Chapter Outline

Two-Dimensional Wave Equations and Wave Characteristics

- Surface Gravity Waves
- Small-Amplitude Wave Theory
- Wave Classification
- Wave Kinematics and Pressure
- Energy, Power, and Group Celerity
- Radiation Stress
- Standing Waves, Wave Reflection
- Wave Profile Asymmetry and Breaking
- Wave Runup

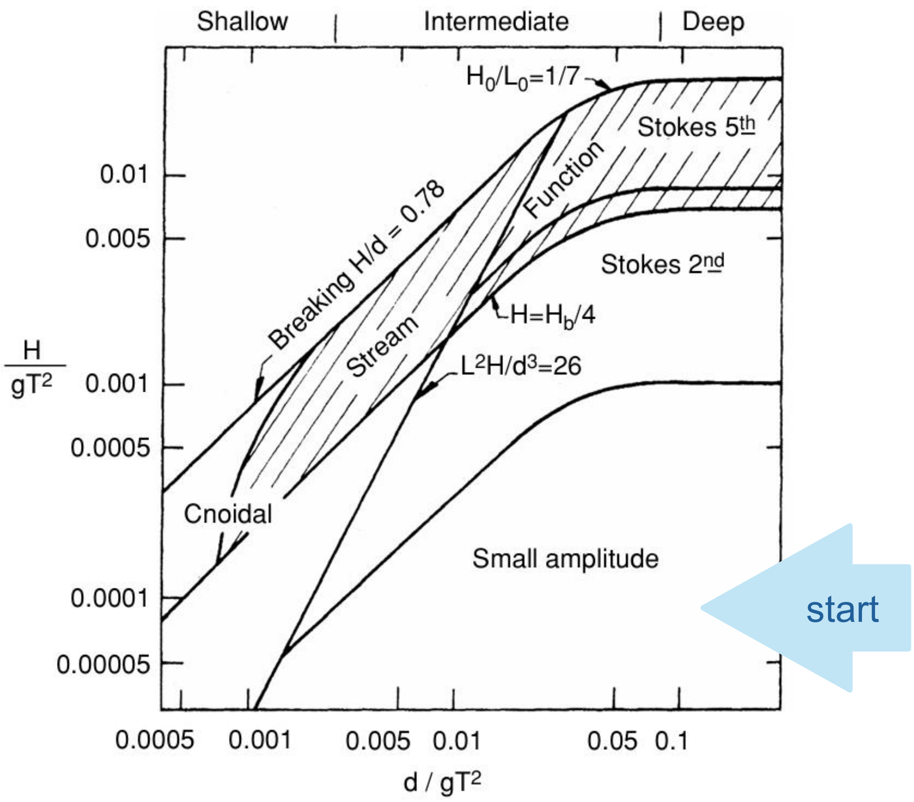
Range of applications

If one is calculating wave characteristics for a given set of input conditions (H, T, and d), these input conditions may be only generally specified or they may be specified for a range of values. This may be due to the fact that these input conditions are only approximately known from wave hindcasts and a shoaling/ refraction analysis for a given return period storm. A very sophisticated wave calculation may not be justified so the easier-to-apply small amplitude theory may be adequate.

However, if it were necessary to calculate the surface profile of a wave in relatively shallow water, say to determine the loading on the underside of a pier deck, cnoidal or stream function wave theory would be more appropriate.

Or, if the surface profile of a wave was being measured in a field experiment on wave forces on a pile structure, the stream function theory might be more appropriate for calculating the wave particle velocity and acceleration fields for that surface profile.

Another factor that compounds the choice of a wave theory for a particular application is that a particular theory may be better at defining some characteristics than others. For example, in fairly shallow water, the small-amplitude wave theory does well at predicting bottom particle velocities, but does not do well at predicting particle velocities near the surface or the surface profile itself.



Surface Gravity Waves

Wave Formation and Propagation

- Waves can be generated by:
- Wind
- Moving vessels
- Seismic disturbances (tsunamis)
- Gravitational attraction of the sun and moon (tides)

- When the water surface is vertically disturbed, **gravity** acts to restore it to equilibrium

- Due to **inertia**, the water overshoots the equilibrium position, creating an oscillation

- This oscillation propagates forward as a **wave**

Surface Gravity Waves

Energy Transmission

Waves transport energy across the water surface

When reaching obstacles (structures or shoreline), energy is:
- Reflected
- Dissipated

During propagation:
- Dynamic pressure gradients develop in the water column
- These are superimposed on hydrostatic pressure

Energy dissipation mainly occurs:
- At the air-water interface
- At the seabed in shallow water

Surface Gravity Waves

Typical Wave Heights

Wind waves:
Typically < **3 m**
During strong storms can exceed **6 m**

Vessel waves:
Rarely exceed **1 m**

Tsunamis:
In deep ocean: about **0.6 m or less**
Near the coast: often > **3 m**

Tides:
Small amplitude in deep ocean
Along some coasts can exceed **6 m**

Wave Periods

Wind-generated waves: 1–30 s
- Ocean storm waves: 5–15 s

Vessel-generated waves: 1–3 s

Tsunamis: 5 minutes to 1 hour

Tides: approximately 12 and 24 hours

Surface Gravity Waves

Complexity of Ocean Waves

Wind-generated waves are **complex**, consisting of many superimposed components with:
- Different heights
- Different periods

For theoretical simplicity, analysis often considers a **two-dimensional monochromatic wave** in constant depth water

Wind-generated waves are complex, consisting of a superimposed multitude of components having different heights and periods.
We consider first the simplest theory for the characteristics and behavior of a two-dimensional mono chromatic wave propagating in water of constant depth.
This will be useful later as a component of the spectrum of waves found at sea. It is also useful for first-order design calculations where the height and period of this monochromatic wave are selected to be representative of a more complex wave spectrum.

Linear (Small-Amplitude) Wave Theory - Airy (1845)

Also known as **linear wave theory**

Describes most important:
- Kinematic properties
- Dynamic properties

Widely used for:
- Practical engineering applications
- Preliminary design calculations
- Laboratory studies (e.g., wave forces on structures, wave breaking)

This theory provides equations that define most of the kinematic and dynamic properties of surface gravity waves and predicts these properties within useful limits for most practical circumstances.

Linear Wave Theory - Assumptions

The theory describes **two-dimensional, periodic gravity waves** by **linearizing the equations** for the free surface and using the bottom boundary condition.
It introduces a **velocity potential** valid throughout most of the water column to derive wave properties (surface shape, wave speed, pressure, particle motion).

