

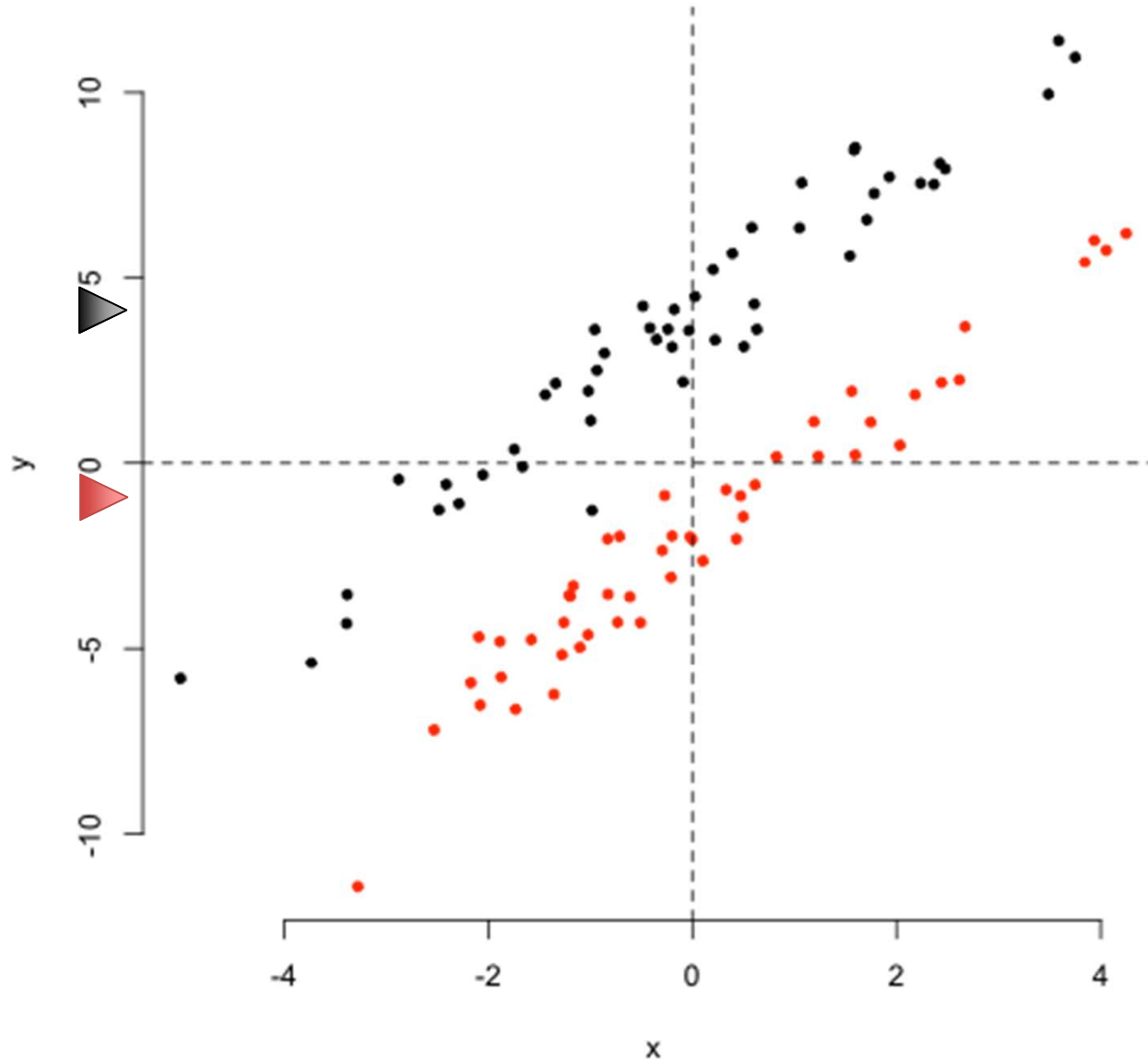
Rappresentazioni grafiche dei modelli di regressione lineare con interazione

Modelli Lineari Applicati

2025/26

(Lezione 2: Sommario ripasso)

Caso 1: due gruppi, differente media



MODELLO:

