Wind energy and fundamentals of nuclear energy (472MI-1) (a module of Alternative energy technologies 2) a.y. 2025/26

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Content of this lecture

1) Sources of tower excitation

2) Main principles of structural dynamics

3) Forced vibrations/Resonance

4) Avoidance of resonance (1P/3P)

5) Natural frequency of a WT tower

Sources of tower excitation

A. Blade passing frequency

Stochastic wind loading (gust slicing)

Each blade "slices through" a localised gust in turn. Dominant effect.

Tower shadow

Load on each blade drops off sharply as it passes behind tower

Wind shear, yaw, shaft tilt

Largely averaged out over three blades – 2^{nd} order

Sources of tower excitation

B. Rotational frequency

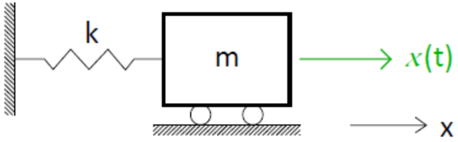
Blade pitch error

+/- 0.3 degrees specified in 2003 GL rules
=> thrust variation of approx. +/- 1% of steady thrust

Rotor mass imbalance

0.005 R eccentricity specified in 1999 GL rules
=> moment variation of approx. +/-1% of max thrust x R

Fundamentals of structural dynamics (one dof, no damping)



$$m\,\ddot{x}(t) + k\,x(t) = 0$$

$$\ddot{x}(t) + \frac{k}{m}\,x(t) = 0$$

$$\ddot{x}(t) + \omega^2\,x(t) = 0$$

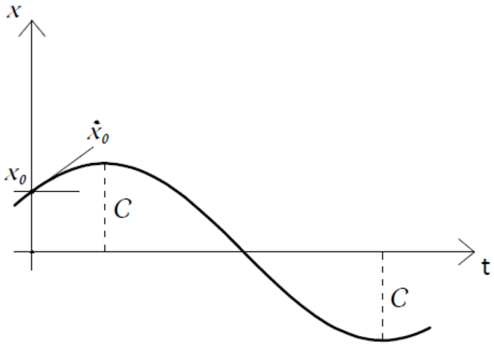
Circular frequency [rad/s]

$$x(t) = A\,\cos(\omega\,t) + B\,\sin(\omega\,t)$$

$$x\,(t=0) = x_0 \quad \Longrightarrow \quad A = x_0$$

$$\dot{x}\,(t=0) = \dot{x}_0 \quad \Longrightarrow \quad B = \frac{\dot{x}_0}{\omega}$$

$$x(t) = x_0\,\cos(\omega\,t) + \frac{\dot{x}_0}{\omega}\,\sin(\omega\,t)$$



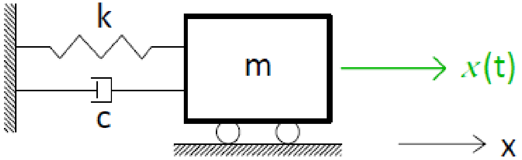
Period [s]

$$T = \frac{2\,\pi}{\omega}$$

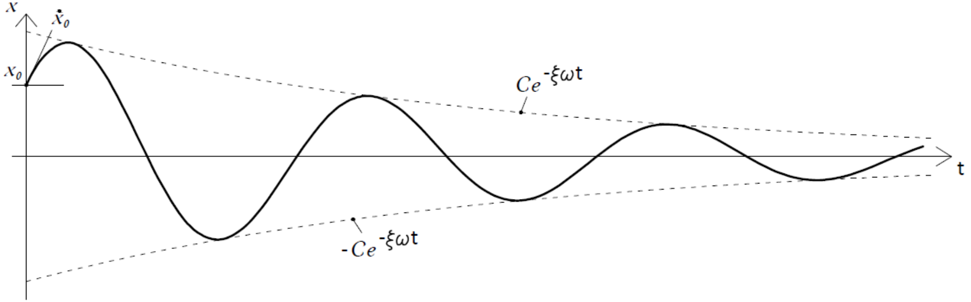
$$f = \frac{1}{T} = \frac{\omega}{2\,\pi}$$

Frequency [Hz]