1. Water properties:
- Homogeneous and incompressible
- Surface tension negligible → only waves longer than ~3 cm considered
- No internal pressure or gravity waves affect the flow

2. Flow is irrotational:
- No shear at surface or bottom
- Wind-generated or damped waves are ignored
- Velocity potential satisfies the **Laplace equation**

3. Bottom boundary:
- Flat, horizontal, impermeable, and stationary
- No energy added or removed from the flow
- Mild slopes can still be approximated as horizontal

4. Surface pressure:
- Constant along air-sea interface
- Wind effects and pressure differences between crest and trough are ignored

5. Small wave height:
- Wave height ≪ wavelength and water depth
- Particle velocities ≪ wave celerity (phase speed)
- Allows **linearization** of boundary conditions and simplifies calculations
- Limits the theory in deep water, very high waves, shallow water, or near breaking waves

Linear Wave Theory - Equations

Let's start from Navier-Stokes equations

- No viscosity
- Incompressible flow
- Irrotational flow

$$\begin{cases} \dfrac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} = -\dfrac{1}{\rho}\nabla p + \mathbf{g} \\ \nabla\cdot\mathbf{u} = \dfrac{\partial u}{\partial x} + \dfrac{\partial w}{\partial z} = 0 \end{cases}$$

$\mathbf{u} = \nabla\phi$

$$\frac{\partial \nabla\phi}{\partial t} + (\nabla\phi\cdot\nabla)\nabla\phi = -\frac{1}{\rho}\nabla p + \mathbf{g}$$

few algebraic steps..

$$\frac{\partial \phi}{\partial t} + \frac{1}{2}|\nabla\phi|^2 + \frac{p}{\rho} + gz = \text{costante}$$

Bernoulli eq.

$\mathbf{u} = \nabla\phi$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

Laplace eq.

Linear Wave Theory - Equations

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial z^2}=0$$

Laplace eq.

Valid throughout the fluid

The wave height is small compared to the wave length and water depth.
Since particle velocities are proportional to the wave height, and wave celerity (phase velocity) is related to the water depth and the wave length, this requires that particle velocities be small compared to the wave celerity.

This assumption allows one to linearize the higher order free surface boundary conditions and to apply these boundary conditions at the still water line rather than at the water surface, to obtain an easier solution.

This assumption means that the small-amplitude wave theory is most limited for high waves in deep water and in shallow water and near wave breaking where the waves peak and wave crest particle velocities approach the wave phase celerity. Given this, the small-amplitude theory is still remarkably useful and extensively used for wave analysis.

$$\frac{\partial\phi}{\partial t}+\cancel{\frac{1}{2}|\nabla\phi|^2}+\cancel{\frac{p}{\rho}}+gz=\text{costante}$$

Bernoulli eq.

$z=\eta(x,t)$

Particle velocities ≪ wave celerity

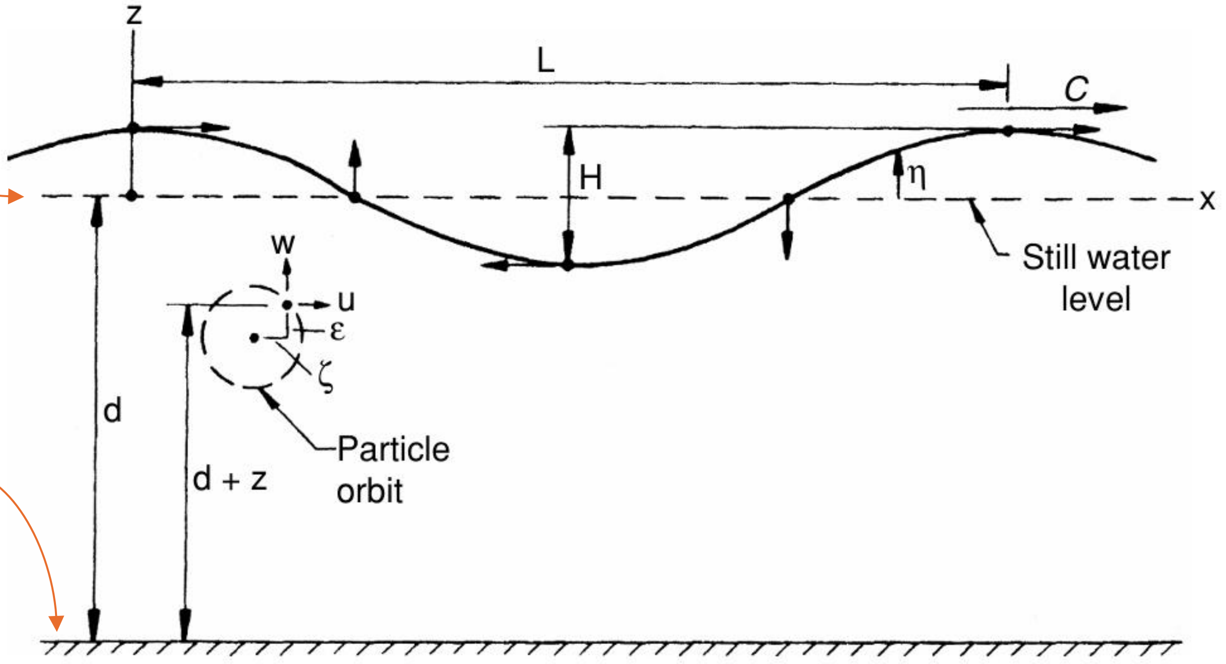
Pressure constant along air-sea interface

$$\frac{\partial\phi}{\partial t}+g\eta=0$$

Boundary Conditions at the surface

Linear Wave Theory - Equations

$$\begin{cases} \dfrac{\partial^2 \phi}{\partial x^2} + \dfrac{\partial^2 \phi}{\partial z^2} = 0 \\ g\eta + \dfrac{\partial \phi}{\partial t} = 0 \text{ at } z = 0 \\ w = \dfrac{\partial \phi}{\partial z} = 0 \text{ at } z = -d \end{cases}$$



This system has solution:

$$\phi = \frac{gH}{2\sigma} \frac{\cosh k(d+z)}{\cosh kd} \sin(kx - \sigma t)$$

Linear Wave Theory - Equations

$k = 2\pi / L$ (wave number)

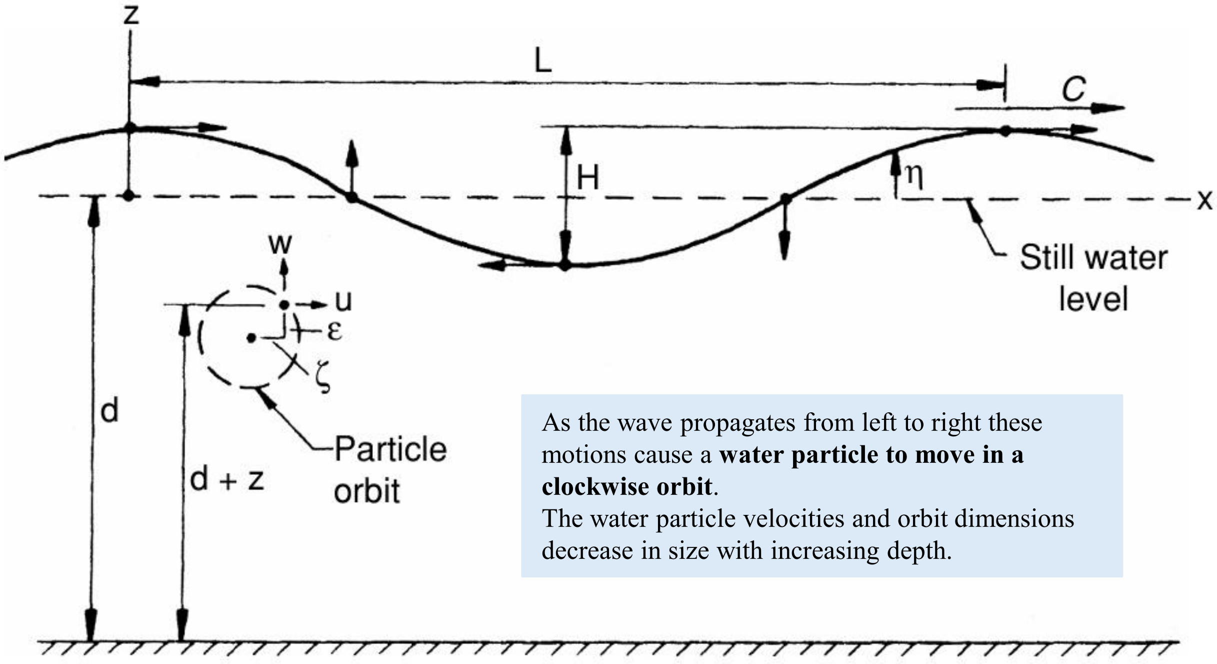
$\sigma = 2\pi / T$ (wave angular frequency)

The figure depicts a monochromatic wave traveling at a phase celerity C on water of depth d in an x, z coordinate system.

