Lezione 24 febbraio 2026

martedì 24 febbraio 2026 11:07

Visto: def SA ; $\forall P \in \mathbb{A}$

$f_P : \mathbb{A} \longrightarrow V$, $f_P(Q) = \overrightarrow{PQ}$

è biiezione

DEF.: Fissiamo $v \in V$;

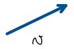
$t_v : \mathbb{A} \to \mathbb{A}$

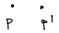
$t_v(P) := Q$ dove Q 1! t.c. $v = \overrightarrow{PQ}$

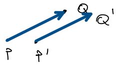
segue da SA1

t_v si dice TRASLAZIONE LUNGO v

es. $\mathbb{A}^2_{\mathbb{R}}$ $(= \mathbb{R}^2)$







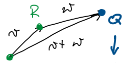
OSS.: Se $v = 0_V \in V$

$t_{0_V} : \mathbb{A} \to \mathbb{A}$, $t_{0_V}(P) = P$

$0_V = \overrightarrow{PP}$

$t_{0_V} = Id_{\mathbb{A}}$

Esercizio: $t_{v+w}(P) = t_w(t_v(P))$ $\forall P \in \mathbb{A}$ $\Rightarrow$ $t_{v+w} = t_w \circ t_v$



$t_v(P) = R$ t.c. $\overrightarrow{PR} = v$

$\overrightarrow{PQ} = v+w$ $t_w(R) = S$?

$\overrightarrow{RS} = w$

(SA2) $\left.\begin{array}{l} \overrightarrow{PR} + \overrightarrow{RS} = \overrightarrow{PS} \\ v + w \quad // \end{array}\right\} \rightarrow S = Q$

ESERCIZIO : L'insieme $\mathcal{T} = \{$traslazioni lungo $v : v \in V\}$

è un GRUPPO RISP. alla composizione $\circ$

ESERCIZIO: $f : V \longrightarrow \mathcal{T}$

$v \longrightarrow t_v$

$f(v) = t_v$

è un ISOMORFISMO DI GRUPPI

tra $(V, +)$ e $(\mathcal{T}, \circ)$

SISTEMI DI RIFERIMENTO AFFINI & COORDINATE AFFINI

Supp. $\dim V = n < +\infty$, $\mathbb{A}$ SA su V

Def.: Un SIST. di RIF. AFFINE (SRA)

è il dato di:

- un punto $O \in \mathbb{A}$, detto ORIGINE
- una base $\{v_1, \dots, v_n\}$ di V

$$SRA = (O, v_1, \dots, v_n)$$

es. $\mathbb{A}^2_{\mathbb{R}}$

$\mathbb{R}^2$
Q
O
$\overrightarrow{OQ}$

Def.: Fissato SRA, $\forall\, Q \in \mathbb{A}$, le COORDINATE di Q (che dipendono da SRA) sono le COORDINATE nella $\{v_1, \dots, v_n\} = \mathcal{B}$ del vettore $\overrightarrow{OQ}$

Cioè: se

$$\overrightarrow{OQ} = a_1 v_1 + \dots + a_n v_n \quad \text{(è unica)}$$

$$F_{\mathcal{B}}(\overrightarrow{OQ}) = \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix} \in \mathbb{K}^n$$

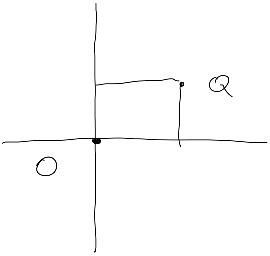
$F_{\mathcal{B}} : V \longrightarrow \mathbb{K}^n$ funzione COORDINATE

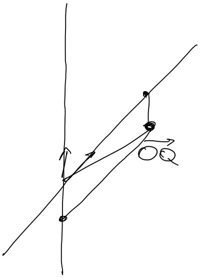
$F_{\mathcal{B}}(v) = \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix}$ coordinate nella $\mathcal{B}$

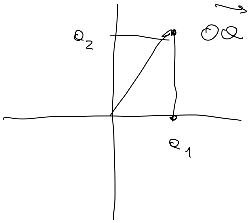
RICHIAMO: $\forall \mathcal{B}$ base, $F_{\mathcal{B}}$ è ISOM. DI SV

