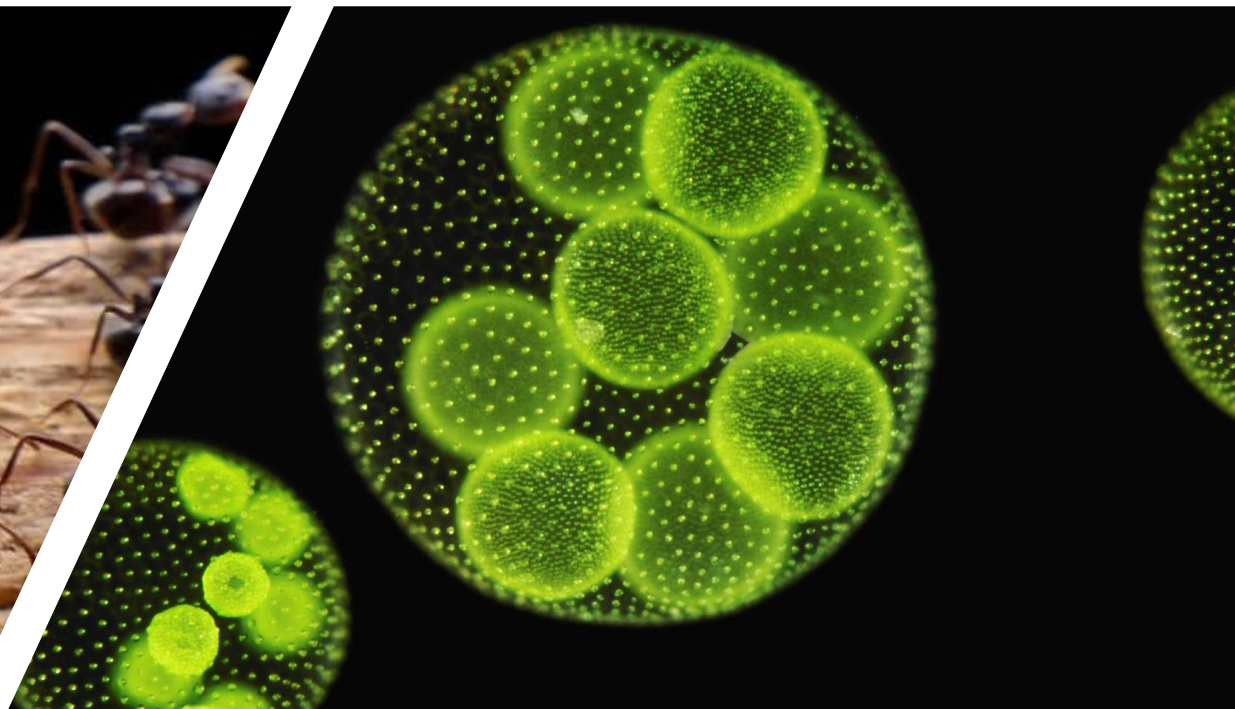


An illustration of a large group of diverse people holding hands in a circle, symbolizing cooperation and community. The people are shown from a top-down perspective, with their arms extended and hands clasped. They are wearing various colorful clothing, including sweaters, shirts, and patterned scarves. The background is a dark blue with faint, lighter blue circular patterns. The overall style is a flat, illustrative art style.

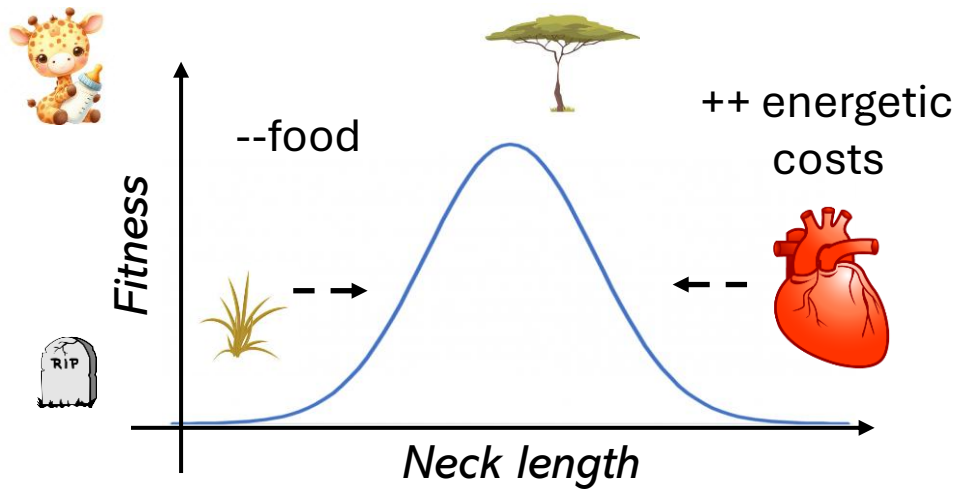
# Evolutionary Game Theory

Part I: The evolution of cooperation



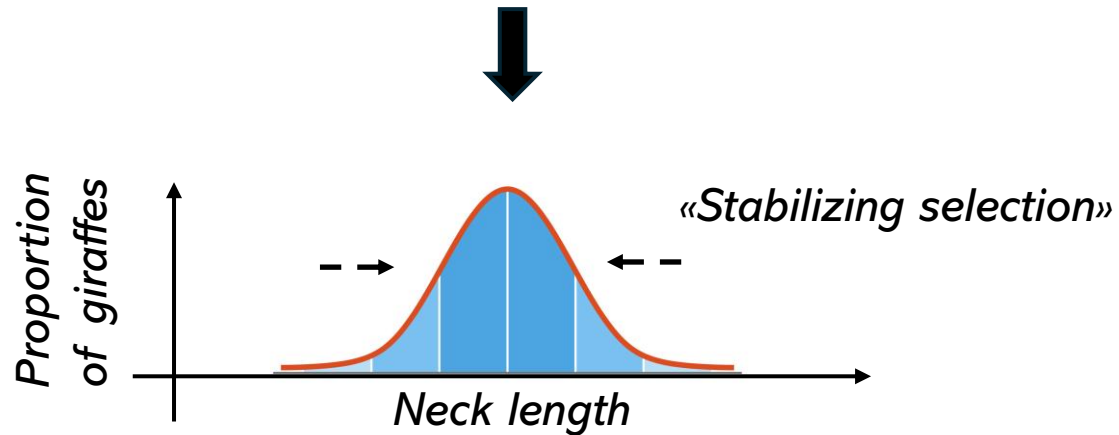
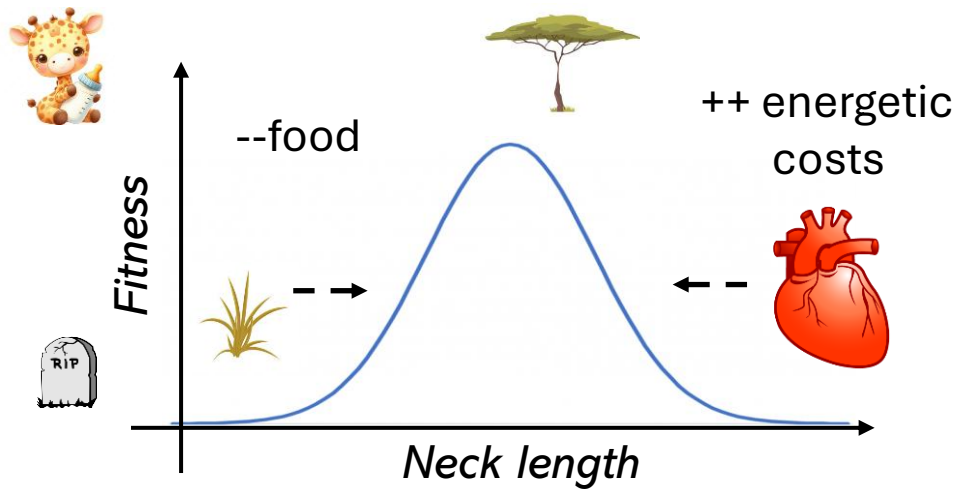
# Evolutionary fitness

**Fitness** represents an organism's ability to survive, reproduce, and pass its genes to the next generation relative to others in its population

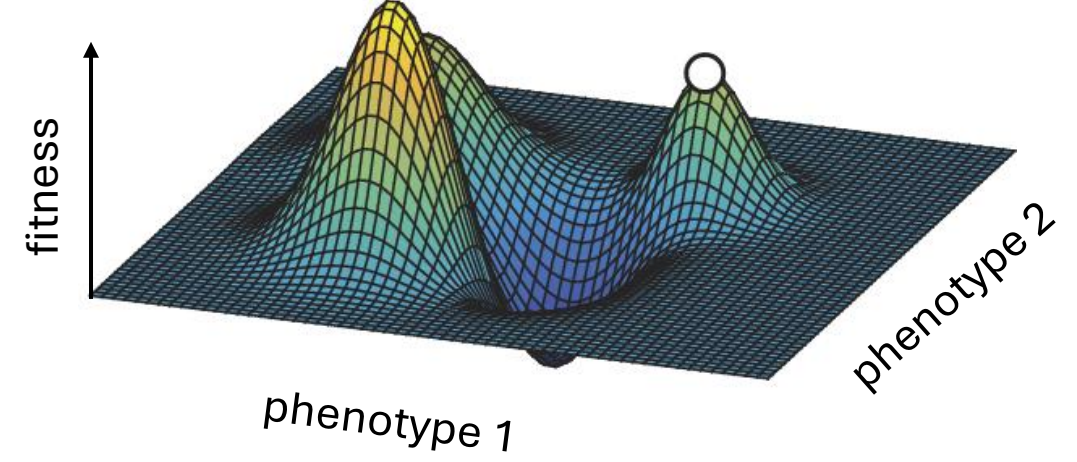


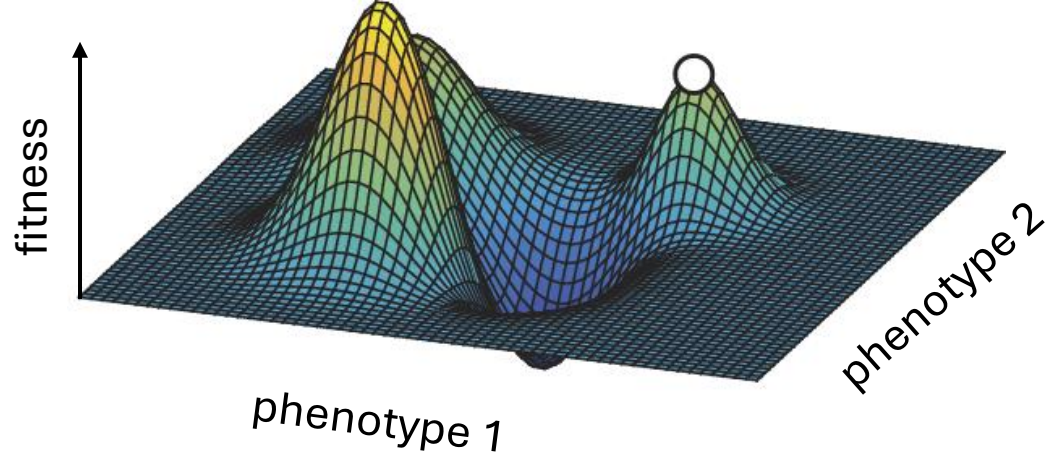
# Evolutionary fitness

**Fitness** represents an organism's ability to survive, reproduce, and pass its genes to the next generation relative to others in its population



Fitness can be represented in a «**fitness landscape**»

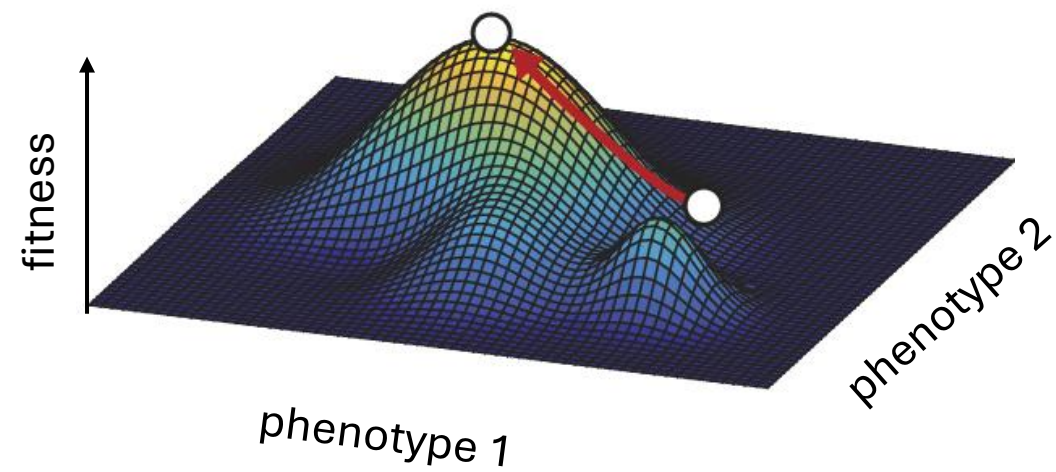




*Environmental changes can modify the «**fitness landscape**»*

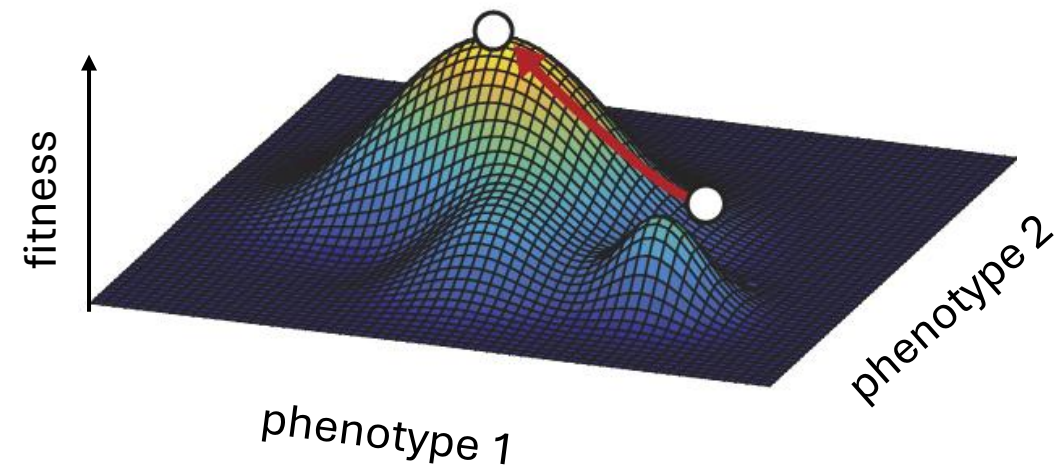
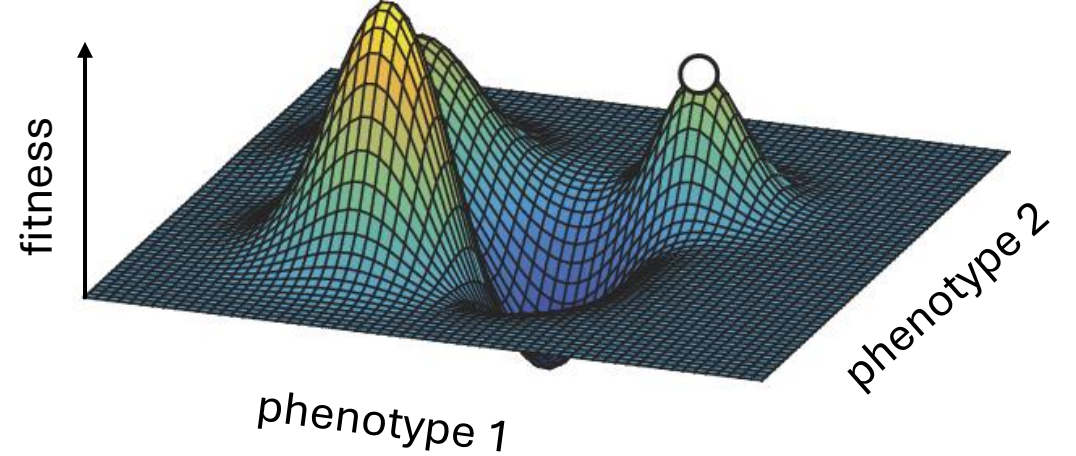
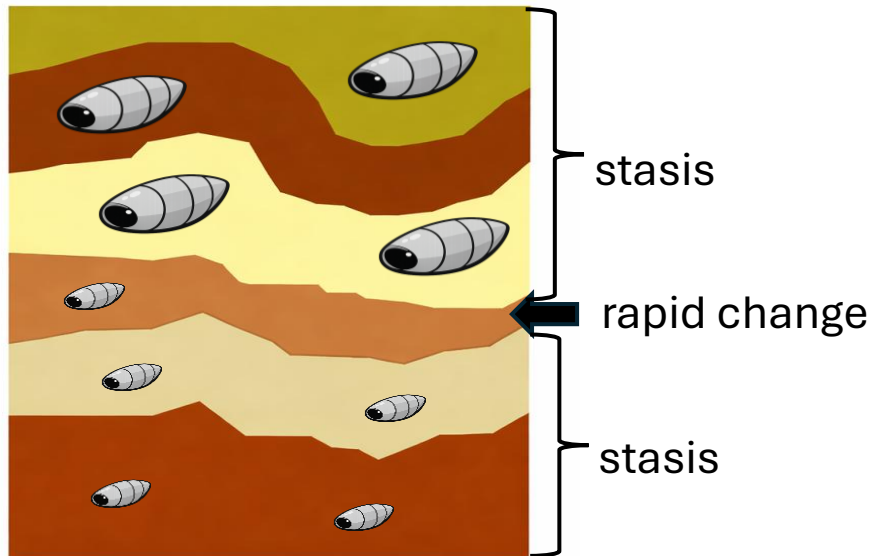