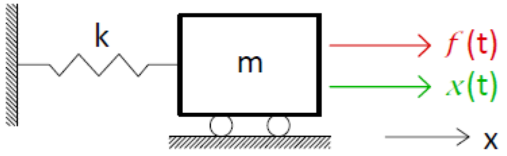
Fundamentals of structural dynamics (solution with damping)



$$m\,\ddot{x}(t) + c\,\dot{x} + k\,x(t) \;=\; 0$$



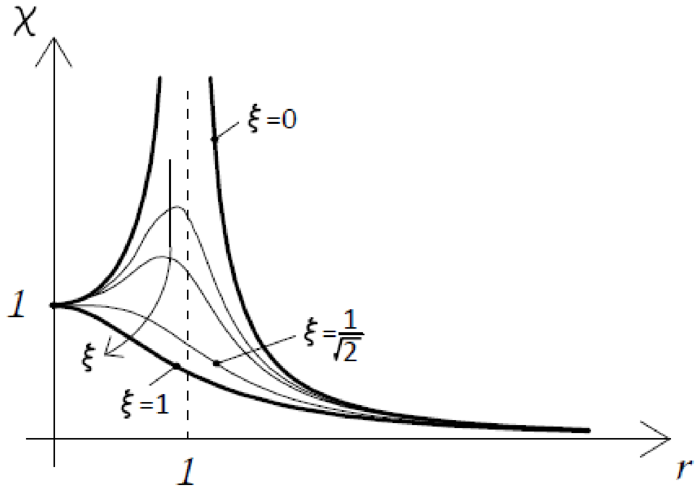
Fundamentals of structural dynamics (one dof, forced vibrations)



$$m\,\ddot{x}(t) + k\,x(t) = \overline{F}\,\sin(\overline{\omega}\,t)$$

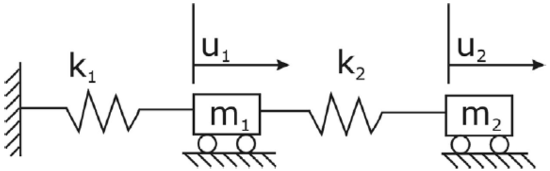
$$\overline{x} = \frac{\overline{F}\,/\,k}{1 - \frac{m}{k}\,\overline{\omega}^2} = \frac{\overline{F}\,/\,k}{1 - \frac{\overline{\omega}^2}{\omega^2}} = \frac{\overline{F}\,/\,k}{1 - r^2}$$

Amplification factor
(w.r.t. to static solution)

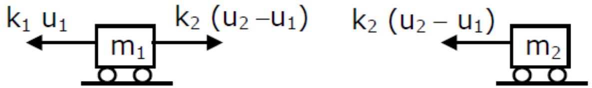


Fundamentals of structural dynamics (vibration modes of multi-dofs)

Esempio (2 gradi di libertà)



- Equazioni del moto per le due masse:



$$m_1\ddot{u}_1 = -k_1 u_1 + k_2(u_2 - u_1) \qquad\qquad \begin{cases} m_1\ddot{u}_1 + k_1 u_1 - k_2(u_2 - u_1) = 0 \\ m_2\ddot{u}_2 + k_2(u_2 - u_1) = 0 \end{cases}$$
$$m_2\ddot{u}_2 = -k_2(u_2 - u_1) \qquad \rightarrow$$

In forma matriciale:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$[\mathbf{M}]\{\ddot{\mathbf{u}}\} + [\mathbf{K}]\{\mathbf{u}\} = \{\mathbf{0}\}.$$

Fundamentals of structural dynamics (vibration modes of multi-dofs)

- Sono in n° pari ai gradi di libertà della struttura
- Ogni modo di vibrare è associato ad una pulsazione (o periodo)

$$\omega_1 \rightarrow \{\mathbf{U}^{(1)}\}, \quad \omega_2 \rightarrow \{\mathbf{U}^{(2)}\}, \quad \omega_3 \rightarrow \{\mathbf{U}^{(3)}\}, \quad \text{ecc.}$$

Pulsazione *naturale* o *fondamentale*

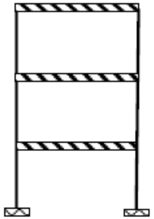
$$\omega_1 \leq \omega_2 \leq \omega_3 \leq \text{ecc.}$$

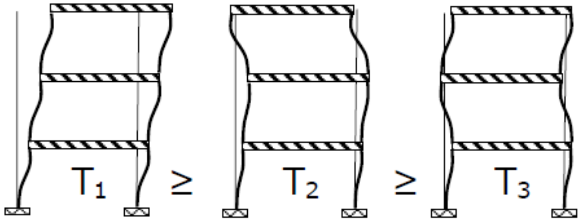
$$T_1 \geq T_2 \geq T_3 \geq \text{ecc.}$$

