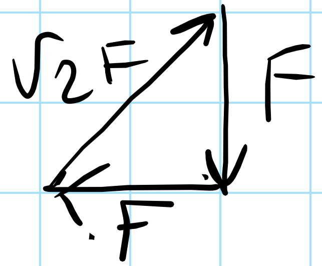
EQUIL. NODO E



$$\left.\begin{array}{l} \overset{+}{\rightarrow}: \; -F - N_{AE}\dfrac{1}{\sqrt{2}} = 0 \\ \uparrow+: \; -F - N_{BE} - N_{AE}\dfrac{1}{\sqrt{2}} = 0 \end{array}\right\} \quad \begin{array}{l} N_{AE} = -\sqrt{2}\,F \quad (AE: \text{PUNTONE}) \\ N_{BE} = -F + \dfrac{\sqrt{2}F}{\sqrt{2}} = 0 \end{array}$$

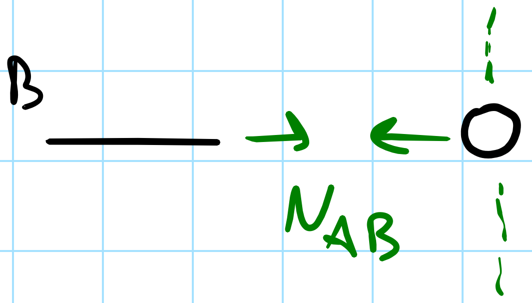


SCL EQUIL DEL NODO E



CHIUSURA DEL TRIANGOLO FORZE

EQUILIBRIO NODO B

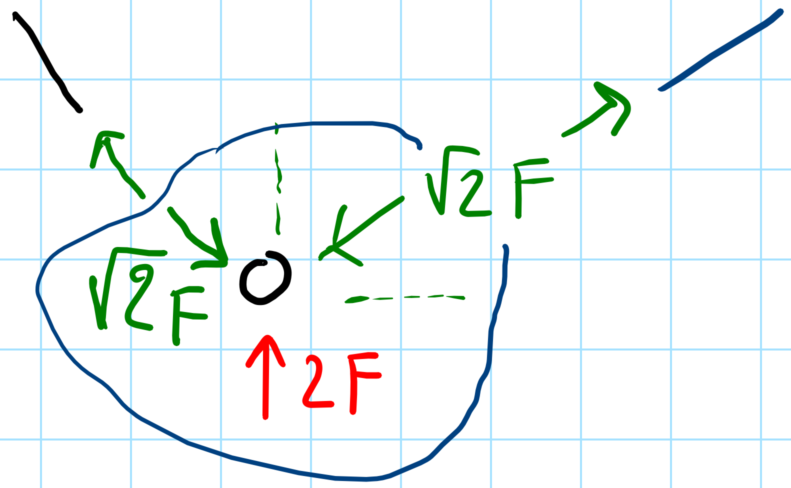


$\xrightarrow{+}: \ N_{AB} = 0$

$+\uparrow: \ 0 = 0$

OSSERVO CHE TUTTI GLI SFORZI SONO STATI CALCOLATI; RIMANE LA VERIFICA DELL'EQUIL. DEL NODO A. SE QUESTO NODO È IN EQUILIBRIO HO OTTIME PROBABILITÀ DI AVER CALCOLATO CORRETT. TUTTI GLI SFORZI. SE LA VERIFICA DEL NODO A NON È SODDISF. HO SICURAMENTE SBAGLIATO.

VERIFICA EQ. NODO A



$$\xrightarrow{+}: \ \sqrt{2}F\frac{1}{\sqrt{2}} - \sqrt{2}F\frac{1}{\sqrt{2}} \overset{?\ \text{sì}}{=} 0$$

$$+\uparrow: \ +2F - \sqrt{2}F\frac{1}{\sqrt{2}} - \sqrt{2}F\frac{1}{\sqrt{2}} \overset{?\ \text{sì}}{=} 0$$