*Environmental  
change*



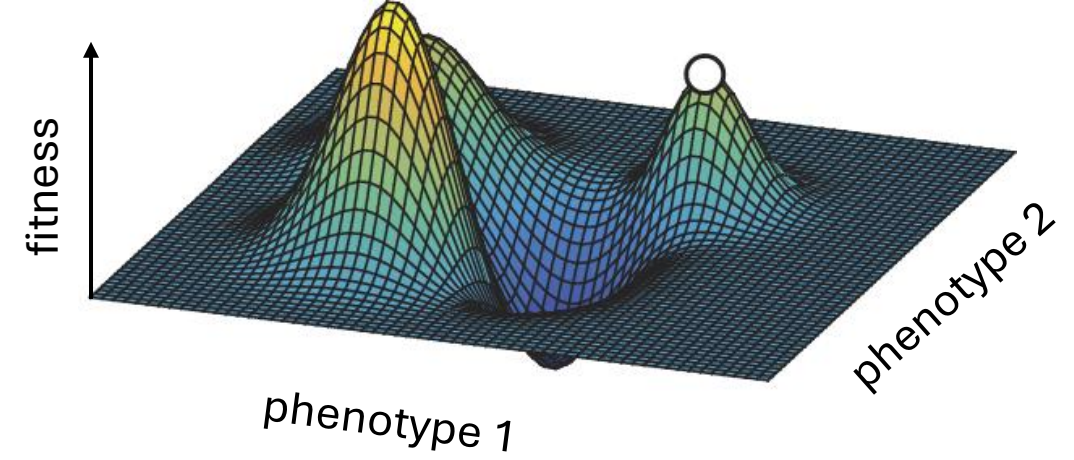
Environmental changes can modify the «**fitness landscape**»

### **Punctuated equilibria**

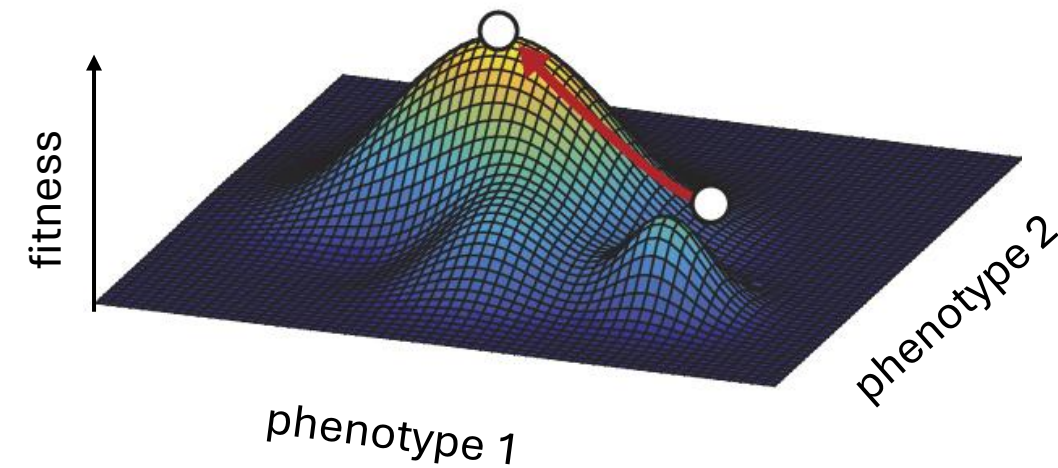


***Even evolution itself can modify the fitness landscape***

***Fitness can be «frequency-dependent»!***



*Changes in  
«allele»  
frequencies*

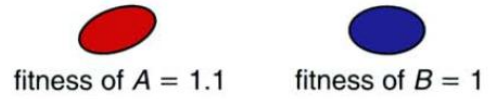




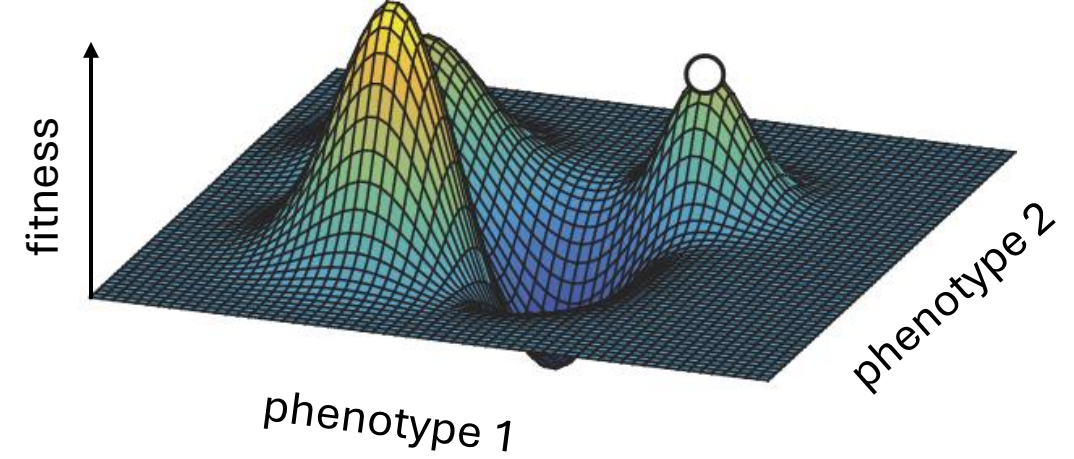
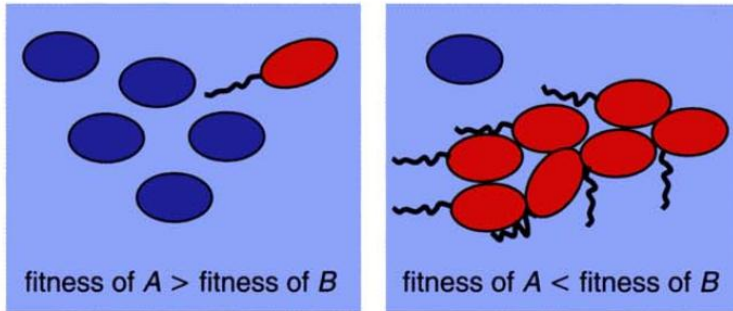
# Evolutionary Game Theory

The study of «frequency-dependent» fitness

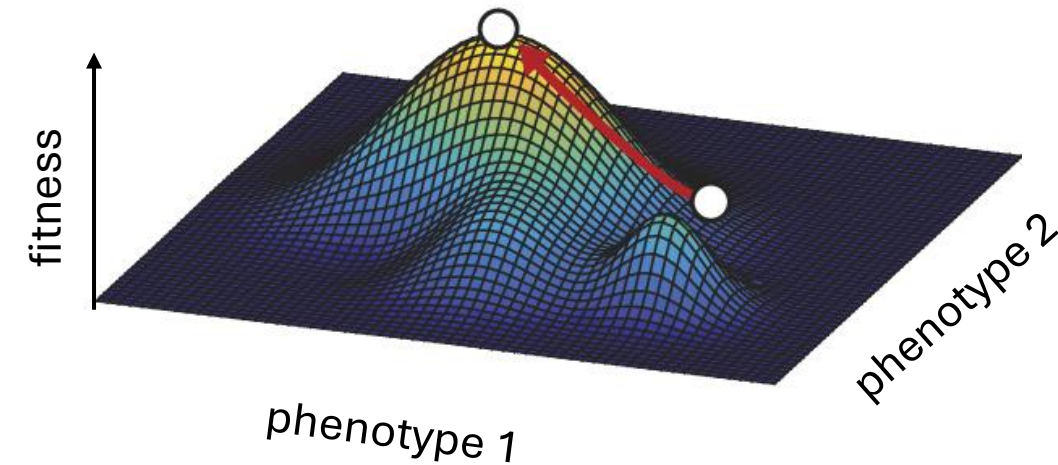
Constant selection:



Frequency-dependent selection:



Changes in  
«allele»  
frequencies



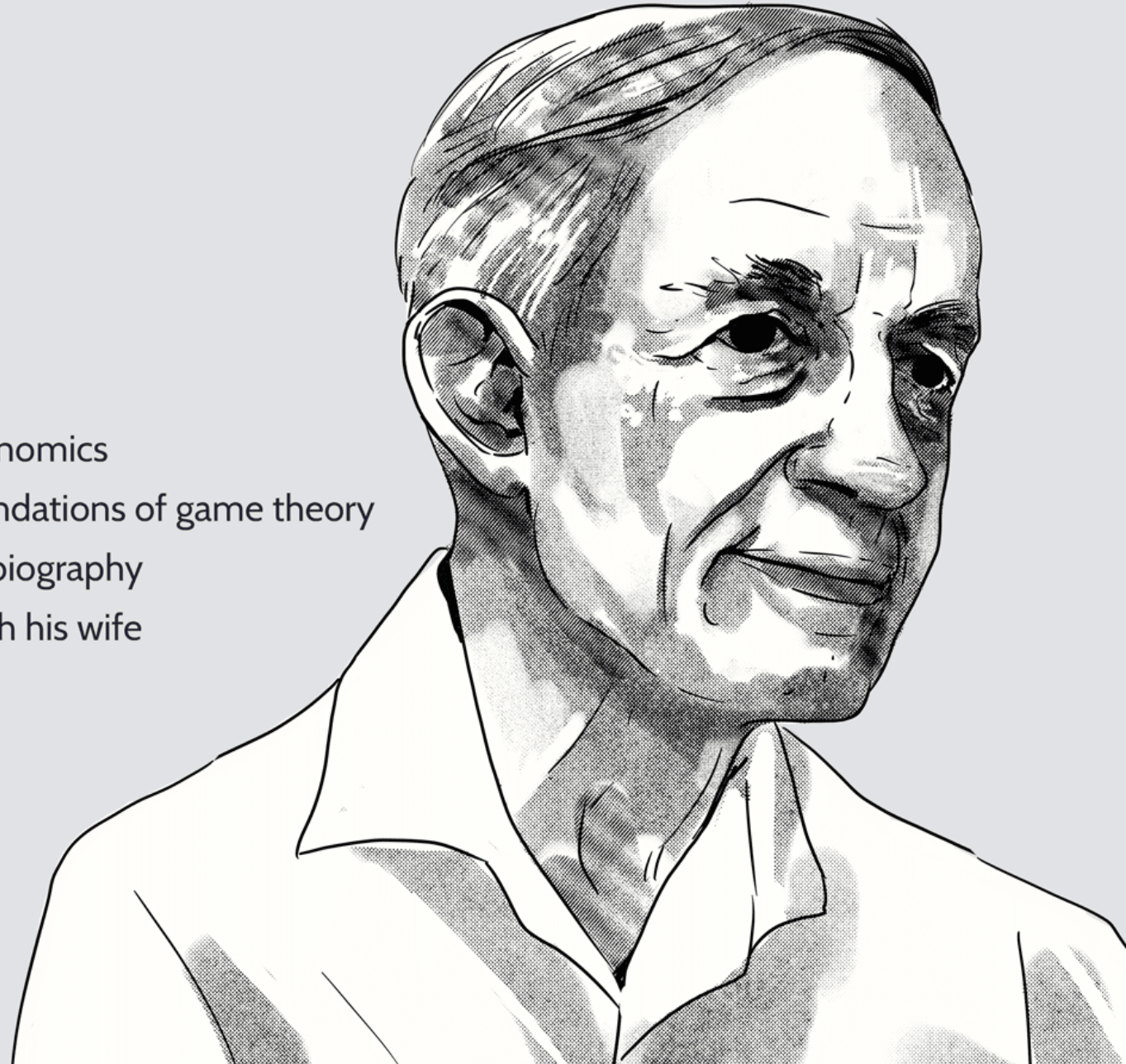
# John F. Nash, Jr.


Born: June 13, 1928

Died: May 23, 2015

## Mathematician

- 1994 Nobel Prize Recipient in Economics
- Developed the mathematical foundations of game theory
- *A Beautiful Mind*, a film based on a biography by Sylvia Nasar, chronicles life with his wife and struggles with mental illness





# A BEAUTIFUL MIND

HE SAW THE  
WORLD IN A WAY  
NO ONE COULD HAVE  
IMAGINED.

