

PSYCHROMETRICS

HVAC SYSTEM DESIGN

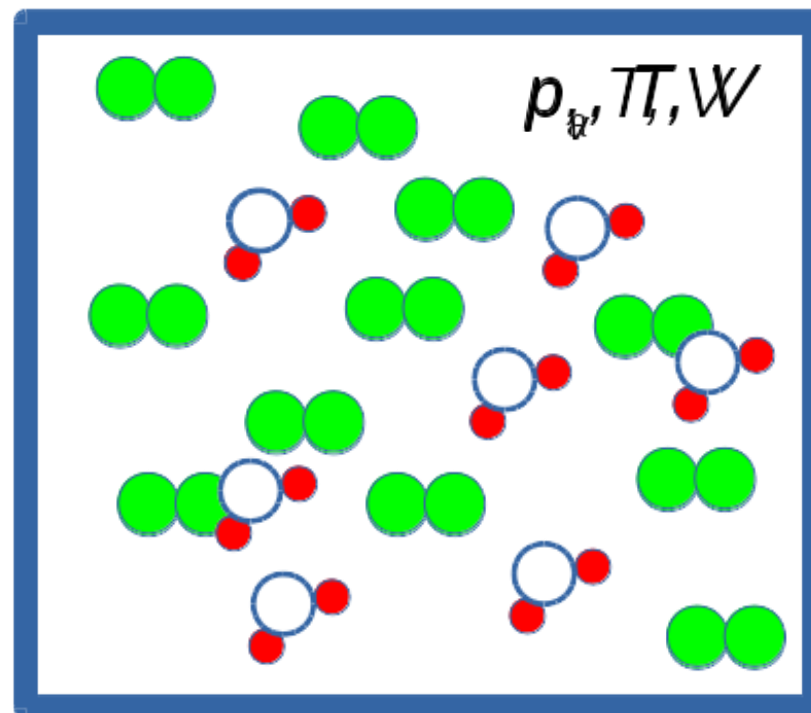
Engineering for the Energy Transition

March 2025

Moist Air

- **Moist Air:** The term moist air is a mixture of dry air and water vapor in which the dry air is treated as a pure substance.
- **Moist air as a perfect gas:** $T_{air} \gg T_{critical}$ e $p \ll p_{critical}$
- **Mixture of perfect gases:** thermodynamic system consisting of a homogeneous mixture of n non-reacting components in the gaseous state, each of which behaves as a perfect gas.
- At equilibrium, the state of the system is determined by the value of two intensive/specific properties and the composition of the system.
- Specific quantities referred to the **mass of dry air**

Moist Air- Dalton



- Partial pressure of a gaseous component i of a mixture: pressure to which the i -th component would be subjected if it alone occupied the entire volume V occupied by the mixture at the same temperature T .

$$p = \frac{n \cdot \bar{R} \cdot T}{V}$$

$$p_{da} = \frac{n_{da} \cdot \bar{R} \cdot T}{V} = \frac{m_{da} \cdot R_{da} \cdot T}{V}$$

$$p_{vap} = \frac{n_{vap} \cdot \bar{R} \cdot T}{V} = \frac{m_{vap} \cdot R_{vap} \cdot T}{V}$$

$$p = p_{da} + p_{vap}$$

Moist Air– mole fraction

Mole fraction: ratio of the number of moles of a component in the mixture to the total number of moles.

$$p_{da} = \frac{n_{da} \cdot \bar{R} \cdot T}{V} = \frac{m_{da} \cdot R_{da} \cdot T}{V}$$

$$p_{vap} = \frac{n_{vap} \cdot \bar{R} \cdot T}{V} = \frac{m_{vap} \cdot R_{vap} \cdot T}{V}$$

$$p = \frac{n \cdot \bar{R} \cdot T}{V}$$

$$n = n_{da} + n_{vap}$$

$$m = m_{da} + m_{vap}$$

Mole fraction of
dry air

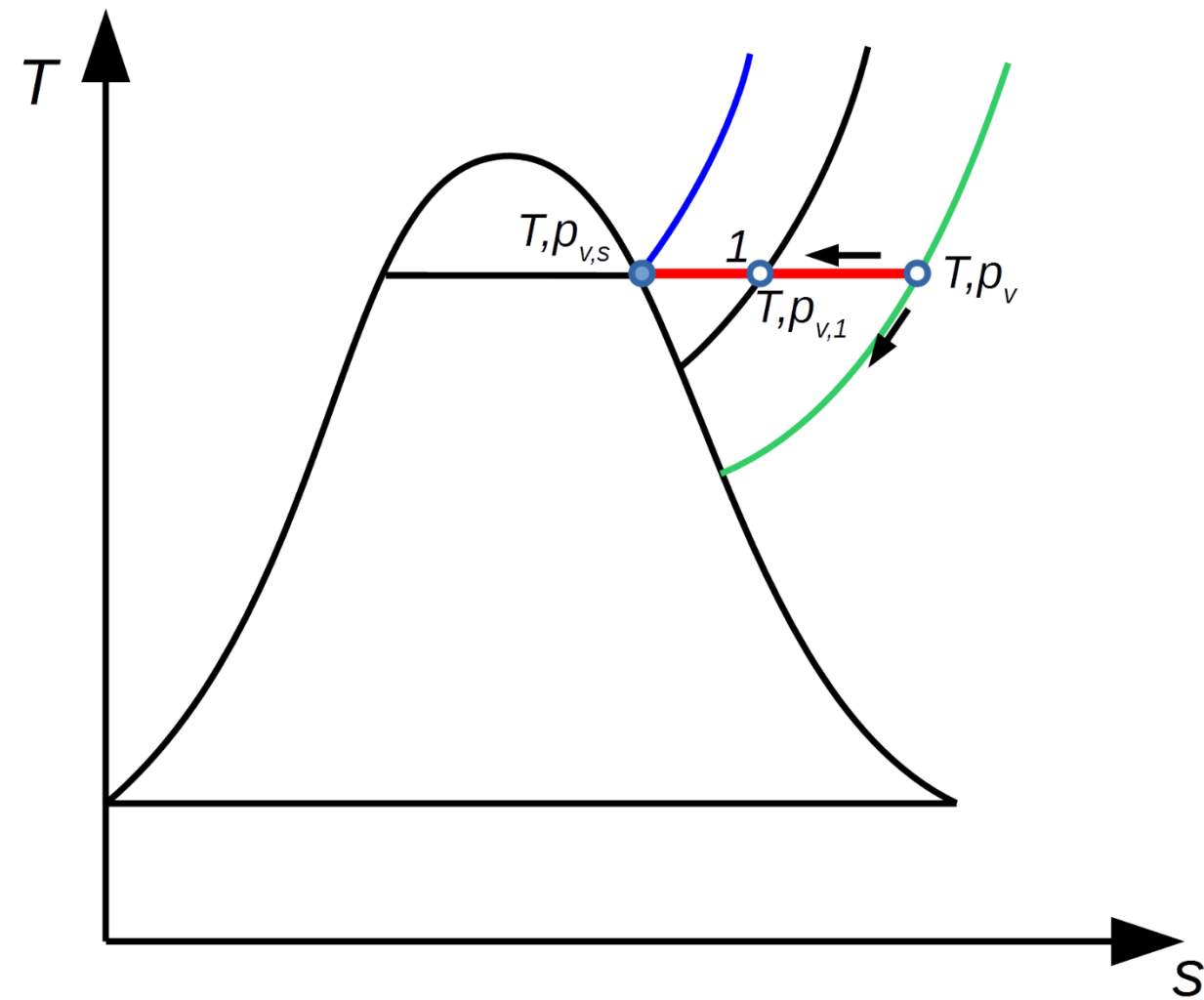
Mole fraction of
water vapor

$$\frac{p_{da}}{p} = \frac{n_{da} \cdot \bar{R} \cdot T / V}{n \cdot \bar{R} \cdot T / V} = \frac{n_{da}}{n} = X_{da}$$

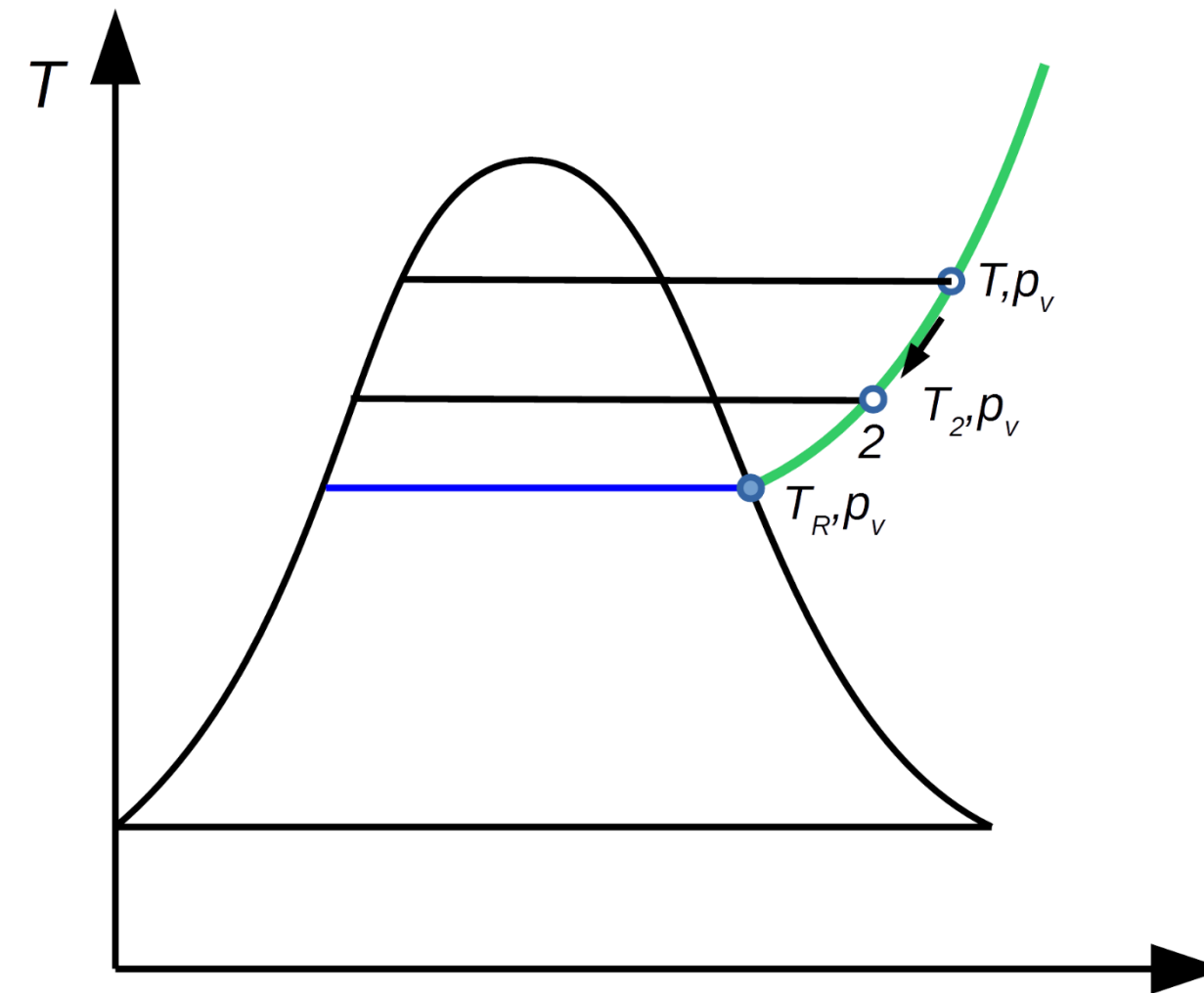
$$\frac{p_{vap}}{p} = \frac{n_{vap} \cdot \bar{R} \cdot T / V}{n \cdot \bar{R} \cdot T / V} = \frac{n_{vap}}{n} = X_{vap}$$

Vapor Condensation

Increase of vapor mole



Air cooling



Humidity Ratio

$$w = \frac{m_{vap}}{m_{da}} = \frac{M_{vap} \cdot \rho_{vap} \cdot V / (\bar{R} \cdot T)}{M_{da} \cdot \rho_{da} \cdot V / (\bar{R} \cdot T)} = \frac{M_{vap} \cdot \rho_{vap}}{M_{da} \cdot \rho_{da}} \quad w = 0.622 \cdot \frac{\rho_{vap}}{p - \rho_{vap}}$$

$$m_{vap} = M_{vap} \cdot n_{vap}$$

$$p_{vap} \cdot V = n_{vap} \cdot \bar{R} \cdot T \Rightarrow n_{vap} = \frac{p_{vap} \cdot V}{\bar{R} \cdot T}$$

$$m_{da} = M_{da} \cdot n_{da}$$

$$p_{da} \cdot V = n_{da} \cdot \bar{R} \cdot T \Rightarrow n_{da} = \frac{p_{da} \cdot V}{\bar{R} \cdot T}$$