$$y_i = \alpha + \beta x_{1i} + \gamma D_{1i}$$

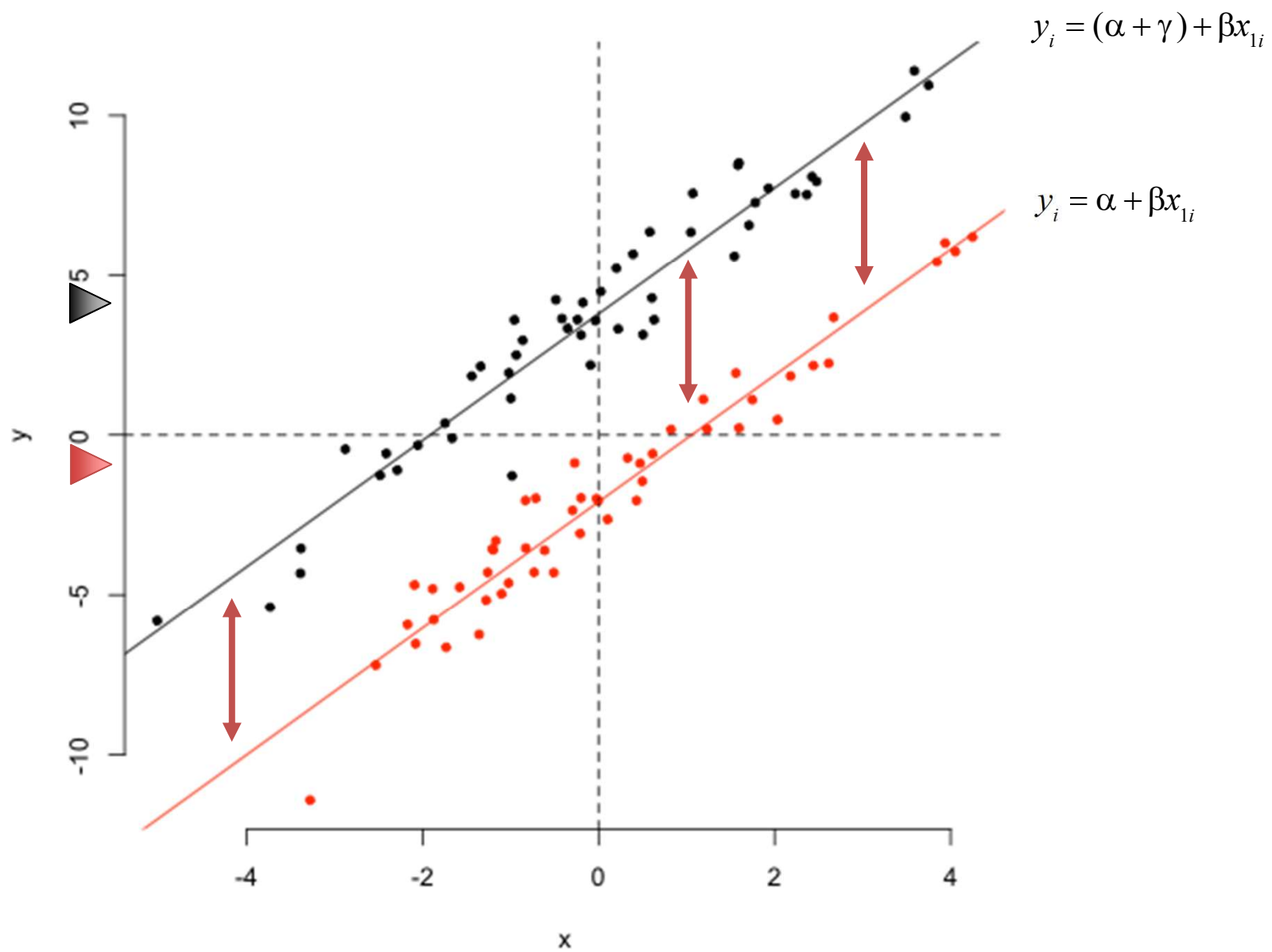
codifica dummy:

se $y_i = \text{MASCHIO}$, allora $D_{1i} = 0$

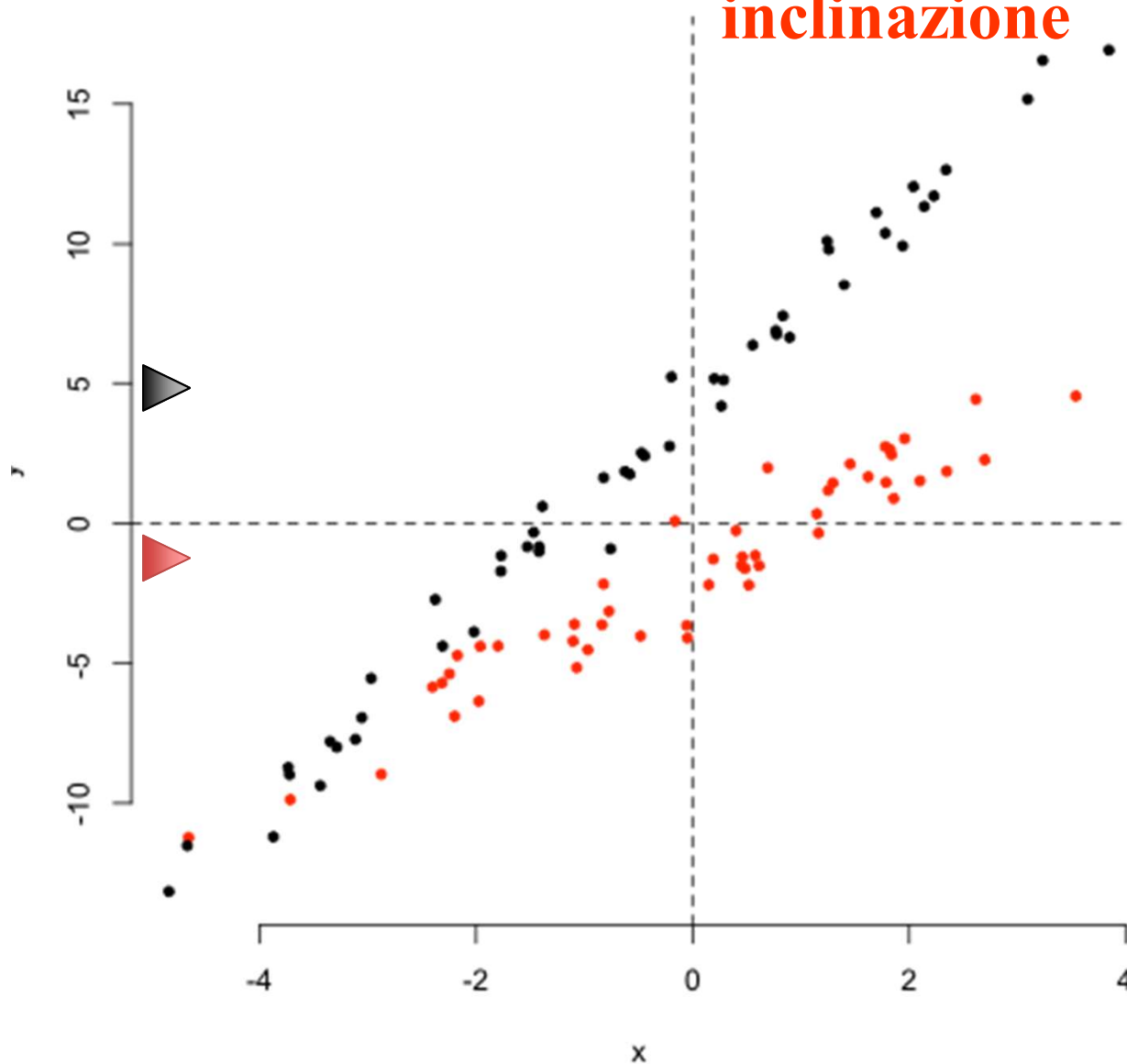
se $y_i = \text{FEMMINA}$, allora $D_{1i} = 1$

Matrice del modello:

$$\mathbf{X} = \begin{bmatrix} 1 & x_{11} & 1 \\ 1 & x_{12} & 1 \\ 1 & x_{13} & 0 \\ \dots & \dots & \dots \\ 1 & x_{1n} & 0 \end{bmatrix}$$



Caso 2: due gruppi, differente media e diversa inclinazione



MODELLO:

$$y_i = \alpha + \beta x_1 + \gamma D_1 + \beta_2 x_1 D_1$$

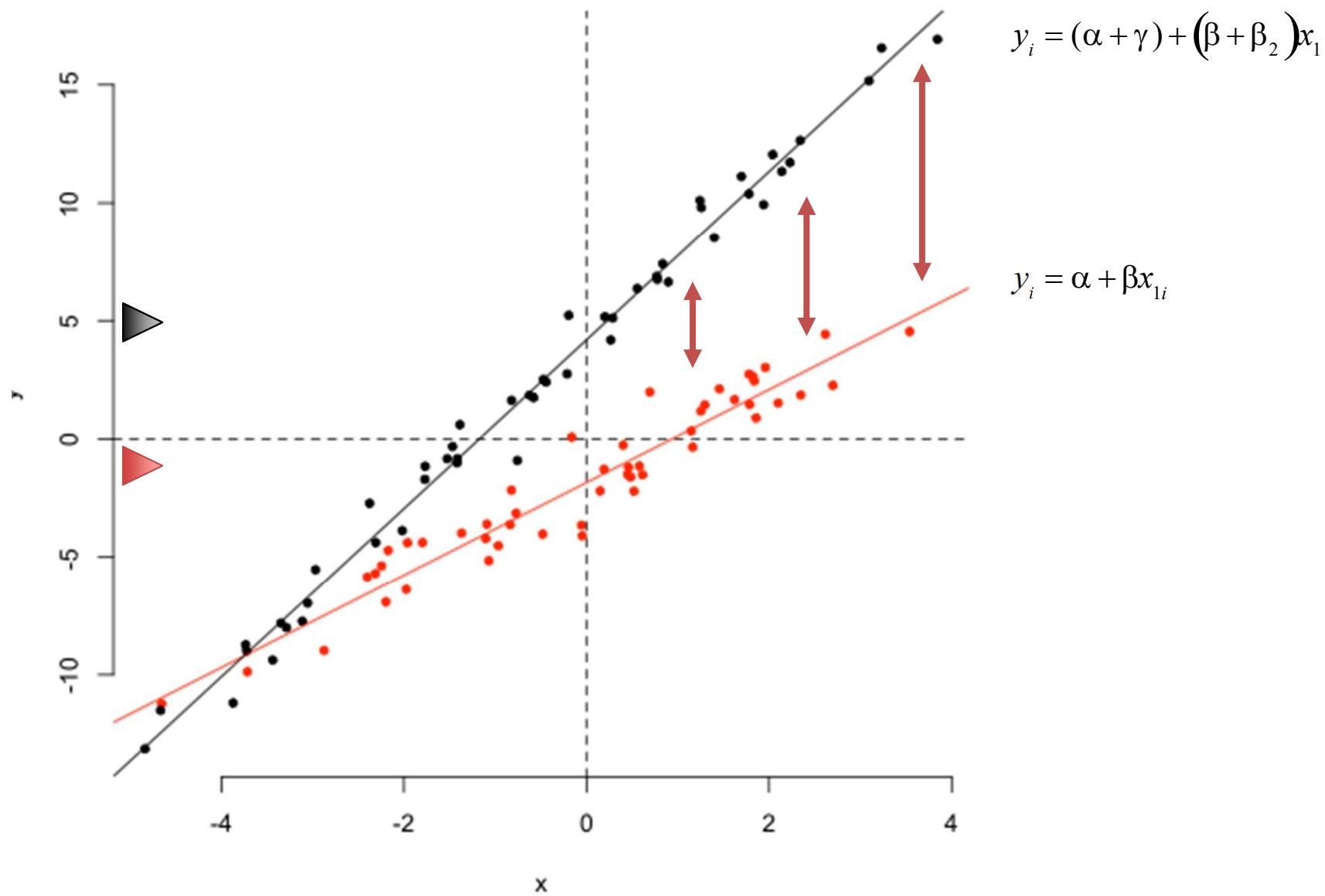
codifica dummy:

se $y_i = \text{MASCHIO}$, allora $D_{1i} = 0$

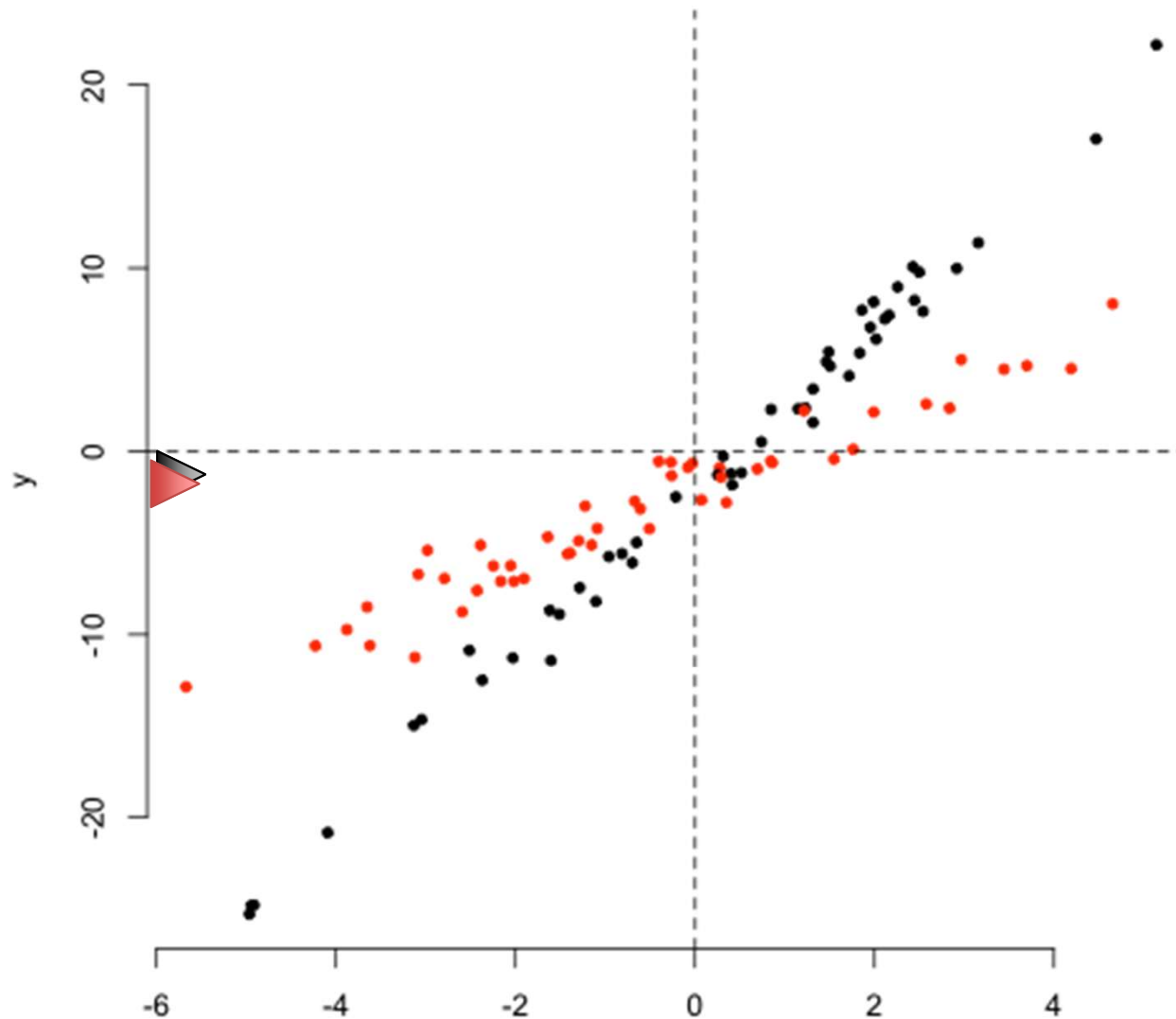
se $y_i = \text{FEMMINA}$, allora $D_{1i} = 1$