# Nash equilibrium

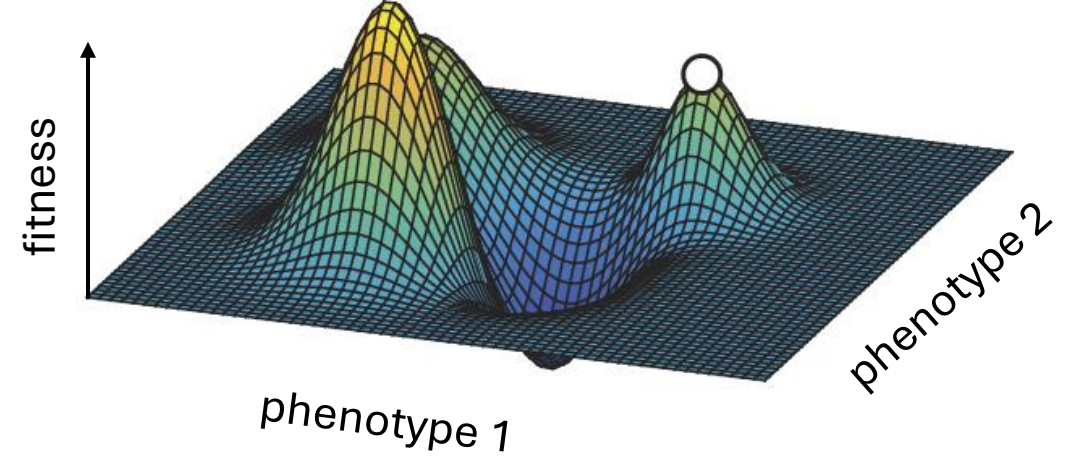
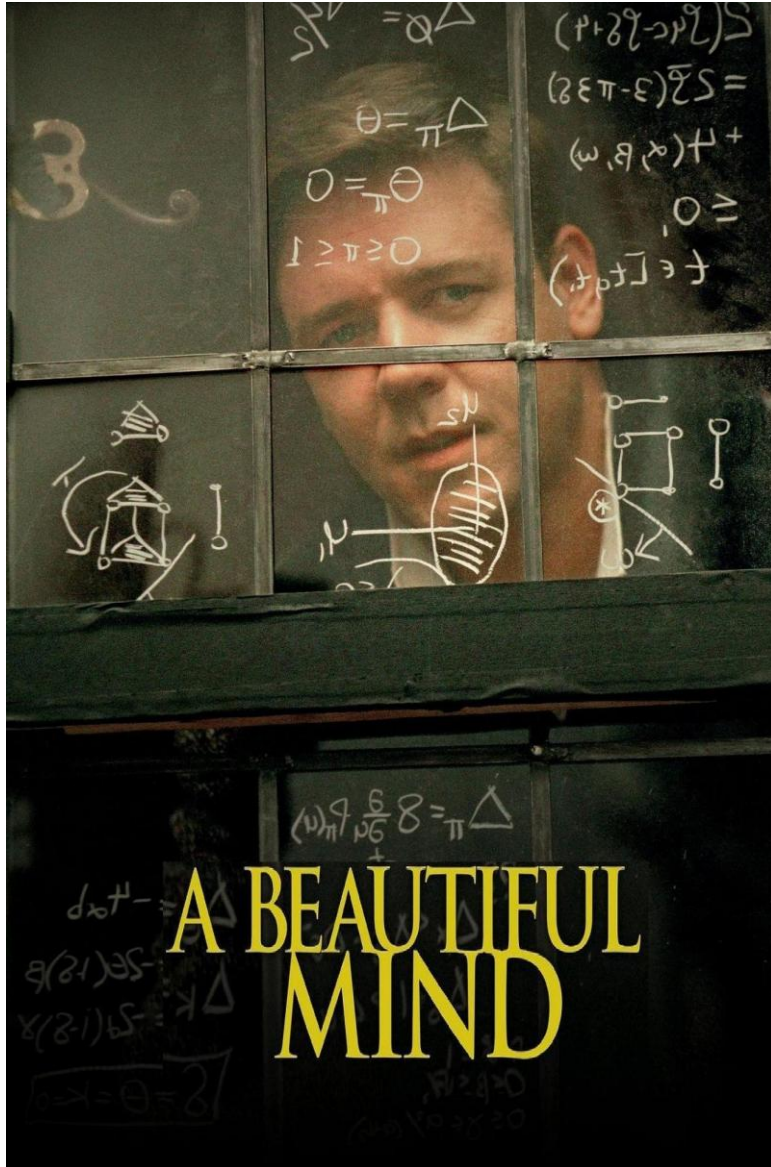
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- In many strategic situations, there exists a set of strategies where **no player can improve their outcome by changing strategy alone.**
- That point is called a **Nash equilibrium.**

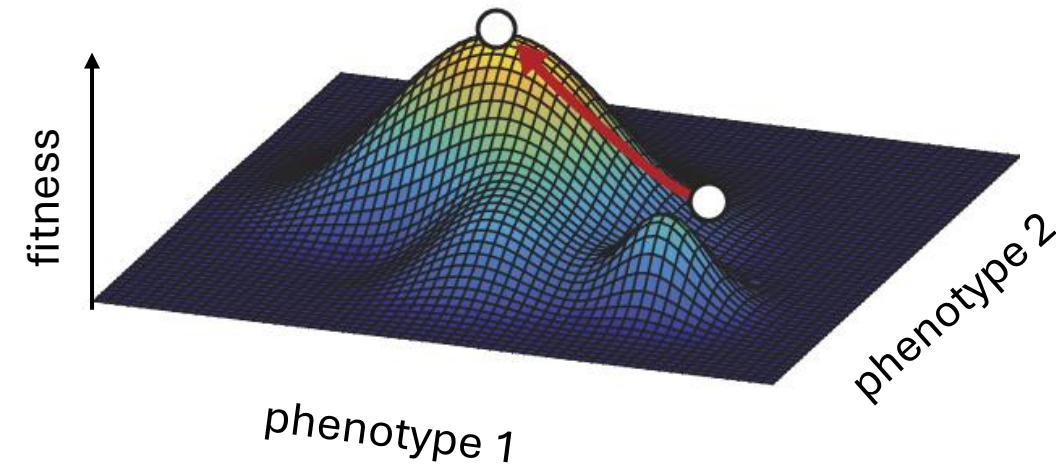


# Evolutionary Game Theory

The study of «frequency-dependent» fitness



Changes in  
«allele»  
frequencies

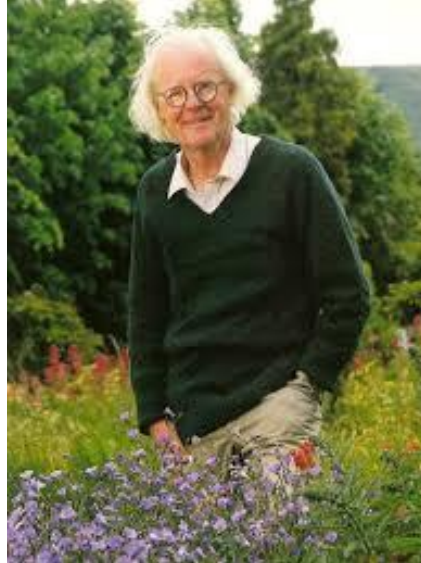


# Evolutionary Game Theory

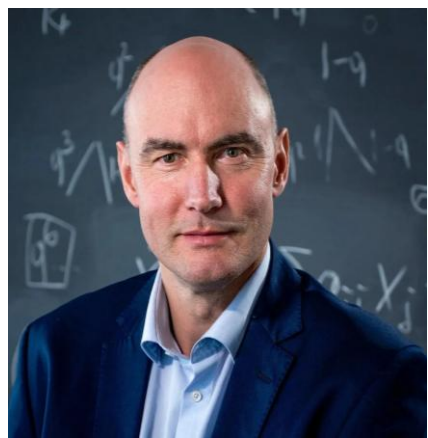
John Nash



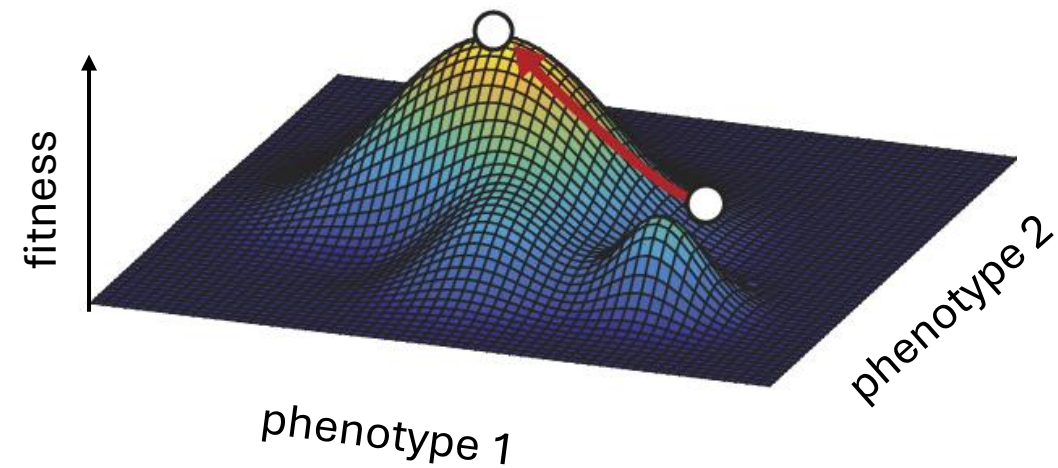
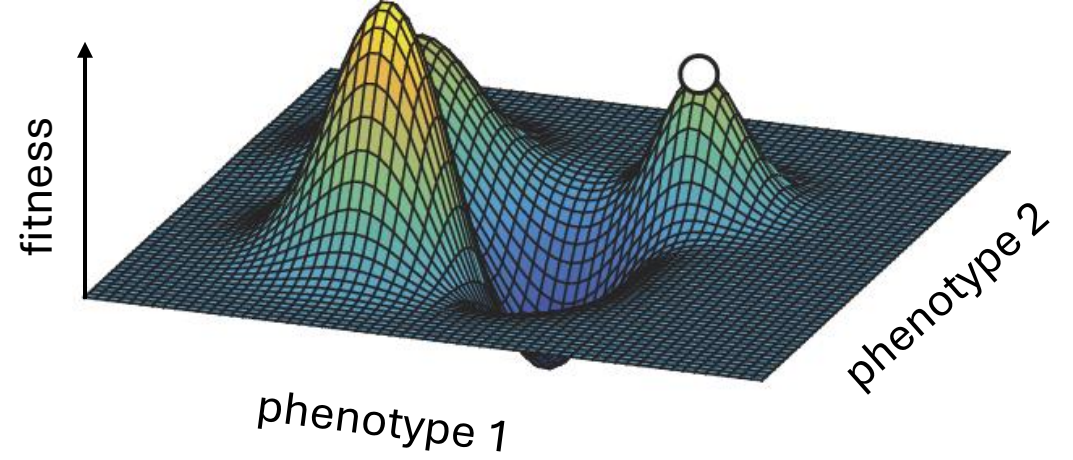
John Maynard-Smith



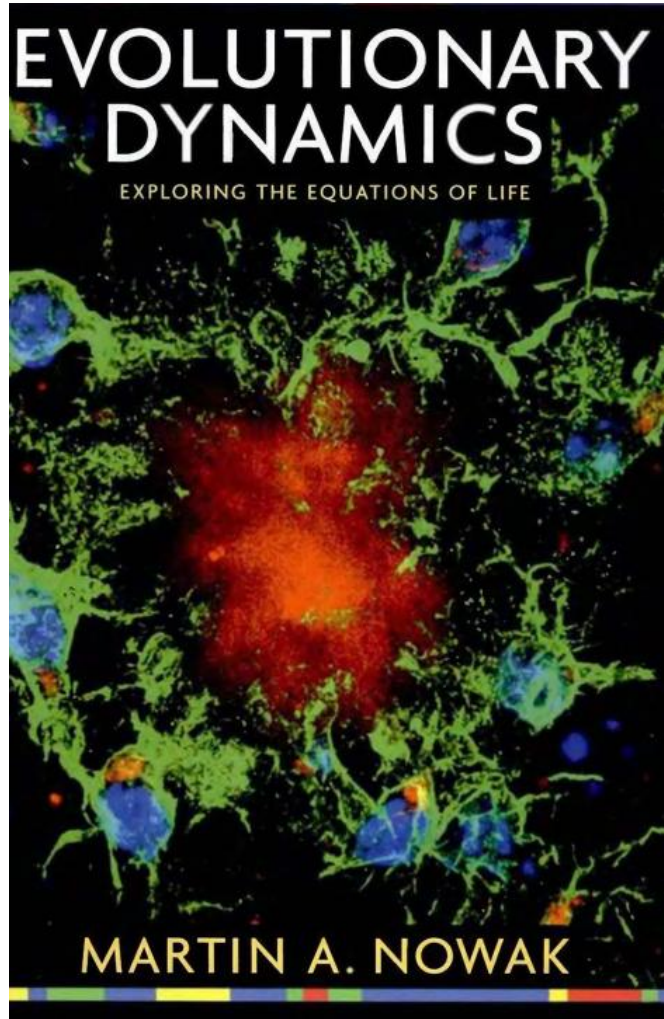
George Price



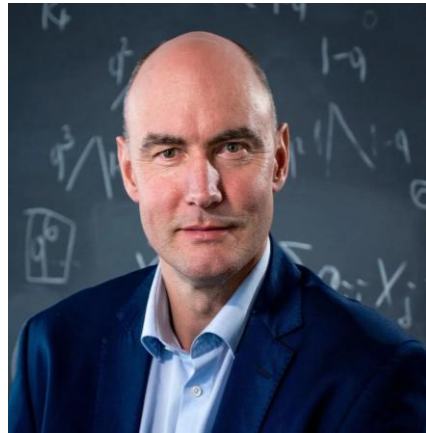
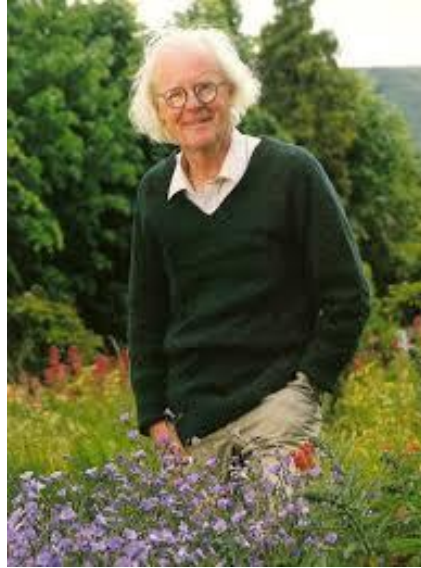
Martin Nowak



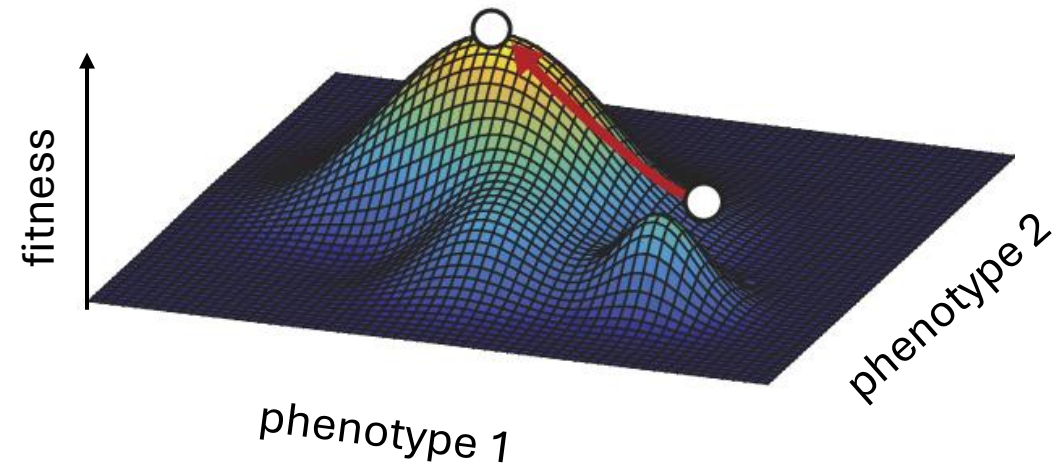
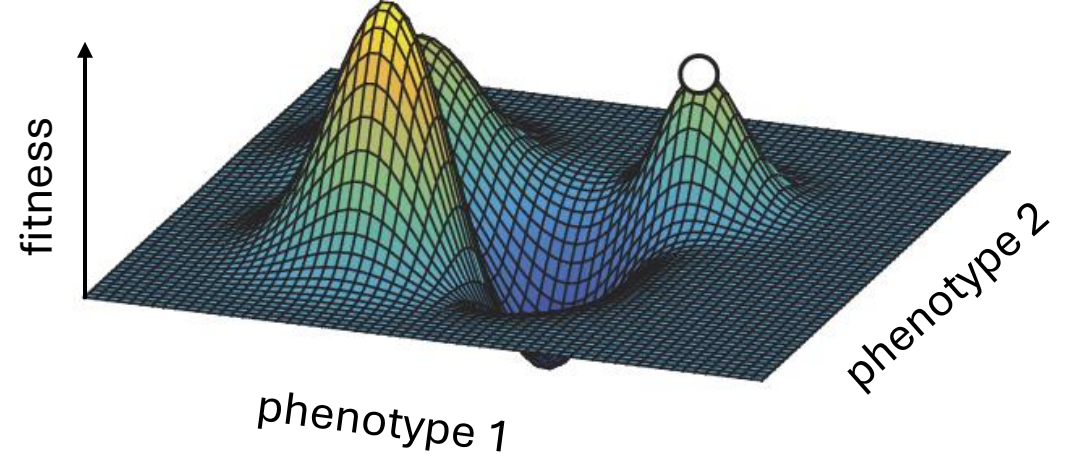
# Evolutionary Game Theory



