Standard Model - Problem sheet

Enrico Morgante*

February 27, 2026

1st Week: Lagrangians and Symmetries

Exercise 1.1: Natural Units

Given the constants used in the natural units convention,

$$c \approx 3 \times 10^8\,\text{m/s} = 1, \quad \hbar c \approx 197.3\,\text{MeV fm} = 1 \tag{1.1}$$

solve the following exercises.

1. The width of a particle is defined as the inverse of its litefime. The mean lifetime for the B^+ meson is $\tau \approx 1.64 \times 10^{-12}\,\text{s}$. What is its width in eV?

2. Find the average distance traveled in the lab frame by a particle with $\gamma = 100$ and a decay width of $\Gamma = 2.3\,\text{eV}$;

3. Quantum gravity effects cannot be neglected at very short distances. This happens when the energy scale is of the order of the Planck mass:

$$M_P = \sqrt{\frac{\hbar c}{G_N}} \tag{1.2}$$

   where G_N is the Newtonian gravitational constant. Express M_P in GeV, and the Planck length $L_P = M_P^{-1}$ in centimeters.

4. In oscillation experiments for neutrinos, it is important to know the oscillation length, $L_{osc} = 4\pi E/\Delta m^2$, where Δm^2 is the mass-squared difference between the two neutrino states. For an experiment conducted with neutrinos of $E = 1.3\,\text{GeV}$, find the value of Δm^2 in units of eV^2 that corresponds to $L_{osc} = 140$ meters.

Exercise 1.2: Weyl & Dirac spinors

Use the chirality projectors to rewrite the following Langragian in function of 4-components Weyl spinors, and then in function of 2-components Weil spinors in the Weil basis:

$$\mathscr{L}_F = i\bar{\psi}\partial\!\!\!/\psi - m_D\bar{\psi}\psi - e\bar{\psi}A\!\!\!/\psi \tag{1.3}$$

*I warmly thank Raffaele Tiede for the help in preparation of this document.

Exercise 1.3: Gamma matrices

1. Prove, without relying on any explicit representation, the following identities
   (a) $\gamma^5 \equiv -\frac{i}{4!}\epsilon^{\mu\nu\rho\sigma}\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma = i\gamma^0\gamma^1\gamma^2\gamma^3$, where $\epsilon^{0123} = +1$ and $\gamma_\mu = \eta_{\mu\nu}\gamma^\nu$
   (b) $(\gamma^5)^2 = \mathbb{1}$
   (c) $\gamma_\mu \not{p} \gamma^\mu = -2\not{p}$
   (d) $\gamma_\mu \not{p}\not{q}\not{p} \gamma^\mu = -2\not{p}\not{q}\not{p}$
   (e) $\{\gamma^5, \gamma^\mu\} = 0$
   (f) $\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = 4(\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho})$

Exercise 1.4: Fierz identities

Consider the two set of matrices

$$\Gamma^A = \{\mathbb{1}, \gamma^\mu, \sigma^{\mu\nu}, i\gamma^\mu\gamma^5, i\gamma^5\}\,, \qquad \Gamma_A = \{\mathbb{1}, \gamma_\mu, \sigma_{\mu\nu}, -i\gamma_\mu\gamma^5, -i\gamma^5\} \tag{1.4}$$

which satisfy the completness relation

$$\sum_A \frac{1}{4}(\Gamma^A)_{ij}(\Gamma_A)_{kl} = \delta_{il}\delta_{jk} \tag{1.5}$$

Using the completeness relation Eq. 1.5, prove the following *Fierz identities*:

a) $(\bar{\psi}_1\gamma^\mu P_L\psi_2)(\bar{\psi}_3\gamma_\mu P_L\psi_4) = -(\bar{\psi}_1\gamma^\mu P_L\psi_4)(\bar{\psi}_3\gamma_\mu P_L\psi_2)$

b) $(\bar{\psi}_1\gamma^\mu\gamma^\alpha\gamma^\beta P_L\psi_2)(\bar{\psi}_3\gamma_\mu\gamma_\alpha\gamma_\beta P_L\psi_4) = -16(\bar{\psi}_1\gamma^\mu P_L\psi_4)(\bar{\psi}_3\gamma_\mu P_L\psi_2)$

c) For generic basis elements

$$(\bar{\psi}_1\Gamma^M\psi_2)(\bar{\psi}_3\Gamma^N\psi_4) = \sum_{PQ}\frac{1}{16}\,\mathrm{Tr}[\Gamma^P\Gamma^Q\Gamma^M\Gamma^N](\bar{\psi}_1\Gamma^P\psi_4)(\bar{\psi}_3\Gamma^Q\psi_2) \tag{1.6}$$