OK

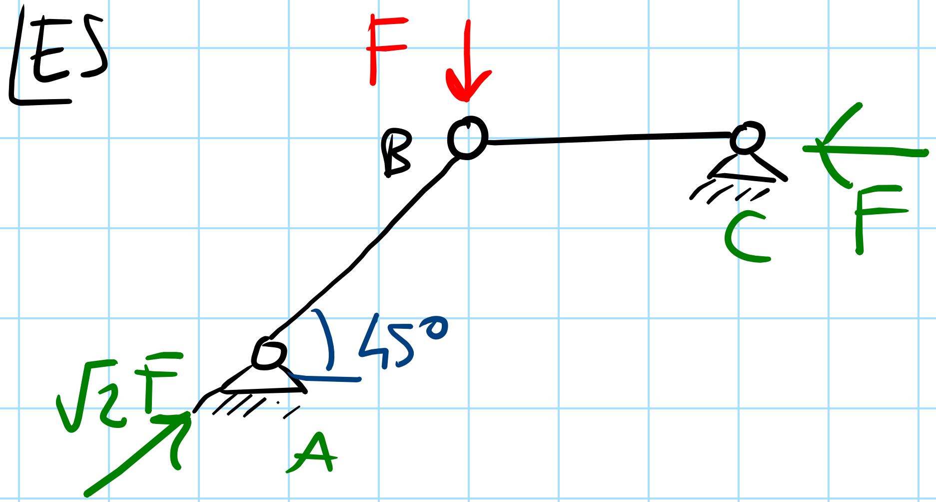
NODO EQUILIBRATO

TABELLA RIASSUNTIVA DELL'ES

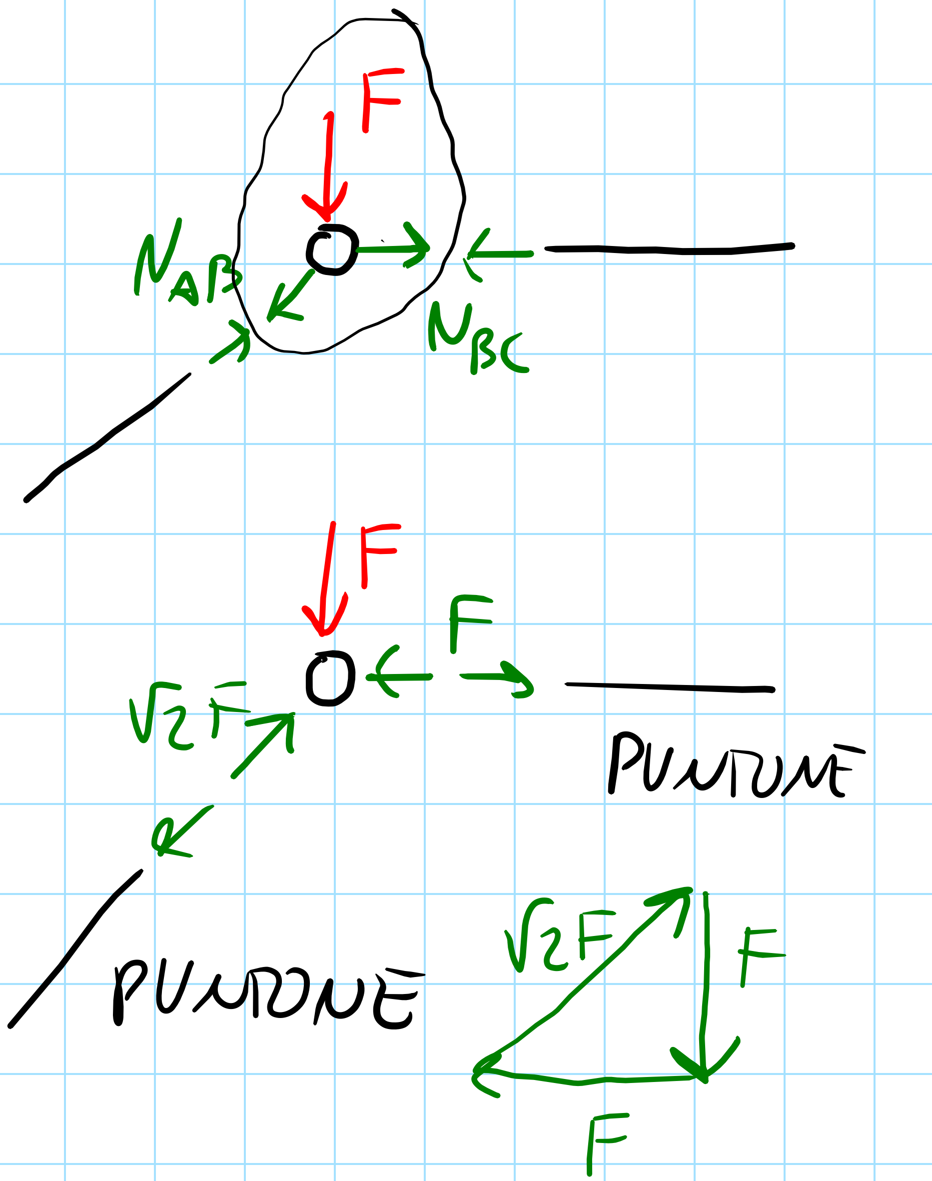
ASTA	AB	AC	AD	AE	BE	CD	DE
N	0	$-\sqrt{2}F$	0	$-\sqrt{2}F$	0	$+F$	$+F$

⊖ : PUNTONE , ⊕ TIRANTE

ES



POSSO VEDERE LA STRUTT. COME UNA SEMPLICISS. RETICOLARE FORMATA DA UN NODO (B) e DUE BIELLE A TERRA



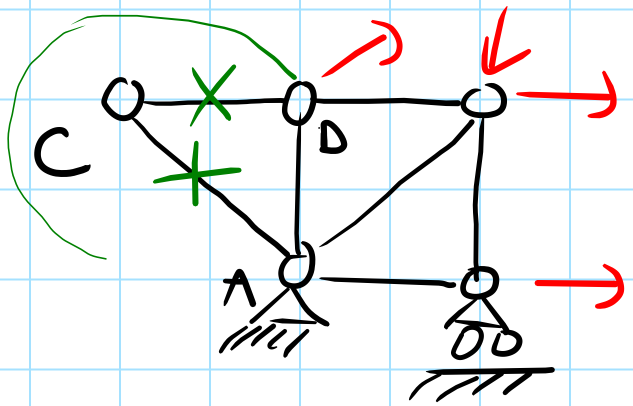
$$\xrightarrow{+} : +N_{BC} - N_{AB}\frac{1}{\sqrt{2}} = 0$$

$$+\uparrow ; -F - N_{AB}\frac{1}{\sqrt{2}} = 0 ;$$

$$N_{BC} = +N_{AB}\frac{1}{\sqrt{2}} = -F$$

$$N_{AB} = -\sqrt{2}F$$

CRITERI PER IL RICONOSCIMENTO DELLE ASTE SCARICHE

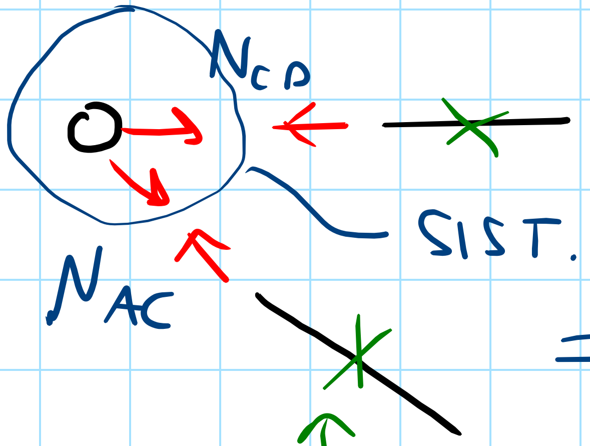


NODO C (SCARICO)

CONVERGONO 2 ASTE

NON ALLINEATE

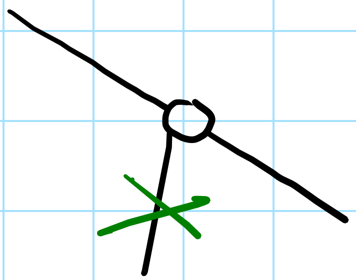
=> GLI SFORZI DELLE 2 ASTE SONO NULLI



SIST. DI 2 EQZ

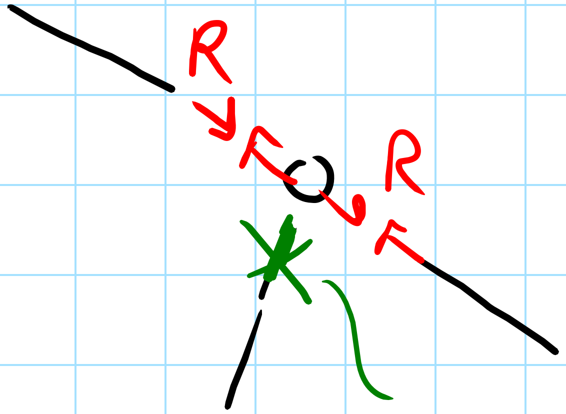
=> $N_{CD} = N_{AC} = 0$

ASTA SCARICA

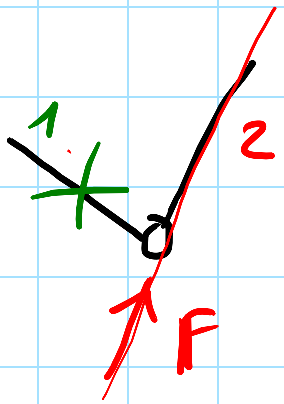


NODO SCARICO, 2 ASTE ALLINEATE, 1 ASTA "OBLIQUA".