John Maynard-Smith



Martin Nowak

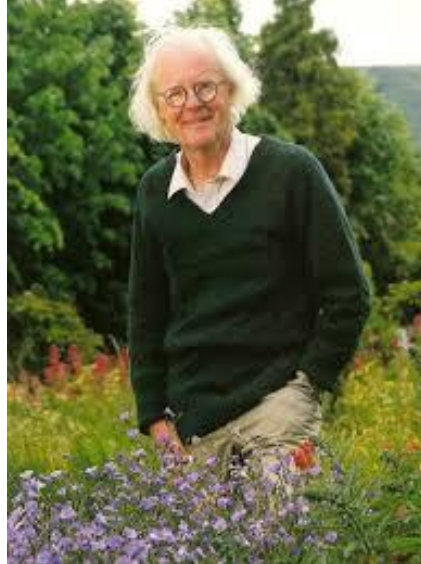


# The evolution of cooperation

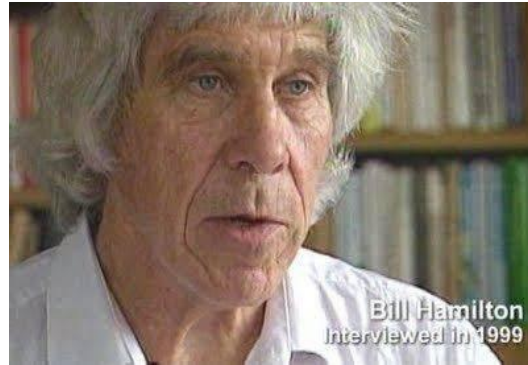
John Nash



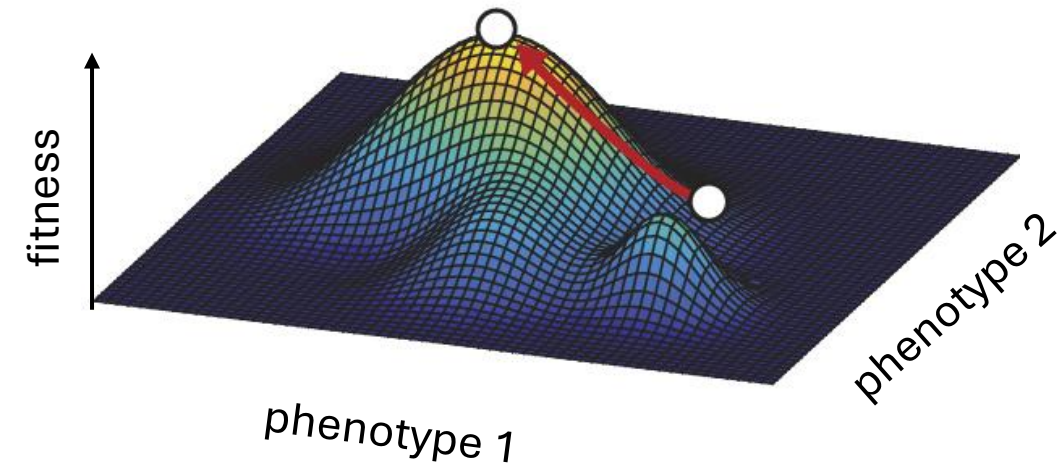
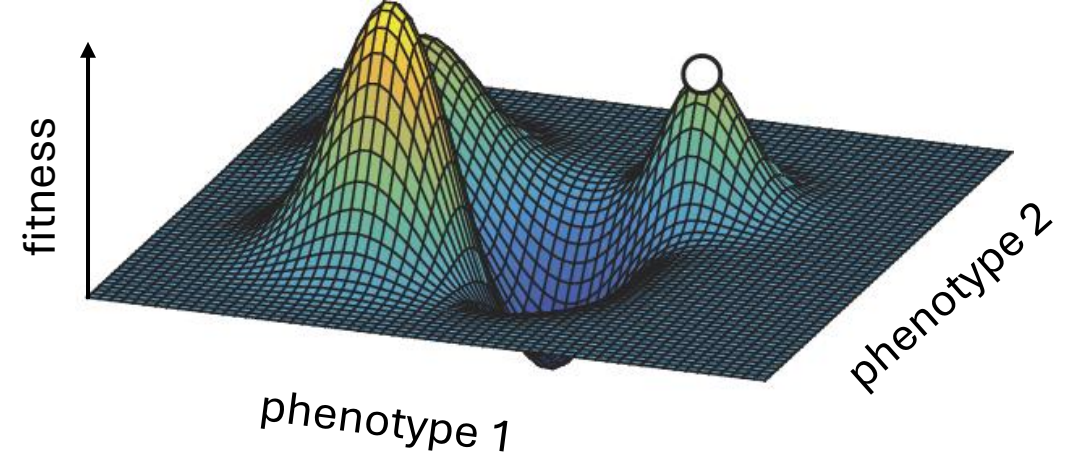
John Maynard-Smith



George Price



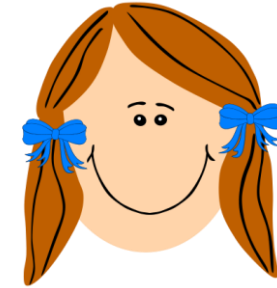
Bill Hamilton





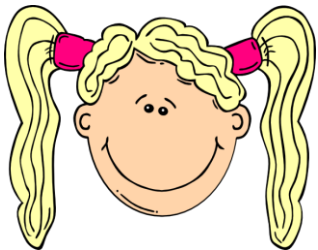
# What is a game? The payoff-matrix

Right player (Bibi)



Left  
player  
(Alice)

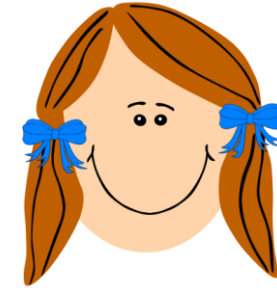
	Strategy A	Strategy B
Strategy A	5	1
Strategy B	3	2



Payoffs obtained by the left player  
(Alice) for each combination of strategies

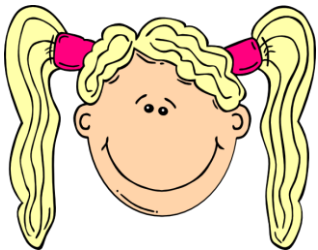
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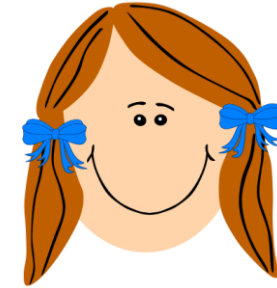
	Strategy A	Strategy B
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*What payoff do I get if I play A and Bibi plays B?*

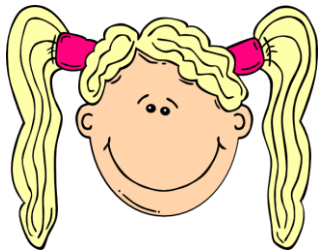
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Right player (Bibi)



Left  
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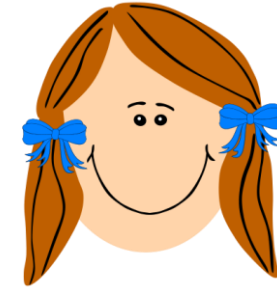
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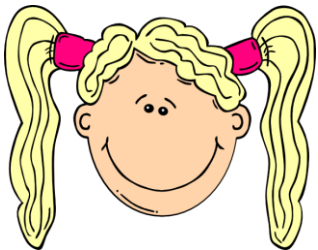
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Right player (Bibi)



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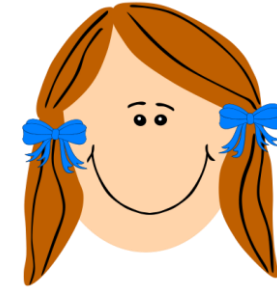
	Strategy A	Strategy B
Strategy A	5	1
Strategy B	3	2



*And if we both play B?*

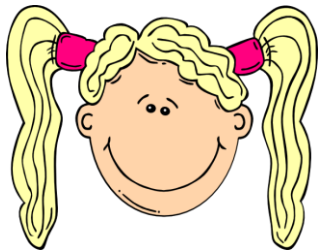
# What is a game? The payoff-matrix

Right player (Bibi)



Left  
player  
(Alice)

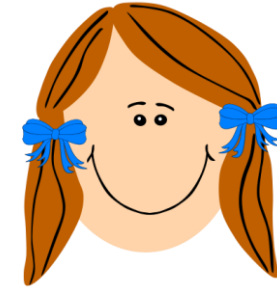
	Strategy A	Strategy B
Strategy A	5	1
Strategy B	3	2



*And if we both play B?*

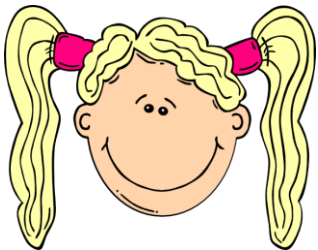
# What is a game? The payoff-matrix

Right player (Bibi)



Left  
player  
(Alice)

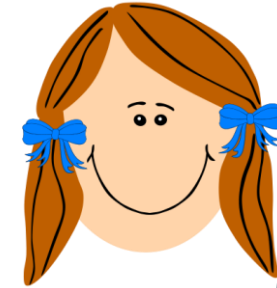
	Strategy A	Strategy B
Strategy A	5	1
Strategy B	3	2



*Last, and if I play B and she plays A?*

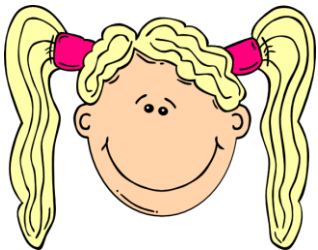
# What is a game? The payoff-matrix

Right player (Bibi)



Left  
player  
(Alice)

	Strategy A	Strategy B
Strategy A	5	1
Strategy B	3	2

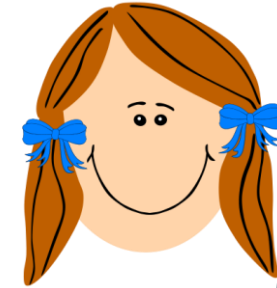


And what do I get in this case?

Last, and if I play B and she plays A?

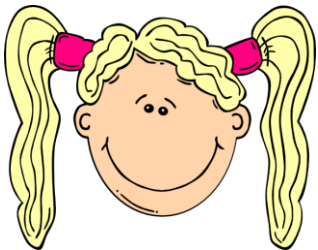
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Left  
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Strategy A	5	1
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And what do I get in this case?

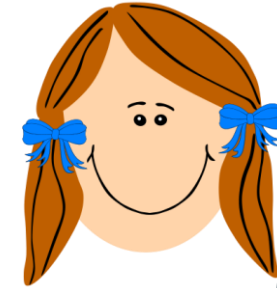
Last, and if I play B and she plays A?



# Sometimes you might see payoff matrices written like this too

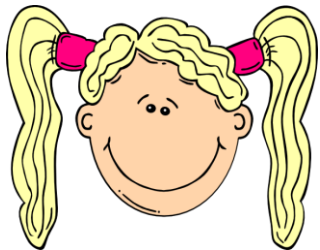
..perhaps it is clearer but redundant when players are symmetric

Right player (Bibi)



Left  
player  
(Alice)

	Strategy A	Strategy B
Strategy A	5,5	1,3
Strategy B	3,1	2,2



And what do I get in this case?

Last, and if I play B and she plays A?

# The evolution of aggressiveness








«The game of (red) deers» or  
«The game of testosterone»



	Fight	Back down
Fight	-1	4
Back down	0	2

# «The game of (red) deers» or «The game of testosterone»




	Fight	Back down
Fight	-1 risk 	4 
Back down	0 	2 

# «The game of (red) deers» or «The game of testosterone»

- Ten rounds of game.
- At each round each of you will choose a random partner
- Write down the payoffs!
- Winner gets special mention and chocolate. Loser does not reproduce.



	Fight	Back down
Fight	-1 risk 	4 
Back down	0 	2 

# Results of the «testosterone game»

- How much did you score?





# Results of the «testosterone game»

- How much did you score?
- What strategy did you use in the first round?

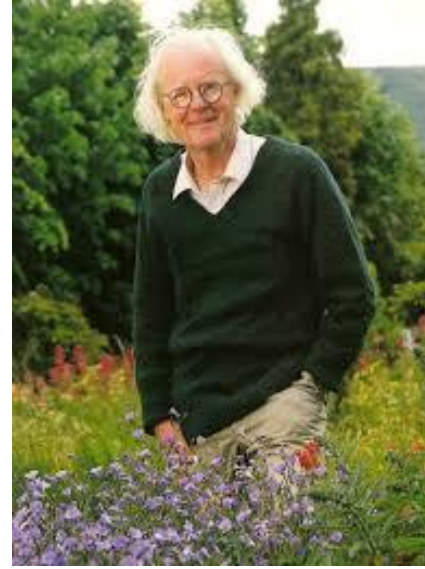


# Results of the «testosterone game»

- How much did you score?
- What strategy did you use in the first round?
- What strategy did you use at the end?



The game we just played is famous as «Hawks vs Doves»



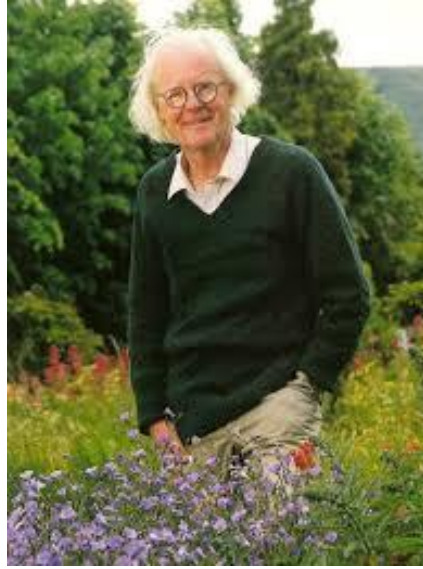
John Maynard-Smith  
(1920-2004)



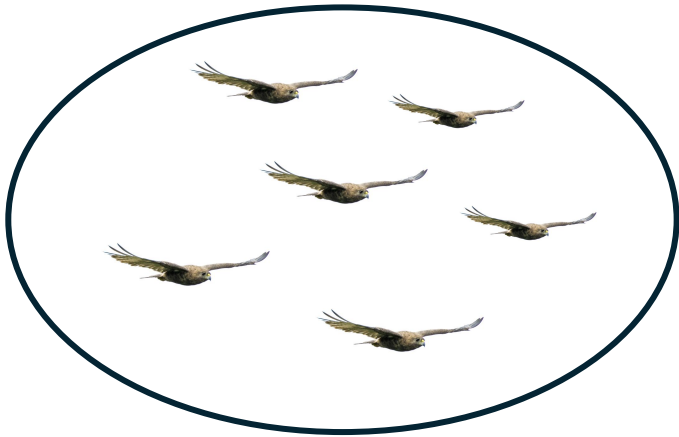
	Hawk	Dove
Hawk	-1	4
Dove	0	2



# What happens if everybody fights/is a Hawk?



John Maynard-Smith  
(1920-2004)



If we play



then we get: ?