The x axis is the still water position and the bottom is at z = - d. The wave surface profile is defined by z = η(x,t).

The wave length L and height H are as shown in the figure. Since the wave travels a distance L in one period T, then C = L/T

- Wave steepness H/L
- Relative depth d/L

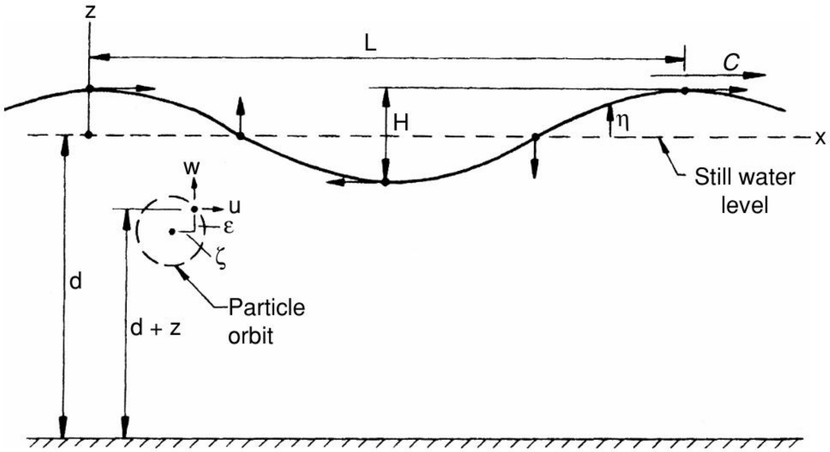


As the wave propagates from left to right these motions cause a **water particle to move in a clockwise orbit**.
The water particle velocities and orbit dimensions decrease in size with increasing depth.

Linear Wave Theory - Equations

$k = 2\pi/L$(wave number)

$\sigma = 2\pi/T$(wave angular frequency)



1) Insert the velocity potential into the linearized BC at z=0 …

$$\eta = \frac{H}{2}\cos(kx - \sigma\ t)$$

The small amplitude wave theory yields a cosine surface profile.
This is reasonable for low amplitude waves, but with increasing wave amplitude the surface profile becomes vertically asymmetric with a more peaked wave crest and a flatter wave trough (as will be shown later).

2) Combining the BC and by eliminating the water surface elevation η, and including ϕ, then differentiate, rearrange …

$$\sigma^2 = gk\tanh kd$$

$$C = \frac{\sigma}{k} = \sqrt{\frac{g}{k}\tanh kd}$$

This quation indicates that for small-amplitude waves, the **wave celerity is independent of the wave height H**.
As the wave height increases there is a small but growing dependence of the wave celerity on the wave height (as will be shown later).

Linear Wave Theory - Equations

$$C = \frac{gT}{2\pi} \tanh \frac{2\pi d}{L}$$

$$L = \frac{gT^2}{2\pi} \tanh \frac{2\pi d}{L}$$

Dispersion relation

From this eq. if the water depth d and the wave period T are known, the wavelength L can be calculated by trial and error. Then the celerity can be determined from C=L/T

For a spectrum of waves having different periods (or lengths), the longer waves will propagate at a higher celerity and move ahead while the shorter waves will lag behind.

It can be demonstrated **that as a wave propagates from deep water into the shore**, **the wave period will remain constant** because the number of waves passing sequential points in a given interval of time must be constant. **Other wave characteristics** including the celerity, length, height, surface profile, particle velocity and acceleration, pressure field, and energy **will all vary** during passage from deep water to the nearshore area.

