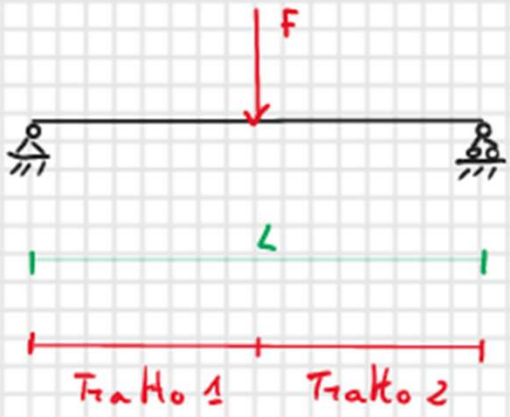
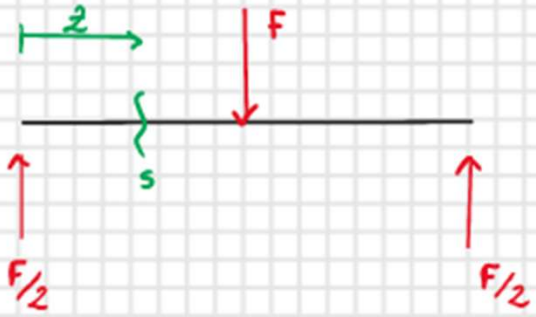
Strutture con discontinuità

- dovute ai carichi
- dovute alla geometria

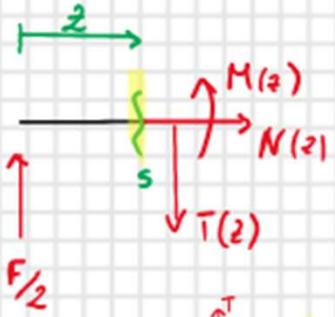
Discontinuità dovuta ai carichi (carico concentrato)





Le espressioni delle azioni interne cambiano in ogni tratto

Tratto 1 $0 \le z \le L/2$



Eq. del tratto

$N(z) = 0$

$T(z) - F/2 = 0$

$M(z) - F/2\, z = 0$

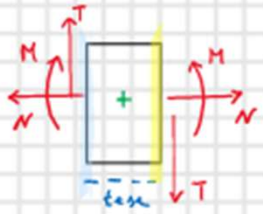
→

Tratto 1

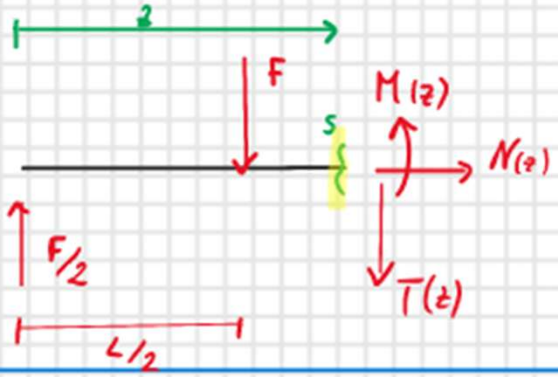
$N(z) = 0$

$T(z) = F/2$

$M(z) = F/2\, z$



Tratto 2 $\frac{L}{2} \le z \le L$ (superiamo il carico concentrato F)