Matrice del modello:

$$\mathbf{X} = \begin{bmatrix} 1 & x_{11} & 1 & x_{11} \cdot 1 \\ 1 & x_{12} & 1 & x_{12} \cdot 1 \\ 1 & x_{13} & 0 & x_{13} \cdot 0 \\ \dots & \dots & \dots & \dots \\ 1 & x_{1n} & 0 & x_{1n} \cdot 0 \end{bmatrix}$$



Caso 3: due gruppi, stessa media ma diversa inclinazione



MODELLO :

$$y_i = \alpha + \beta x_1 + \beta_2 x_1 D_1$$

$(\gamma = 0)$

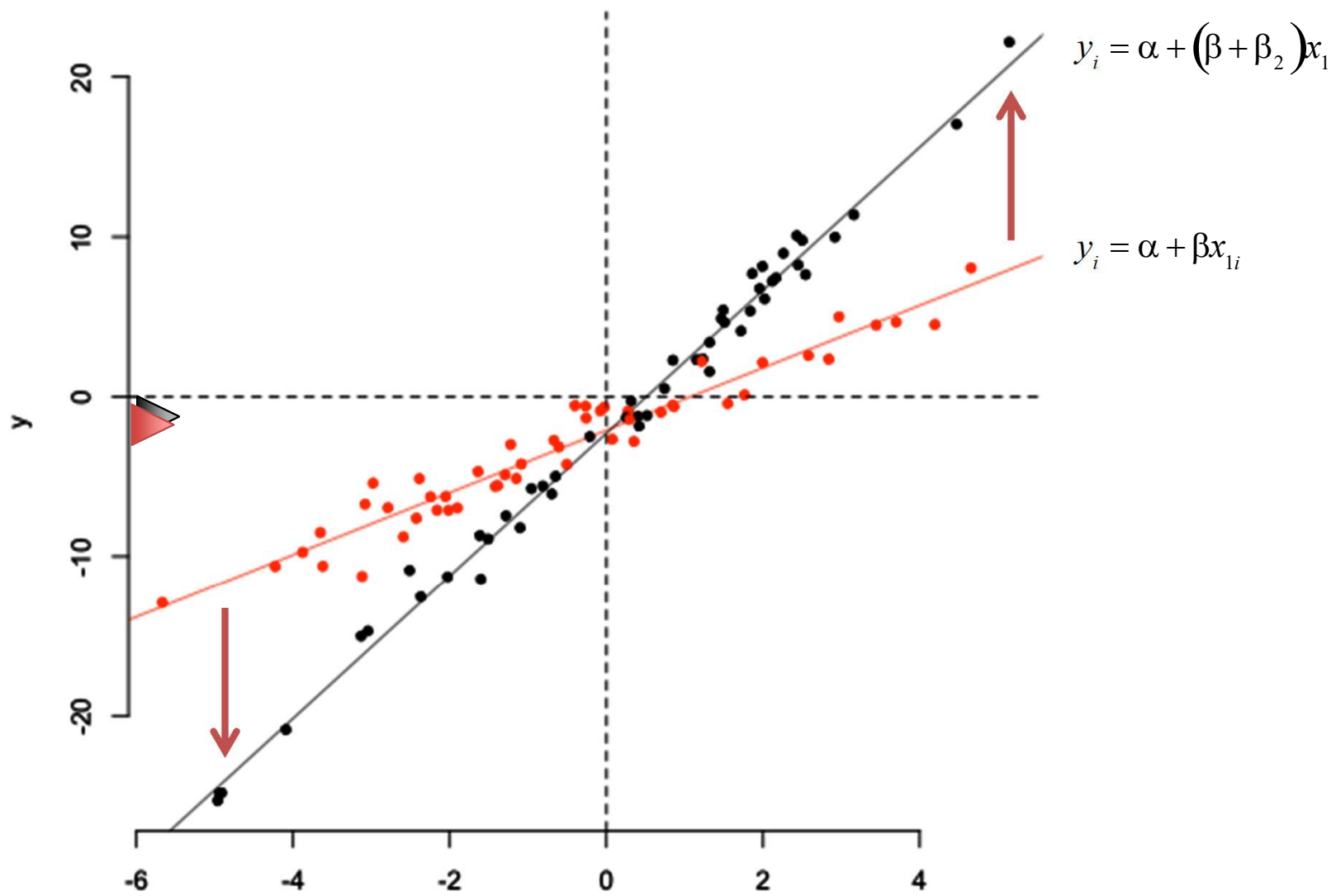
codifica dummy:

se $y_i = \text{MASCHIO}$, allora $D_{1i} = 0$

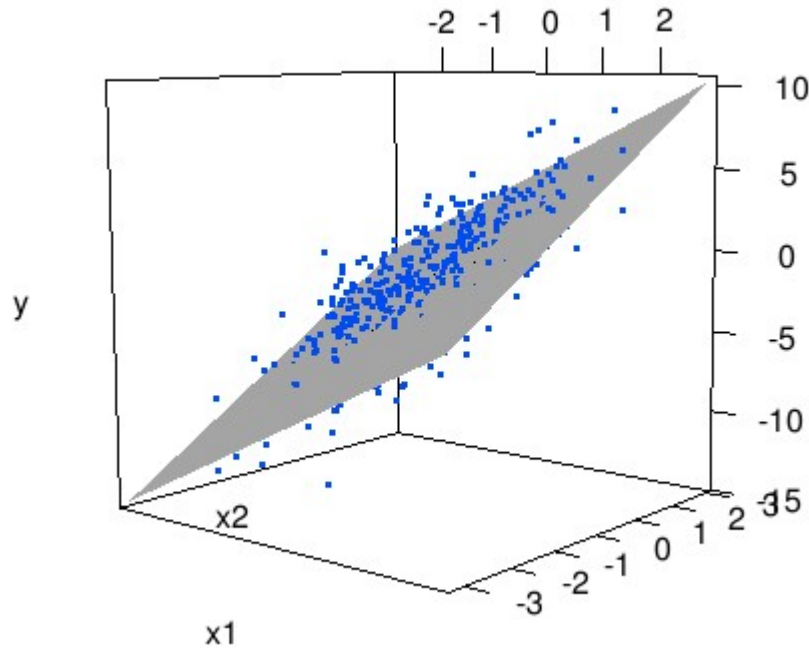
se $y_i = \text{FEMMINA}$, allora $D_{1i} = 1$

Matrice del modello:

$$\mathbf{X} = \begin{bmatrix} 1 & x_{11} & x_{11} \cdot 1 \\ 1 & x_{12} & x_{12} \cdot 1 \\ 1 & x_{13} & x_{13} \cdot 0 \\ \dots & \dots & \dots \\ 1 & x_{1n} & x_{1n} \cdot 0 \end{bmatrix}$$



Caso 4: piano con coefficienti β equivalenti



MODELLO:

$$y_i = \alpha + \beta_1 x_1 + \beta_2 x_2;$$

ipotizzando che i due effetti marginali siano uguali ($\beta_1 = \beta_2 = \beta$), avremo:

$$y_i = \alpha + \beta x_1 + \beta x_2$$

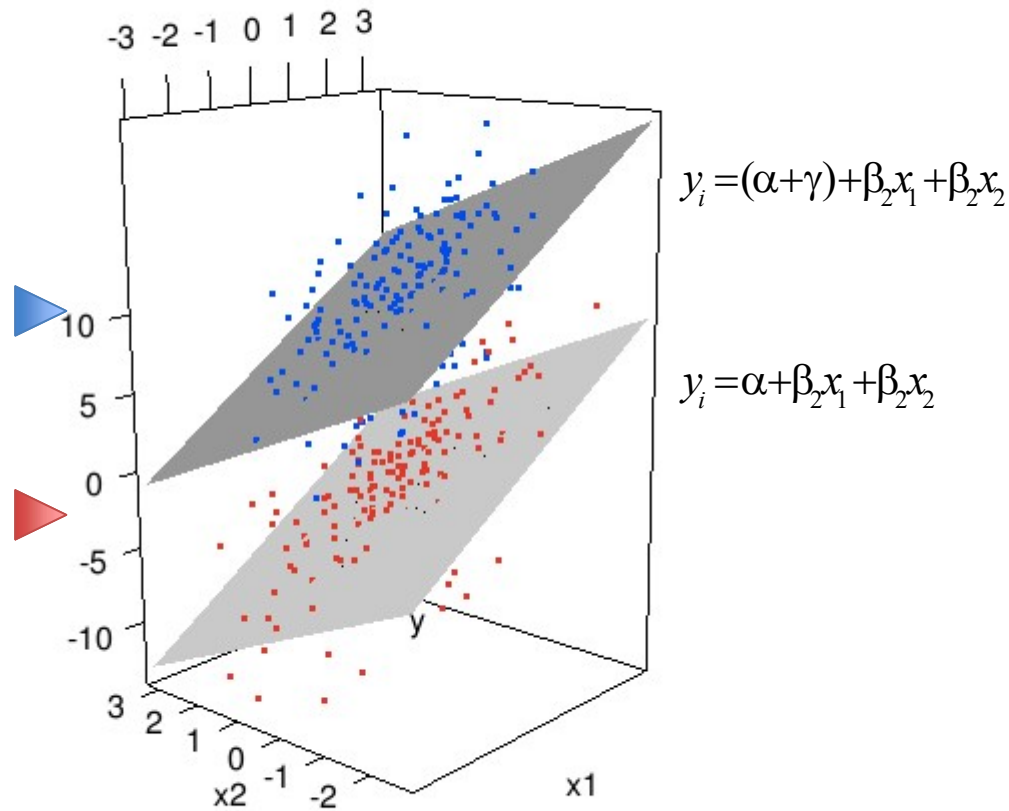
$$y_i = \alpha + \beta (x_1 + x_2)$$

e la matrice del modello diventa:

$$\mathbf{X} = \begin{bmatrix} 1 & x_{11} + x_{21} \\ 1 & x_{12} + x_{22} \\ 1 & x_{13} + x_{23} \\ \dots & \dots \\ 1 & x_{1n} + x_{2n} \end{bmatrix}$$

come nella regressione bivariata.

Caso 5: regressione lineare multipla, due gruppi, differente media



MODELLO:

$$y_i = \alpha + \beta_1 x_1 + \beta_2 x_2 + \gamma D_1$$

codifica dummy:

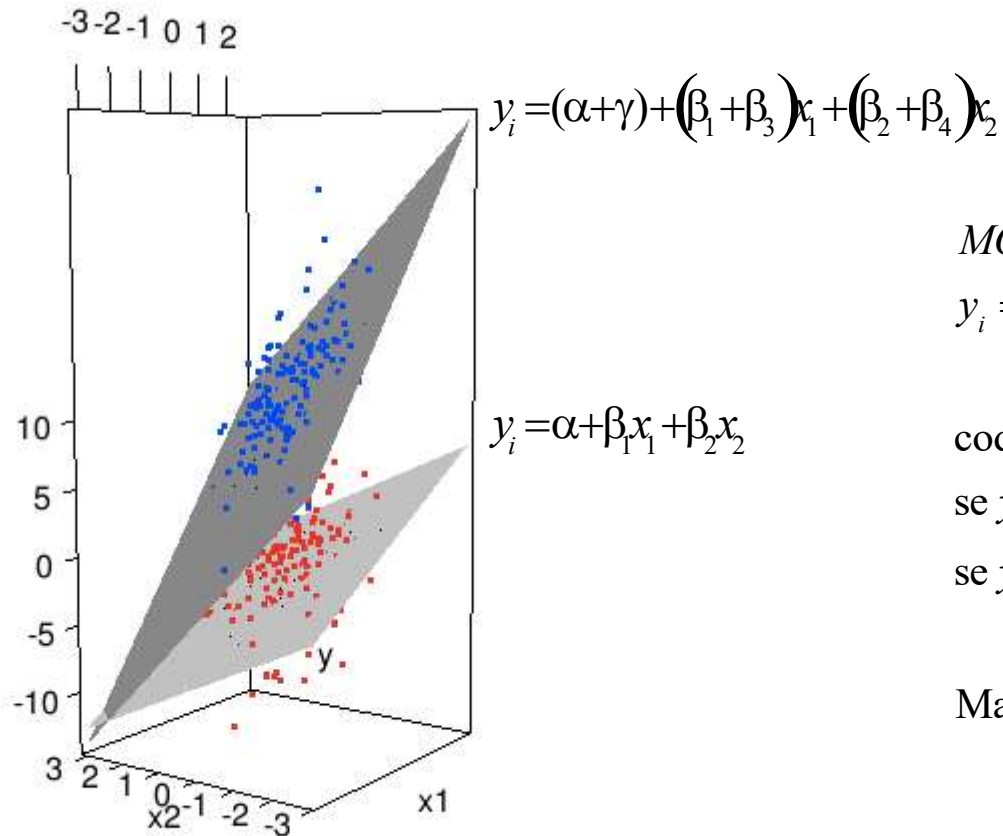
se $y_i = \text{MASCHIO}$, allora $D_{1i} = 0$

se $y_i = \text{FEMMINA}$, allora $D_{1i} = 1$

Matrice del modello:

$$\mathbf{X} = \begin{bmatrix} 1 & x_{11} & x_{21} & 1 \\ 1 & x_{12} & x_{22} & 1 \\ 1 & x_{13} & x_{23} & 0 \\ \dots & \dots & \dots & \dots \\ 1 & x_{1n} & x_{2n} & 0 \end{bmatrix}$$

Caso 6: regressione lineare multipla, due gruppi, differente media e diversa inclinazione