L'ASTA OBLIQUA è SCARICA



FORZA CHE NON PUÒ ESSERE EQUIL. SUL NODO => NULLA

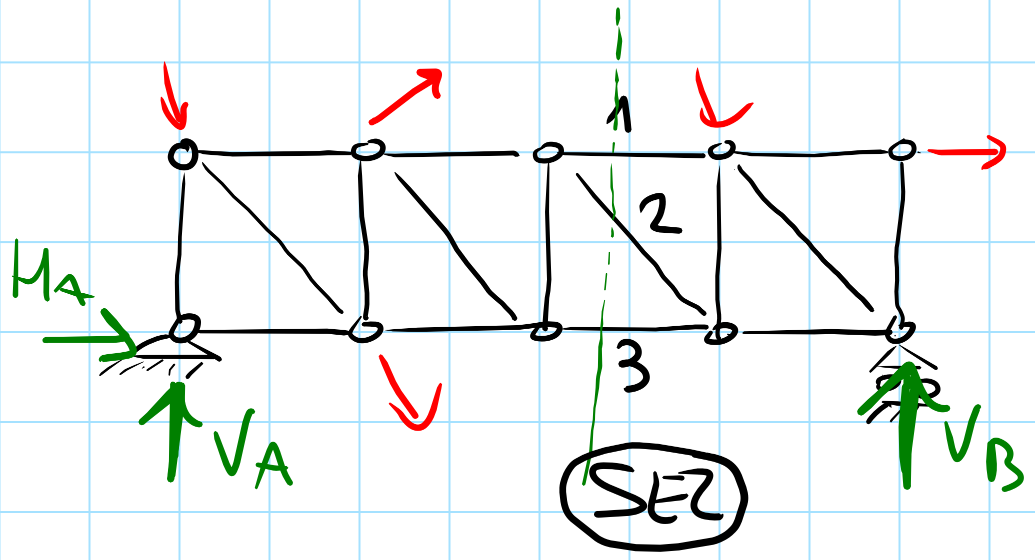


F è ALLINEATA ALL'ASTA 2

ASTA 1 SCARICA

METODO DELLE SEZIONI (o DI RITTER)

(PRIMA CALCOLO SEMPRE LE REAZ VINCOLARI)

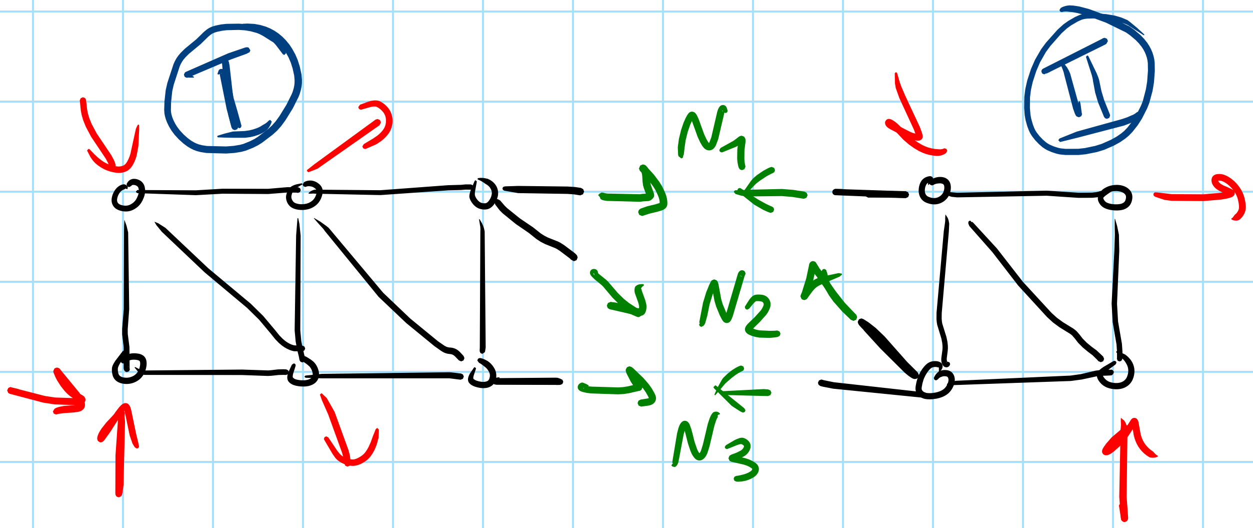


VER. ISOST.

PROBLEMA: COME CALCOLARE N_1, N_2, N_3?

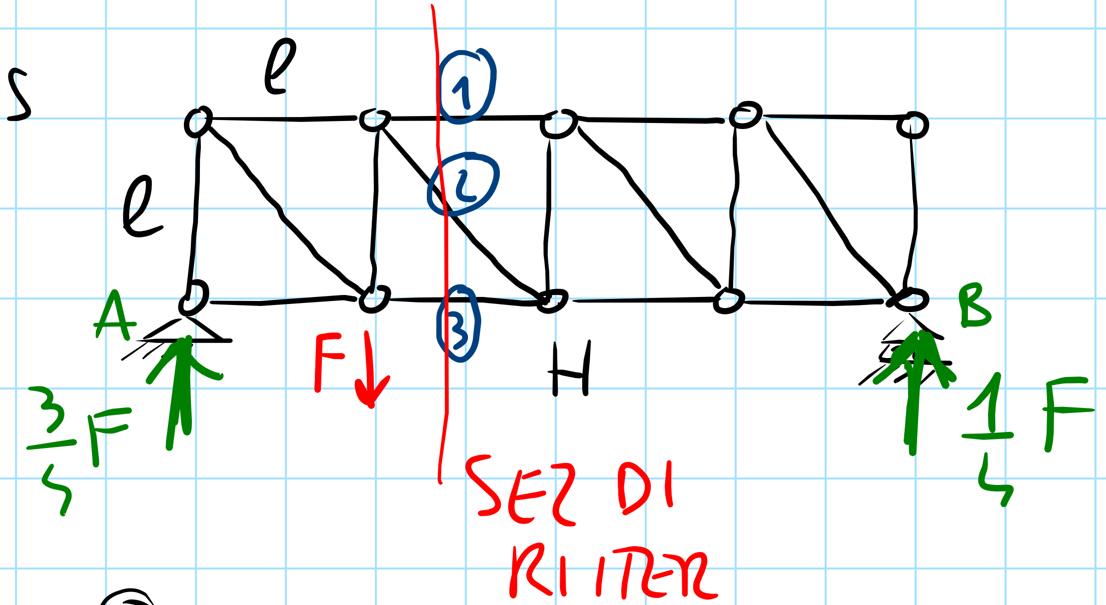
COL METODO DEI NODI DOVREI PRIMA CALCOLARE LE ASTE A PARTIRE DAI PUNTI A o B.