Eq. Equilibrio

$N(z) = 0$

$T(z) - F/2 + F = 0$

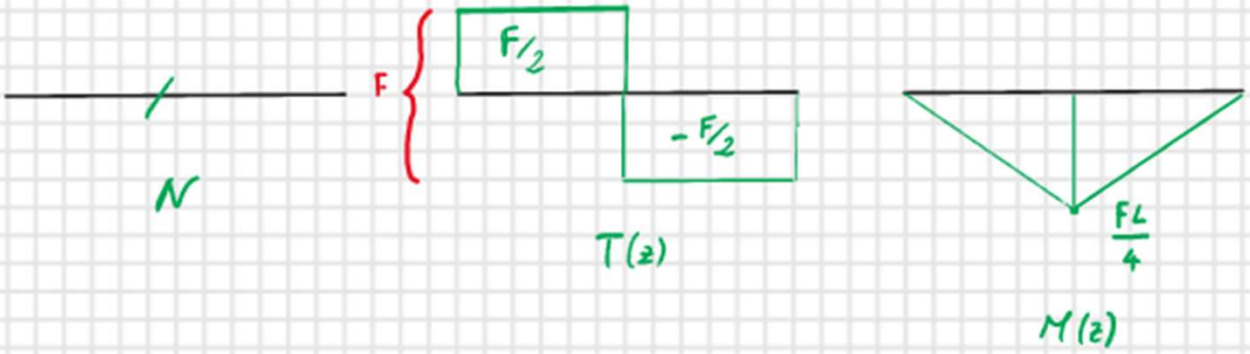
$M(z) - F/2\, z + F\left(z - \frac{L}{2}\right) = 0$

$N(z) = 0 \qquad T(z) = -\frac{F}{2} \qquad M(z) = \frac{F}{2}(L - z)$

Tratto 2

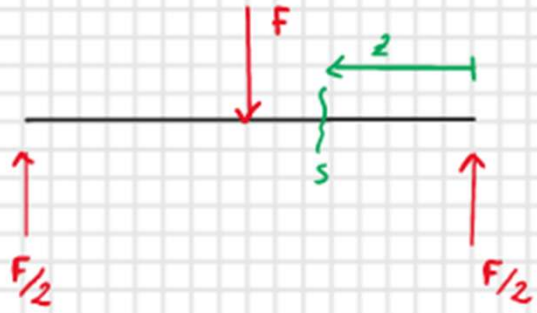
(1)

Diagrammi



Discontinuità nel passaggio dal tratto 1 a 2, il valore della discontinuità coincide col valore del carico concentrato applicato

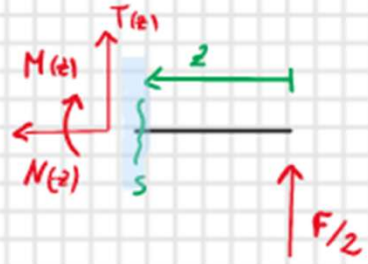
Alternativa calcolo azioni interne nel tratto 2



Fisso l'origine della coordinata z nell'estremità destra

Tratto 2 $\quad 0 \le z \le \frac{L}{2}$

Eq. di equilibrio



$N(z) = 0$

$T(z) + \frac{F}{2} = 0$

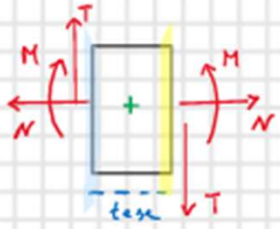
$M(z) - \frac{F}{2} z = 0$

Tratto 2

$N(z) = 0$

$T(z) = -F/2$ (2)

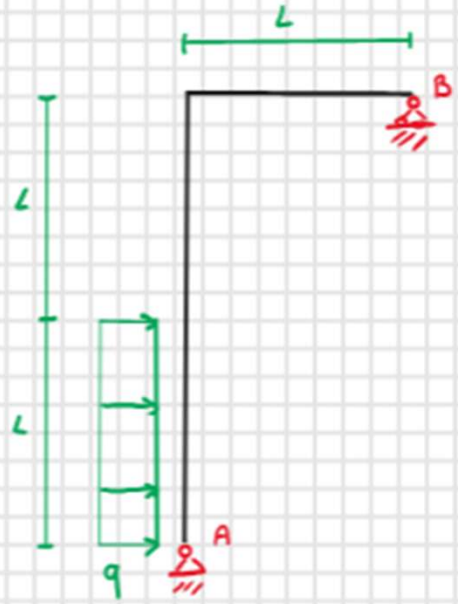
$M(z) = \frac{F}{2} z$



Le equazioni (1) e (2) del tratto 2 sono diverse perché la coordinata z è diversa

Le equazioni sono diverse, i diagrammi delle azioni interne sono gli stessi.