Periodo *proprio* o *fondamentale*

$$T = \frac{2\pi}{\omega}$$

3 piani,
3 modi di vibrare





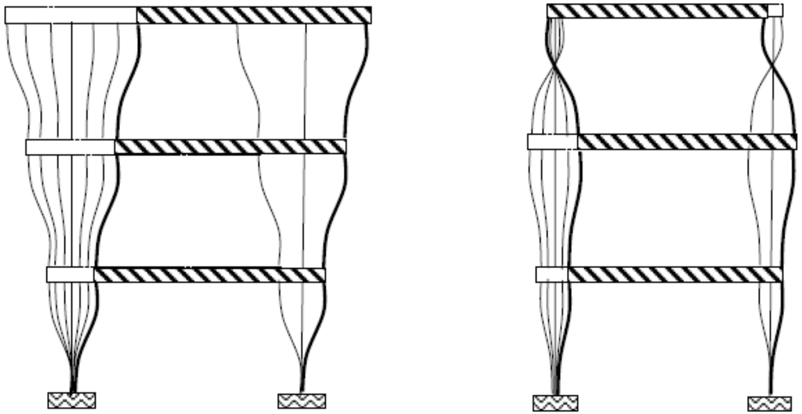
1° modo 2° modo 3° modo

Fundamentals of structural dynamics (vibration modes of multi-dofs)

- Una vibrazione generica di una struttura è una combinazione dei modi di vibrare della struttura (dipende dalle condizioni iniziali).

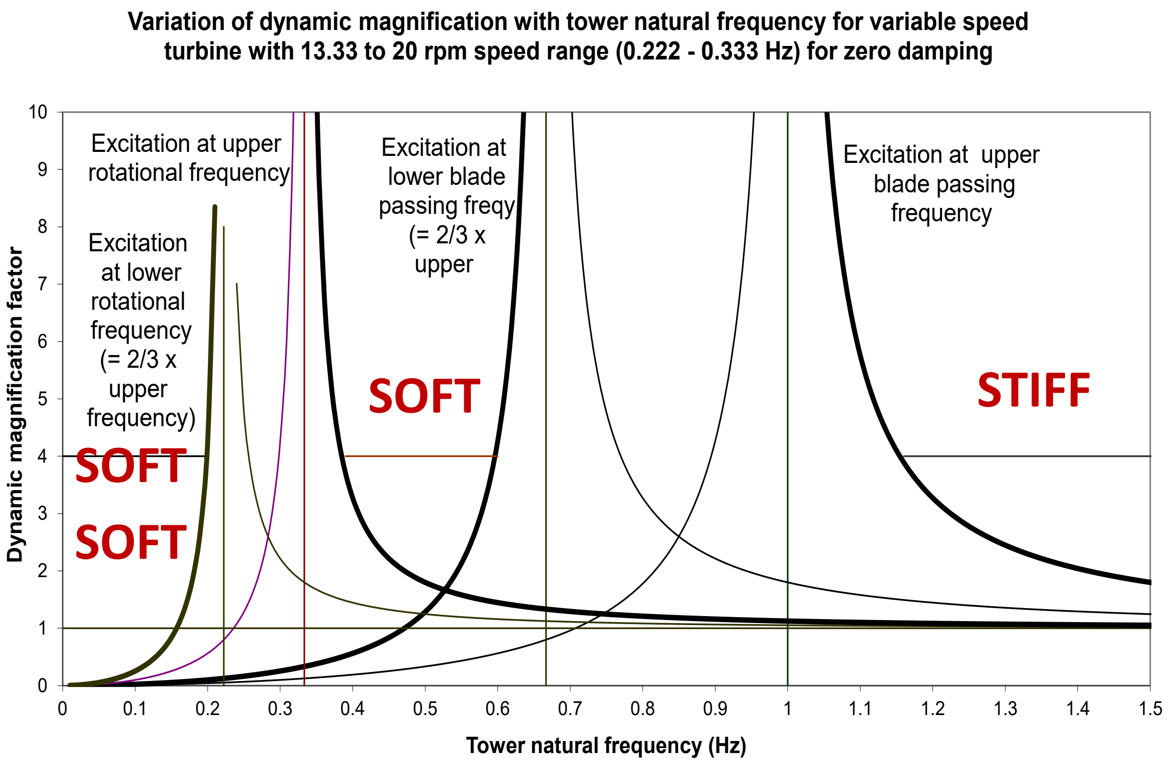
- Se le condizioni iniziali corrispondono ad un modo di vibrare allora la struttura vibrerà liberamente secondo il modo stesso.