> coordinate di Q = coordinate nella base $\mathcal{B}$ di $\overrightarrow{OQ}$
>
> $= (q_1, \dots, q_n)$

Es.:







Def.: Se $\mathbb{A} = \mathbb{A}^n_{\mathbb{K}}$ $(= \underset{V}{\overset{\shortparallel}{\mathbb{K}^n}})$

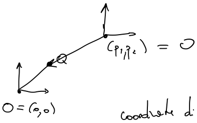
in $\mathbb{K}^n$ possiamo scegliere la BASE CANONICA

$$\mathcal{E} = \{e_1, \dots, e_n\} = \left\{ \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \dots \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix} \right\}$$

come $O \in \mathbb{A}^n_{\mathbb{K}}$ scegliamo $O = (0, 0, \dots, 0)$

$\Longrightarrow (O, e_1, \dots, e_n)$ si dice SRA CANONICO

es.



coordinate di Q = coord. di $\overrightarrow{OQ}$

=

$\mathbb{R}^2 = \mathbb{A}^2_{\mathbb{R}} \ni O = (0,0)$

$Q = (q_1, q_2)$

$$\mathbb{R}^2 \ni \overrightarrow{OQ} = \begin{pmatrix} q_1 - 0 \\ q_2 - 0 \end{pmatrix} = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}, \text{ coordinate = componenti}$$

$$\overset{!}{=} q_1 \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} + q_2 \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

OSS.: Se $(O, v_1, \dots, v_n)$ SRA

coordinate di O = coordinate di $\overrightarrow{OO} = 0_V$

$= (0, \dots, 0)$

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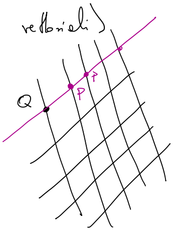
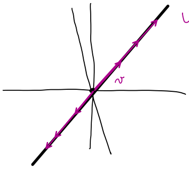
SOTTOSPAZI AFFINI

"Idea": negli SV visto SSV (sottospazi vettoriali)





$W = \mathrm{Span}(v)$

SSV generato da v

$W = \{\lambda v : \lambda \in \mathbb{R}\}$

Def.: Dato $\mathbb{A}$ SA su V, un sottoinsieme $S \subseteq \mathbb{A}$ si dice

SOTTOSP. AFFINE se:

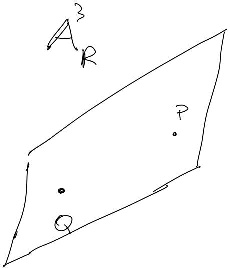
$\exists\ Q \in S$, e $\exists\ W \subseteq V$ SSV

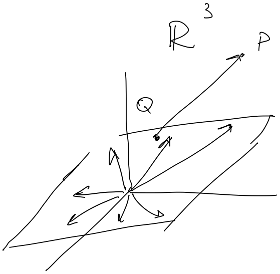
t.c.

$$\boxed{S = \{P \in \mathbb{A} : \overrightarrow{QP} \in W\}}$$

Es.: in $\mathbb{A}^3_{\mathbb{R}}$, $W = \mathrm{Span}(v_1, v_2)$, Q fissato

$$S = \{P \in \mathbb{A}^3_{\mathbb{R}} : \overrightarrow{QP} \in \mathrm{Span}(v_1, v_2)\}$$





Def.: Se $\dim W = 1$, S si dice RETTA AFFINE

se $\dim W = 2$, S si dice PIANO AFFINE

Def.: Se S è SSA (sottosp. affine) determinato da W, allora poniamo

$$\dim S := \dim W$$

e W si dice **GIACITURA** di S

o anche **SOTTOSP. DELLE DIREZIONI** di S

* * *

EQUAZIONI per SSA:

Sia $S \subseteq \mathbb{A}$ con giacitura $W \subseteq V$

$\dim \mathbb{A} = n = \dim V$

$Q \in S$

Fissiamo SRA in $\mathbb{A}$: $(O, v_1, .., v_n)$

$Q = (q_1, .., q_n)$

$\dim W = k < n$

in W fissiamo base $\{w_1, .., w_k\}$

$W = \mathrm{Span}(w_1, .., w_k)$

$$S = \{P \in \mathbb{A} : \overrightarrow{QP} \in W\}$$

$$\boxed{\overrightarrow{QP} = t_1 w_1 + .. + t_k w_k}$$

EQ. VETT. di S

$Q = (q_1, .., q_n) \rightsquigarrow$ coordinate di $\overrightarrow{OQ}$

$P = (p_1, .., p_n) \rightsquigarrow$ coordinate di $\overrightarrow{OP}$

$$\overrightarrow{QP} = \overrightarrow{QO} + \overrightarrow{OP} = -\underset{\begin{pmatrix}-q_1\\ \vdots\\ -q_n\end{pmatrix}}{\underset{\downarrow}{\overrightarrow{OQ}}} + \underset{\begin{pmatrix}p_1\\ \vdots\\ p_n\end{pmatrix}}{\underset{\downarrow}{\overrightarrow{OP}}} \longrightarrow \begin{pmatrix}p_1 - q_1\\ \vdots\\ p_n - q_n\end{pmatrix}$$

Ora scrivo anche $w_1, .., w_k$ in coord. risp. $\mathcal{B}$ base fissata di V

$$w_1 \rightsquigarrow \begin{pmatrix} w_{11} \\ \vdots \\ w_{n1} \end{pmatrix}$$

$$\vdots$$

$$w_k \rightsquigarrow \begin{pmatrix} w_{1k} \\ \vdots \\ w_{nk} \end{pmatrix}$$

$$\boxed{\overrightarrow{QP} = t_1 w_1 + \dots + t_k w_k}$$

IN COORDINATE

$$\begin{pmatrix} p_1 - q_1 \\ \vdots \\ p_n - q_n \end{pmatrix} = t_1 \cdot \begin{pmatrix} w_{11} \\ \vdots \\ w_{n1} \end{pmatrix} + \dots + t_k \begin{pmatrix} w_{1k} \\ \vdots \\ w_{nk} \end{pmatrix}, \quad t_i \in \mathbb{R}$$

$$= \begin{pmatrix} t_1 w_{11} \\ \vdots \\ t_1 w_{n1} \end{pmatrix} + \dots + \begin{pmatrix} t_k w_{1k} \\ \vdots \\ t_k w_{nk} \end{pmatrix}$$

$$= \begin{pmatrix} t_1 w_{11} + \dots + t_k w_{1k} \\ \vdots \\ t_1 w_{n1} + \dots + t_k w_{nk} \end{pmatrix}$$

$$\iff \quad \circledast \quad \begin{cases} x_1 = q_1 + t_1 w_{11} + \dots + t_k w_{1k} \\ x_2 = q_2 + t_1 w_{21} + \dots + t_k w_{2k} \\ \vdots \\ x_n = q_n + t_1 w_{n1} + \dots + t_k w_{nk} \end{cases}$$

EQUAZIONI PARAMETRICHE di S

$t_1, \dots, t_k$ PARAMETRI

$(x_1, \dots, x_n)$ sono le coordinate di un punto di S

$$\iff \exists\, t_1, \dots, t_k \text{ t.c. vale } \circledast$$

$\circledast$ EQ PARAMETRICHE per S

$t_1, \dots, t_k \in \mathbb{K}$ PARAMETRI

Esempi: ① Rette affini: $k = 1$

$Q = (q_1, \dots, q_n)$, $W = \mathrm{Span}(v)$

$$\begin{cases} x_1 = q_1 + t\,b_1 \\ x_2 = q_2 + t\,b_2 \\ \vdots \qquad \vdots \\ x_n = q_n + t\,b_n \end{cases} \qquad v \rightsquigarrow \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

Es.: $n = 2$

$$\begin{cases} x = q_1 + t \cdot b_1 \\ y = q_2 + t b_2 \end{cases}$$

$$\begin{cases} x = 3t - 2 \\ y = 1 - t \end{cases}$$

rette in $\mathbb{A}^2_{\mathbb{R}}$

$Q = (-2, 1)$, $v = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

$$\begin{cases} x = -2 + 3t \\ y = 1 - t \end{cases} \xrightarrow[\text{eq. cartesiana}]{} x = -2 + 3(1-y)$$

$$\Downarrow$$

$$t = 1 - y$$

$$x = -2 + 3 - 3y$$

$$\boxed{x + 3y - 1 = 0}$$