Discontinuità nei carichi e nella geometria



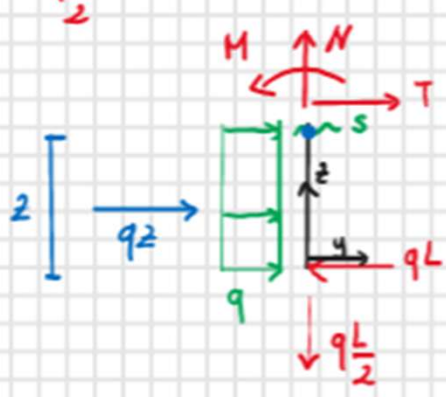
$H_A + qL = 0 \qquad H_A = -qL$

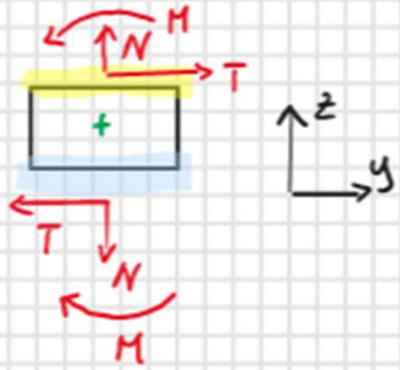
$V_A + V_B = 0 \qquad V_A = -qL/2$

$V_B L - qL\left(\frac{L}{2}\right) = 0 \qquad V_B = \frac{qL}{2}$

Tre tratti

Tratto n° 1 $0 \le z \le L$



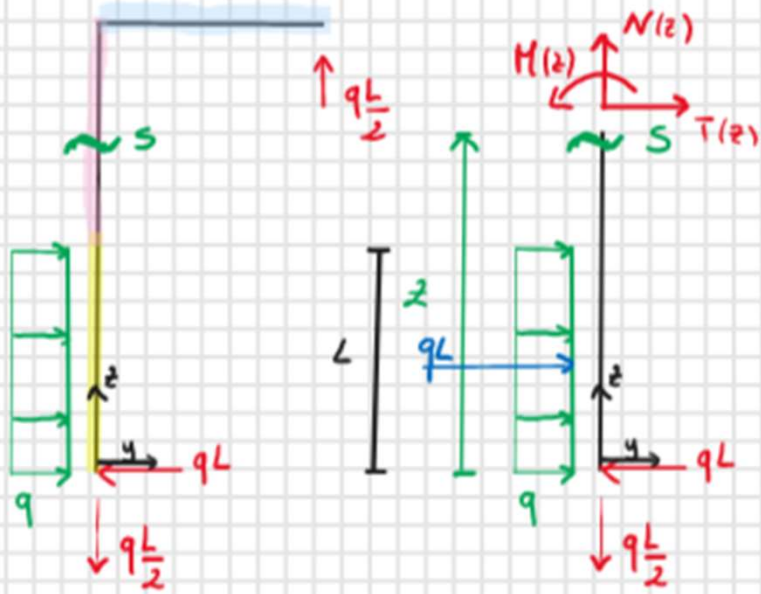


Eq. equilibrio tratto n° 1

1) $N(z) - q\frac{L}{2} = 0$ 2) $T(z) - qL + qz = 0$ 3) $M(z) - qLz + \frac{qz^2}{2} = 0$