COL METODO DELLE SEZ. POSSO CALCOLARE DIRETT. LE INCOGNITE



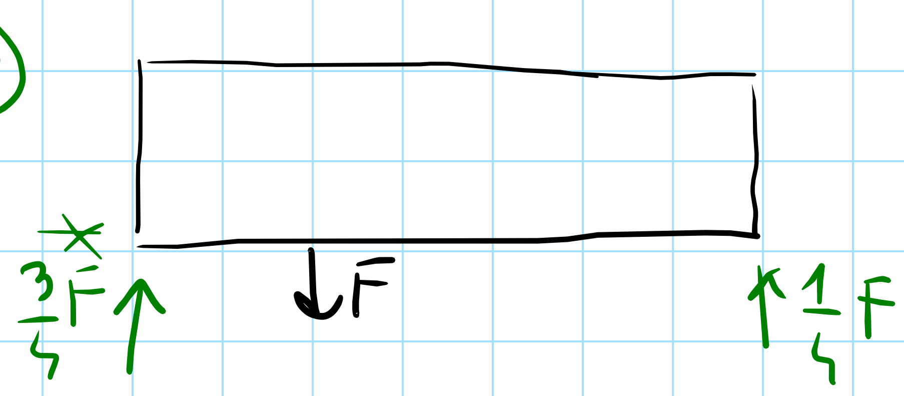
PER OGNI TRONCO (o I o II) POSSO SCRIVERE 3 EQZ DI EQUILIBRIO NELLE 3 INCOGNITE N_1, N_2, N_3

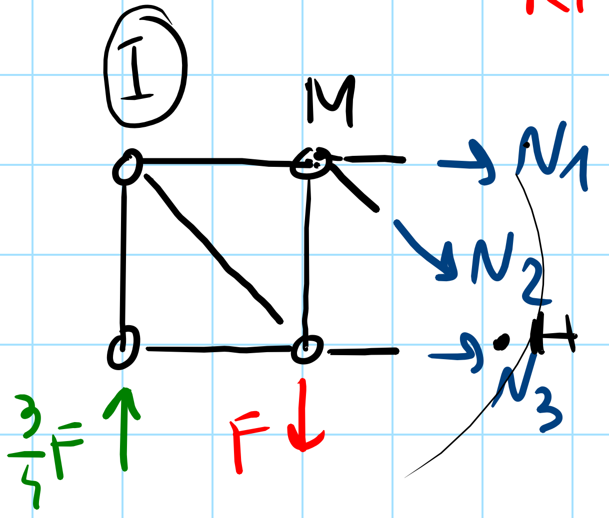
ES



N_1, N_2, N_3?

o)





N_3? $\overset{+}{\curvearrowleft} M$ (I) : $-\frac{3}{4}F\cancel{\ell} + N_3\cancel{\ell} = 0$; $\boxed{N_3 = \frac{3}{4}F}$

(3: TIRANTE)

N_2? EQUIL. VERTICALI (I)

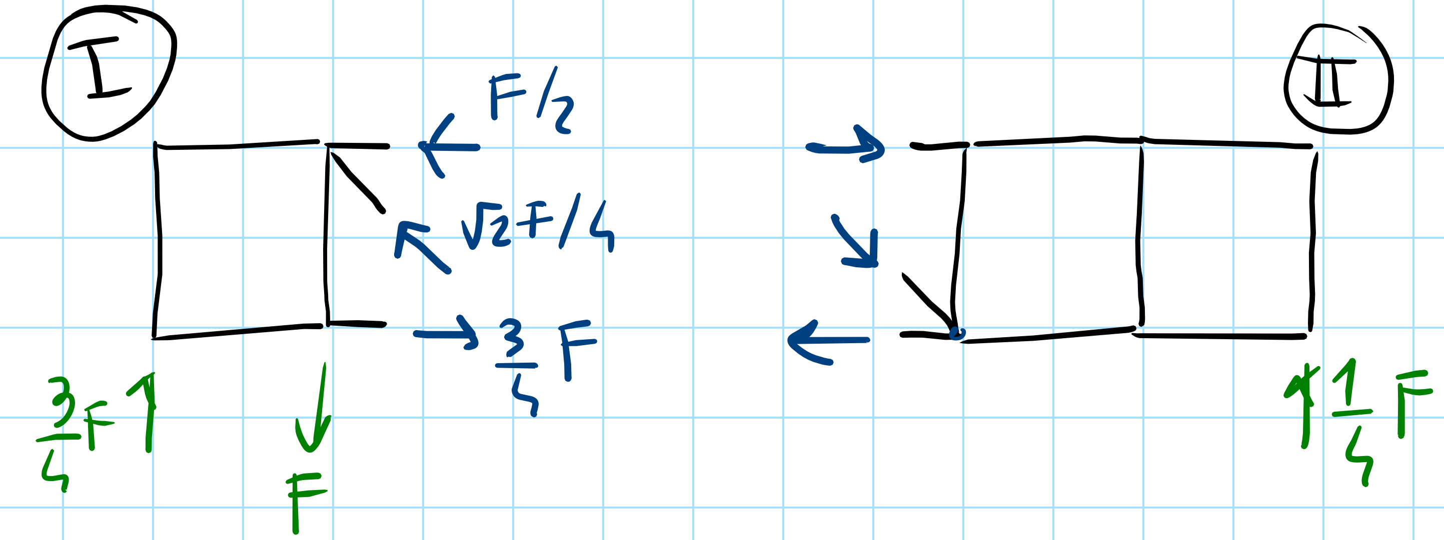
$+\uparrow$: $+\frac{3}{4}F - F - N_2\frac{1}{\sqrt{2}} = 0$; $-\frac{F}{4} = N_2\frac{1}{\sqrt{2}}$; $\boxed{N_2 = -\frac{F}{4}\sqrt{2}}$

(2: PUNTONE)

POSSO SCRIVERE UNA SOLA EQZ PER CALCOLARE DIRETT. N_1?