1° modo



2° modo

Avoidance of resonance (1P/3P)



Avoidance of resonance (1P/3P)

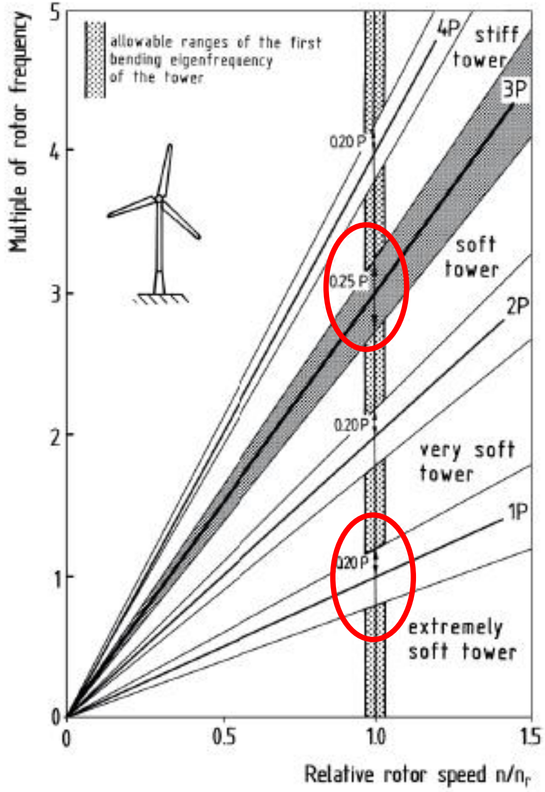


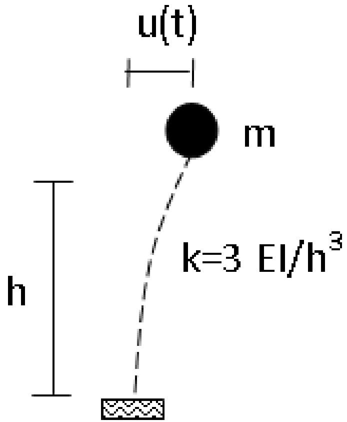
Fig. 7.20. Tower stiffness in the resonance diagram of a wind turbine with a three-bladed rotor

Hau (2006)

Avoidance of resonance

An estimate of the natural frequency

A wind turbine and its tower can be idealised as a mass on a weightless, uniform cantilever.



Equation of motion (free vibr)

$\ddot{u}(t) + \omega^2 u(t) = 0$, $\omega = \sqrt{k/m}$

ω: radian frequency

T: period $T = 2\pi/\omega$

Avoidance of resonance

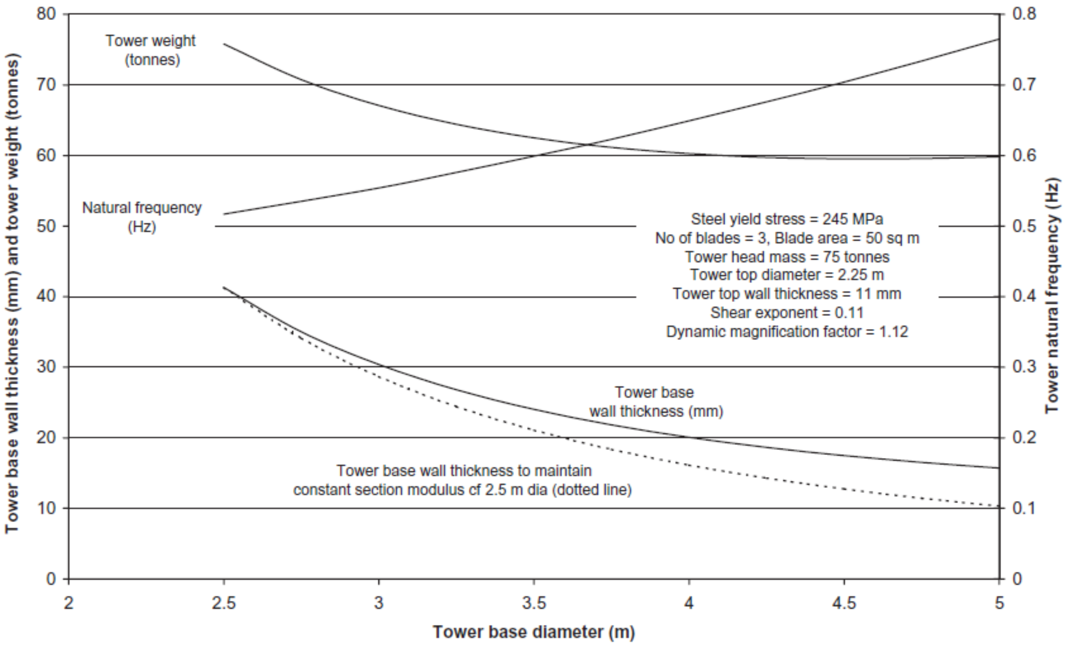


Figure 7.40 Variation in Tower Base Wall Thickness with Diameter Required for Support of 60 m Diameter Stall-regulated Wind Turbine at 50 m Height in 70 m/s Extreme Wind

Design options

Second moment of area ($\pi R^3 t$) restricted by natural frequency limitations.

Initially, increasing the R/t ratio gives more efficient use of material, but at high R/t the buckling reduction factor penalty increases.