$N(z) = q\frac{L}{2}$	$T(z) = qL - qz$	$M(z) = qLz - qz^2/2$	Tratto 1

Tratto n°2 $L \le z \le 2L$



eq. di equilibrio

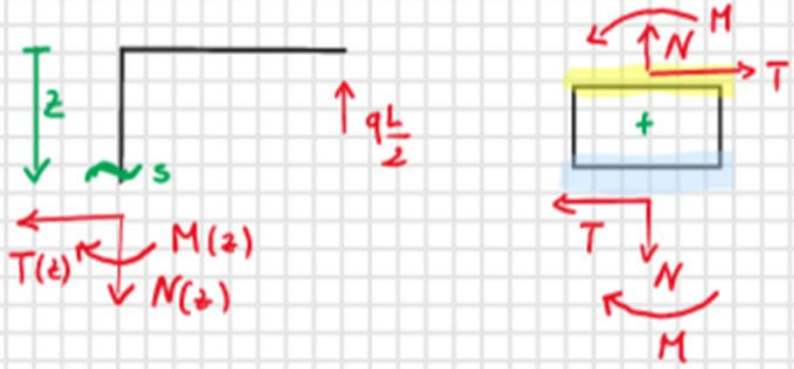
1) $N(z) - q\frac{L}{2} = 0$

2) $T(z) - qL + qL = 0$

3) $M(z) - qLz + qL\left(z - \frac{L}{2}\right) = 0$

$N(z) = q\frac{L}{2}$ $\quad T(z) = 0$ $\quad M(z) = q\frac{L^2}{2}$ Tratto n°2

Alternativa calcolo tratto n°2 (z dall'alto)

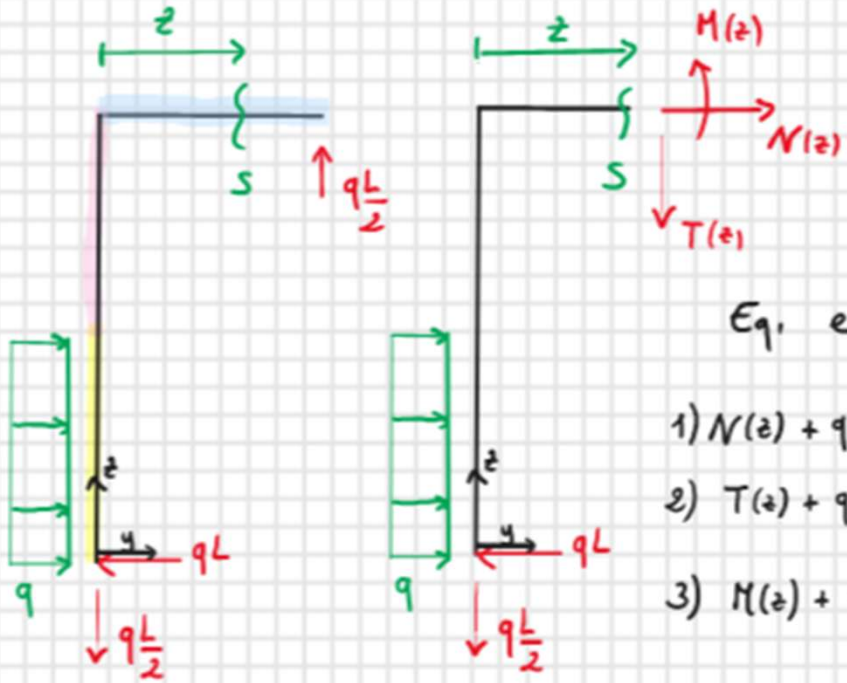


Eq. di equilibrio

1) $N(z) - q\frac{L}{2} = 0$ $\quad$ 2) $T(z) = 0$ $\quad$ 3) $M(z) - q\frac{L^2}{2} = 0$

$N(z) = q\frac{L}{2}$ $\quad T(z) = 0$ $\quad M(z) = q\frac{L^2}{2}$ Tratto 2

Tratto n° 3



Eq. equilibrio

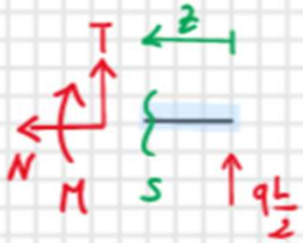
1) $N(z) + qL - qL = 0$

2) $T(z) + q\frac{L}{2} = 0$

3) $M(z) + q\frac{L}{2}z - qL(zL) + qL\left(\frac{3}{2}L\right) = 0$

$N(z) = 0 \qquad T(z) = -q\frac{L}{2} \qquad M(z) = q\frac{L^2}{2} - q\frac{L}{2}z$ Tratto n° 3