2) Combining the BC and by eliminating the water surface ele ... n

$$\sigma^2 = gk \tanh kd$$

$$C = \frac{\sigma}{k} = \sqrt{\frac{g}{k} \tanh kd}$$

Deep water

$$kd \gg 1$$

$$\tanh(kd) \simeq 1$$

$$c = \sqrt{\frac{g}{k}}$$

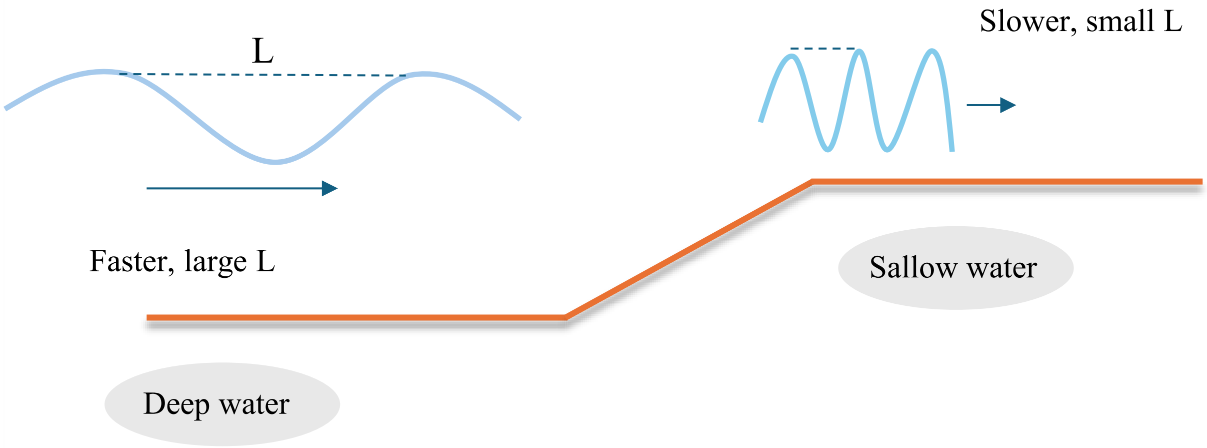
Shallow water

$$kd \ll 1$$

$$\tanh(kd) \simeq kd$$

$$c = \sqrt{gd}$$

Wave Classification



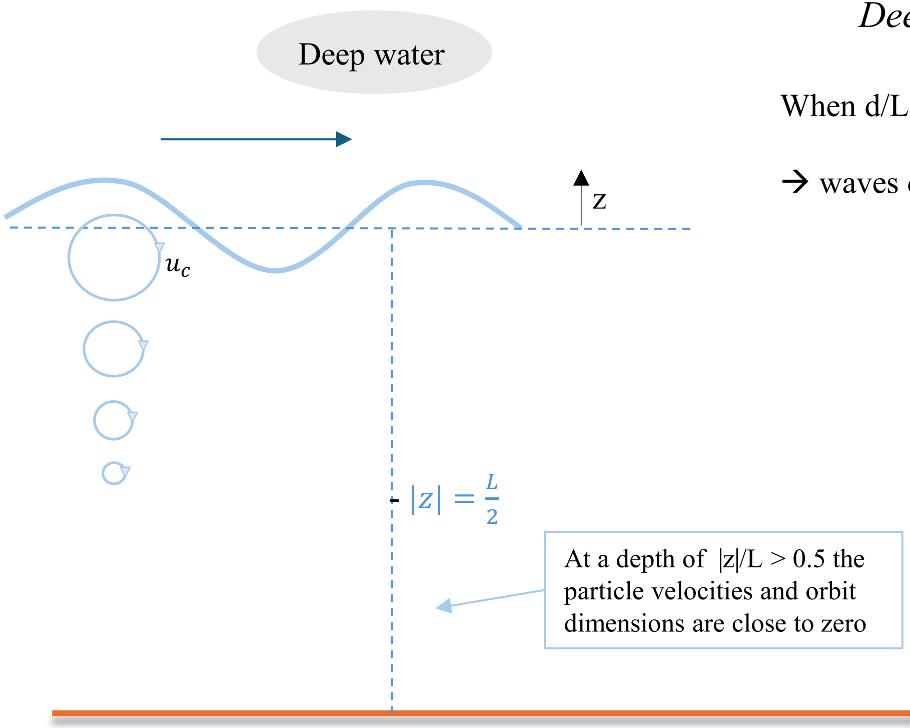
Relative depth d/L

Approaching the shore

- D decreases
- L decreases (but slower than d)

- Thus, d/L decreases

Wave Classification



Deep water waves [d/L > 0.5]

When d/L is greater than approximately 0.5, tanh(2πd/L) ≈ 1

→ waves do not interact with the bottom $C_o = \sqrt{\frac{gL_o}{2\pi}}$

For deep water the particle orbits are circular having adiameter at the surface equal to the waveheight. Since a particle completes one orbit in one wave period, the particle speed at the crest would be the orbit circumference divided by the period or $u_c = \pi \frac{H_0}{T}$

→ Note that this is much less than Co !

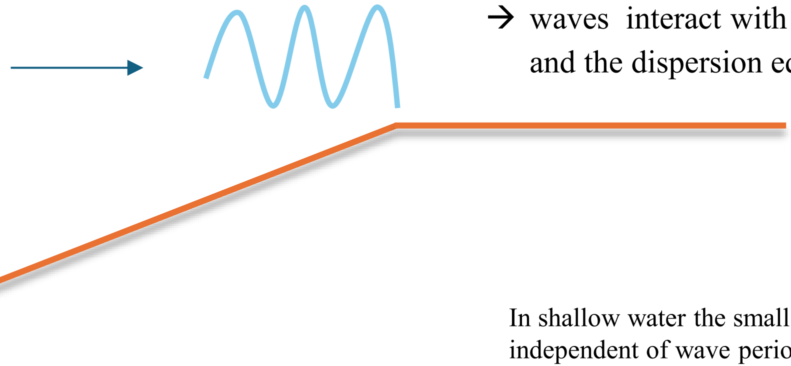
Wave Classification

Shallow water waves [d/L < 0.05]

Sallow water

When the d/L is less than approximately 0.05, $\tanh(2\pi d/L) \approx 2\pi d/L$

→ waves interact with the bottom
and the dispersion equation yields $C = \sqrt{gd} \quad L = \sqrt{gd}\, T$



Waves propagating in the range of relative depths from 0.5 to 0.05 are called intermediate or transitional water waves.

In shallow water the small-amplitude wave theory gives a wave **celerity** that is independent of wave period T and **dependent only on the water depth d**.
We'll see, the finite-amplitude wave theories show that the shallow water wave celerity is a function of the water depth and the wave height H so that in shallow water waves are amplitude-dispersive.

Remember that it is the relative depth d/L, not the actual depth alone, that defines deep, intermediate, and shallow water conditions. For example, the tide is a very long wave that behaves as a shallow water wave in the deepest parts of the ocean

A wave in water 100 m deep has a period of 10 s and a height of 2 m. Determine the wave celerity, length, and steepness. What is the water particle speed at the wave crest?

Assume that this is a deep water wave. Then,

$$L_o = \frac{9.81(10)^2}{2\pi} = 156\,\mathrm{m}$$

Since the depth is greater than half of the calculated wave length, the wave is in deep water and the wave length is 156 m.

The wave celerity is

$$C_o = \frac{156}{10} = 15.6\,m/s$$

and the steepness is

$$\frac{H_o}{L_o} = \frac{2}{156} = 0.013$$

For deep water the particle orbits are circular having a diameter at the surface equal to the wave height. Since a particle completes one orbit in one wave period, the particle speed at the crest would be the orbit circumference divided by the period or

$$u_c = \frac{\pi H_o}{T} = \frac{3.14(2)}{10} = 0.63\,m/s$$

Example of deep water wave and shallow water wave

Consider the wave from Example 2.3-1 when it has propagated in to a nearshore depth of 2.3 m. Calculate the wave celerity and length.

Assuming this is a shallow water wave,

$$C = \sqrt{9.81(2.3)} = 4.75\,\mathrm{m/s}$$

and

$$L = 4.75(10) = 47.5\,\mathrm{m}$$

So $d/L = 2.3/47.5 = 0.048 < 0.05$ and the assumption of shallow water was correct.

→ Check the code for the numerical solution

Wave Kinematics and Pressure

Calculation of the wave conditions that will cause the initiation of bottom sediment motion, for example, requires a method for calculating water particle velocities in a wave.

The water **particle velocity and acceleration** as well as **the pressure field** in a wave are all needed to determine **wave-induced forces** on various types of coastal structures.

$$u = \frac{\partial \phi}{\partial x}, \quad w = \frac{\partial \phi}{\partial z}$$

$$u = \frac{\pi H}{T} \left[\frac{\cosh k(d+z)}{\sinh kd} \right] \cos(kx - \sigma t)$$

$$w = \frac{\pi H}{T} \left[\frac{\sinh k(d+z)}{\sinh kd} \right] \sin(kx - \sigma t)$$

(1) surface deep water particle speed $\pi H/T$

(2) particle velocity variation over the vertical water column

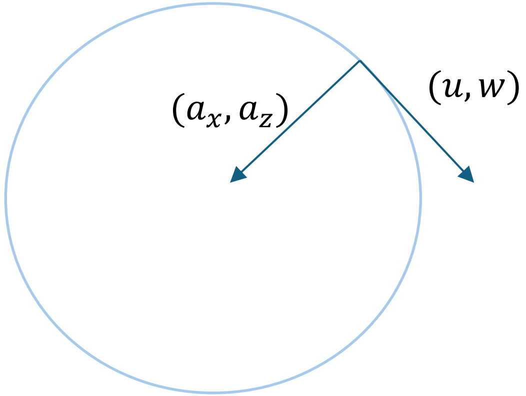
(3) a phasing term dependent on position in the wave and time.