Relative humidity

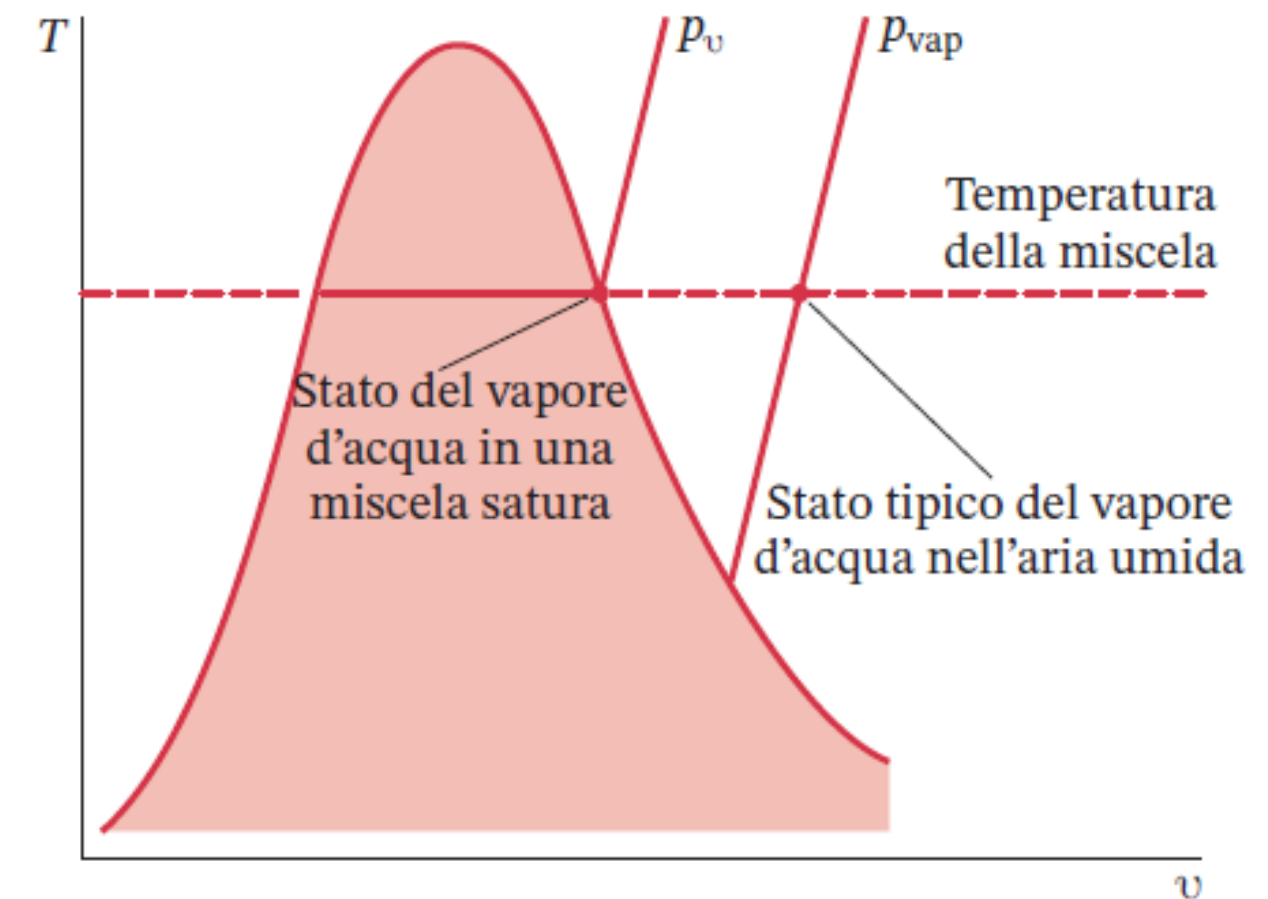
$$\phi = \frac{m_{vap}}{m_{vs}} = \frac{M_{vap} \cdot p_{vap} \cdot V / (\bar{R} \cdot T)}{M_{vap} \cdot p_{vs} \cdot V / (\bar{R} \cdot T)} = \frac{p_{vap}}{p_v}$$

$$m_{vap} = M_{vap} \cdot n_{vap}$$

$$p_{vap} \cdot V = n_{vap} \cdot \bar{R} \cdot T \Rightarrow n_{vap} = \frac{p_{vap} \cdot V}{\bar{R} \cdot T}$$

$$m_{vs} = M_{vap} \cdot n_{vs}$$

$$p_{vs} \cdot V = n_{vs} \cdot \bar{R} \cdot T \Rightarrow n_{vs} = \frac{p_{vs} \cdot V}{\bar{R} \cdot T}$$



$$\phi = \frac{p_{vap}}{p_{vs}(T)} \Big|_T \rightarrow w = 0,622 \cdot \frac{\phi \cdot p_{vs}(T)}{p - \phi \cdot p_{vs}(T)}$$

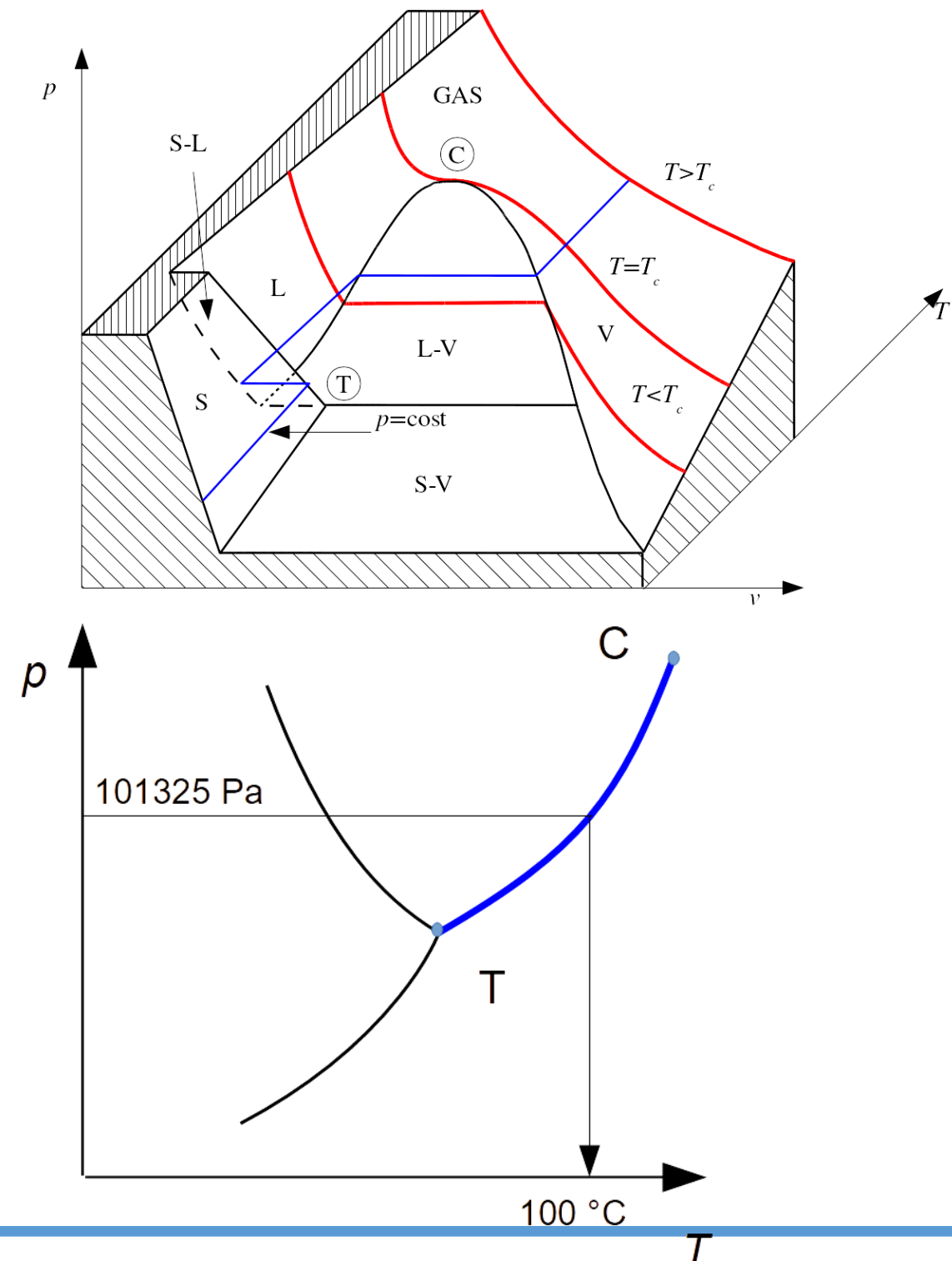
Saturation pressure

- Saturation pressure depends only on temperature
- In Italian standards the saturation pressure can be computed with the following expressions

$$p_v(t) = 610,5 \cdot e^{\frac{17,269 \cdot t}{237,3+t}} \text{ per } t \geq 0 \text{ } ^\circ\text{C}$$

$$p_v(t) = 610,5 \cdot e^{\frac{21,875 \cdot t}{265,5+t}} \text{ per } t < 0 \text{ } ^\circ\text{C}$$

- t [$^\circ\text{C}$], p_v [Pa]



Enthalpy of moist air

Enthalpy of mixture (H)

$$H = H_{da} + H_{vap} = m_{da} \cdot h_{da} + m_{vap} \cdot h_{vap}$$

Specific enthalpy (h)

$$h = \frac{H}{m_{da}} = h_{da} + \frac{m_{vap}}{m_{da}} \cdot h_{vap} = h_{da} + w \cdot h_{vap} \left(\frac{\text{kJ}}{\text{kg}_{da}} \right)$$

$$h_{da} = c_{p,da}(T - 273.15) = c_{p,da} \cdot \theta$$

Conditions:

- Superheated vapor
- Low pressures (<10kPa)
- Specific values referred to **dry air**

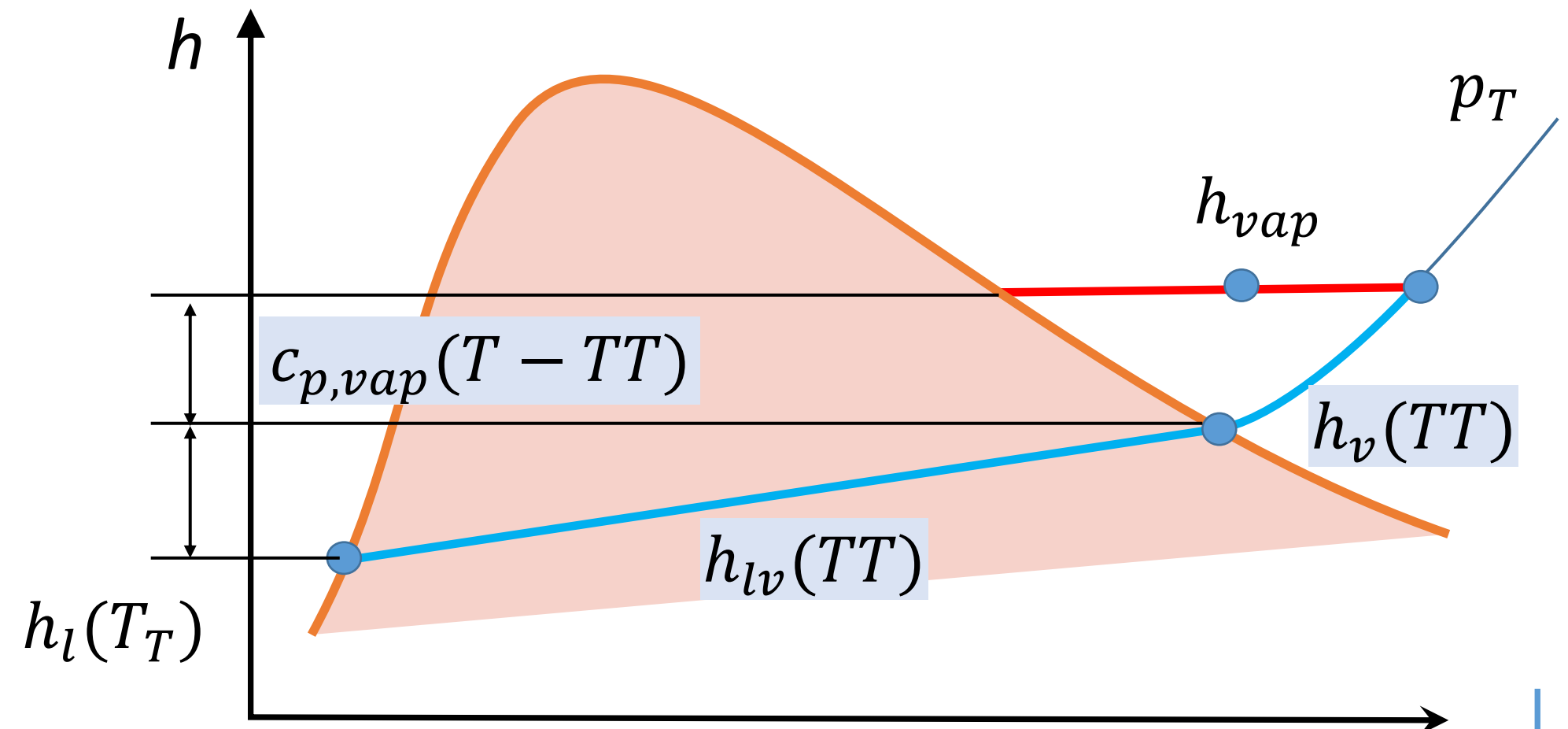
Vapor enthalpy

- The behavior is of an ideal gas
- The enthalpy is a function of temperature
- I construct a transformation from a reference state to a state at the same temperature
- $h_l(0^\circ\text{C}) = 0$ enthalpy on the lower limit curve
- $h_{lv}(0^\circ\text{C})$ latent heat of vaporization
- $c_{p,vap}$ specific heat of vapor

$$h_{lv} = 2501,3 \text{ kJ/kg}$$

$$c_{p,vap} = 1,875 \text{ kJ/(kg K)}$$

$$h_{lv} = h_v - h_l$$



$$h_{vap} = h_l(T_T) + h_{lv}(T_T) + c_{p,vap}(T - T_T) \quad h_l(T_T) = 0 \quad \longrightarrow \quad h_{vap} = h_{lv} + c_{p,vap} \cdot t$$

Specific enthalpy of moist air

$$h = \frac{H}{m_{as}} = h_{da} + \frac{m_{vap}}{m_{da}} \cdot h_{vap} = h_{da} + w \cdot h_{vap} \left(\frac{\text{kJ}}{\text{kg}_{as}} \right)$$

$$h = c_{p,as} \cdot \theta + w \cdot (c_{p,vap} \cdot \theta + h_{lv})$$

$$h = 1,005 \cdot t + w \cdot (1,875 \cdot t + 2501,3) \frac{\text{kJ}}{\text{kg}}$$

Condizioni:

- Superheated vapor
- Low vapor pressure (<10kPa)
- Specific quantities referred to **dry air**

Dew point temperature

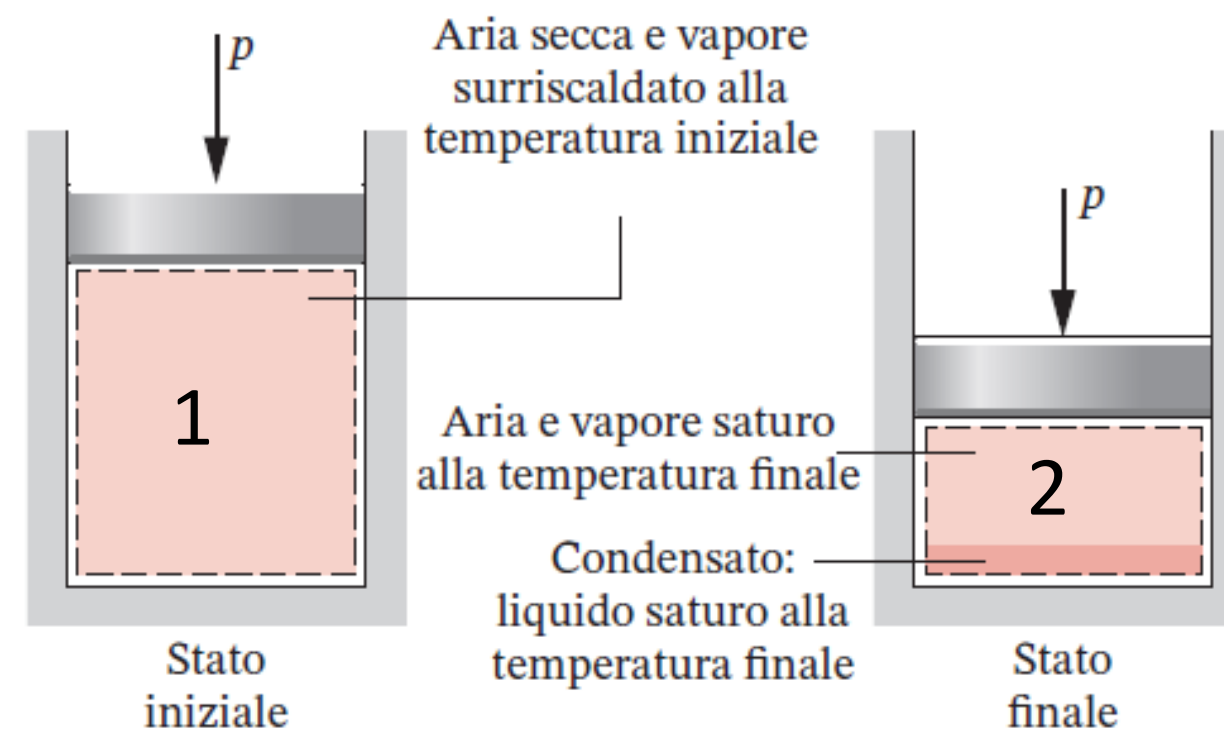
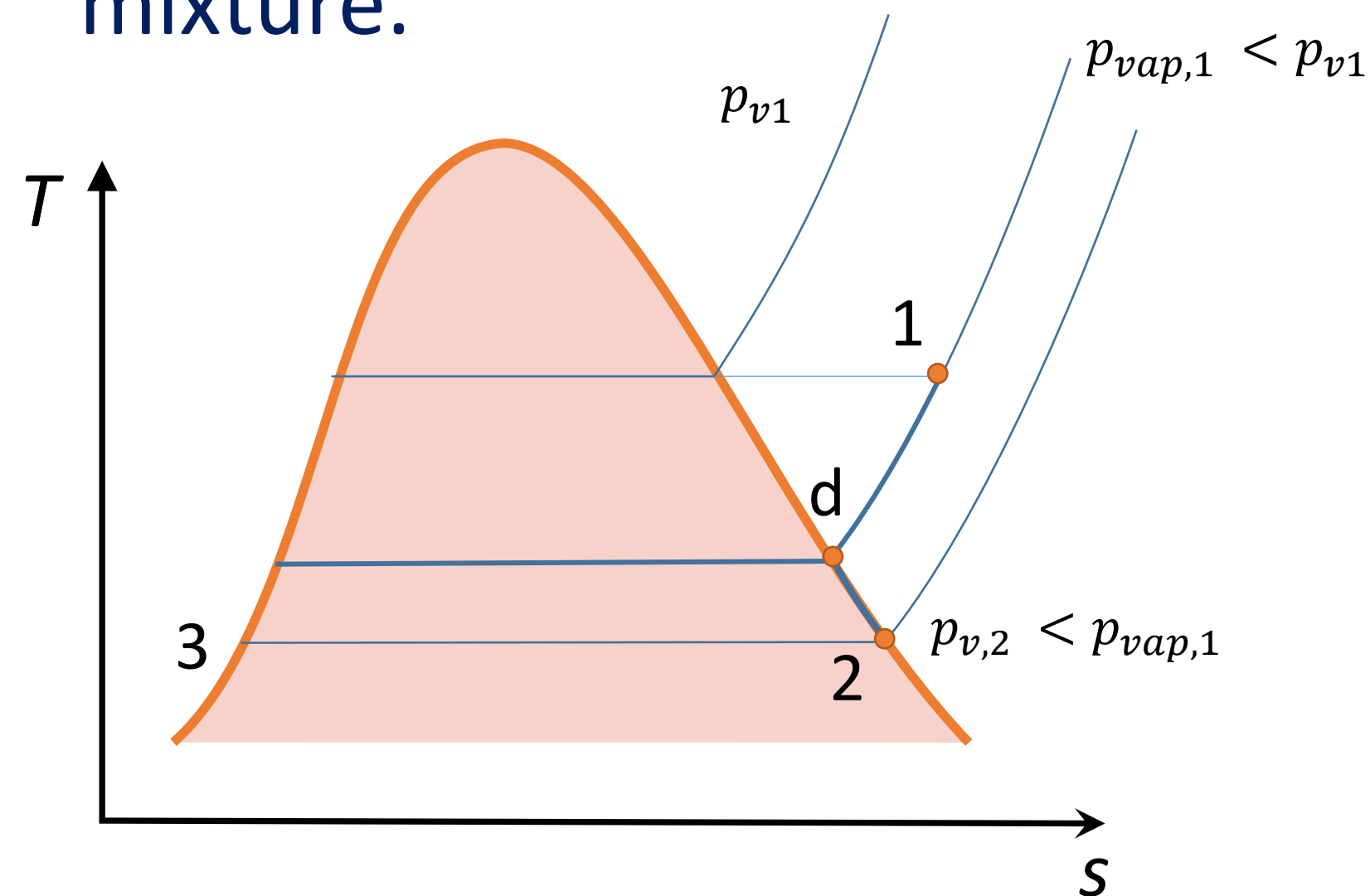
- **Dew point temperature** is the temperature of saturated moist air of the same moist air sample, having the same humidity ratio, and at the same atmospheric pressure of the mixture.
- Two moist air samples at the same T_{dew} will have the same humidity ratio w and the same partial pressure of water vapor p_w .

$$w_s(p_{at}, T_{dew}) = w$$

- where w_s humidity ratio of saturated moist air. At a specific atmospheric pressure,
- the dew-point temperature determines the humidity ratio w and the water vapor pressure p_w of the moist air.

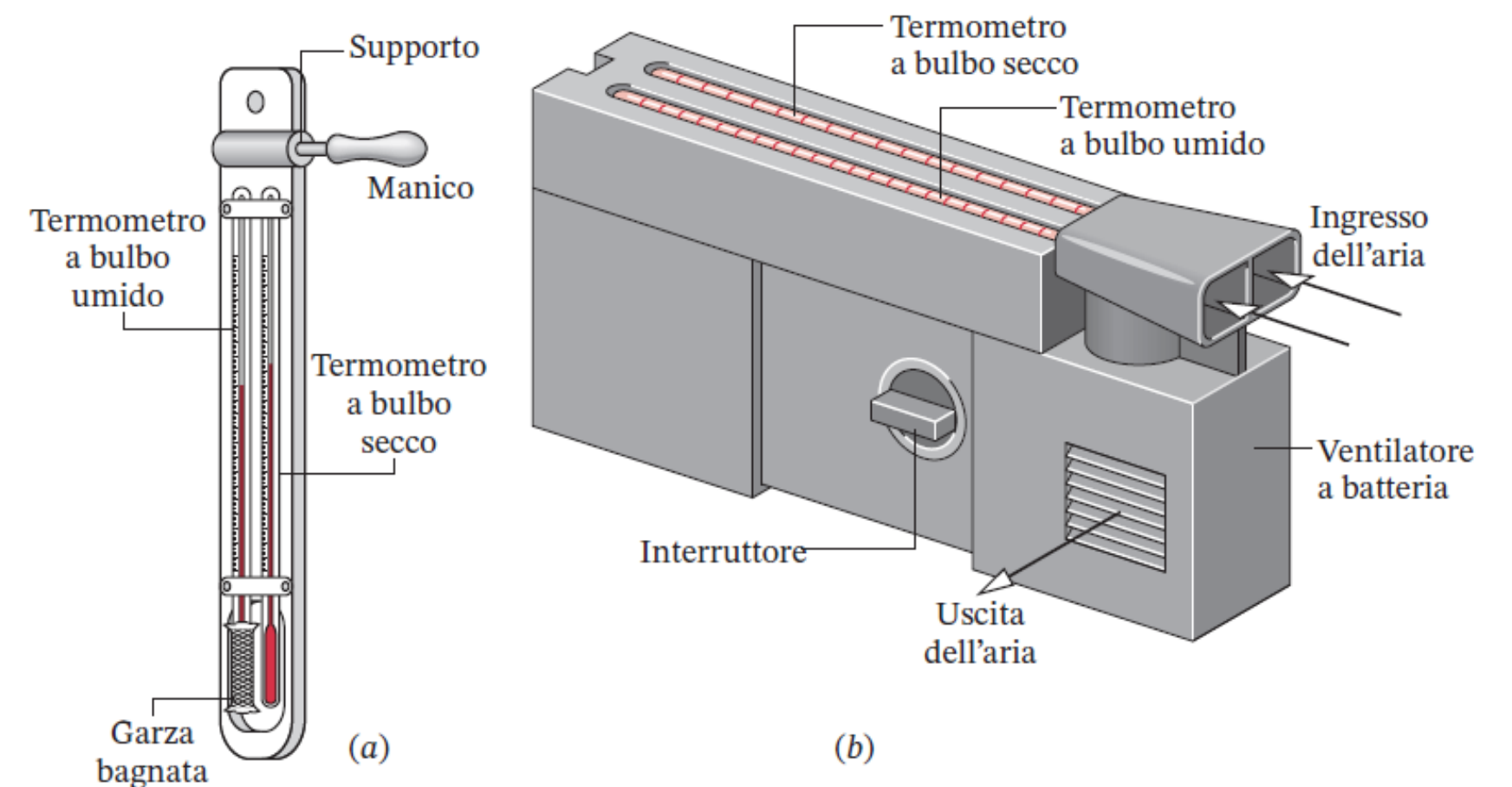
Dew point temperature

- The dew point temperature is the saturation temperature of water vapor corresponding to the partial pressure of vapor in the mixture.



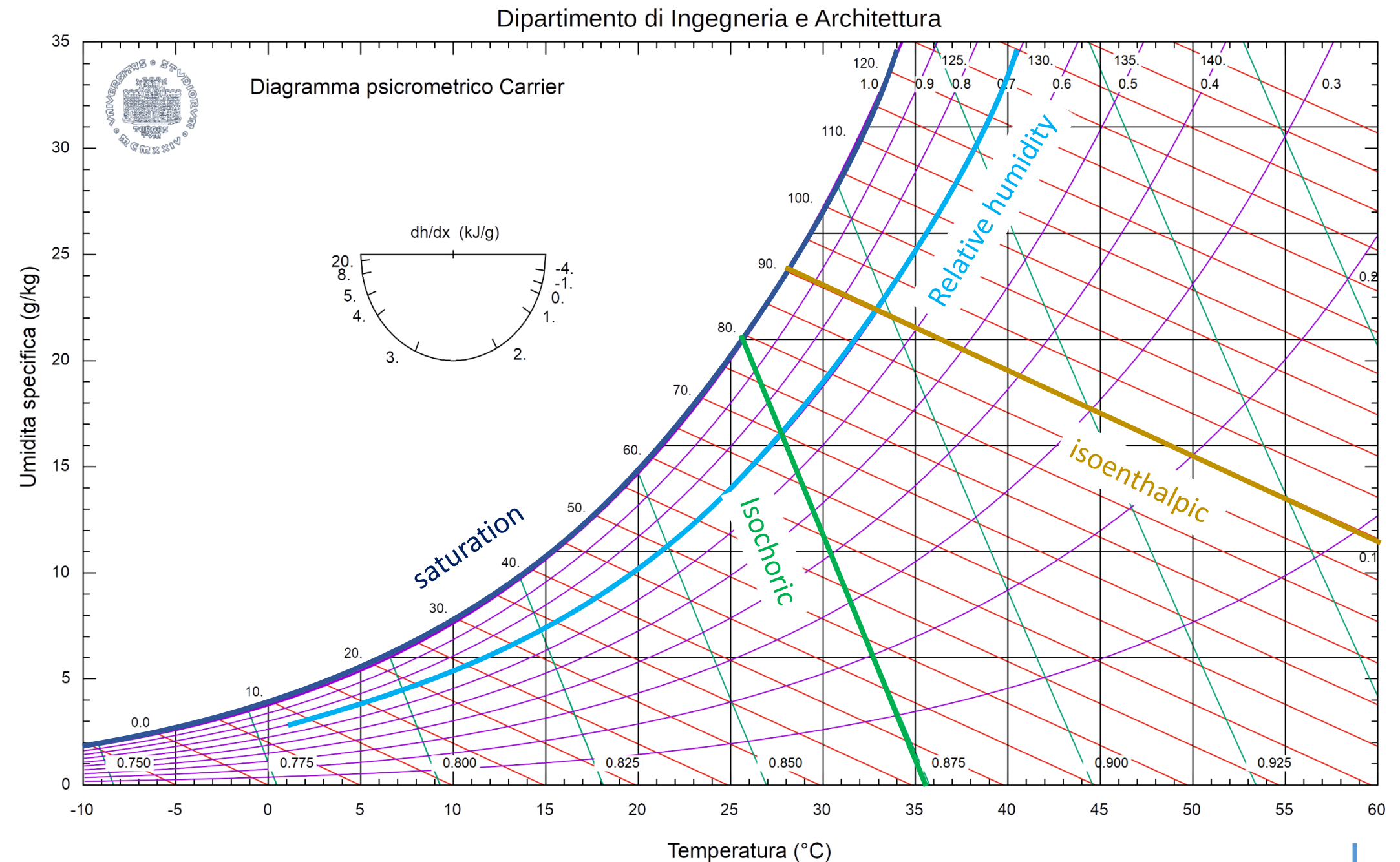
Dry and wet bulb temperatures

- **Wet bulb temperature:** temperature measured with a thermometer whose sensor, wrapped in a moist gauze, is ventilated.
- **Dry bulb temperature:** temperature measured with a thermometer immersed in the air-vapor mixture.