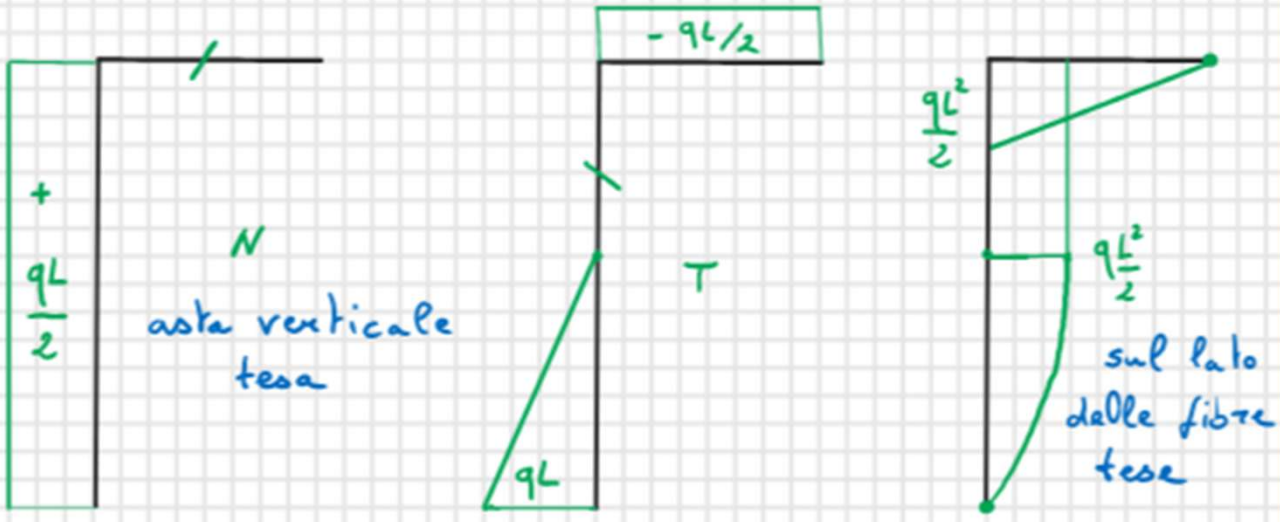
Alternativa tratto n° 3 (z da destra)