If we play

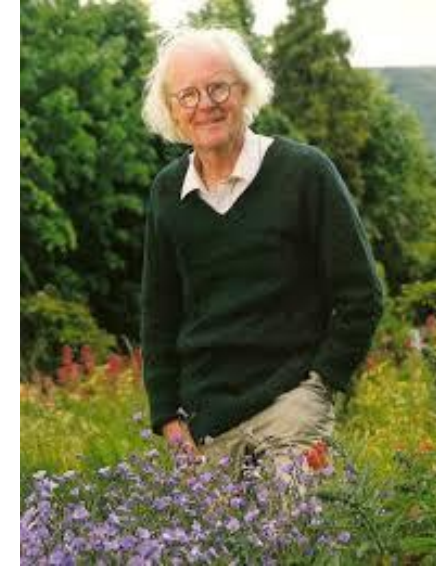


then we get: ?

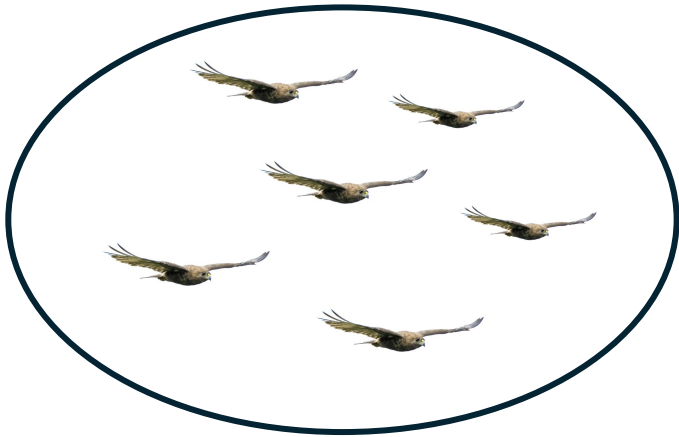


	Hawk	Dove
Hawk	-1	4
Dove	0	2

# What happens if everybody fights/is a Hawk?



John Maynard-Smith  
(1920-2004)



If we play  then we get: **-1**

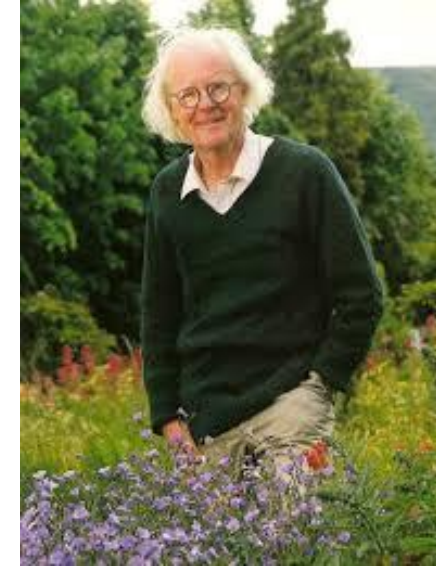
If we play  then we get: **0**

Better being a **dove!**

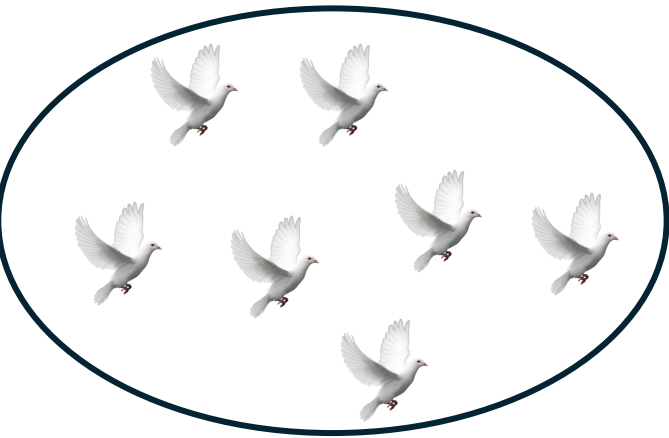


	Hawk	Dove
Hawk	-1	4
Dove	0	2

# What happens if everybody is a dove?



John Maynard-Smith  
(1920-2004)



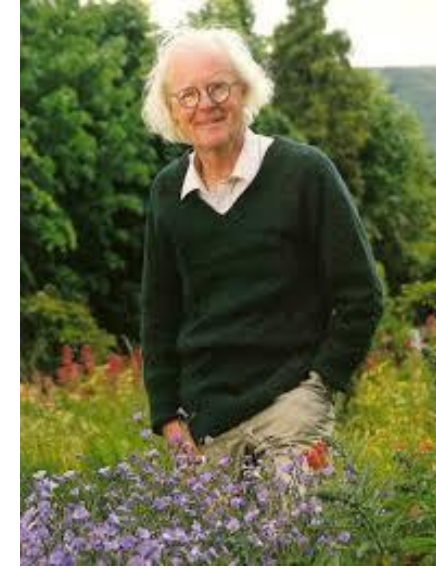
If we play  then we get: ?

If we play  then we get: ?

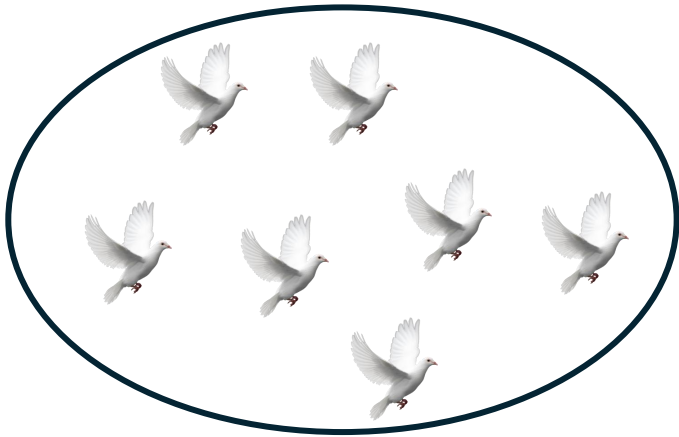


	Hawk	Dove
Hawk	-1	4
Dove	0	2

# What happens if everybody is a dove?



John Maynard-Smith  
(1920-2004)



If we play  then we get: **4**

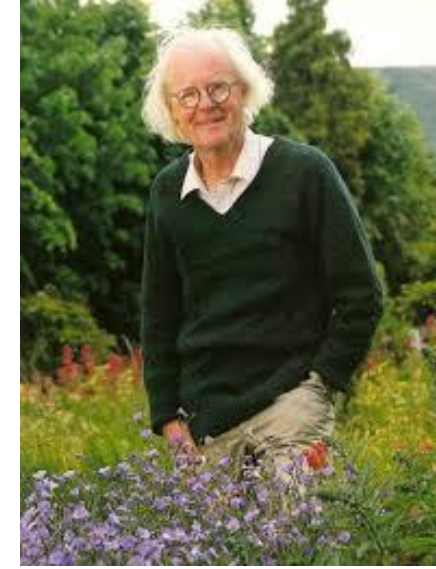
If we play  then we get: **2**

Better being a **hawk!**

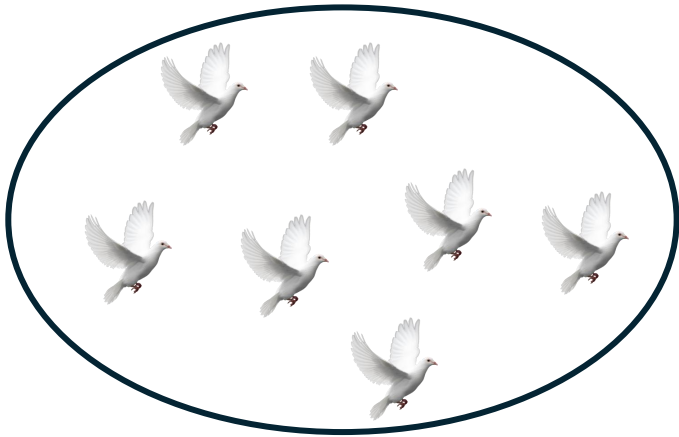


	Hawk	Dove
Hawk	-1	4
Dove	0	2

# What happens if everybody is a dove?



John Maynard-Smith  
(1920-2004)



If we play  then we get: **4**

If we play  then we get: **2**

Better being a **hawk!**

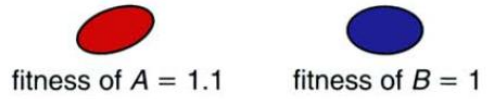


	Hawk	Dove
Hawk	-1	4
Dove	0	2

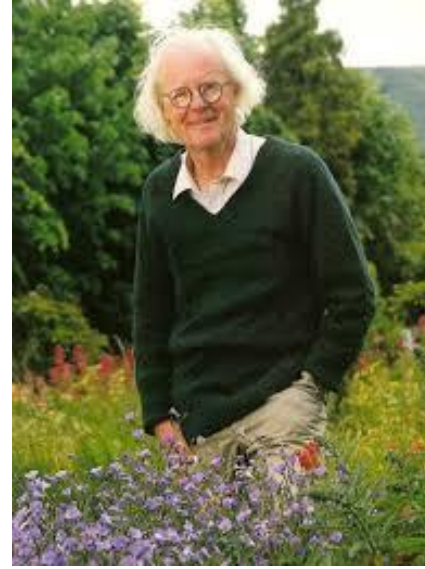
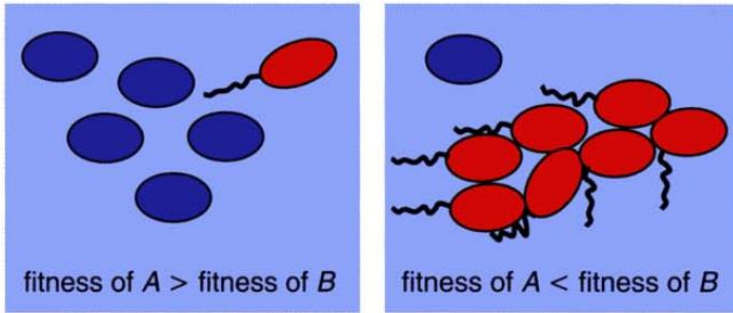


# It is not always best to be aggressive or shy! It depends on who is around!

Constant selection:



Frequency-dependent selection:

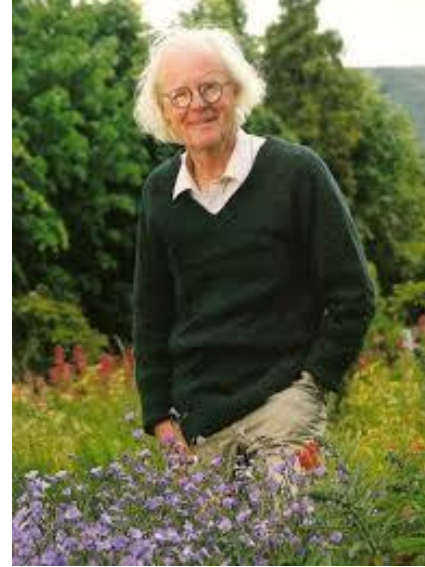


John Maynard-Smith  
(1920-2004)

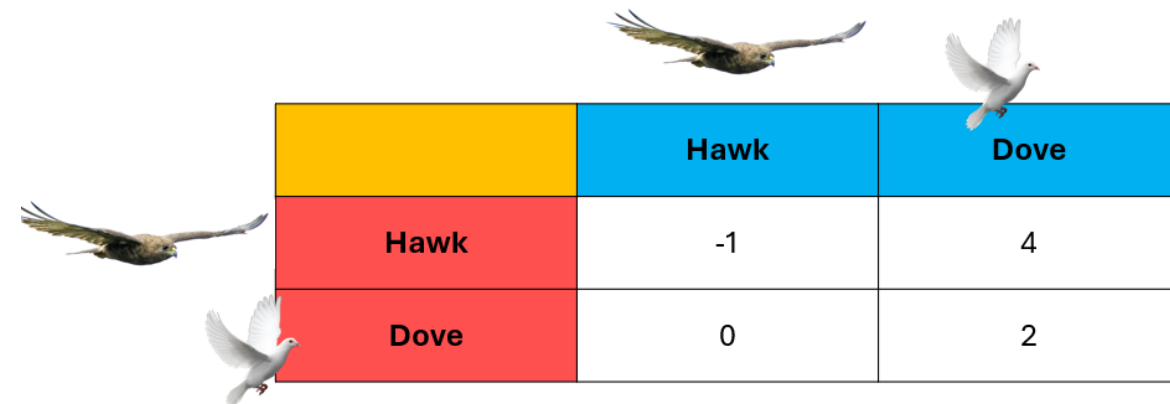


	Hawk	Dove
Hawk	-1	4
Dove	0	2

So what strategy should one choose?

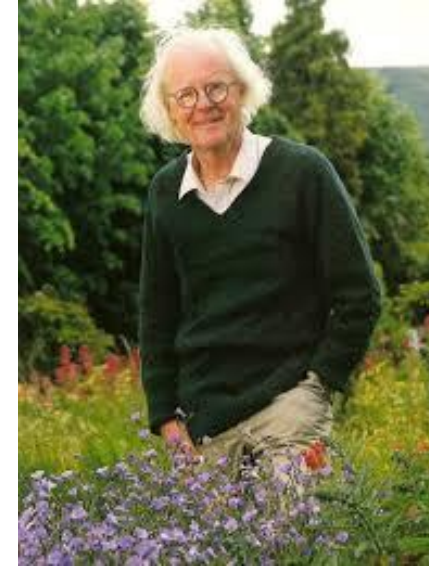
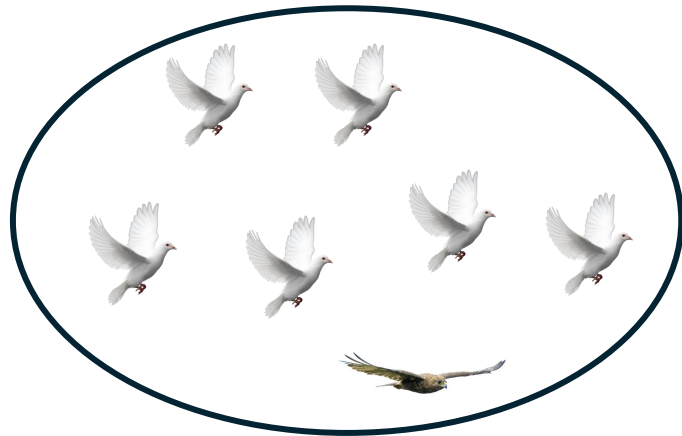


John Maynard-Smith  
(1920-2004)

Illustrations of a hawk and a dove are placed around the payoff matrix table. A hawk is shown above the Hawk column, a dove above the Dove column, a hawk to the left of the Hawk row, and a dove to the left of the Dove row.



	Hawk	Dove
Hawk	-1	4
Dove	0	2

# Mixed strategies







John Maynard-Smith  
(1920-2004)

Since a «Hawk» mutant will always spread/invade a population of Doves and a «Dove» mutant will always spread/invade a population of Hawks let's explore «**Mixed strategies**» in which an individual play Hawk with a probability  $p$  (Dove with probability  $1-p$ )

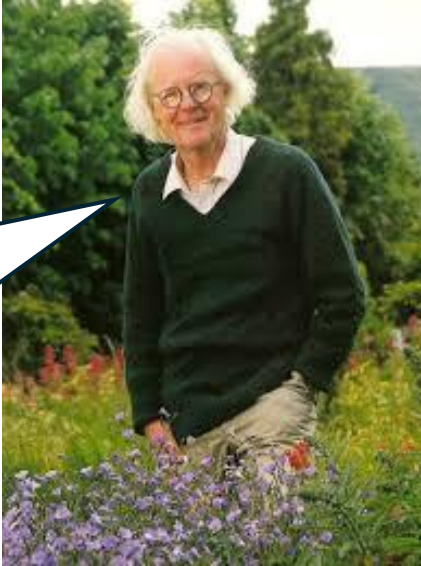
Pure strategies: always  or 

Mixed strategies:  $p$   and  $1-p$  

		
	Hawk	Dove
	-1	4
	0	2

# Mixed strategies



What other famous game do you know where mixed strategies are convenient?