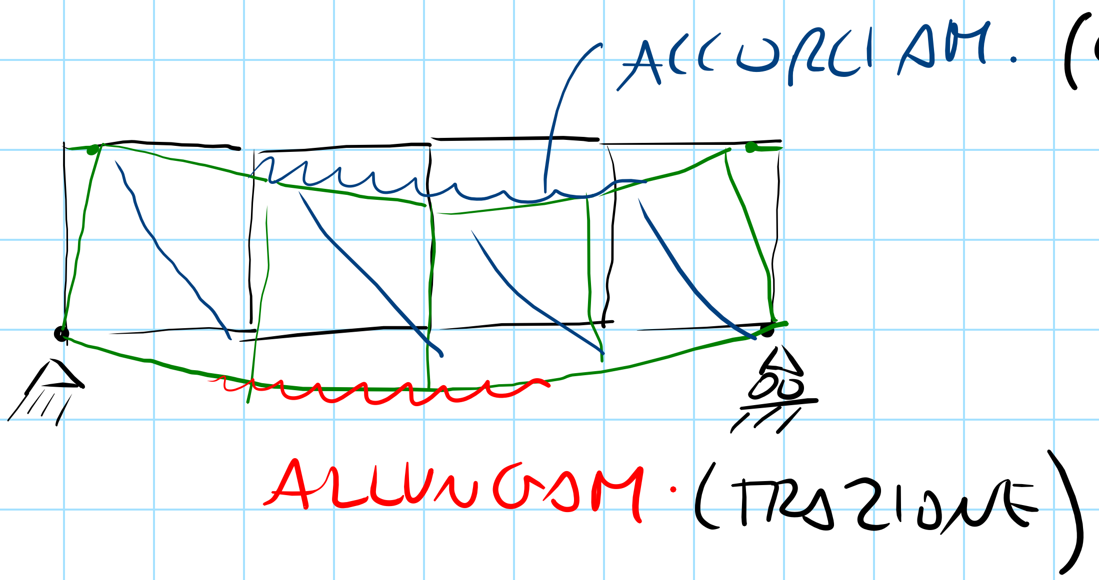
SI $\overset{+}{\curvearrowleft} H$ (I) : $-\frac{3}{4}F\,2\cancel{\ell} + F\cancel{\ell} - N_1\cancel{\ell} = 0 \Rightarrow \boxed{N_1 = -\frac{F}{2}}$

(1: PUNTONE)



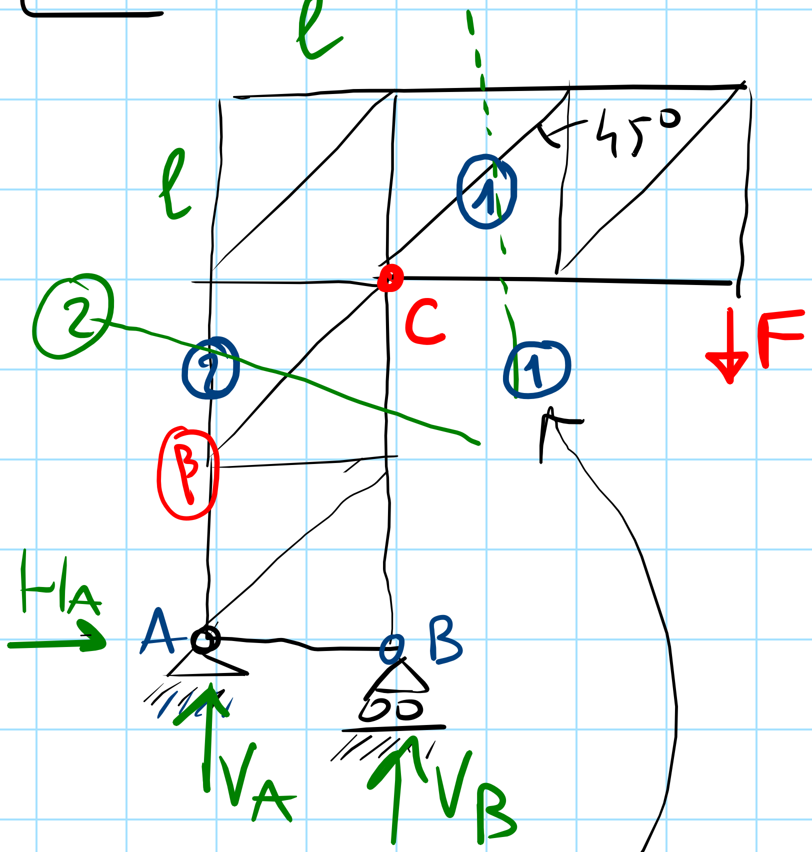
ENTRAMBI I TRONCHI DEVONO SODDISF. L'EQUIL. ASSEGNATI GLI SFORZI N_1, N_2, N_3 CALCOLATI PRECEDENTEMENTE

DEFORM. QUALITATIVA



ES

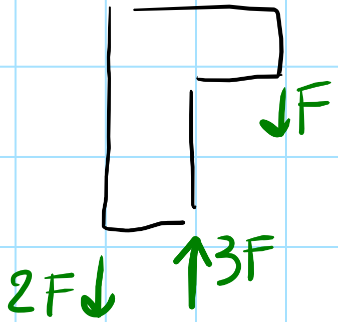
(VERIF. L'ISOST. DELLA STR)



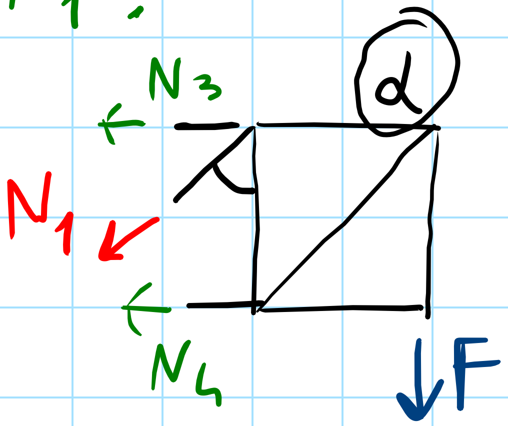
CALCOLARE N_1, N_2.

0) CALCOLO REAZ. VINCOLARI

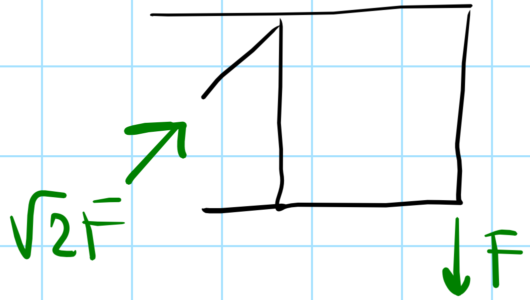
$$\begin{cases} \xrightarrow{+} : +H_A = 0 \\ \uparrow + : +V_A + V_B - F = 0 \\ \circlearrowleft A^{+} : +V_B \ell - F\,3\ell = 0 \end{cases} \qquad \begin{aligned} H_A &= 0 \\ V_A &= -2F \\ V_B &= 3F \end{aligned}$$



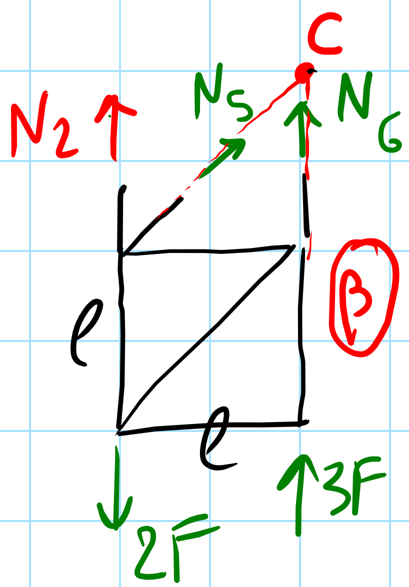
1) N_1 ?



$$+\uparrow \alpha : -N_1 \frac{1}{\sqrt{2}} - F = 0 \quad ; \quad \boxed{N_1 = -\sqrt{2}\,F}$$