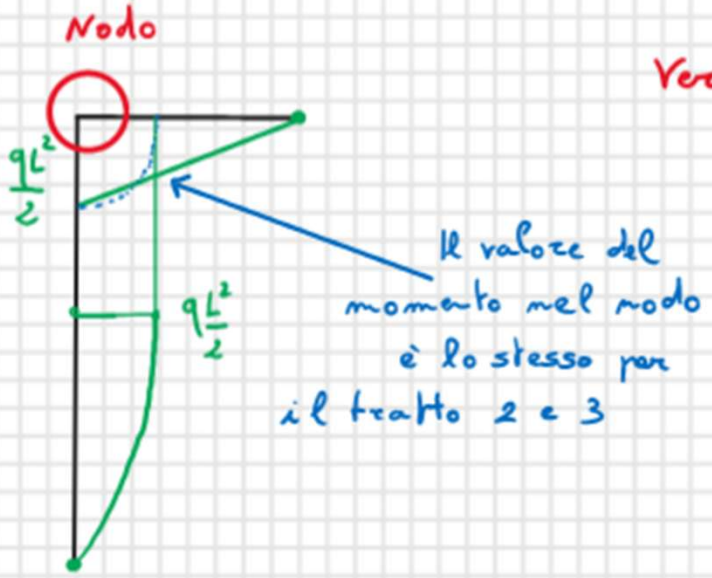


$N(z) = 0 \qquad T(z) = -q\frac{L}{2} \qquad M(z) = q\frac{L}{2}z$

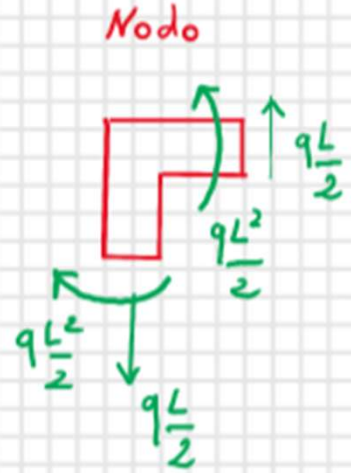
Tratto n° 3

Diagrammi





Verifica del nodo



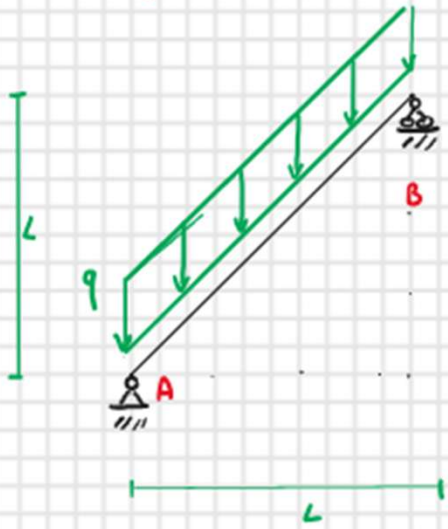
Il nodo deve essere equilibrato alla traslazione

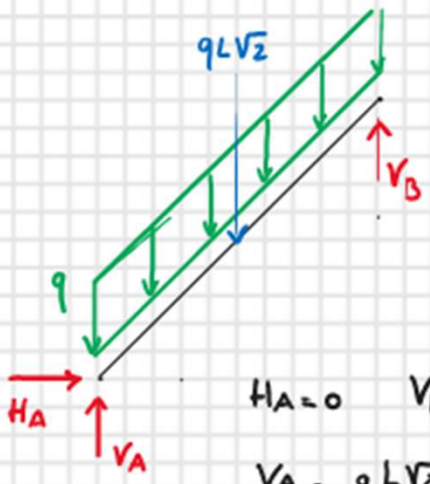
La somma dei momenti che agiscono sul nodo sia nullo $\sum_{nodo} M = 0$

Se nel nodo è presente un momento concentrato $\bar{M}$

$$\sum_{nodo} M = \bar{M}$$

Esempio: Trave inclinata (Peso proprio)





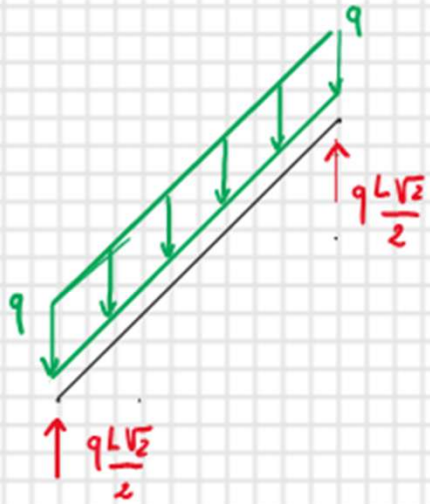
1) $H_A = 0$

2) $V_A + V_B = qL\sqrt{2}$

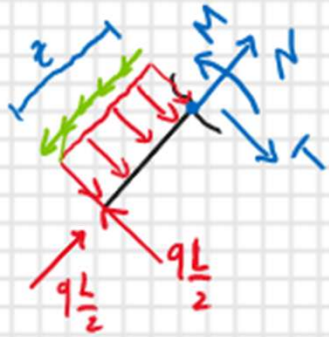
3) $V_B L - qL\sqrt{2}\left(\frac{L}{2}\right) = 0$

$H_A = 0 \qquad V_B = qL\frac{\sqrt{2}}{2}$

$V_A = qL\frac{\sqrt{2}}{2}$



Proietto lungo l'asse e perpendicolarmente all'asse della trave



$$q\frac{L}{2} - q\frac{\sqrt{2}}{2}z + N(z) = 0$$

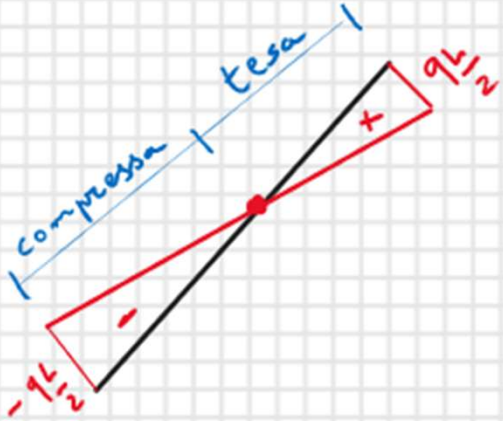
$$q\frac{L}{2} - q\frac{\sqrt{2}}{2}z - T(z) = 0$$

