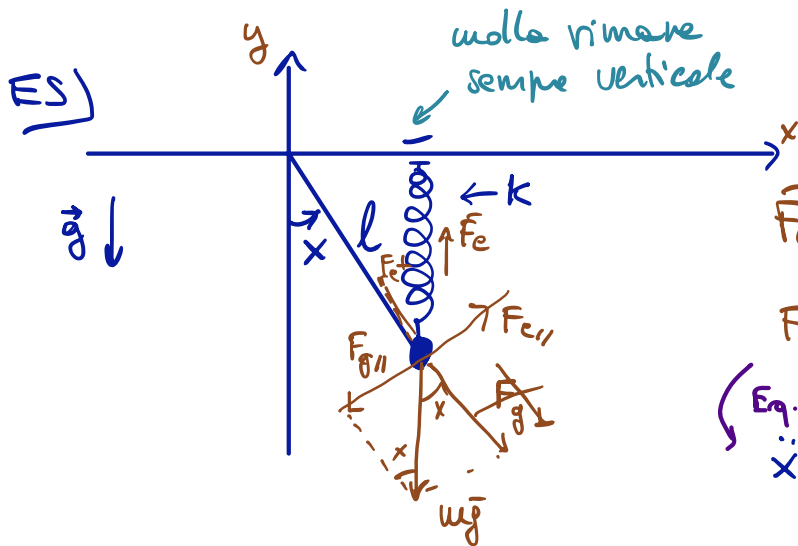


BIFURCAZIONI



$$\vec{F}_e = Kl \cos x \vec{e}_y \quad \vec{F}_g = -mg \vec{e}_y$$

$$F_{e||} = Kl \cos x \sin x \quad F_{g||} = -mg \sin x$$

Eq. Newton:

$$\ddot{x} = -\omega^2 \sin x + \Omega^2 \cos x \sin x$$

$$\omega^2 \equiv \frac{g}{l}$$

$$\Omega^2 \equiv \frac{k}{m}$$

$$V(x) = mg(1 - \cos x)l + \frac{1}{2}kl^2 \cos^2 x$$

$$\tilde{V}(x) = \frac{V(x)}{ml^2} = \omega^2(1 - \cos x) + \frac{1}{2}\Omega^2 \cos^2 x$$

Disegniamo il grafico di questo potenziale

- $V(x)$ è periodico per $x \rightarrow x + 2\pi$

$$\tilde{V}'(x) = \omega^2 \sin x - \Omega^2 \sin x \cos x = \sin x (\omega^2 - \Omega^2 \cos x)$$

$$\tilde{V}''(x) = \omega^2 \cos x + \Omega^2 \sin^2 x - \Omega^2 \cos^2 x = \omega^2 \cos x + \Omega^2 - 2\Omega^2 \cos^2 x$$

PTI estremali: $x = 0, \pi \leftarrow c_1, c_2$

$$x = \pm \arccos\left(\frac{\omega^2}{\Omega^2}\right) \leftarrow \exists \text{ se } \frac{\omega^2}{\Omega^2} < 1$$

$$V''(0) = \omega^2 - \Omega^2$$

$$V''(\pi) = -\omega^2 - \Omega^2$$

$$V''\left(\pm \arccos\left(\frac{\omega^2}{\Omega^2}\right)\right) = \frac{\omega^4}{\Omega^2} + \Omega^2 - 2\Omega^2 \frac{\omega^4}{\Omega^4} = \frac{1}{\Omega^2}(\Omega^2 - \omega^2)$$

→ Ci sono due potenziali qualitativamente distinti:

Quando $\omega^2 > \Omega^2$

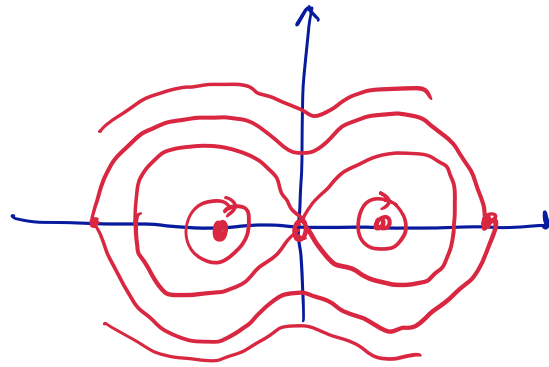
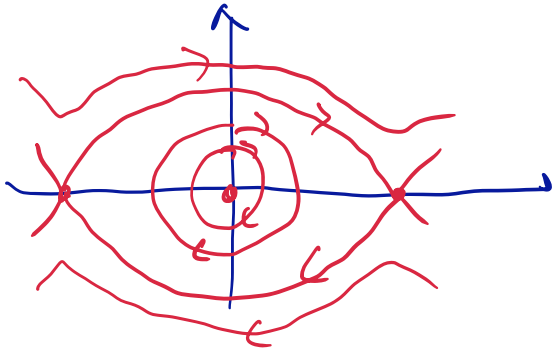
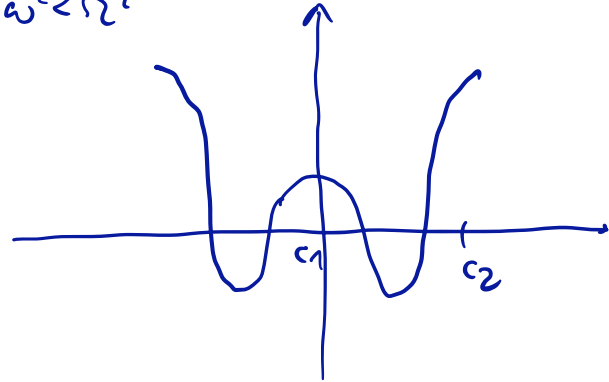
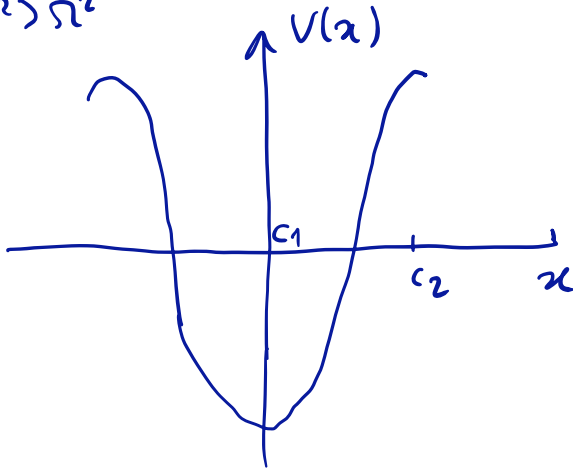
c_1	e^-	MIN	← stab.
c_2	e^-	MAX	← instab.
$c_{3,4}$	non sono soluz.		

Quando $\omega^2 < \Omega^2$

c_1	e^-	MAX	← inst.
c_2	e^-	MAX	← inst.
$c_{3,4}$	son	MINIMI	← stab.

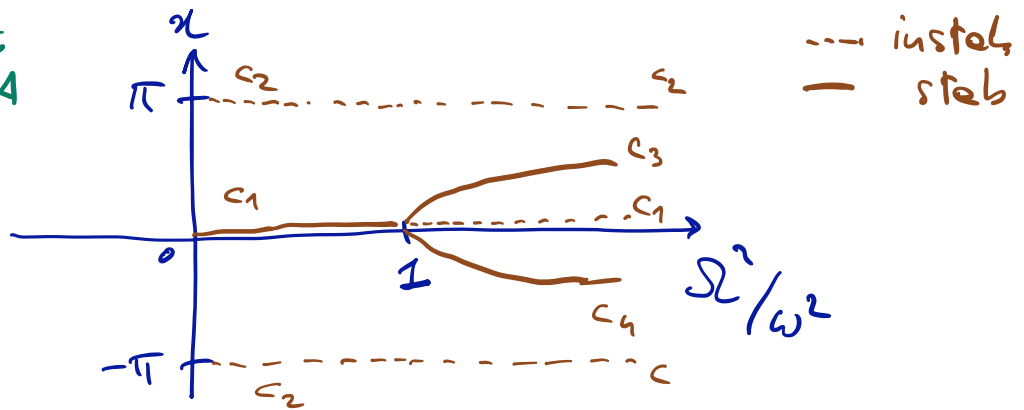
$\omega^2 < \Omega^2$

$\omega^2 > \Omega^2$



BIFORCAZIONE
A FORCHETTA

"Grafico di →
biforcazione"



Al variare del parametro, cambia il numero e il tipo di pti di equilibrio.