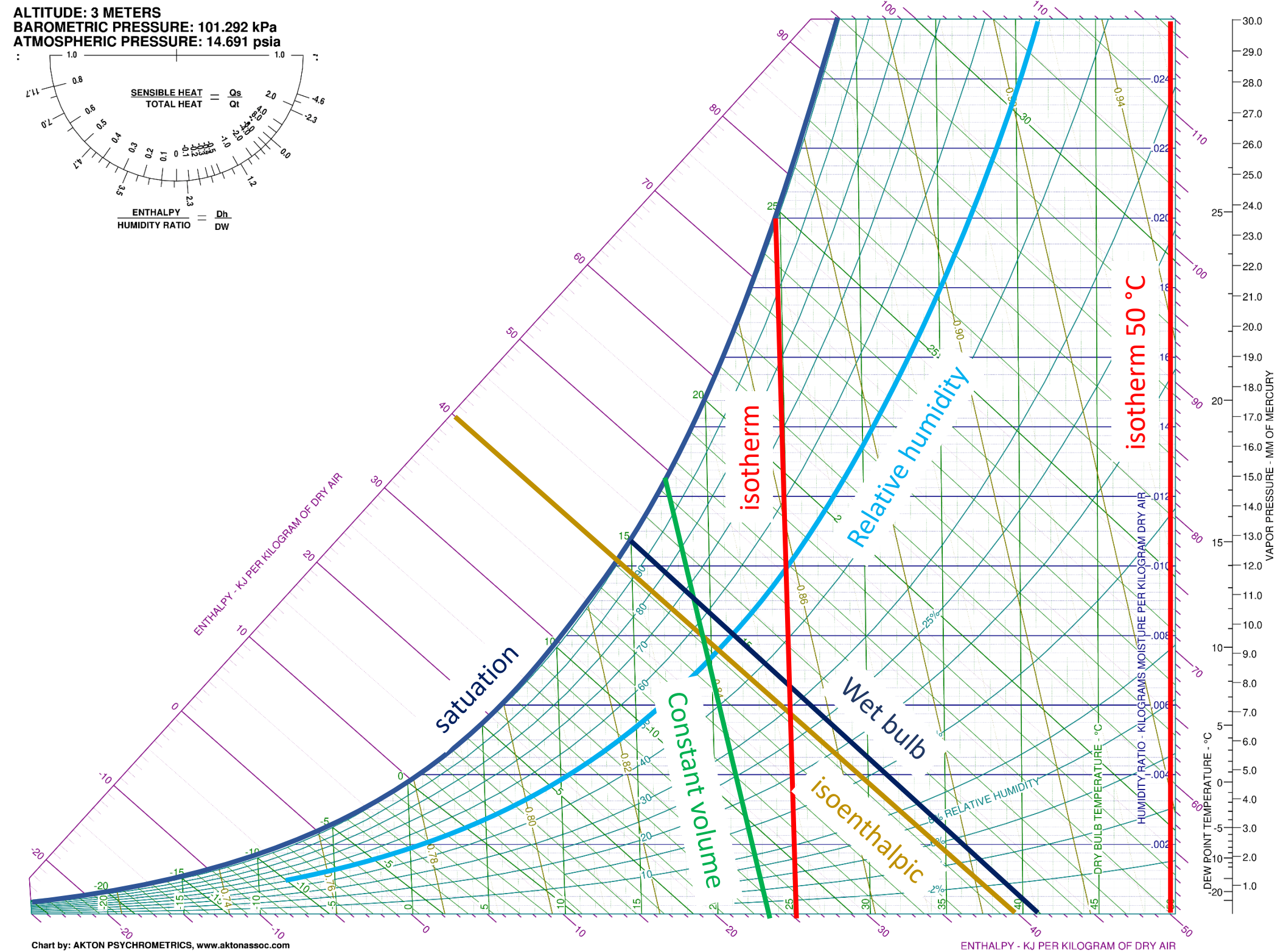
Carrier Psychrometric chart

- Constant relative humidity
- Saturation curve $\phi = 1$
- Isoenthalpic
- Constant volume
- In the Carrier diagram, enthalpies and constant wet bulb curves coincide



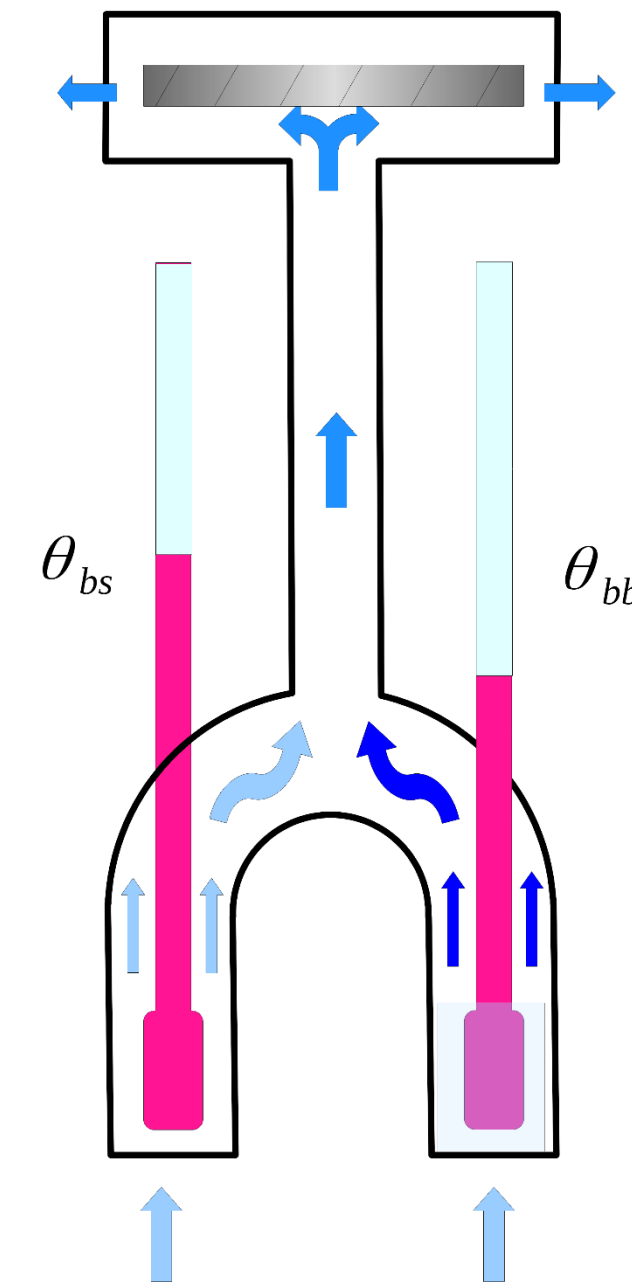
Ashrae chart

- American Society of Heating, Refrigerating and Air-Conditioning Engineers
- Isotherm
- Not vertical, only 50 °C isotherm is vertical
- Constant relative humidity
- Saturation curve $\phi=1$
- Isoenthalpic
- Wet bulb
- Isochoric



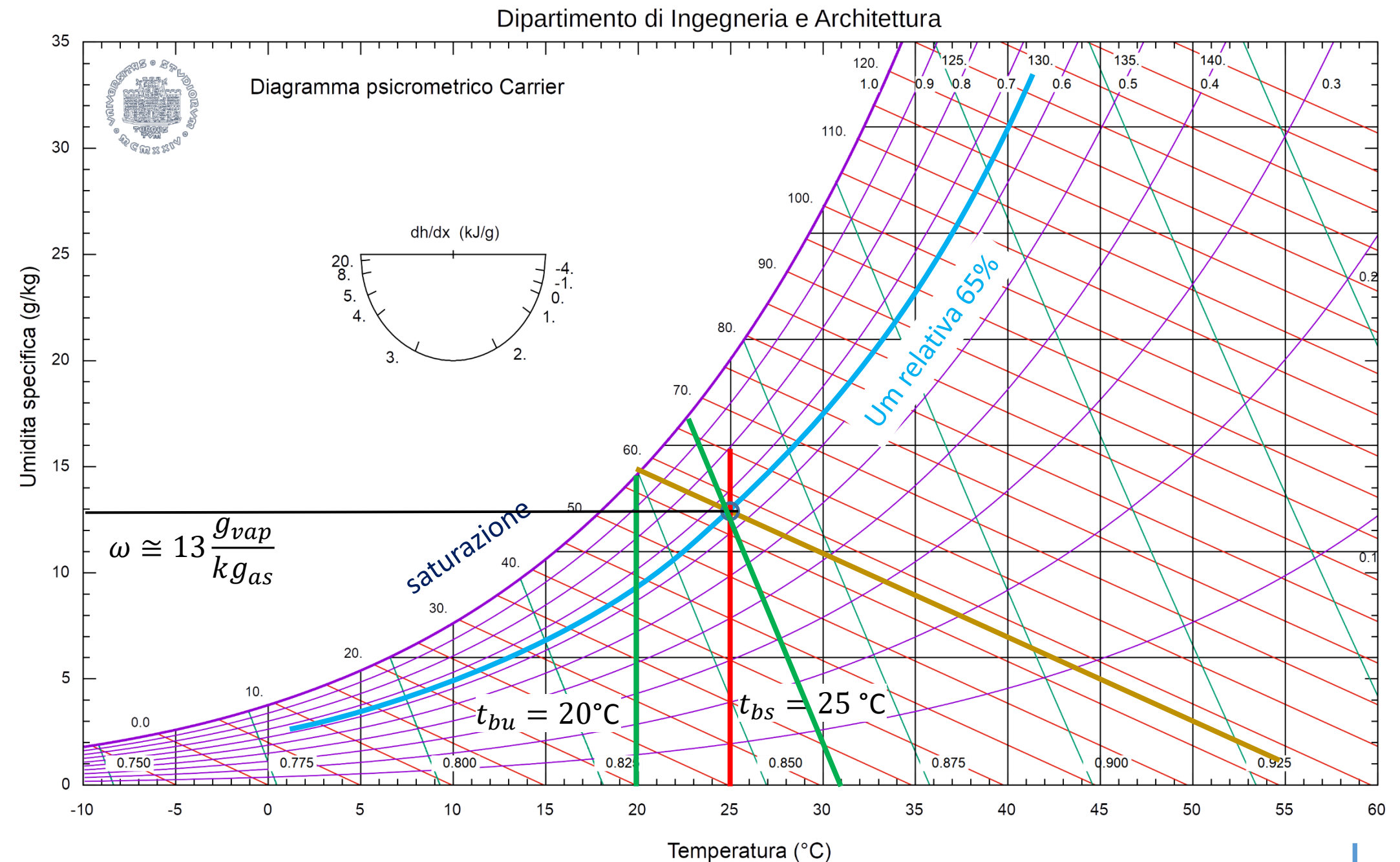
Hasman psychrometer

- Simple method to obtain the state of the system
- measures the dry bulb and wet bulb temperature
- The temperature difference allows the determination of the state of moist air