$$M(z) - q\frac{L}{2}z + \left(q\frac{\sqrt{2}}{2}\right)\frac{z^2}{2} = 0$$

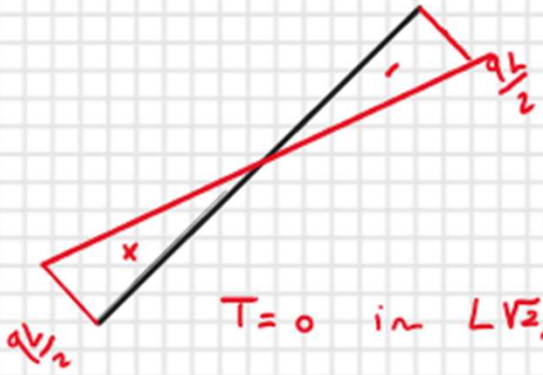
$N(z) = -q\frac{L}{2} + q\frac{\sqrt{2}}{2}z \qquad T(z) = q\frac{L}{2} - q\frac{\sqrt{2}}{2}z \qquad M(z) = q\frac{L}{2}z - \left(q\frac{\sqrt{2}}{2}\right)\frac{z^2}{2}$

Diagrammi

Azione assiale



Taglio



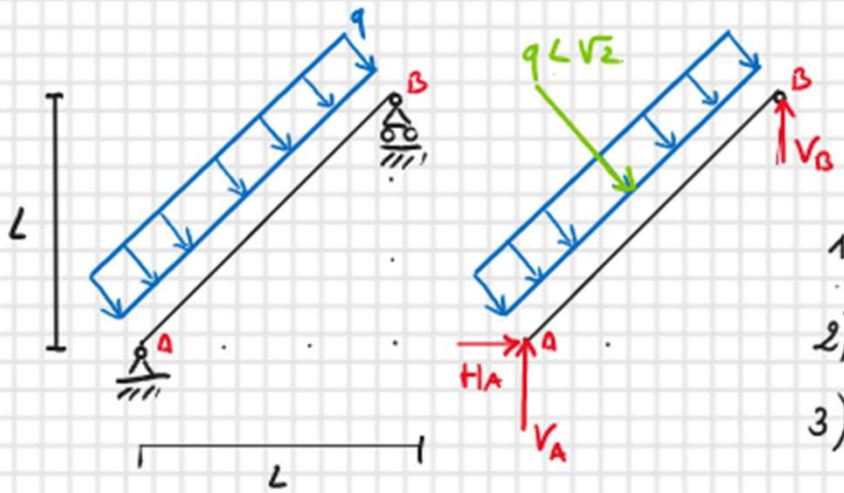
$T = 0$ in $L\sqrt{2}/2$

Momento flettente

$Max \longrightarrow T = 0$

$M_{max} = M\left(\frac{L\sqrt{2}}{2}\right) = q\frac{L^2\sqrt{2}}{8}$

Esempio: Trave inclinata (vento)