Wave Kinematics and Pressure

$$a_x = \frac{2\pi^2 H}{T^2}\left[\frac{\cosh k(d+z)}{\sinh kd}\right]\sin(kx-\sigma t)$$

$$a_z = -\frac{2\pi^2 H}{T^2}\left[\frac{\sinh k(d+z)}{\sinh kd}\right]\cos(kx-\sigma t)$$

The cosine/sine terms indicate that the particle velocity components are 90° out of phase with the **acceleration components**. This is easily seen by considering a particle following a circular orbit.
The velocity is tangent to the circle and the acceleration is toward the center of the circle or normal to the velocity.

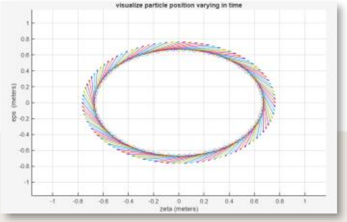


The horizontal and vertical coordinates of the **particle position** relative to the mean position are given by ζ and ε, respectively. These components can be found by integrating the particle velocity components with time.

$$\zeta = \frac{-H}{2}\left[\frac{\cosh k(d+z)}{\sinh kd}\right]\sin(kx-\sigma t)$$

$$\varepsilon = \frac{H}{2}\left[\frac{\sinh k(d+z)}{\sinh kd}\right]\cos(kx-\sigma t)$$

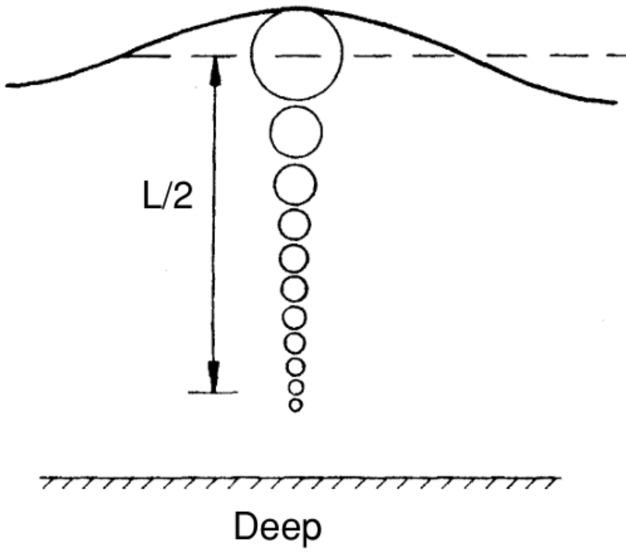
Check the code →



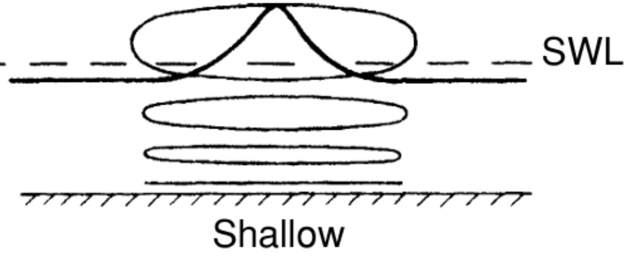
H/2 is the orbit radius for a particle at the surface of a deep water wave

Wave Kinematics and Pressure

As a wave propagates from deep water into shallow water, the particle orbit geometries undergo a transformation.

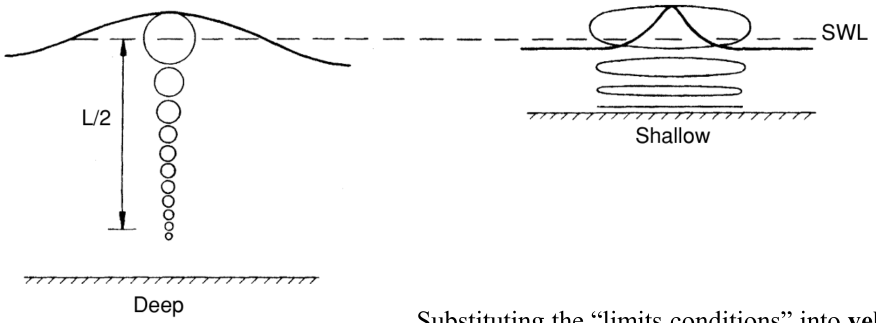


In deep water the orbits are **circular throughout the water column** but **decrease in diameter** with increasing distance below the water surface, to approximately die out at a distance of L/2.



In transitional to shallow water, **the orbits reach the bottom and become elliptical**, with the ellipses becoming flatter near the bottom. At the bottom the particles follow a reversing horizontal path.
(This is for the assumed irrotational motion, for real conditions a bottom boundary layer develops and the horizontal dimension of the particle orbit reduces to zero at the bottom.)

Wave Kinematics and Pressure



Deep water

$kd \gg 1$

Shallow water

$kd \ll 1$

Substituting the "limits conditions" into **velocity field and orbit coordinates** indicates that, in **deep water**, the particle velocity, acceleration, and orbit displacement **decay exponentially with increasing distance** below the still water line. At z = -L/2 they are reduced to 4.3% of their value at the surface.

$$\text{Deep water:} \quad \frac{\cosh k(d+z)}{\sinh kd} = \frac{\sinh k(d+z)}{\sinh kd} = e^{kz}$$

$$\text{Shallow water:} \quad \frac{\cosh k(d+z)}{\sinh (kd)} = \frac{1}{kd}$$

$$\frac{\sinh k(d+z)}{\sinh kd} = 1 + \frac{z}{d}$$

While the following equations describe **particle velocity in shallow water**

$$u = \frac{H}{2}\sqrt{\frac{g}{d}}\cos(kx - \sigma t)$$

Note u doesn't depend on z, while w does.

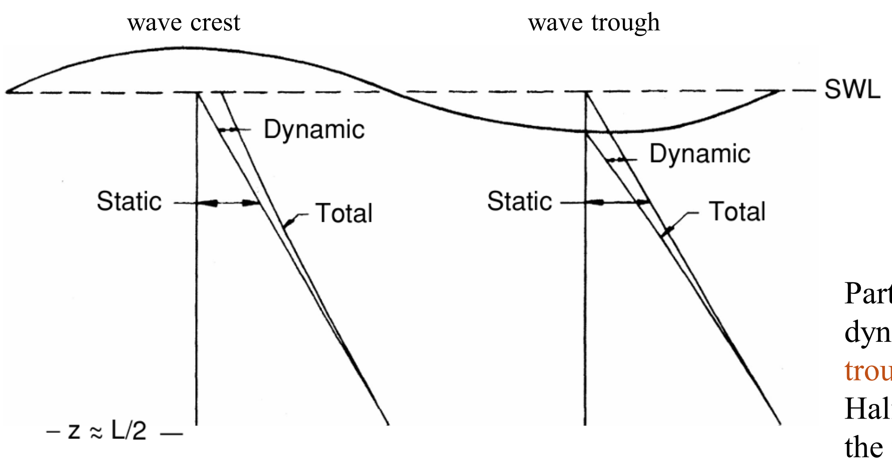
$$w = \frac{\pi H}{T}\left(1 + \frac{z}{d}\right)\sin(kx - \sigma t)$$

Wave Kinematics and Pressure

Substitution of the velocity potential into the linearized form of the unsteady Bernoulli equation, yields the following equation for the pressure field in a wave:

$$p = -\rho g z + \frac{\rho g H}{2}\left[\frac{\cosh k(d+z)}{\cosh kd}\right]\cos(kx - \sigma t)$$

The first term on the right gives the normal hydrostatic pressure variation and the second term is the dynamic pressure variation owing to the wave-induced particle acceleration.