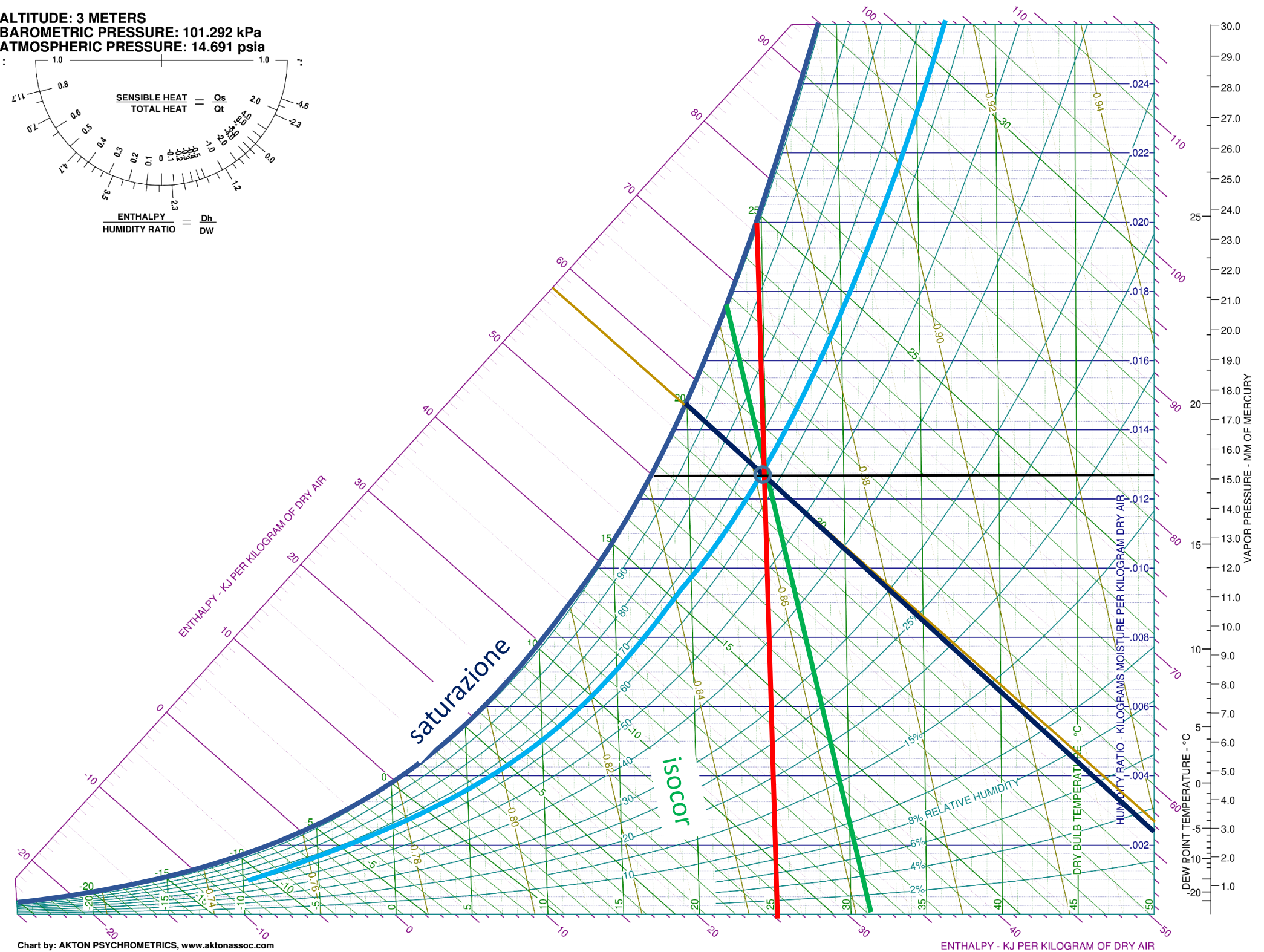
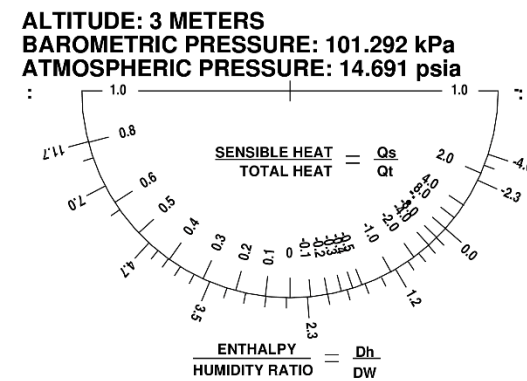
Assman Psychrometer

- example
 - $t_{db} = 25\text{ }^{\circ}\text{C}$
 - $t_{wb} = 20\text{ }^{\circ}\text{C}$
- I measure the dry bulb temperature t_{db}
- I measure the wet bulb temperature t_{wb}
- consider the isenthalpic and wet bulb temperature to be coincident
- Intersection identifies the state
- Then can be read the
 - Relative humidity
 - Specific humidity
 - Specific volume



Assman Psychrometer

- example
 - $t_{bs} = 25\text{ }^{\circ}\text{C}$
 - $t_{bu} = 20\text{ }^{\circ}\text{C}$
- I measure the dry bulb temperature t_{db}
- I measure the wet bulb temperature t_{wb}
- consider the isenthalpic and wet bulb temperature to be coincident
- Intersection identifies the state
- Then can be read the
 - Relative humidity
 - Specific humidity
 - Specific volume



Moist air balance equations

Dry air mass balance

$$\dot{m}_{da} = \dot{m}_{da1} = \dot{m}_{da2}$$

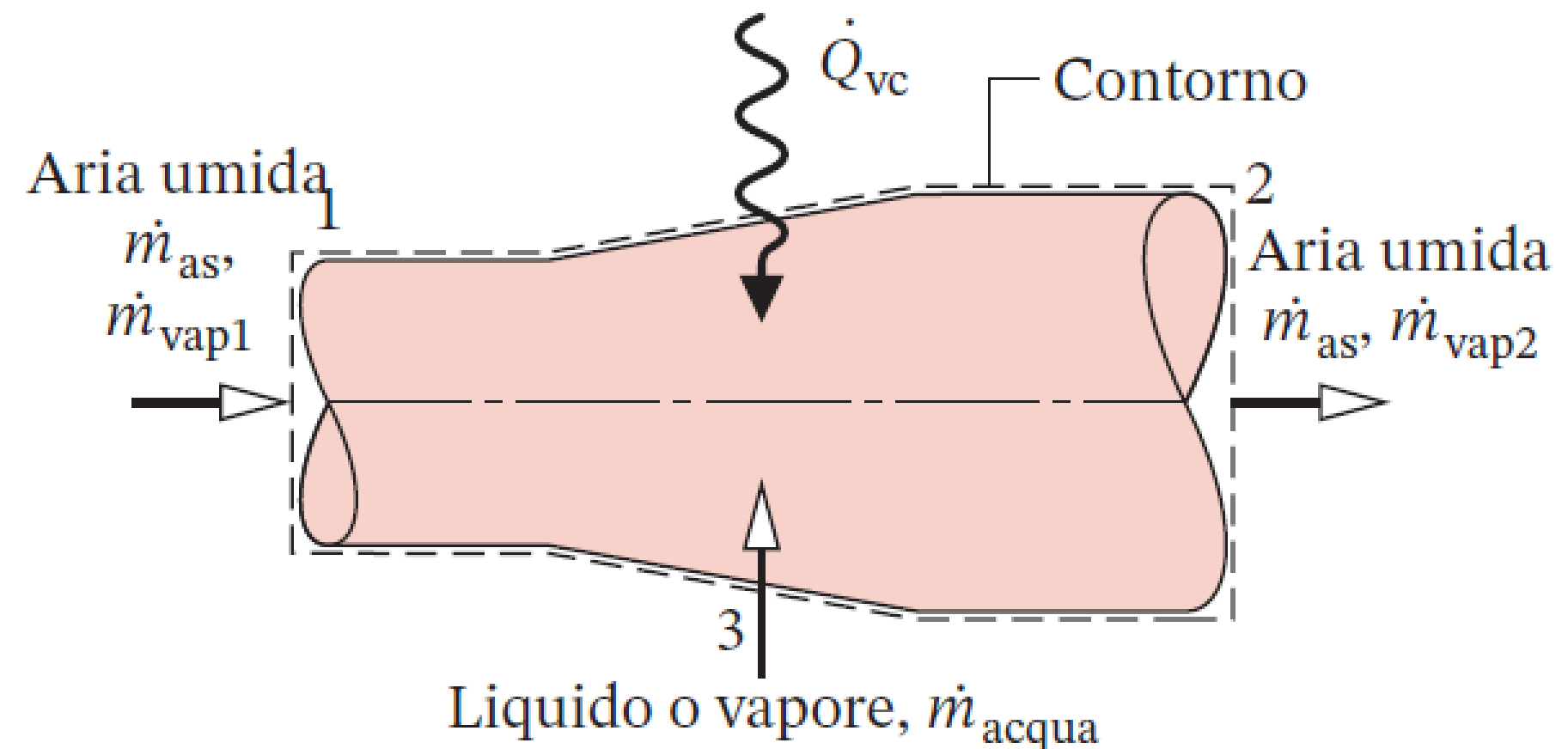
Vapor mass balance

$$\dot{m}_{vap2} = \dot{m}_{water} + \dot{m}_{vap1}$$

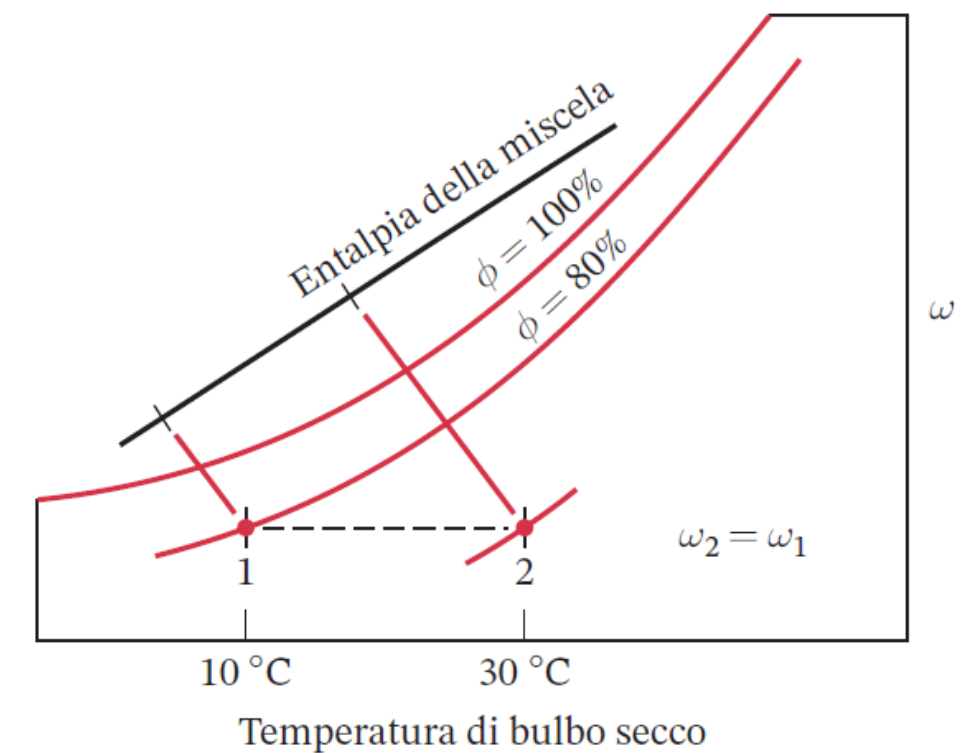
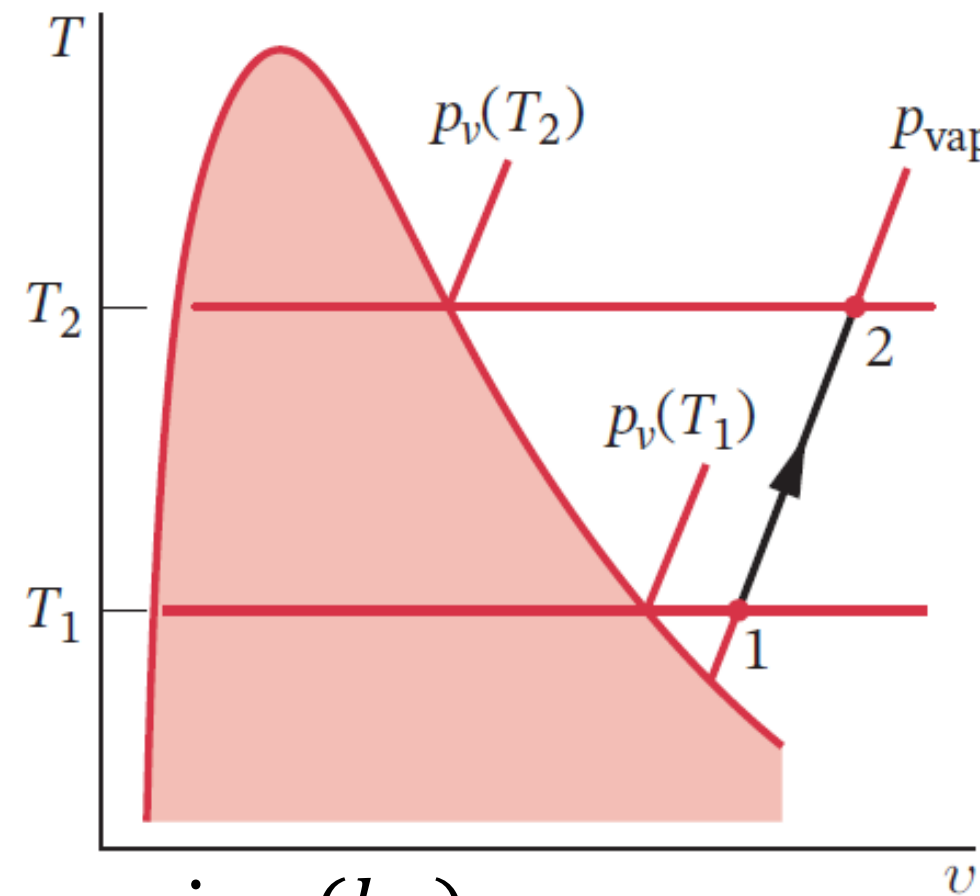
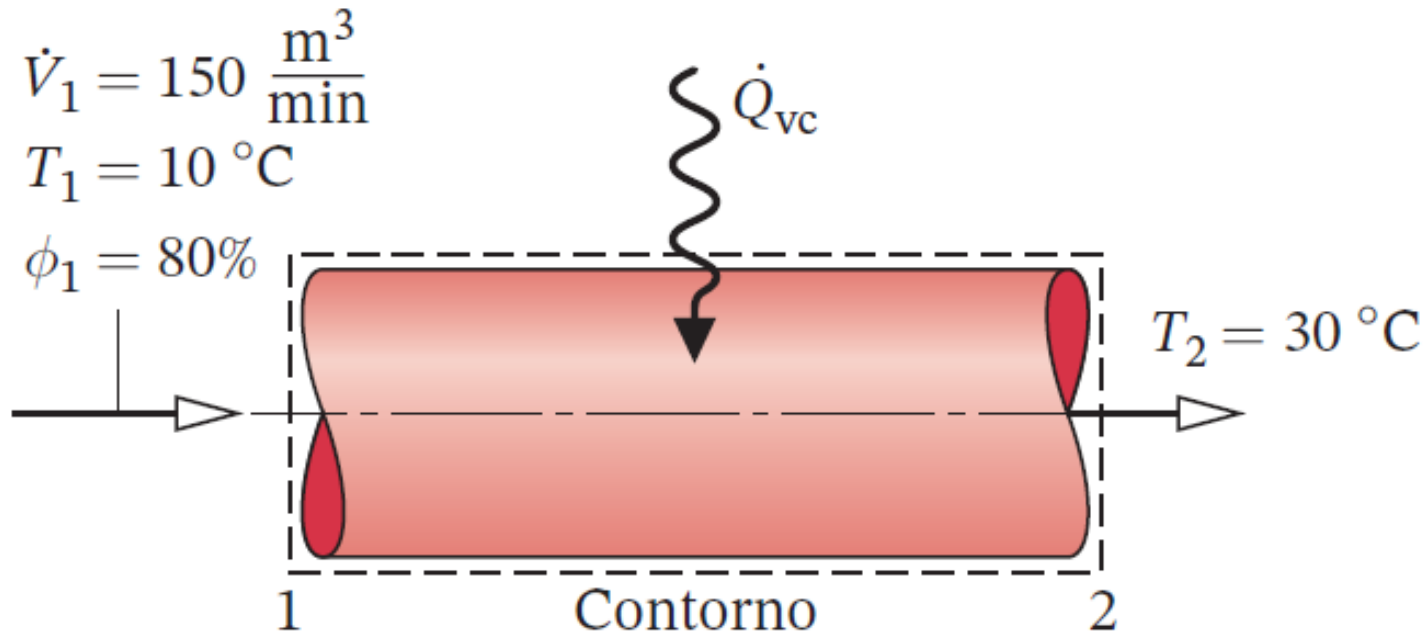
$$\dot{m}_{vap1} = w_1 \cdot \dot{m}_{da}$$