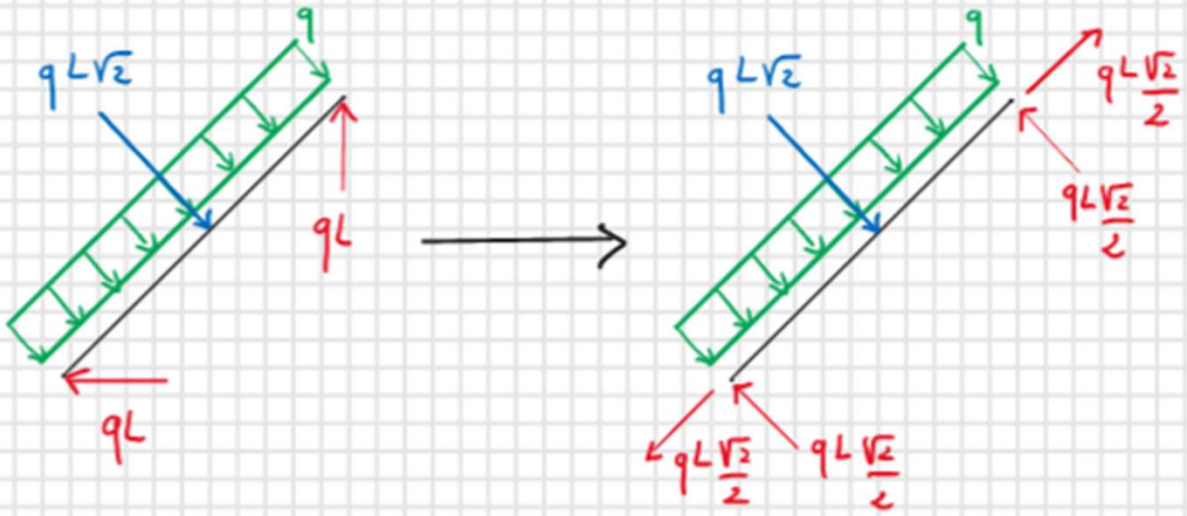


1) $H_A + qL = 0$

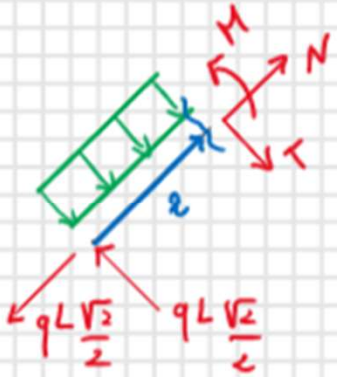
2) $V_A + V_B = qL$

3) $V_B L - qL\sqrt{2}\left(L\frac{\sqrt{2}}{2}\right) = 0$

$V_B = qL \quad V_A = 0 \quad H_A = -qL$



Calcolo azioni interne

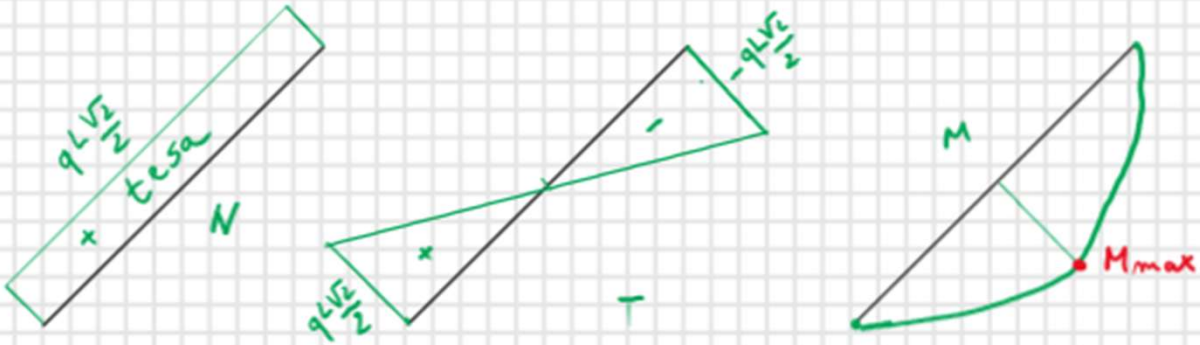


$$N(z) = qL\frac{\sqrt{2}}{2}$$

$$T(z) = qL\frac{\sqrt{2}}{2} - qz$$

$$M(z) = qL\frac{\sqrt{2}}{2}z - q\frac{z^2}{2}$$

Diagrammi



$$M_{max} = M\left(z = L\frac{\sqrt{2}}{2}\right)$$