Particles under the crest are accelerating downward, a downward dynamic pressure gradient is required. The reverse is true under a wave trough.
Halfway between the crest and trough the acceleration is horizontal so the vertical pressure distribution is hydrostatic.

Wave Kinematics and Pressure

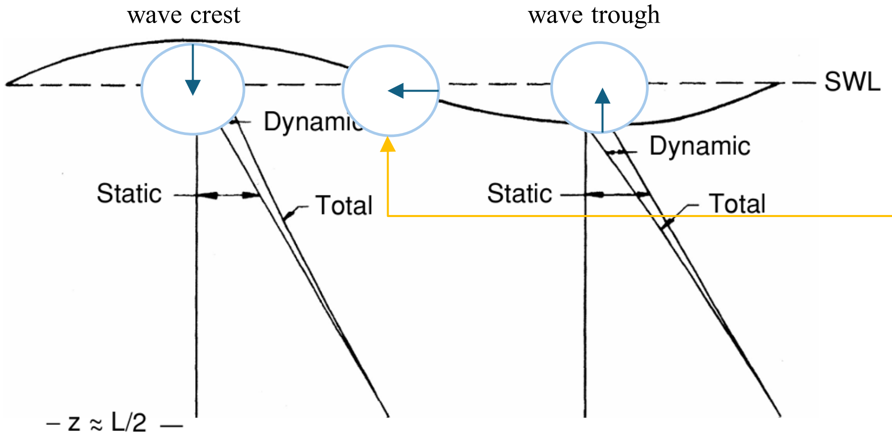
Dynamic pressure accelerates the fluid particle $\rho a_z = -\frac{\partial p}{\partial z}$

Under the crest a downward force is required to "push" the particle downward.

The pressure is greater than the hydrostatic value.

Under the trough the opposite occurs:
the particle is beginning to move upward - it has upward acceleration.

The pressure gradient is now such that it produces an upward force. The dynamic pressure is smaller than the hydrostatic pressure.



Halfway acceleration is horizontal $a_z = 0$ so the vertical pressure distribution is hydrostatic

A pressure gage (located above L/2) can be used as a wave gage. The period of the pressure fluctuation is the wave period which can be used to calculate the wave length from the dispersion equation.

Energy, Power, and Group Celerity

An important characteristic of gravity waves is that they have **mechanical energy** and that **this energy is transmitted forward** as they propagate. It is important to be able to quantify this energy level and the rate of energy transmission (energy flux or power) for a given wave height and period and water depth. We compute the energy for a unit width (2D) and for one wave length.

$$E = E_k + E_p$$

Kinetic Energy

$$E_k = \int_o^L \int_{-d}^o \frac{1}{2} \rho dxdz(u^2 + w^2)$$

$$E_k = \frac{\rho g\, H^2 L}{16}$$

Potential Energy

$$E_p = \int_o^L \rho g(d+\eta)\left(\frac{d+\eta}{2}\right)dx - \rho g L d\left(\frac{d}{2}\right)$$

$$E_k = \frac{\rho g\, H^2 L}{16}$$

Potential energy is the energy needed to:
lift water above the average level against gravity

In time, the wave is a continuous exchange:
crest → maximum potential energy
mid-descent → maximum kinetic energy
trough → potential energy again

$$E = E_k + E_p = \frac{\rho g\, H^2 L}{8}$$

Energy, Power, and Group Celerity

Wave power P is the wave **energy per unit time** transmitted in the **direction of wave propagation**.

Wave power can be written as the product of the **force acting on a vertical plane** normal to the direction of wave propagation **times the particle flow velocity** across this plane $P = \int F \, x \, u \, dzdt$: the wave-induced force is provided by the dynamic pressure and the flow velocity is the horizontal component of the particle velocity:

$$p = \frac{1}{T}\int_{o}^{T}\int_{-d}^{o}(p + \rho g z)u dz dt$$

Inserting the dynamic pressure and the horizontal component of velocity

$$P = \frac{\rho g H^2 L}{16T}\left(1 + \frac{2kd}{\sinh 2kd}\right)$$

or

$$P = \frac{E}{2T}\left(1 + \frac{2kd}{\sinh 2kd}\right)$$

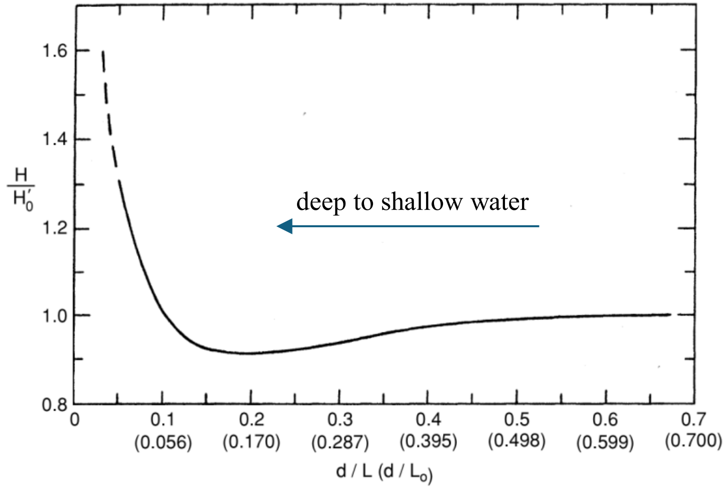
$$n = \frac{1}{2}\left(1 + \frac{2kd}{\sinh 2kd}\right)$$

$$P = \frac{nE}{T}$$

The value of n increases as a wave propagates toward the shore from 0.5 in deep water to 1.0 in shallow water.

The term n can be interpreted as the *fraction of the mechanical energy in a wave that is transmitted forward each wave period*.

Energy, Power, and Group Celerity



The figure is a plot of H/H'_0 versus d/L (and d/L_0) from deep to shallow water.

- Initially, as a wave enters intermediate water depths the wave height decreases because n increases at a faster rate than L decreases.
- H/H'_0 reaches a minimum value of 0.913 at d/L=0.189 (d/L_0= 0.157).
- Shoreward of this point the wave height grows at an ever-increasing rate until the wave becomes unstable and breaks.

Shoaling effect: As the wave enter to shallow water and the seafloor forces its height to grow, potential energy grows, so kinetic energy decreases (assume total energy is conserved). This means that wave slow down, and so - for the dispersion effects, the wavelength must decrease.

Here we neglect energy transfer to and from waves by surface and bottom effects. The nature of these effects is discussed briefly in the following slide.

Bottom effects, of course, require that the water depth be sufficiently shallow for a strong interaction between the wave train and the bottom.

Energy, Power, and Group Celerity

Wave Reflection

If the bottom is other than horizontal, a portion of the incident wave energy will be reflected seaward. This reflection is generally negligible for wind wave periods on typical nearshore slopes. However, for longer period waves and steeper bottom slopes wave reflection would not be negligible. Any sharp bottom irregularity such as a submerged structure of sufficient size will also reflect a significant portion of the incident wave energy.