$$\dot{m}_{vap2} = w_2 \cdot \dot{m}_{da}$$

$$\dot{m}_{wate} = (w_2 - w_1) \cdot \dot{m}_{da}$$



Heating of a moist air



$$w_2 = w_1 \Rightarrow \dot{m}_{water} = 0$$

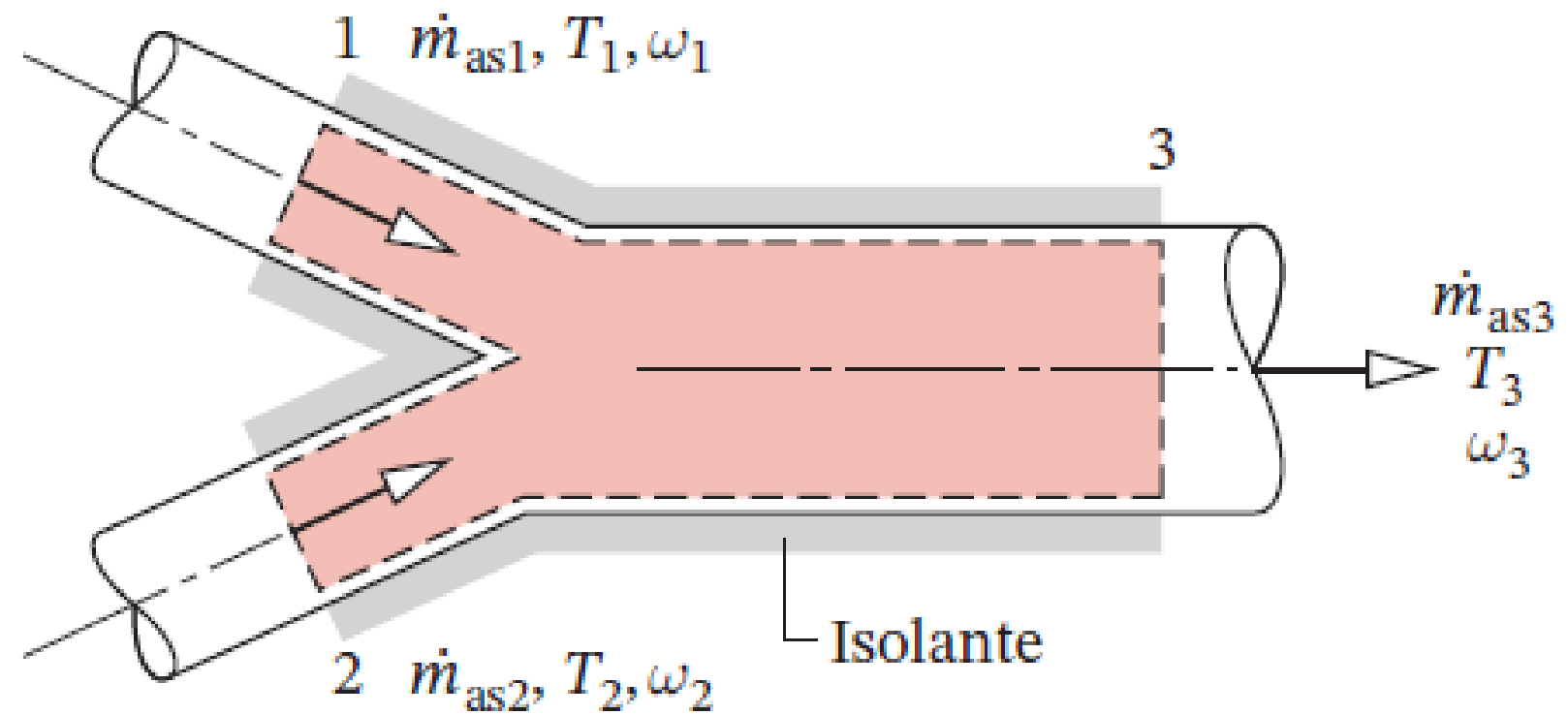
$$0 = \dot{Q}_{vc} + \dot{m}_{da}(h_1) + \cancel{\dot{m}_{water} \cdot h_{acqua}} - \dot{m}_{da}(h_2)$$

$$\dot{Q}_{vc} = \dot{m}_{da} \cdot (h_2 - h_1) = \dot{m}_{da} \cdot [c_{p,da} \cdot t_2 + w_2 \cdot (c_{p,v} \cdot t_2 + \cancel{h_{lv}}) - c_{p,da} \cdot t_1 - w_1 \cdot (c_{p,v} \cdot t_1 + \cancel{h_{lv}})]$$

$$\dot{Q}_{vc} = \dot{m}_{da} \cdot (c_{p,da} + w_2 \cdot c_{p,v}) \cdot (t_2 - t_1) = \dot{m}_{da} \cdot c_{p,a} \cdot (t_2 - t_1)$$

specific heat of moist air

Adiabatic mixing



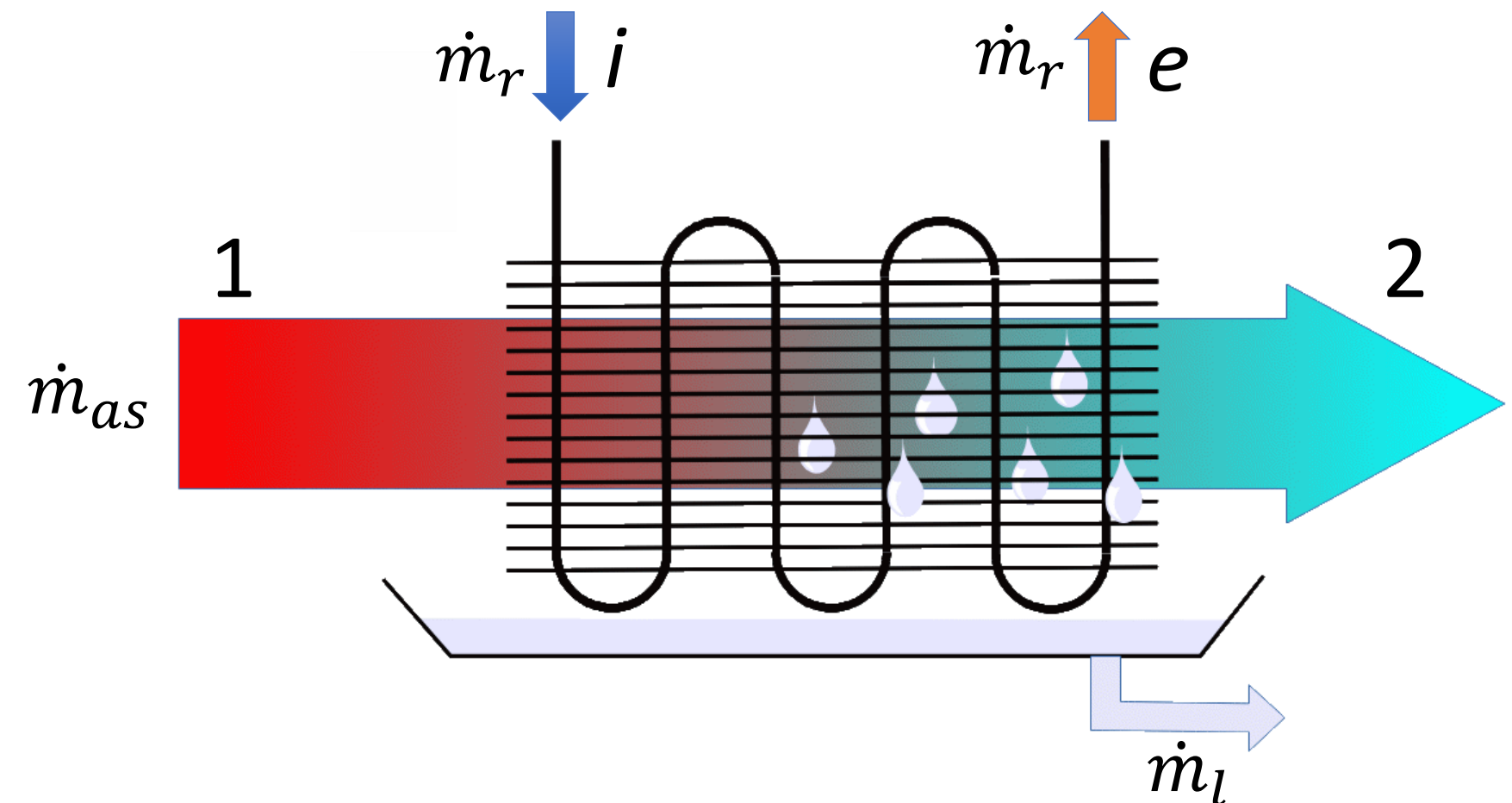
$$\dot{m}_{da1} \cdot (h_{da1} + w_1 h_{vap,1}) + \dot{m}_{da2} \cdot (h_{da2} + w_2 h_{vap,2}) = \dot{m}_{da3} \cdot (h_{da3} + w_3 \cdot h_{v3})$$

$$w_1 \dot{m}_{da1} + w_2 \dot{m}_{da2} = w_3 \dot{m}_{da3}$$

$$\dot{m}_{da1} + \dot{m}_{da2} = \dot{m}_{da3}$$

Cooling and dehumidification

- The air flow, in condition 1, pass through a cold surface (coil)
- The coil is at an average temperature t_s lower than the dew point temperature of the air
- The air flow cools and dehumidifies
- The condensed water \dot{m}_l must be collected and drained
- The coil is cooled by a cold fluid flow that enters at condition i and exits at condition e
- The heat flow to be removed is q_{cool}



Cooling and dehumidification

- The exit condition can be represented as a point positioned in a line joining the air inlet condition 1 and the point S placed on the saturation curve at the average surface temperature of the cold battery t_s
- the thermal flux needed is $|\dot{Q}_{12}^-|$

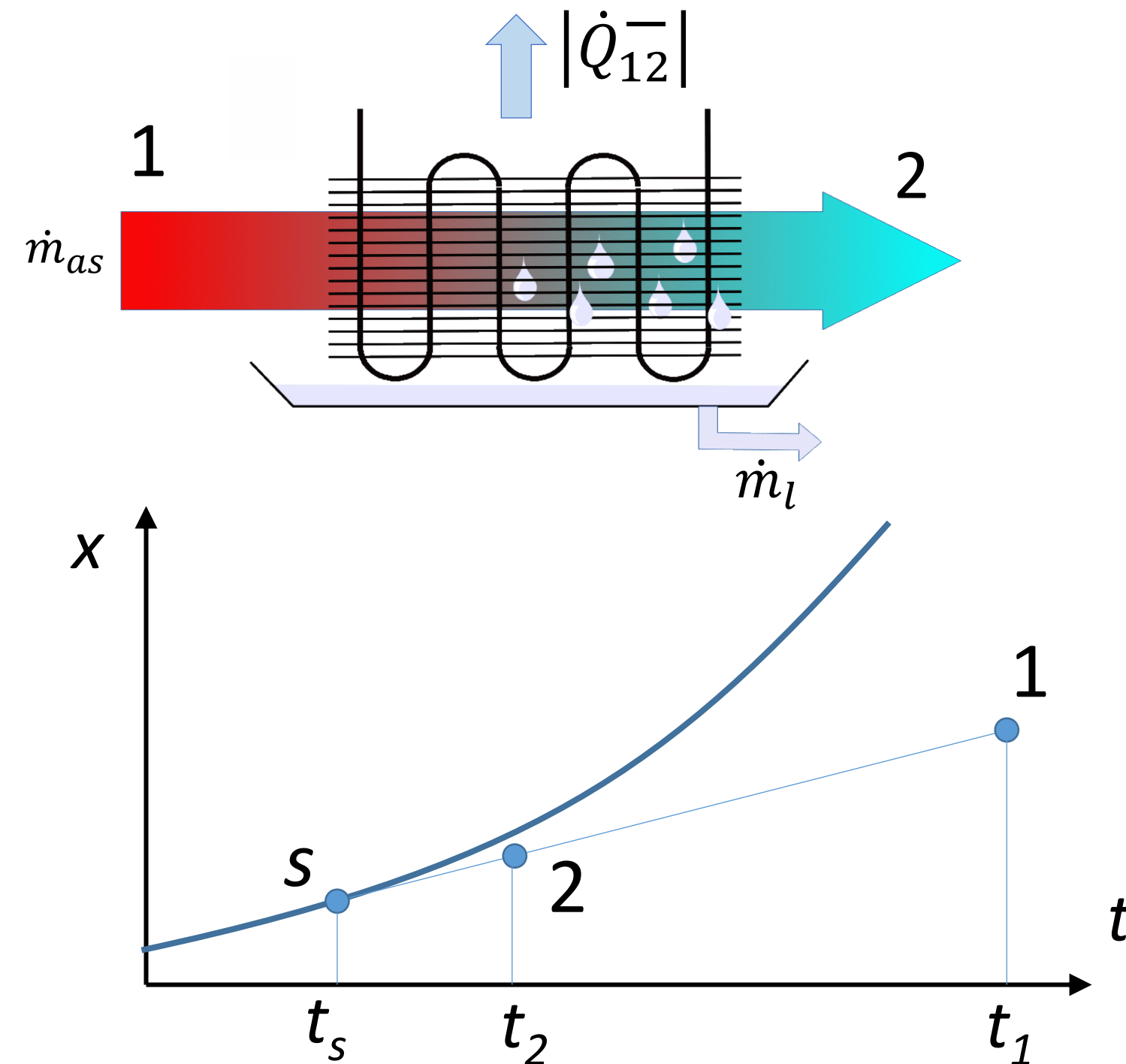
$$|\dot{Q}_{12}^-| = \dot{m}_{da} \cdot (h_1 - h_2) + \dot{m}_l \cdot h_s$$