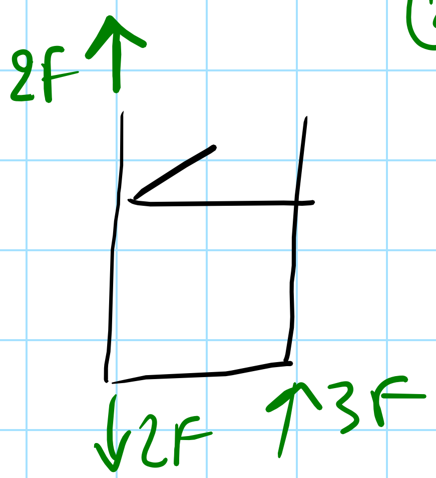


ASTA 1: PUNTONE

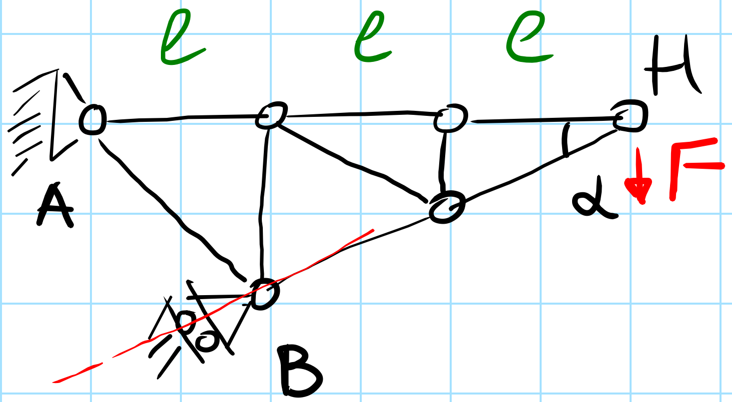


N_2 ? $\overset{+}{\circlearrowleft} C)\ \beta$: $+2F\ell - N_2\ell = 0$; $N_2 = +2F$

2 : TIRANTE



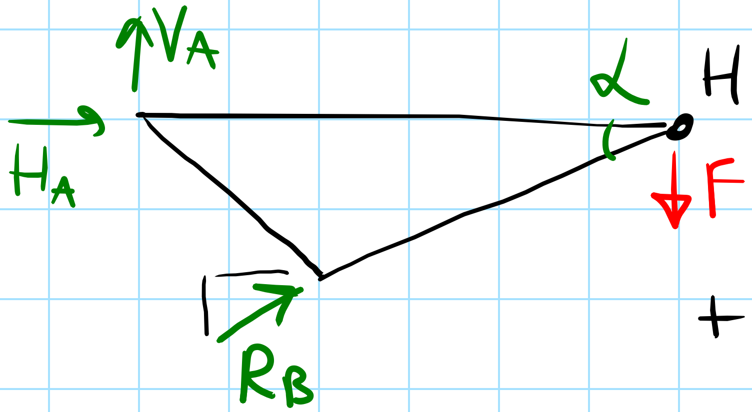
LES



$$2\,n_{NODI} = 2 \cdot 6 = 12$$

$$\left.\begin{array}{l} n_{ASTE} = 9 \\ n_{EST} = 3 \end{array}\right\} 12$$

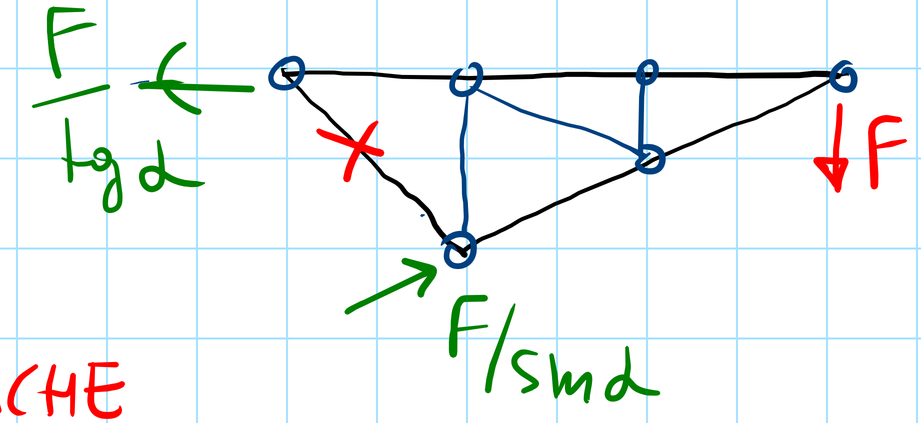
o) CALCOLO REAZ. ESTERNE



$\xrightarrow{+}$: $H_A + R_B \cos\alpha = 0$

$+\uparrow$: $+V_A + R_B \sin\alpha - F = 0$

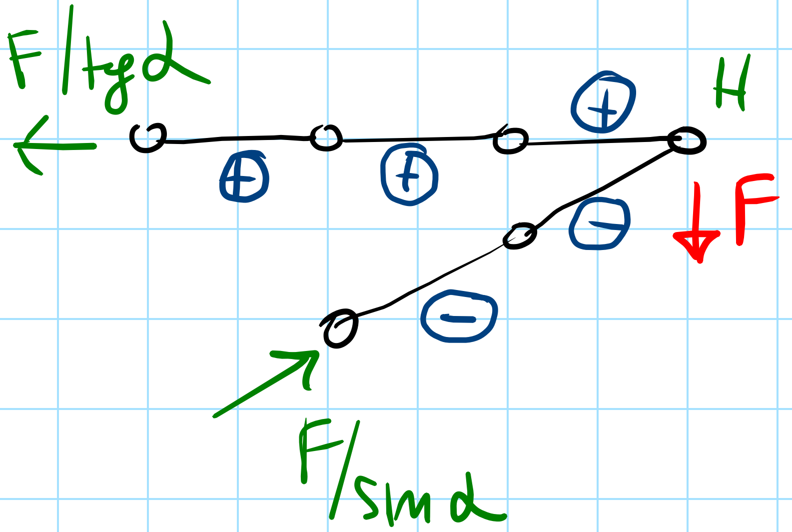
$+\circlearrowleft(H)$: $-V_A\, 3l = 0$ $\quad \boxed{V_A = 0}$