Wind effects

Nominally, if the wind has a velocity component in the direction of wave propagation that exceeds the wave celerity the wind will add energy to the waves. If the velocity component is less than the wave celerity or the wind blows opposite to the direction of wave propagation the wind will remove energy from the waves. For typical non-stormy wind conditions and the distances from deep water to the nearshore zone found in most coastal locations, the wind effect can be neglected in the analysis of wave conditions nearshore.

Bottom Friction

As the water particle motion in a wave interacts with a still bottom, an unsteady oscillatory boundary layer develops near the bottom. For long period waves in relatively shallow water this boundary layer can extend up through much of the water column. But, for typical wind waves the boundary layer is quite thin relative to the water depth, and if propagation distances are not too long and the bottom is not too rough, bottom friction energy losses can be neglected.

Bottom Percolation

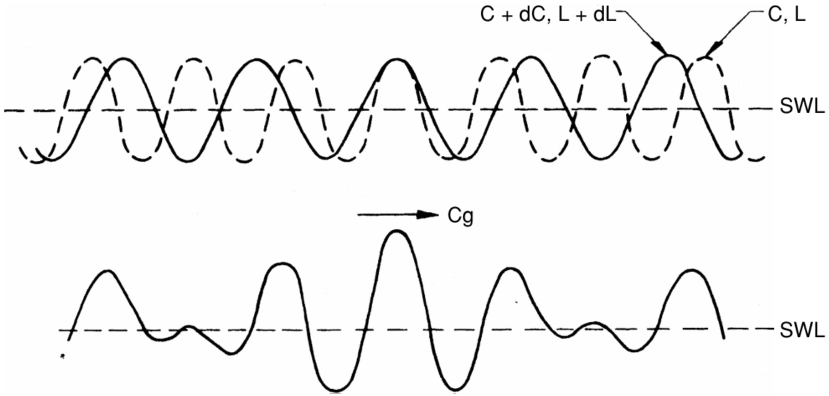
If the bottom is permeable to a sufficient depth, the wave-induced fluctuating pressure distribution on the bottom will cause water to percolate in and out of the bottom and thus dissipate wave energy.

Bottom Movement

When a wave train propagates over a bottom consisting of soft viscous material (such as the mud) the fluctuating pressure on the bottom can set the bottom in motion. Viscous stresses in the soft bottom dissipate energy provided by the waves.

Energy, Power, and Group Celerity

Wave group velocity



It results $C = n\, C_g$

So n is also the ratio of the wave group celerity to the phase celerity.

Another way to look at this is that the wave energy is propagated forward at the group celerity.

- The crests travel at speed C, but the energy travels at speed Cg.
- To predict when the storm surge will reach the coast, you need to use Cg.

Radiation stress

The **horizontal flux of momentum** at a given location consists of the **pressure force acting on a vertical plane** normal to the flow plus the **transfer of momentum** through that vertical plane.

The latter is the product of the momentum in the flow and the flow rate across the plane. From classical fluid mechanics, the momentum flux from one location to another will remain constant unless there is a force acting on the fluid in the flow direction to change the flux of momentum.

$$p + \rho u^2$$

For a wave, we want the excess momentum flux owing to the wave, so the radiation stress S_{xx} for a wave propagating in the x direction becomes

$$S_{xx} = \int_{-d}^{\eta} \overline{(p + \rho u^2) dz} - \int_{-d}^{o} \rho g dz$$

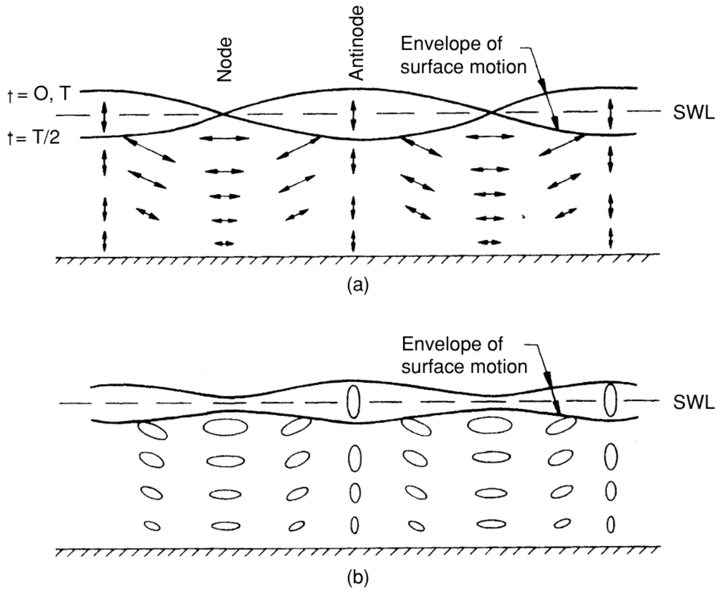
The static pressure must be subtracted to obtain the radiation stress for only the wave. The overbar denotes that the first term on the right must be averaged over the wave period.

Deep water → $S_{xx} = \frac{\bar{E}}{2}$ $S_{yy} = 0$

Shallow water → $S_{xx} = \frac{3\bar{E}}{2}$ $S_{yy} = \frac{\bar{E}}{2}$

Standing waves

A solid structure such as a **vertical wall will reflect an incident wave**, the amplitude of the reflected wave depends on the wave and wall characteristics. When the reflected wave passes through the incident wave a standing wave will develop.



Consider two waves having the same height and period but propagating in opposite directions along the x axis. When these two waves are super-imposed the resulting motion is a standing wave as depicted in Figure.

The arrows indicate the paths of water particle oscillation. Under a nodal point particles oscillate in a horizontal plane while under an antinodal point they oscillate in a vertical plane.

The velocity potential for a standing wave can be obtained by adding the velocity potentials for the two component waves that move in opposite directions. This yields

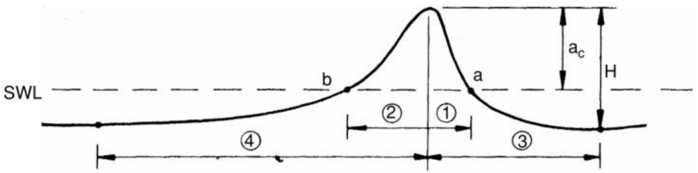
$$\phi = \frac{gH}{\sigma}\left[\frac{\cosh k(d+z)}{\cosh kd}\right]\cos kx \sin \sigma t$$

With this, we can derive the various standing wave characteristics (u,w,p,E..) in the same way as for a progressive wave. This yields, for examples, a surface profile given by $\eta = \mathrm{H}\cos(kx)\sin(\sigma t)$

→ Check the code for visualizing standing waves

Profile Asymmetry and Breaking

As a wave propagates into intermediate and shallow water an initial profile asymmetry develops around the horizontal axis as the wave crest steepens and the wave trough flattens. Further on an asymmetry also develops around a vertical axis through the wave crest (neither asymmetry is defined by the small amplitude wave-theory). These asymmetries ultimately lead to wave instability and breaking.