$$\dot{m}_l = \dot{m}_{da} \cdot (w_1 - w_2)$$

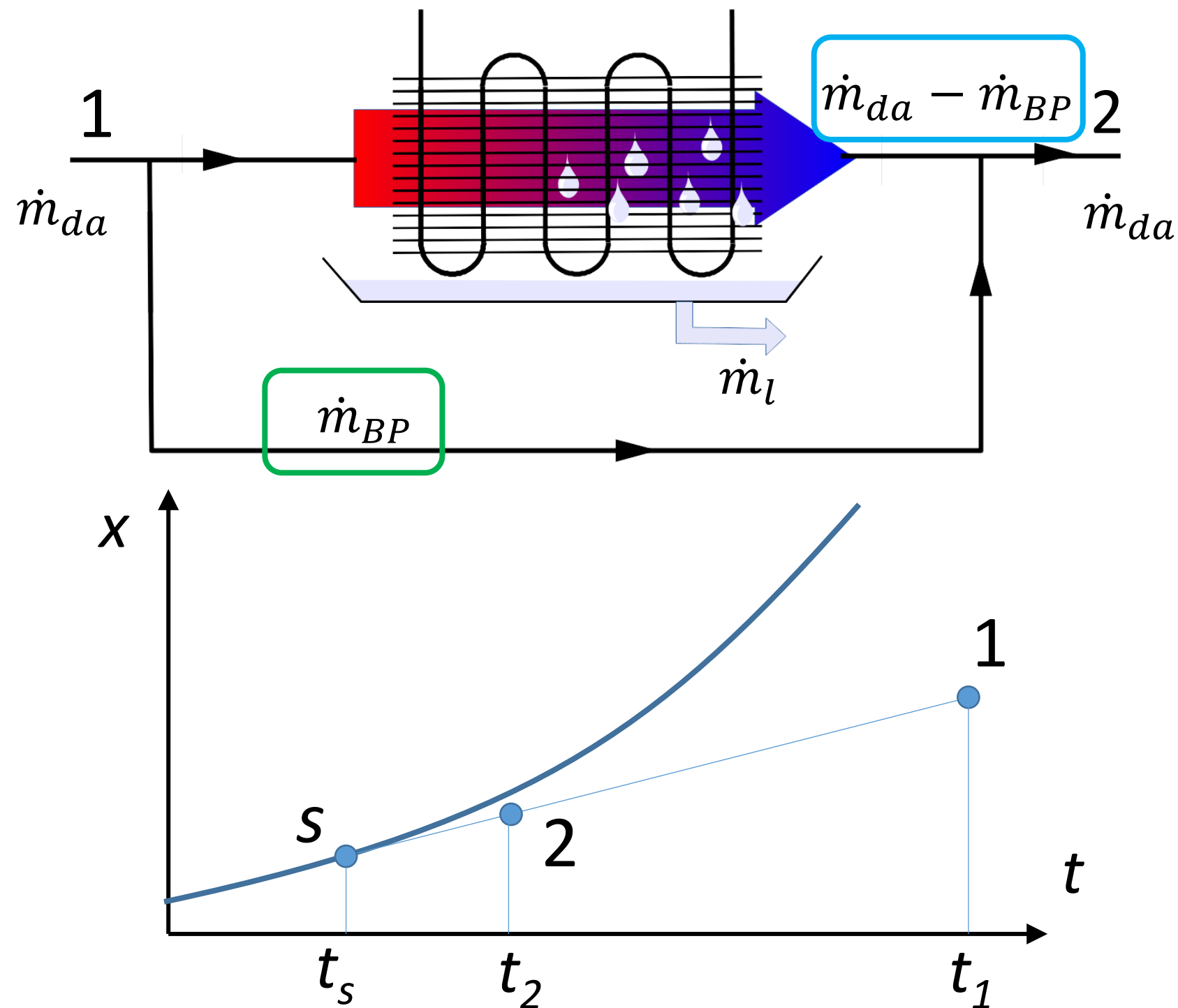
$$|\dot{Q}_{12}^-| = \dot{m}_{da} \cdot (h_1 - h_2) + \dot{m}_{da} \cdot (w_1 - w_2) \cdot h_s$$

$$h_s = c_l \cdot t_s, h_1, h_2 \text{ same order of magnitude}$$

$$|\dot{Q}_{12}^-| \cong \dot{m}_{da} \cdot (h_1 - h_2)$$

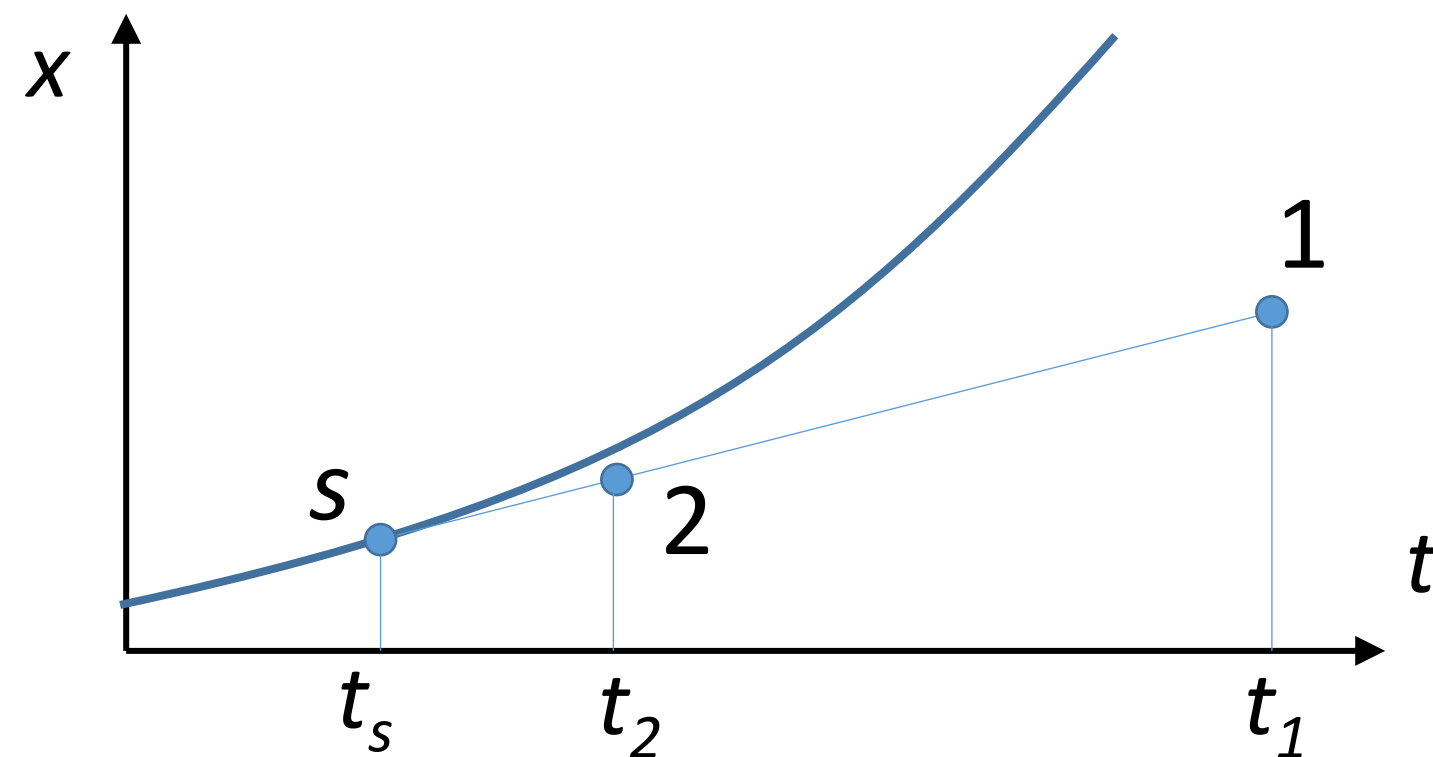
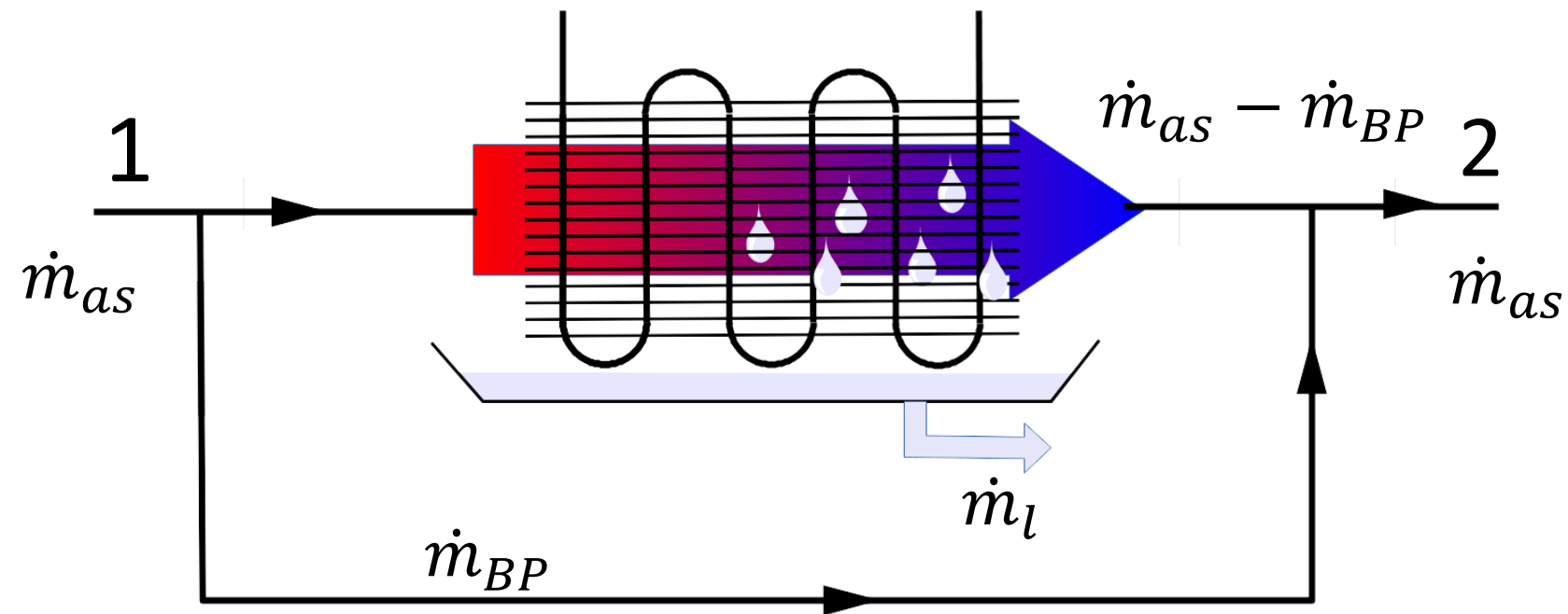


Bypass Factor



- The process can be represented as the mixing of two air flows
 - One mass flow bypasses the coil \dot{m}_{BP}
 - The mass flow $\dot{m}_{da} - \dot{m}_{BP}$ exits the coil in saturation condition s at temperature t_s
- Bypass Factor $BPF = \frac{\dot{m}_{BP}}{\dot{m}_{da}}$