John Maynard-Smith  
(1920-2004)

Pure strategies: always  or 

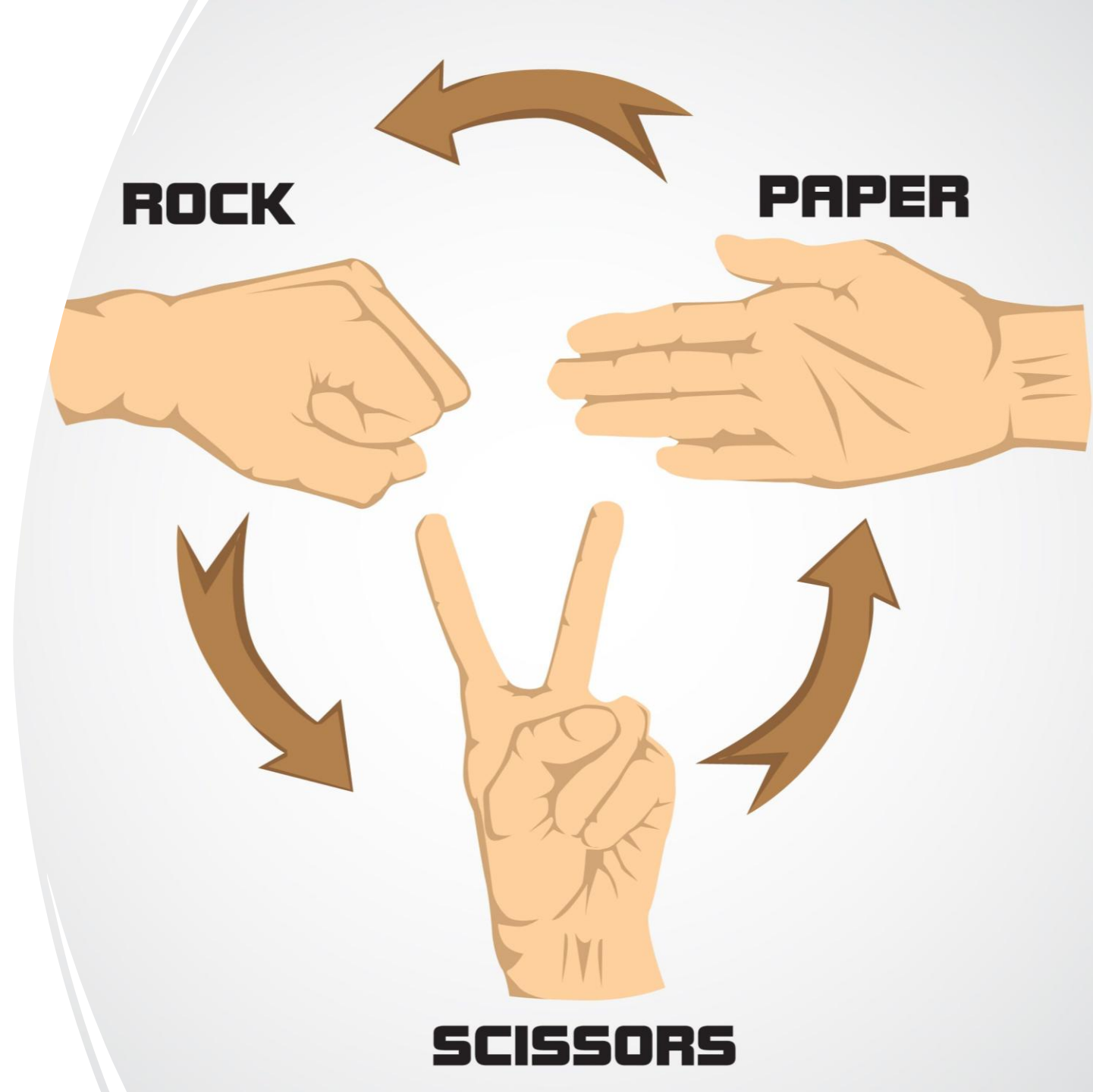
Mixed strategies:  $p$   and  $1-p$  



	Hawk	Dove
Hawk	-1	4
Dove	0	2

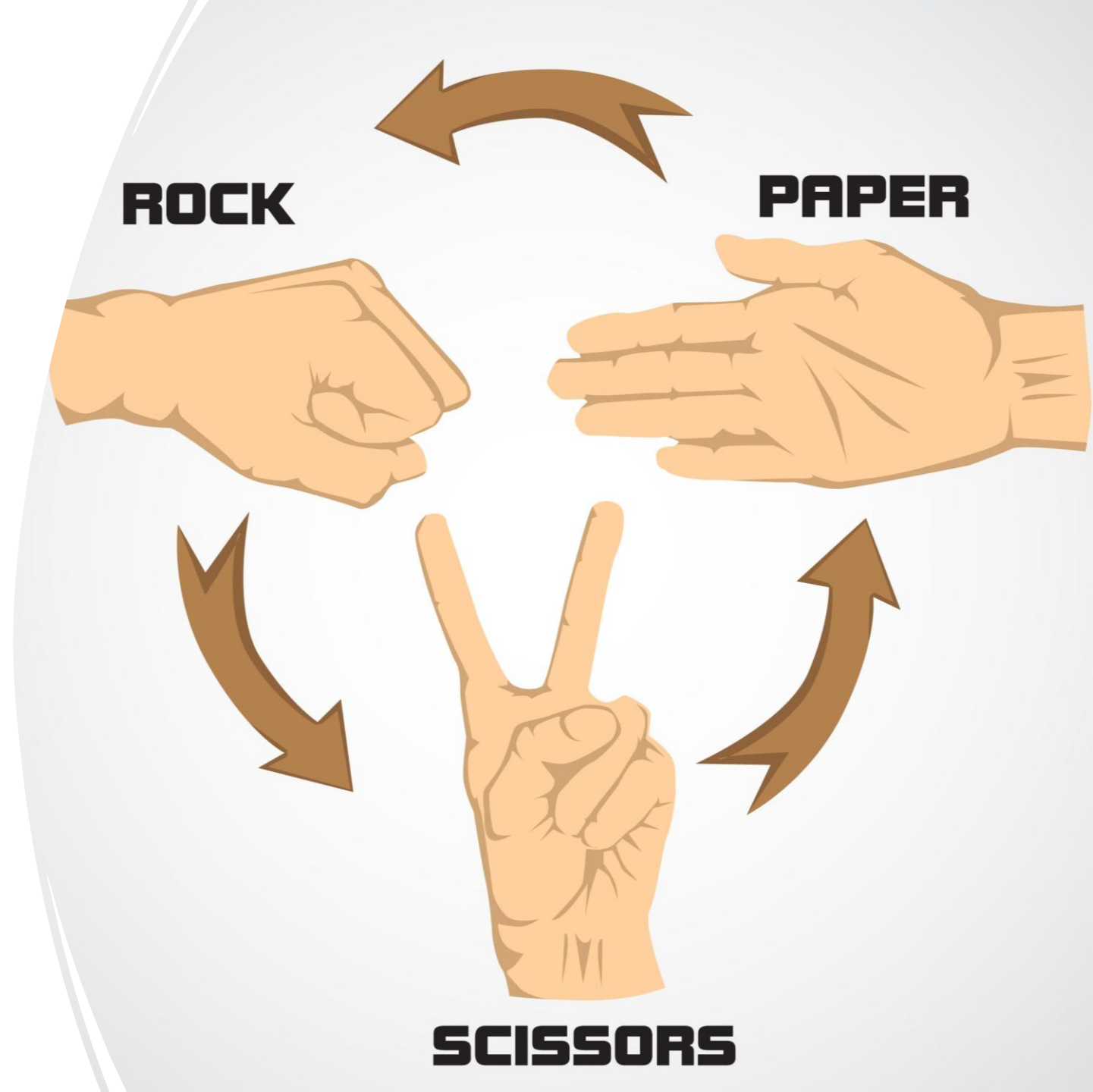


What is the  
payoff matrix?



# What is the payoff matrix?

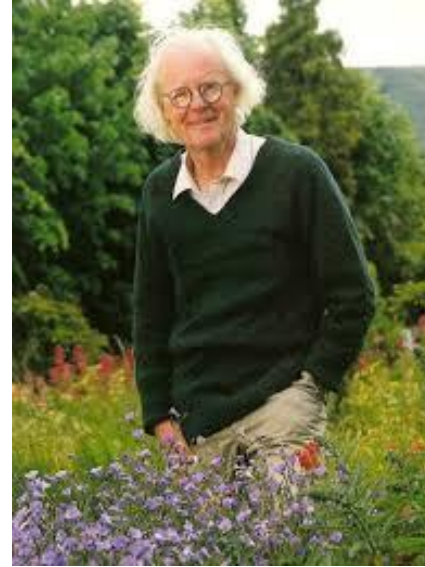
Gains of Player 1		Player 2 chooses...		
				
Player 1 chooses...		0	-1	1
		1	0	-1
		-1	1	0



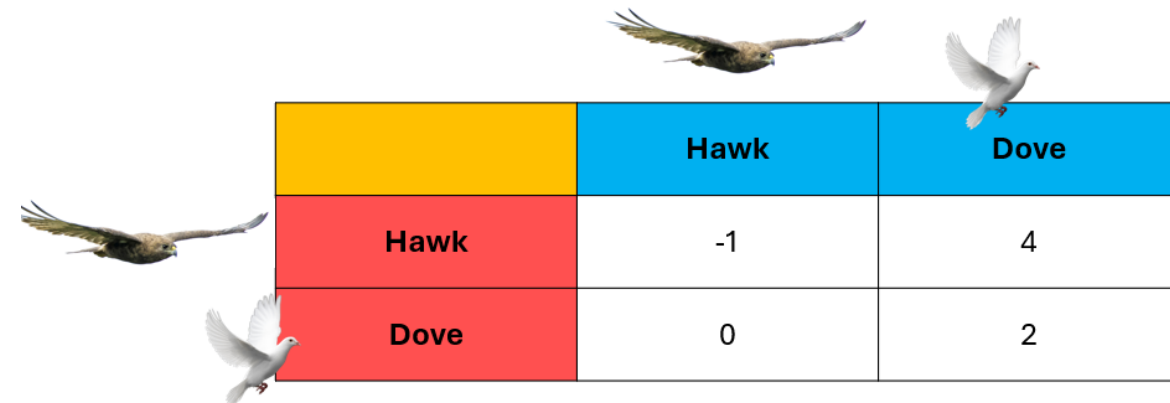
Let's look at how the fitness ( $W$ ) of Hawks and Doves change with the frequencies of Hawks ( $p$ )

$W_H = ?$

$W_D = ?$



John Maynard-Smith  
(1920-2004)

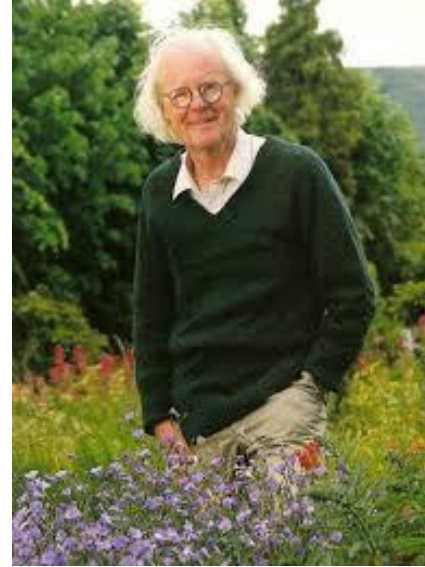
Illustrations of a hawk and a dove are placed around the payoff matrix table. A hawk is shown above the Hawk row, a dove above the Dove row, a hawk to the left of the Hawk row, and a dove to the left of the Dove row.

	Hawk	Dove
Hawk	-1	4
Dove	0	2

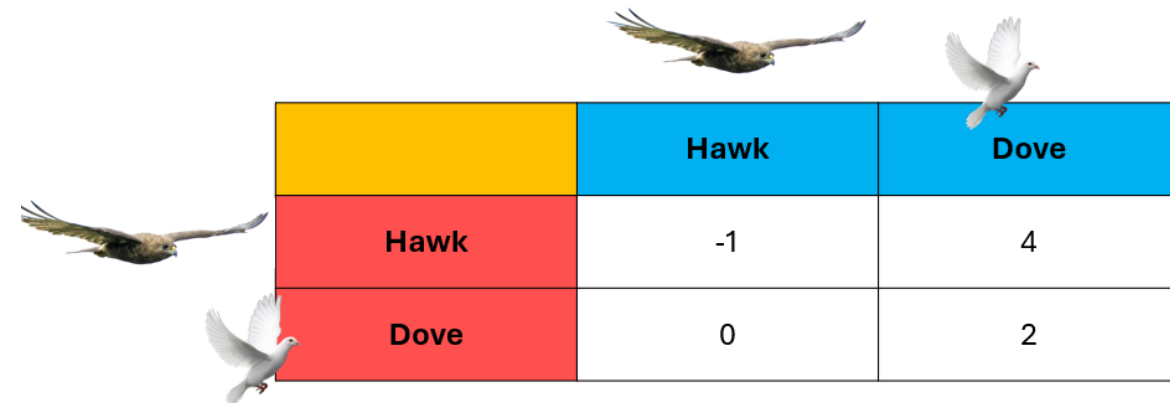
Let's look at how the fitness ( $W$ ) of Hawks and Doves change with the frequencies of Hawks ( $p$ )

$$W_H = (-1) * p + 4 * (1-p) = 4 - 5p$$

$$W_D = 0 * p + 2 * (1-p) = 2 - 2p$$



John Maynard-Smith  
(1920-2004)

Illustrations of a hawk and a dove are placed around the payoff matrix table. A hawk is shown above the Hawk column, a dove above the Dove column, a hawk to the left of the Hawk row, and a dove to the left of the Dove row.