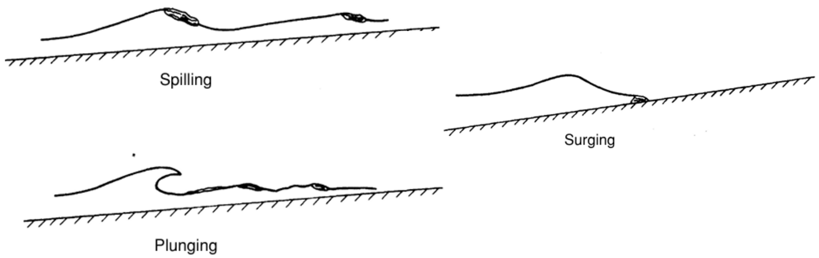
Deep water → $\left(\frac{H_0}{L_0}\right)_{max} = \frac{1}{7}$

Shallow water → $\left(\frac{H}{L}\right)_{max} = 0.9$

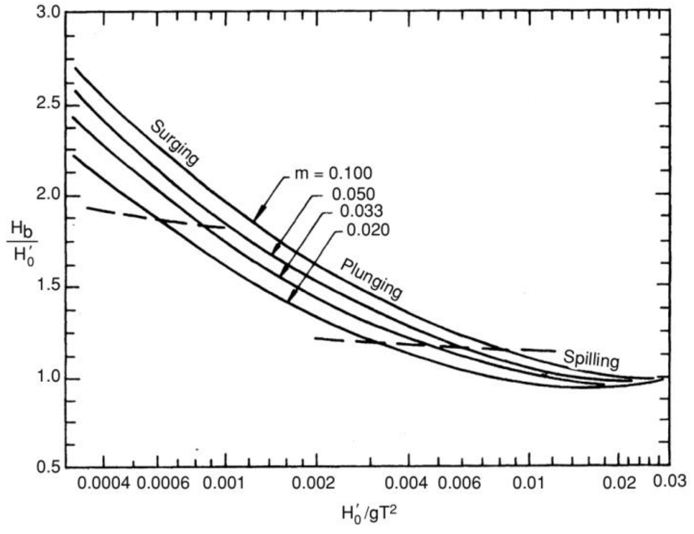
Only gives an approximate rule of thumb for wave breaking in shallow water.
A number of experimenters have investigated near-shore breaking conditions in the laboratory and presented procedures for predicting the **breaking height Hb** and **water depth at breaking db** as a function of incident wave characteristics and bottom slope m.

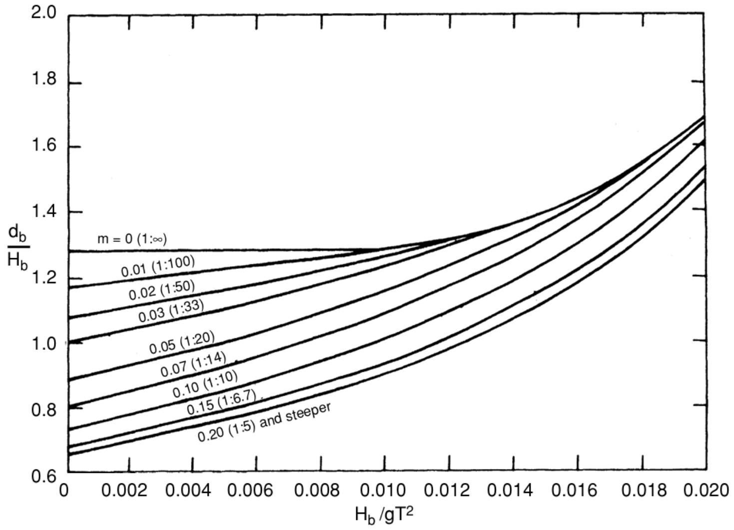
Waves breaking on a beach are commonly classified into three categories (U.S. Army Coastal Engineering Research Center, 1984):

- Spilling
- Plunging
- Surging



Profile Asymmetry and Breaking

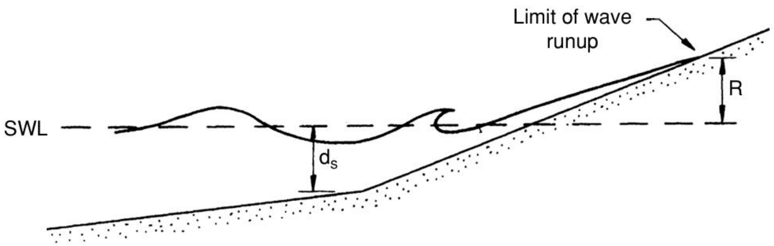




Based on studies by Goda(1970) and Weggel(1972), are commonly used for estimating breaking conditions. Given the beach slope, the unrefracted deep water wave height, and the wave period one can calculate the deep water wave steepness and then determine the breaker height.

Douglass (1990) conducted limited laboratory tests on the effect of inline following and opposing winds on nearshore wave breaking. He found that offshore directed winds retarded the growth of wave height toward the shore and consequently caused the waves to break in shallower water than for the no wind condition. Onshore winds had the opposite effect but to a lesser extent. Note that the design of some coastal structures is dependent on the higher wave that breaks somewhat seaward of the structure and plunges forward to hit the structure.

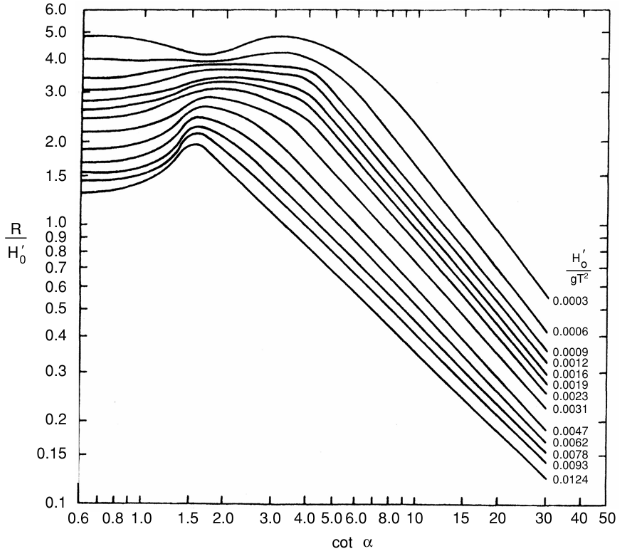
Wave runup



After a wave breaks, a portion of the remaining energy will energize a bore that will run up the face of a beach or sloped shore structure.

The runup R is the maximum vertical elevation above the still water level to which the water rises on the beach or structure. Prediction of the wave runup is important, for example, for the determination of the required crest elevation for a sloping coastal structure or to establish a beach setback line for limiting coastal construction.

The runup depends on the incident deep water wave height and period, the surface slope and profile form if not planar, the depth d_s fronting the slope, and the roughness and permeability of the slope face.



Typical plot of experimental data from a laboratory wave runup study with monochromatic waves.
These data are for a smooth, planar, impermeable slope with d_s/H_0 between 1 and 3.
Indicates that, for a given structure slope, steeper waves (higher H_0/gT^2) have a lower relative runup (R/H_0). Ù
Also, for most beach and revetment slopes (which are flatter than 1 on 2), the wave runup increases as the slope becomes steeper.