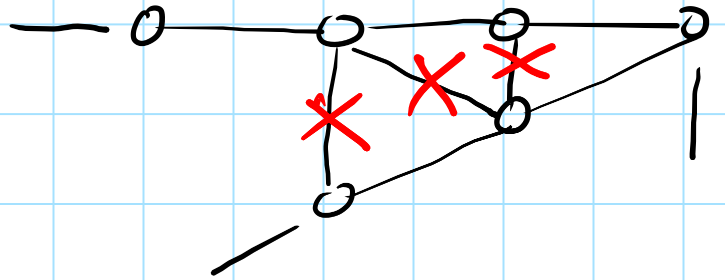


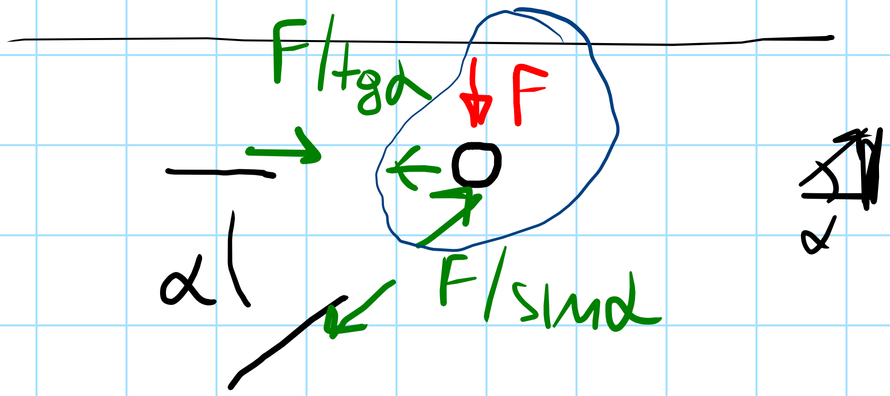
$$R_B = \frac{F}{\sin\alpha}$$

$$H_A = -R_B \cos\alpha = -\frac{F}{\sin\alpha}\cos\alpha = -\frac{F}{tg\alpha}$$

X: ASTE SCARICHE





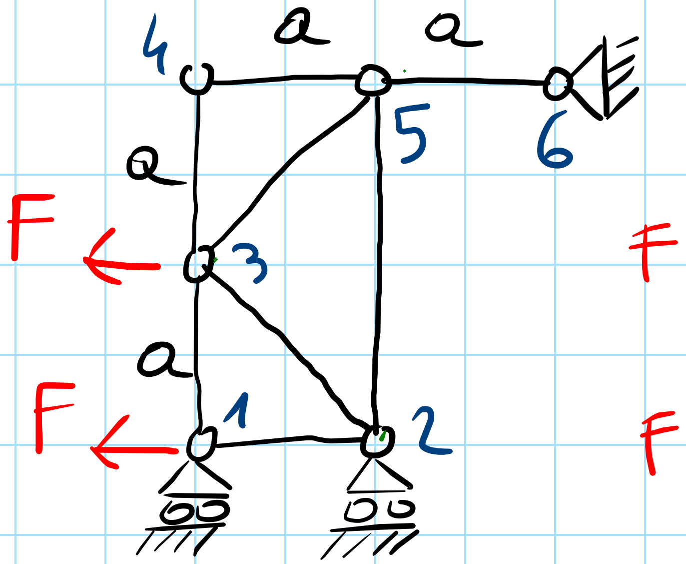


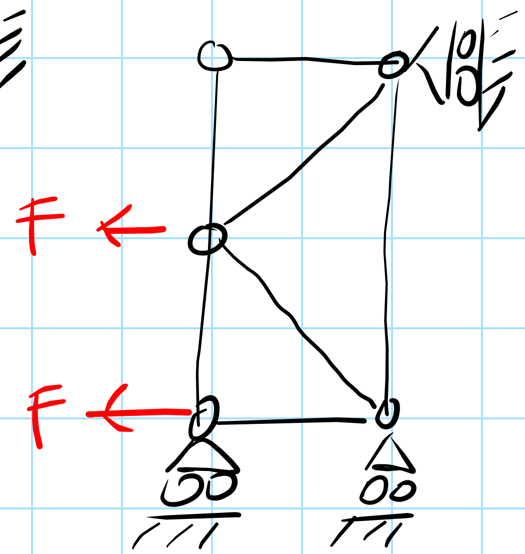
VERIFICA EQUIL NODO H:

$\xrightarrow{+}$: $-\frac{F}{tg\alpha} + \frac{F}{\sin\alpha}\cos\alpha = 0$ OK

$\uparrow +$: $-F + \frac{F}{\cancel{\sin\alpha}}\cancel{\sin\alpha} = 0$ OK

ES





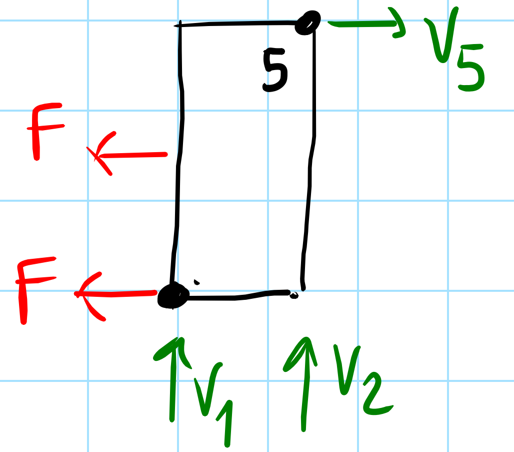
$2\, m_{NODI} = 2 \cdot 5 = 10$

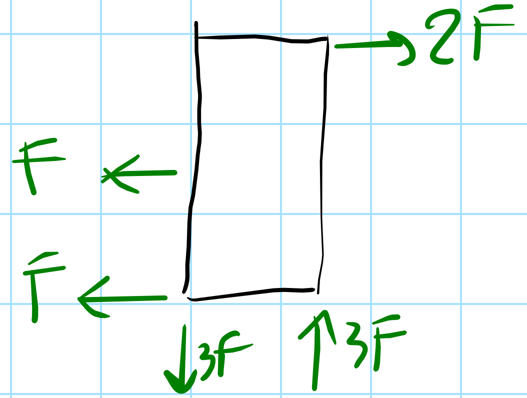
$m_{ASTE} = 7$

$m_{EST} = 3$

$\left.\begin{matrix} 7 \\ 3 \end{matrix}\right\} 10$

OK





0) $\xrightarrow{+}: \; -2F + V_5 = 0$

$+\uparrow: \; V_1 + V_2 = 0$

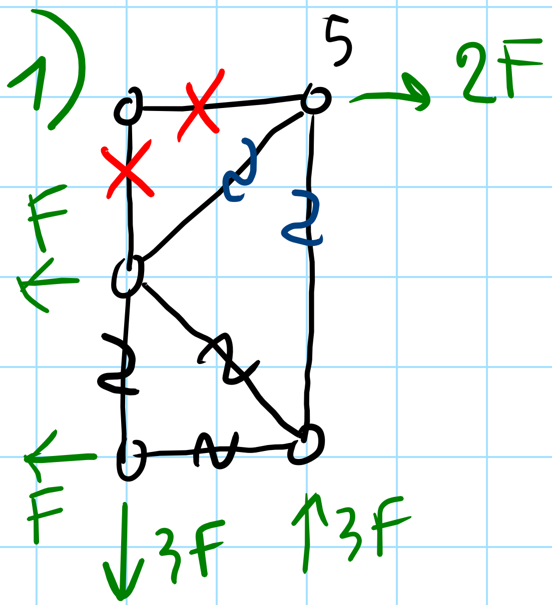
$\circlearrowleft 5^{+}: \; -V_1 a - F2a - Fa = 0$