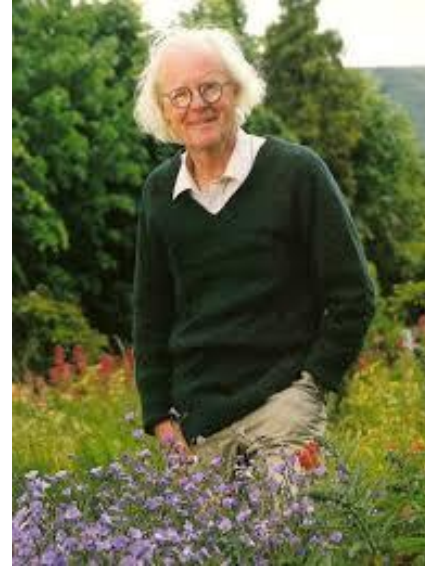
	Hawk	Dove
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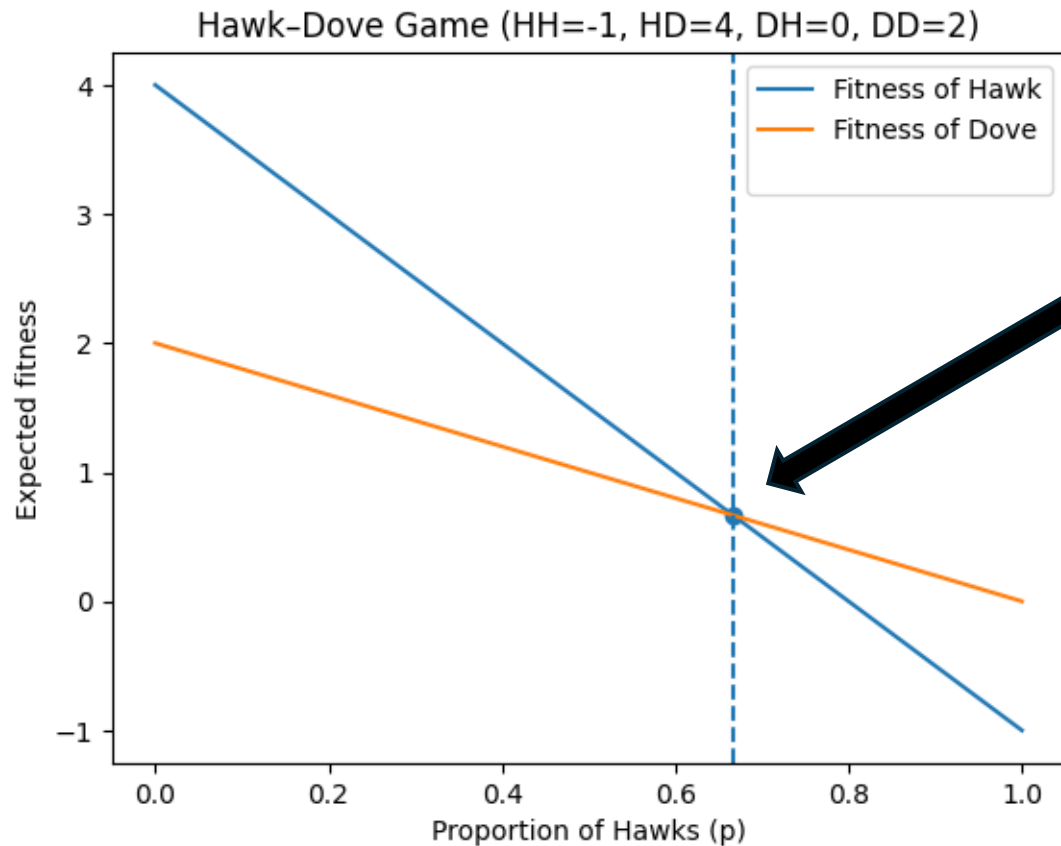
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John Maynard-Smith  
(1920-2004)



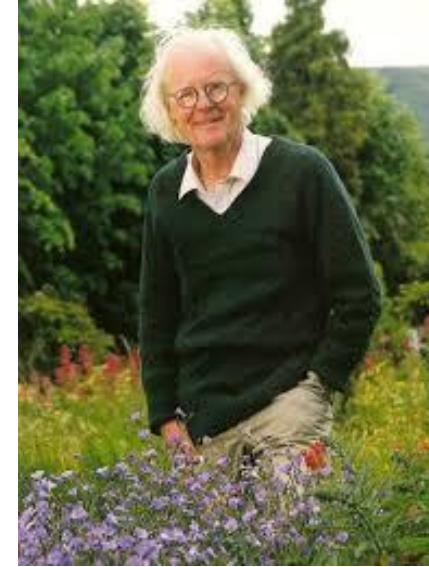
What is this point?

		
	<b>Hawk</b>	<b>Dove</b>
<b>Hawk</b>	-1	4
<b>Dove</b>	0	2

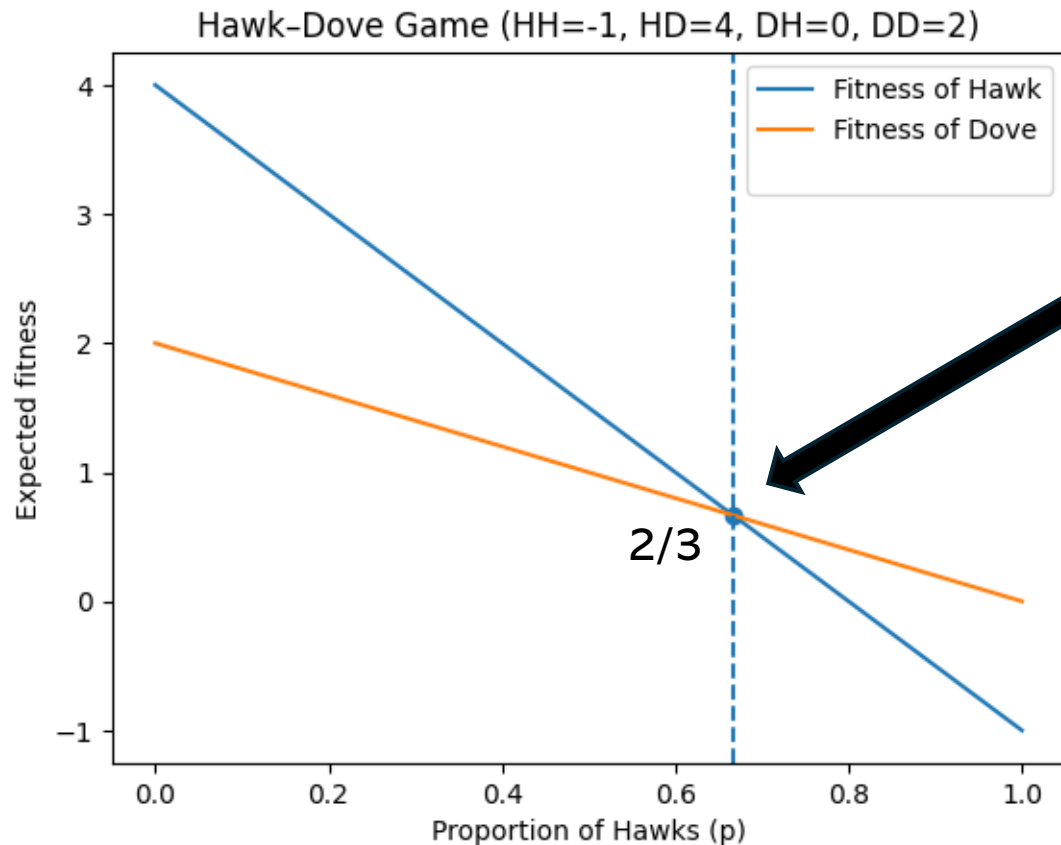
# Let's look at how the fitness ( $W$ ) of Hawks and Doves change with the frequencies of Hawks ( $p$ )

$$W_H = (-1) \cdot p + 4 \cdot (1-p) = 4 - 5p$$

$$W_D = 0 \cdot p + 2 \cdot (1-p) = 2 - 2p$$



John Maynard-Smith  
(1920-2004)







What is this point?

$$W_H = W_D$$

$$4 - 5p = 2 - 2p$$

$$p^* = 2/3$$

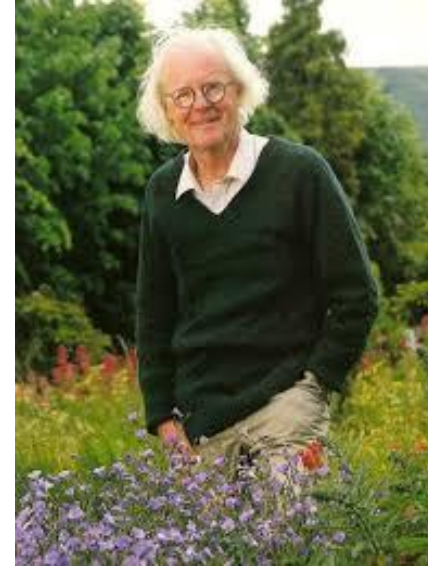
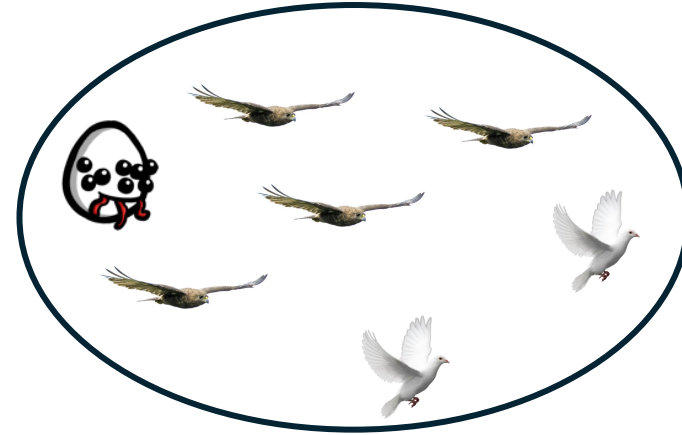
		
	-1	4
	0	2

An **Evolutionarily Stable Strategy (ESS)** is a strategy that, if adopted by almost all individuals, cannot be invaded by a rare alternative strategy.





Let's think in our case of a **rare mutant** playing  $q$  times Hawk in a population with 2/3 Hawks and 1/3 Doves.

The **fitness of the mutant** will be

$$E(q,p) = q * W_H + (1-q) * W_D$$



John Maynard-Smith  
(1920-2004)

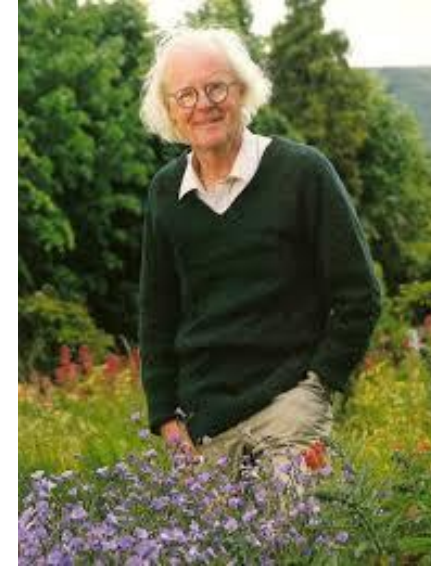
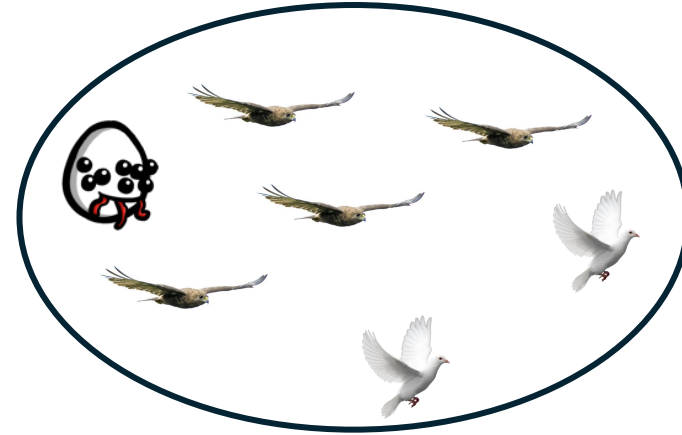
		
	Hawk	Dove
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John Maynard-Smith  
(1920-2004)

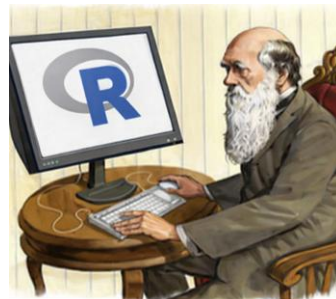
$$p < 2/3$$

$$W_H < p * (-1) + (1-p) * 4$$

$$W_D < p * 0 + (1-p) * 2$$

$$W_q < q * W_H + (1-q) * W_D$$

#fitness Hawks  
#fitness Doves  
#fitness mutant



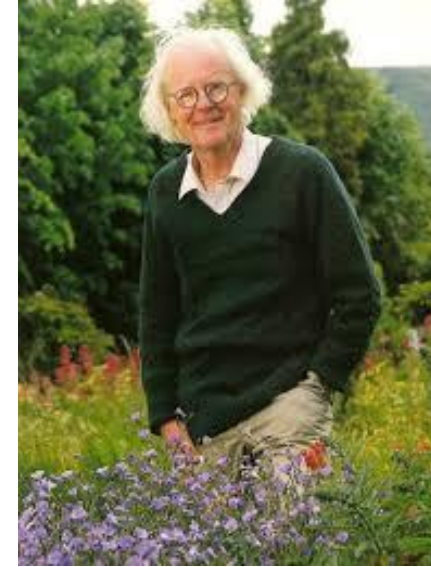
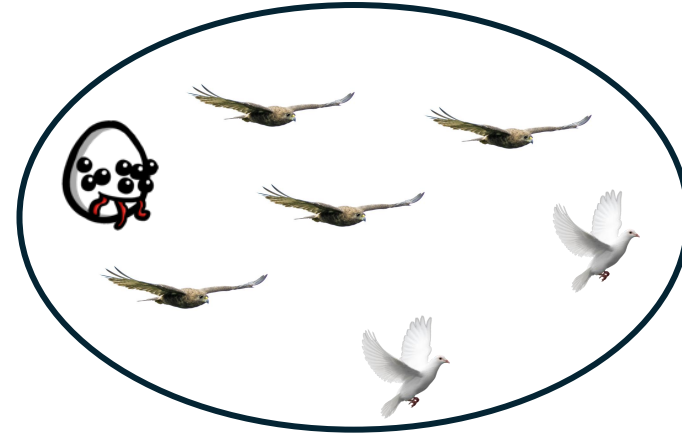
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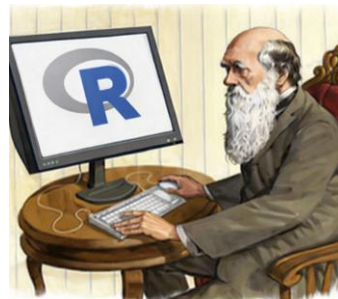
The **fitness of the mutant** will be

$$E(q,p) = q \cdot W_H + (1-q) \cdot W_D$$



John Maynard-Smith  
(1920-2004)

```
p <- 2/3
W_H <- p*(-1) + (1-p)*4 #fitness Hawks
W_D <- p*0 + (1-p)*2 #fitness Doves
W_q <- q*W_H + (1-q)*W_D #fitness mutant
plot(q, W_q, type="l", lwd=2,
      xlab="Probability mutant plays Hawk (q)",
      ylab="Expected fitness",
      main="Fitness of mutant strategy vs ESS
population")
abline(h=W_H, lty=2)
```



	Hawk	Dove
Hawk	-1	4
Dove	0	2

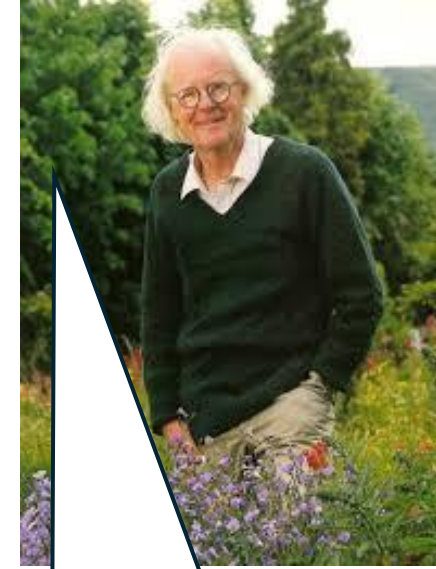
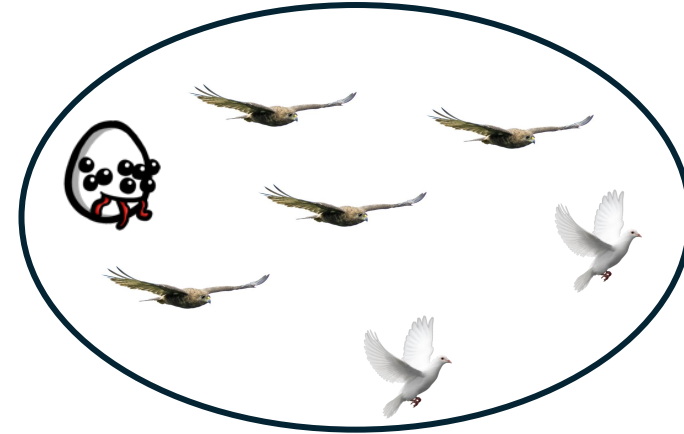
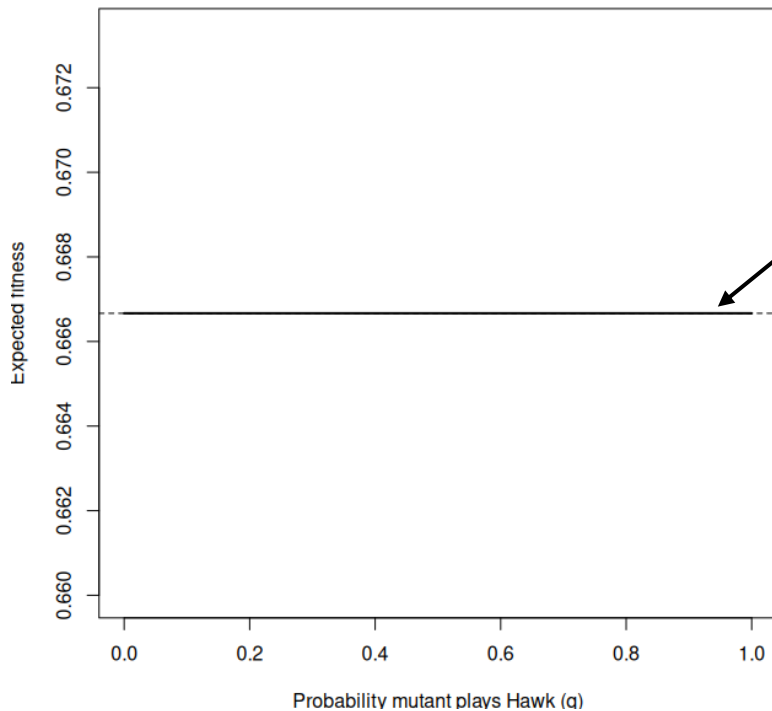
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$$E(q,p) = q \cdot W_H + (1-q) \cdot W_D$$