Bypass Factor



$$h_2 = \frac{(\dot{m}_{da} - \dot{m}_{BP}) \cdot h_S + \dot{m}_{BP} \cdot h_1}{\dot{m}_{da}}$$

$$h_2 = (1 - BPF) \cdot h_S + BPF \cdot h_1$$

$$BPF = \frac{h_2 - h_S}{h_1 - h_S}$$

Same result with humidity ratio

$$BPF = \frac{\omega_2 - \omega_S}{\omega_1 - \omega_S}$$

Steam Humidifiers

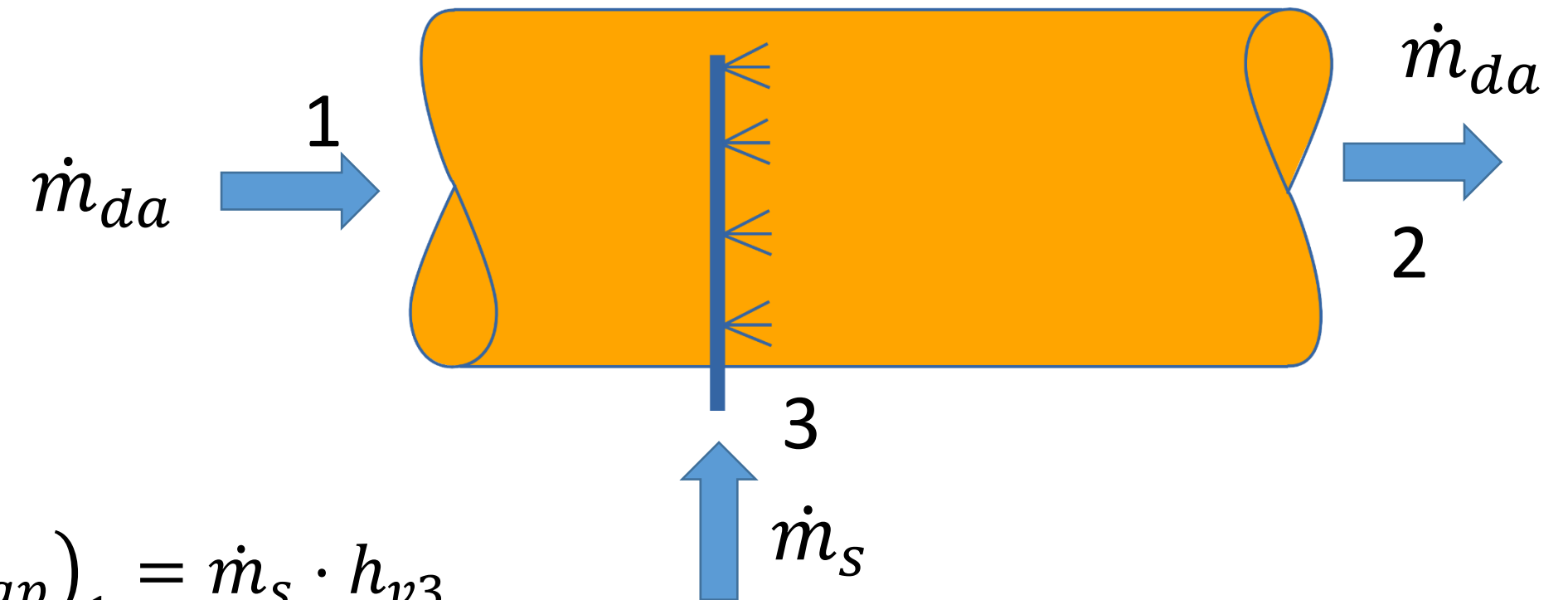
$$\dot{m}_{da} \cdot (w_2 - w_1) = \dot{m}_s$$

$$w_2 = w_1 + \frac{\dot{m}_s}{\dot{m}_{da}}$$

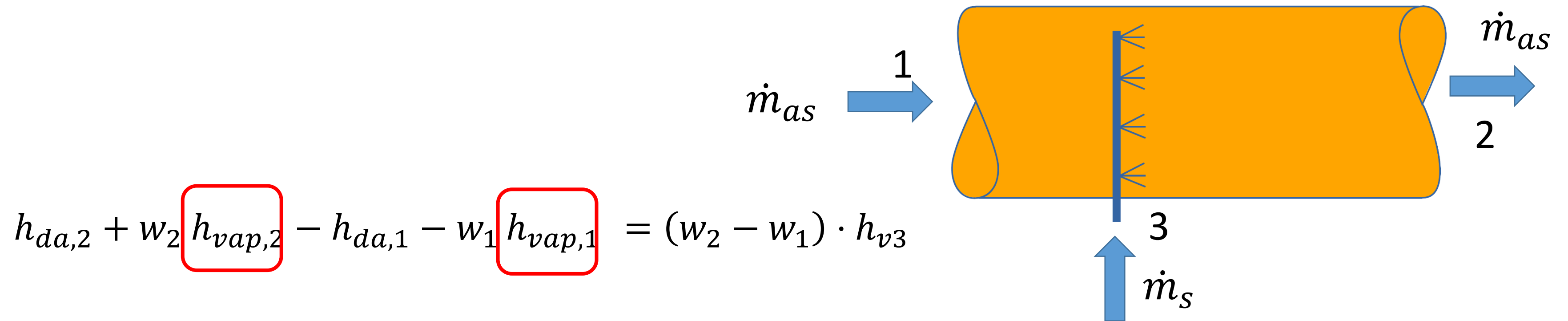
$$\dot{m}_{da} \cdot (h_{da} + w h_{vap})_2 - \dot{m}_{da} \cdot (h_{da} + w h_{vap})_1 = \dot{m}_s \cdot h_{v3}$$

$$(h_{da} + w h_{vap})_2 - (h_{da} + w h_{vap})_1 = (w_2 - w_1) h_{v3}$$

$$h_{da,2} + w_2 h_{vap,2} - h_{da,1} - w_1 h_{vap,1} = (w_2 - w_1) h_{v3}$$



Steam Humidifiers



$$h_{da,2} + w_2 h_{vap,2} - h_{da,1} - w_1 h_{vap,1} = (w_2 - w_1) \cdot h_{v3}$$

$$h_{da,2} - h_{da,1} + \overline{h_{vap,12}} \cdot (w_2 - w_1) = (w_2 - w_1) h_{v3}$$

$$h_{da,2} - h_{da,1} = (h_{v3} - \overline{h_{vap,12}}) \cdot (w_2 - w_1)$$

$$c_{p,da} \cdot (t_2 - t_1) = (h_{v3} - \overline{h_{vap,12}}) \cdot (w_2 - w_1) = c_{p,v} \cdot (t_3 - \overline{t_{12}}) \cdot (w_2 - w_1)$$

$$(t_2 - t_1) = c_{p,v} / c_{p,da} \cdot (t_3 - \overline{t_{12}}) \cdot (w_2 - w_1) \cong 0$$

Air Washer

$$\dot{m}_{da} \cdot (w_2 - w_1) = \dot{m}_s$$

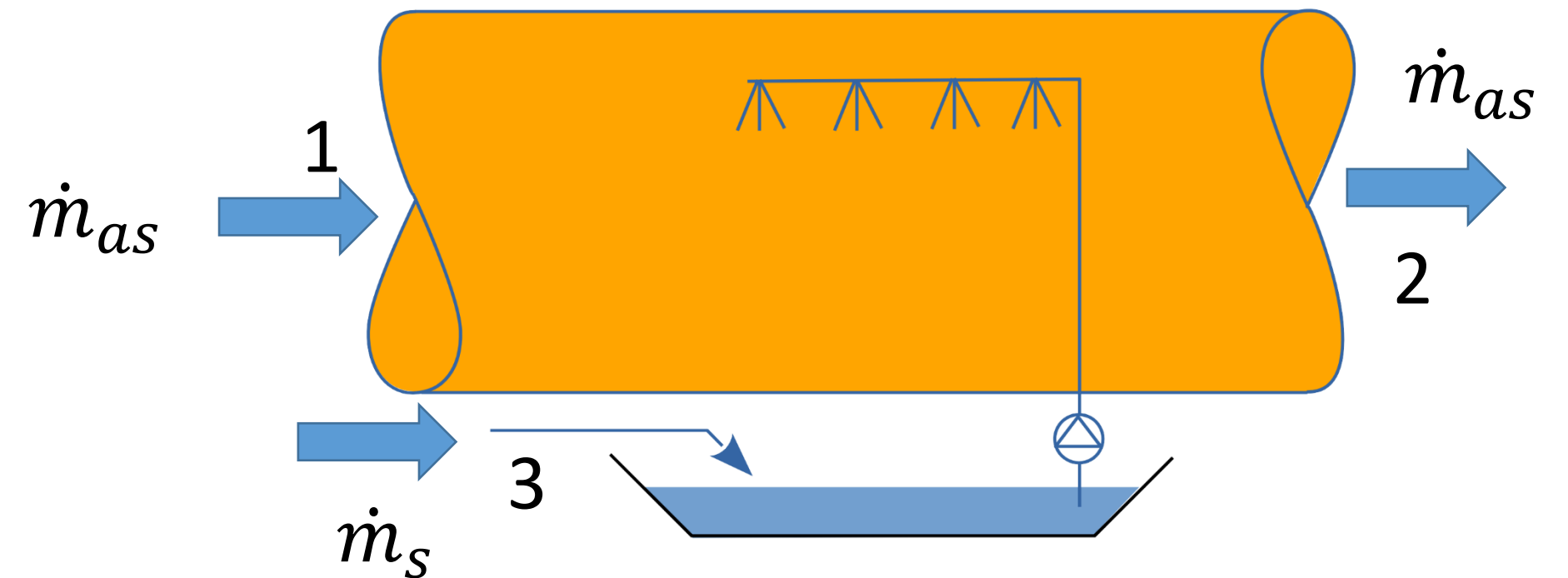
$$w_2 = w_1 + \frac{\dot{m}_s}{\dot{m}_{da}}$$

$$\dot{m}_{da}(h_{da} + w h_{vap})_2 - \dot{m}_{da}(h_{da} + w h_{vap})_1 = \dot{m}_s \cdot h_{l3}$$

$$(h_{da} + w h_{vap})_2 - (h_{da} + w h_{vap})_1 = (w_2 - w_1)h_{l3} \cong 0$$

$$(h_{da} + w h_{vap})_2 - (h_{da} + w h_{vap})_1 = (w_2 - w_1)h_{l3} \cong 0$$

$$h_2 - h_1 = (w_2 - w_1)h_l \cong 0$$



Performance of an Air Washer

- For a humidification process along the thermodynamic wet-bulb temperature line, the performance of an air washer can be illustrated by the saturation

$$\epsilon_{sat} = \frac{w_2 - w_1}{w_s^* - w_1} \approx \frac{T_2 - T_1}{T_s^* - T_1}$$

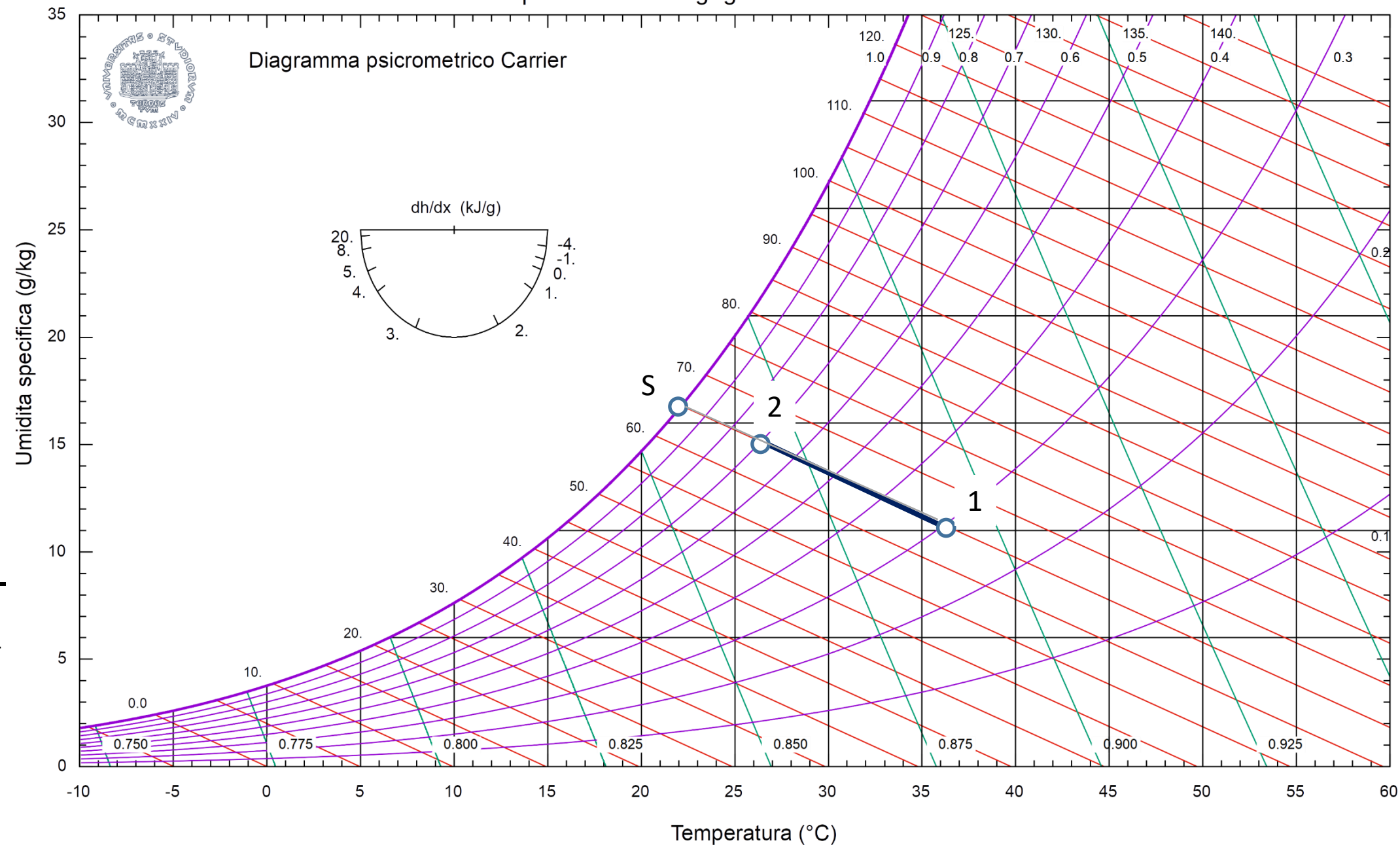
where T_1, T_2 temperature of air entering and leaving air washer

w_1, w_2 humidity ratio of air entering and leaving air washer

w_s^*, T_s^* humidity ratio and temperature of saturated air at thermodynamic wet-bulb temperature,

Air Washer

Dipartimento di Ingegneria e Architettura



$$h_2 = (w_2 - w_1)h_l + h_1$$

Enthalpy of liquid.

$$\epsilon_{sat} = \frac{w_2 - w_1}{w_s^* - w_1} \approx \frac{T_2 - T_1}{T_s^* - T_1}$$