$V_5 = +2F$

$V_1 = -3F$

$V_2 = +3F$

1)



$N_{13} = +3F$ (TIRANTE)

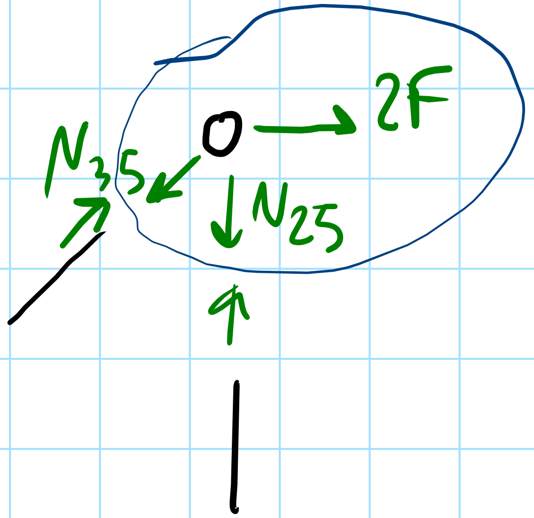
$N_{12} = +F$ (TIRANTE)

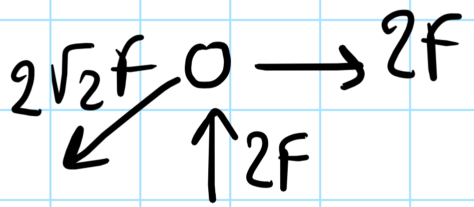
$N_{35} = +2\sqrt{2}F$

$N_{25} = -2F$

$N_{23} = -\sqrt{2}F$

NODO 5:



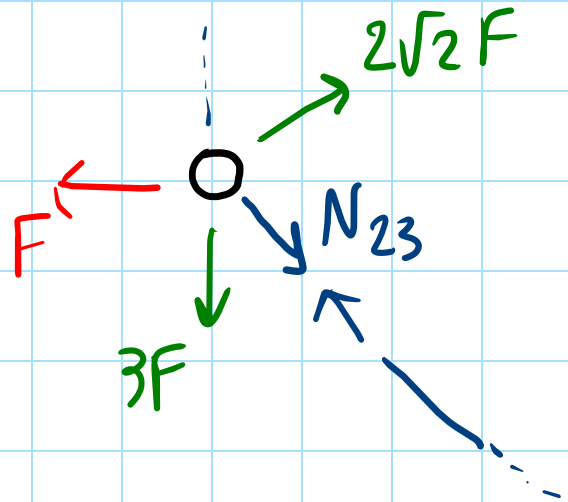


$\xrightarrow{+}: \; 2F - N_{35}\frac{1}{\sqrt{2}} = 0$

$+\uparrow: \; -N_{25} - N_{35}\frac{1}{\sqrt{2}} = 0 \; ; \; N_{25} = -N_{35}\frac{1}{\sqrt{2}} = -\frac{2\cancel{\sqrt{2}}F}{\cancel{\sqrt{2}}} = -2F$

NODO 3

$$\xrightarrow{+}: -F + 2\cancel{\sqrt{2}}F\frac{1}{\cancel{\sqrt{2}}} + N_{23}\frac{1}{\sqrt{2}} = 0 \; ; \; +F + N_{23}\frac{1}{\sqrt{2}} = 0 \; ; \; \boxed{N_{23} = -F\sqrt{2}}$$



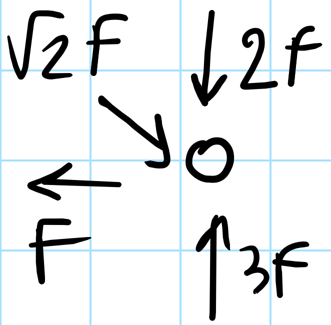
VERIFICA $\uparrow +$:

$$2\cancel{\sqrt{2}}F\frac{1}{\cancel{\sqrt{2}}} - 3F - N_{23}\frac{1}{\sqrt{2}} \overset{?}{=} 0$$

$$2F - 3F + F \overset{?}{=} 0 \quad \text{OK}$$

VERIFICA EQUIL NODO 2 :

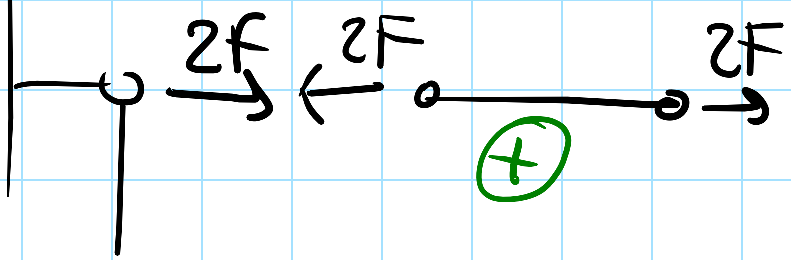
$$\xrightarrow{+}: -F + \cancel{\sqrt{2}}F\frac{1}{\cancel{\sqrt{2}}} \overset{?}{=} 0 \quad \text{OK}$$



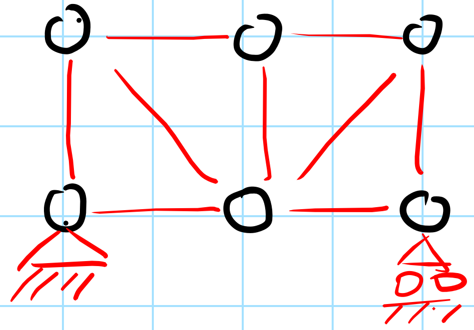
$$+\uparrow: 3F - 2F - \cancel{\sqrt{2}}F\frac{1}{\cancel{\sqrt{2}}} \overset{?}{=} 0 \quad \text{OK}$$

NOTA:

L'ASTA 5G, ESCLUSA DAL CALCOLO PERCHÉ EQUIV. AL CARRELLO, È UN TIRANTE



CON IL METODO DEI RIMANE SEMPRE UN NODO CHE PUÒ ESSERE USATO COME VERIFICA FINALE. PERCHÈ?



6 NODI ⟹ EQZ DI EQUIL TOTALI DISPONIBILI SONO 12

9 ASTE INCOGNITE e 3 REAZ. ESTERNE INCOGNITE

UTILIZZO 3 EQZ PER L'EQUIL. ESTERNO, POI UTILIZZO 9 EQZ PER CALCOLARE LE 9 INCOGNITE DELLE ASTE.

ALLA FINE RIMANGONO 3 EQZ DI EQUILIBRIO NODALE CHE NON USO PER LE INCOGNITE; ESSE POSSONO ESSERE UTILE PER LA VERIFICA DELL'EQUILIBRIO.