MODELLO:

$$y_i = \alpha + \beta_1x_1 + \beta_2x_2 + \gamma D_1 + \beta_3 D_1x_1 + \beta_4 D_1x_2$$

codifica dummy:

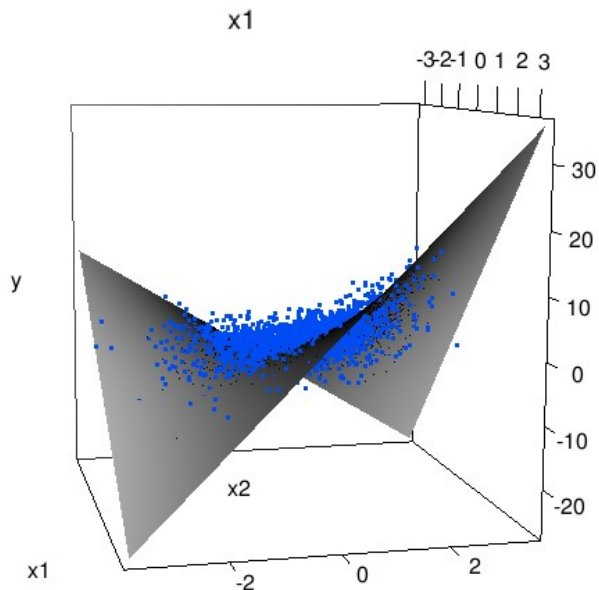
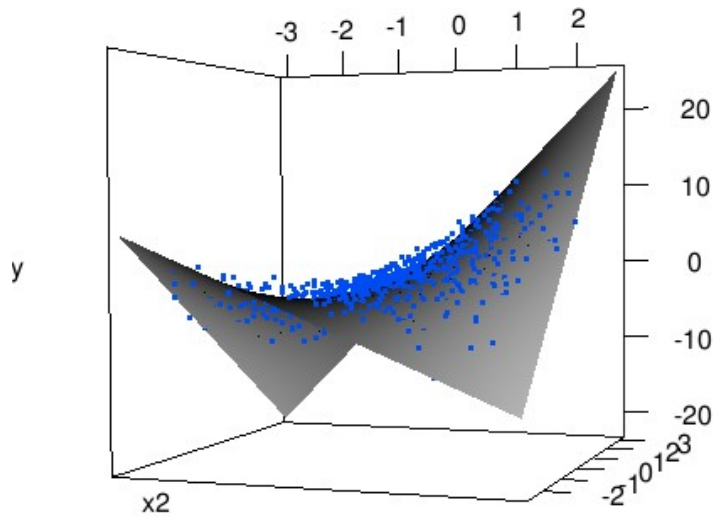
se $y_i = \text{MASCHIO}$, allora $D_{1i} = 0$

se $y_i = \text{FEMMINA}$, allora $D_{1i} = 1$

Matrice del modello:

$$\mathbf{X} = \begin{bmatrix} 1 & x_{11} & x_{21} & 1 & x_{11} \cdot 1 & x_{21} \cdot 1 \\ 1 & x_{12} & x_{22} & 1 & x_{12} \cdot 1 & x_{22} \cdot 1 \\ 1 & x_{13} & x_{23} & 0 & x_{13} \cdot 0 & x_{23} \cdot 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & x_{1n} & x_{2n} & 0 & x_{1n} \cdot 0 & x_{2n} \cdot 0 \end{bmatrix}$$

Caso 7: regressione lineare multipla, interazione tra variabili continue



MODELLO :

$$y_i = \alpha + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2;$$

raggruppando

$$y_i = \alpha + (\beta_1 + \beta_3 x_2) x_1 + \beta_2 x_2,$$

$$y_i = \alpha + \beta_1 x_1 + (\beta_2 + \beta_3 x_1) x_2.$$

– Al variare di x_2 l'inclinazione del piano determinata da x_1 diventa: $(\beta_1 + \beta_3 x_2)$.

– Al variare di x_1 l'inclinazione del piano determinata da x_2 diventa: $(\beta_2 + \beta_3 x_1)$.

La matrice del modello diventa:

$$\mathbf{X} = \begin{bmatrix} 1 & x_{11} & x_{21} & x_{11} \cdot x_{21} \\ 1 & x_{12} & x_{22} & x_{12} \cdot x_{22} \\ 1 & x_{13} & x_{23} & x_{13} \cdot x_{23} \\ \dots & \dots & \dots & \dots \\ 1 & x_{1n} & x_{2n} & x_{1n} \cdot x_{2n} \end{bmatrix}$$

Compito 2

- Ripassare (studiare) la regola per calcolare le seguenti derivate (prime):

- $\frac{\Delta \exp(a)}{\Delta a} = ?$, $\frac{\Delta \ln(a)}{\Delta a} = ?$

- $\frac{\Delta \ln(2b^3)}{\Delta b} = ?$ (Chain Rule)

- $\frac{\Delta \{ \ln(2b^3) \cdot \exp(b) \}}{\Delta b} = ?$ (Product Rule)