Fitness of mutant strategy vs ESS population



John Maynard-Smith (2004)

First condition for an ESS:  
 $E(p^*, p^*) \geq E(q, p^*)$

	Hawk	Dove
Hawk	-1	4
Dove	0	2

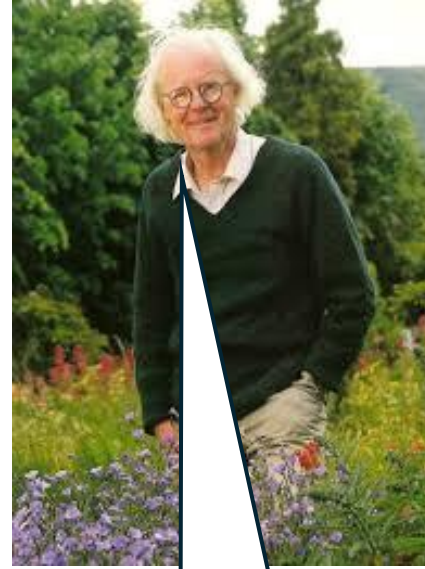
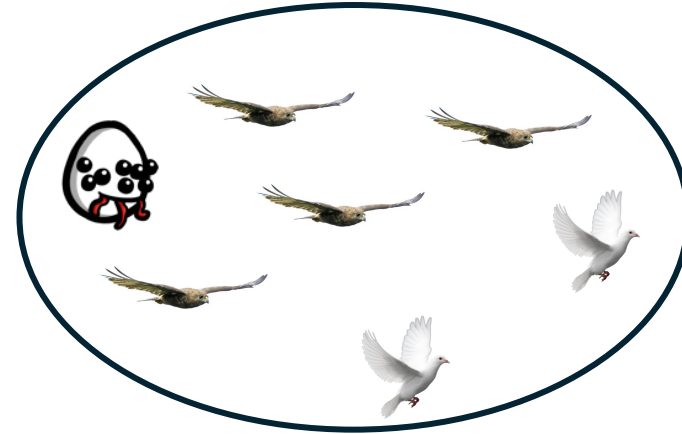
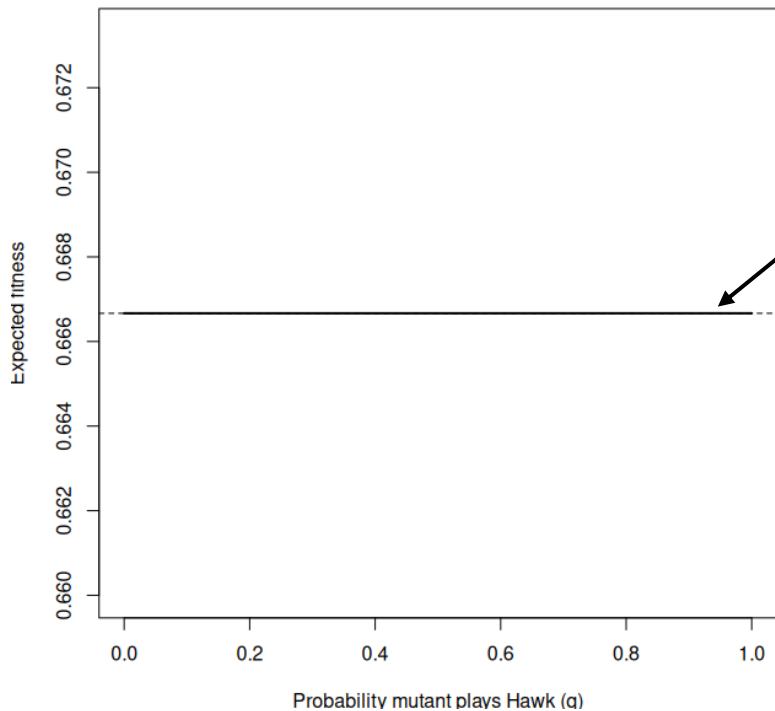
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John Maynard Smith (1920-2004)

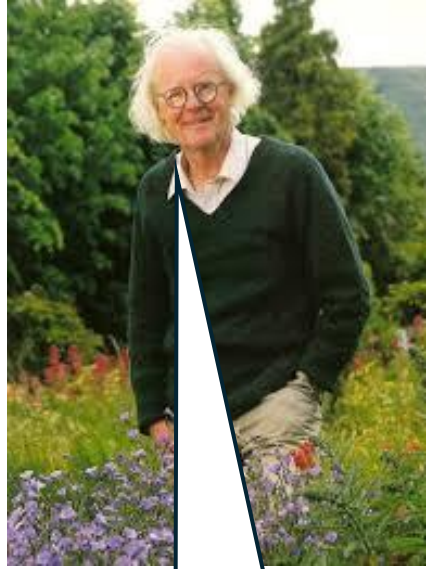
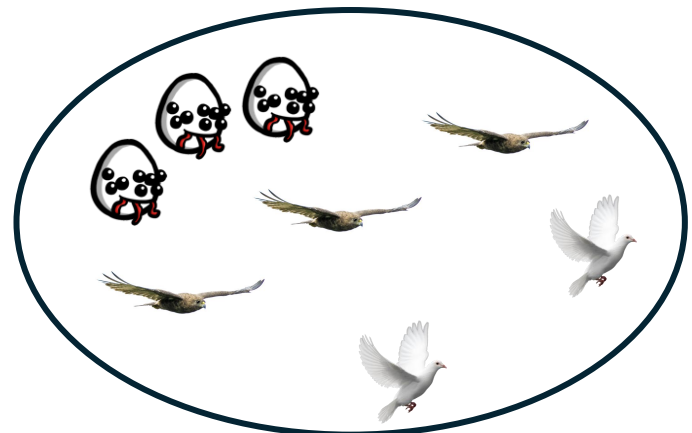
Are we sure then that  $p^* = 2/3$  is an ESS?

	Hawk	Dove
Hawk	-1	4
Dove	0	2

An **Evolutionarily Stable Strategy (ESS)** is a strategy that, if adopted by almost all individuals, cannot be invaded by a rare alternative strategy.

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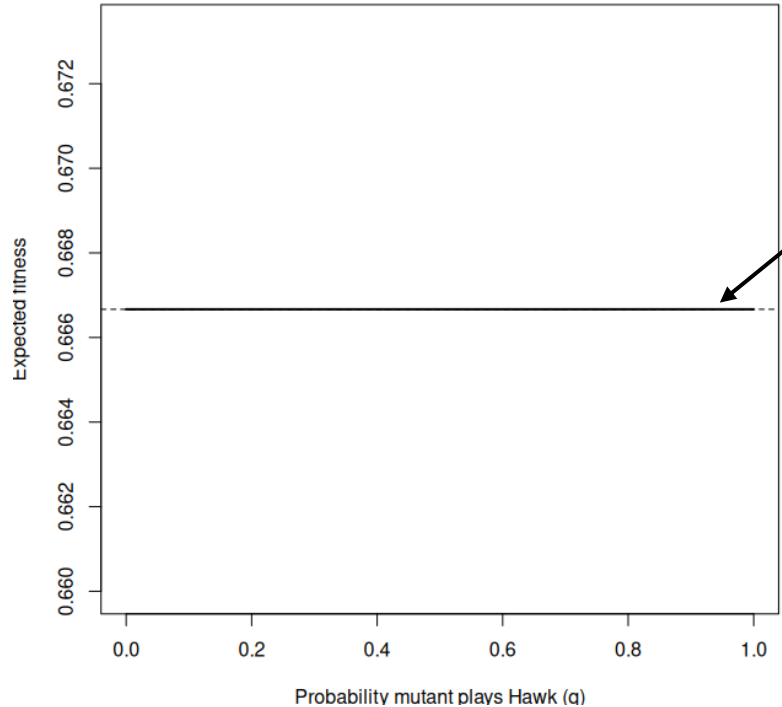
The **fitness of the mutant** will be  
 $E(q,p) = q * W_H + (1-q) * W_D$







John Maynard Smith (1920-2004)

It could still initially spread in the population by chance!

Fitness of mutant strategy vs ESS population



			
		<b>Hawk</b>	<b>Dove</b>
	<b>Hawk</b>	-1	4
	<b>Dove</b>	0	2



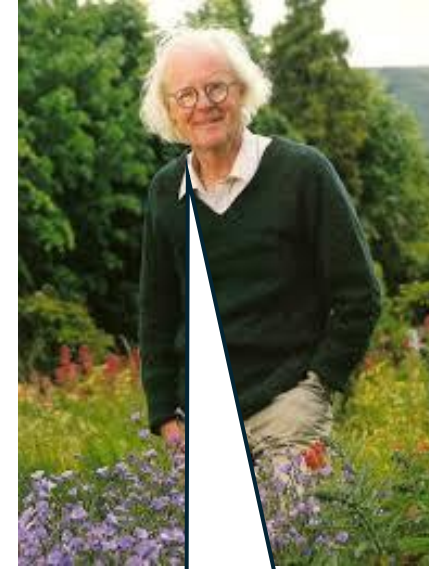
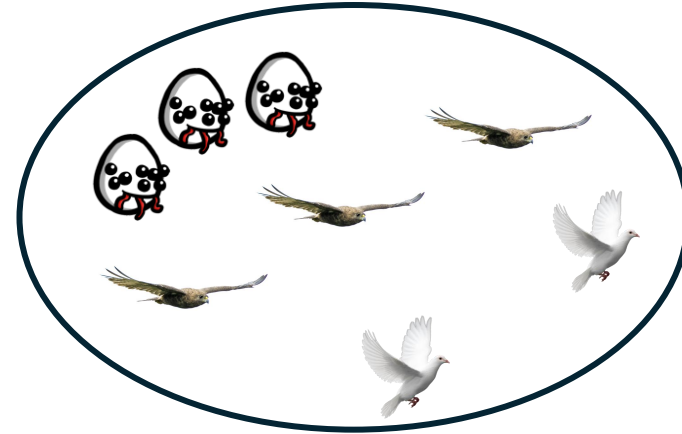
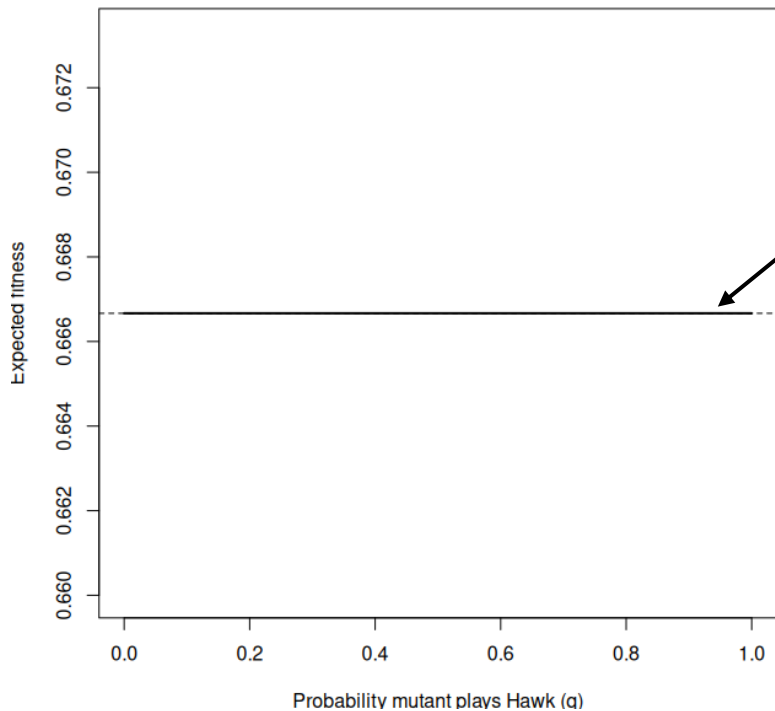
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Fitness of mutant strategy vs ESS population



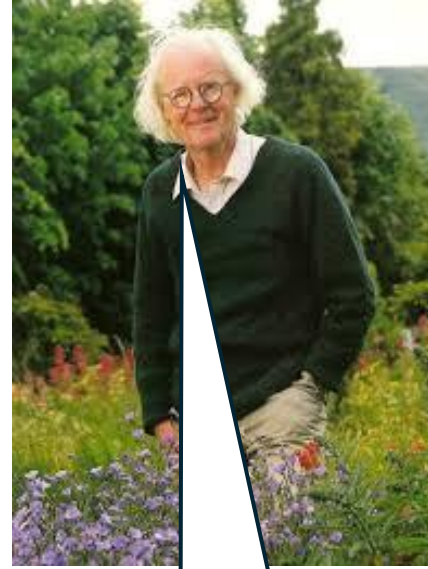
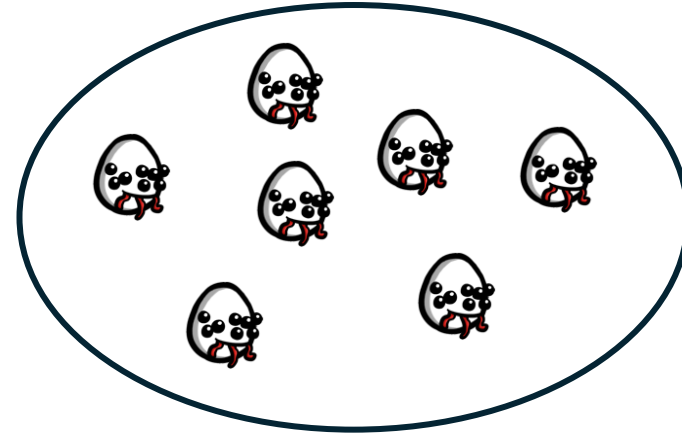
John Maynard Smith  
(1920 - 2004)

How can we verify that it doesn't?

		
	Hawk	Dove
	Hawk	Dove
	-1	4
	0	2





An **Evolutionarily Stable Strategy (ESS)** is a strategy that, if adopted by almost all individuals, cannot be invaded by a rare alternative strategy.

Let's calculate what happens when **almost all individuals are mutants**, and let's calculate the **fitness of the mutant**  $E(q,q)$  and that of our focal strategy  $p^*$ ,  $E(p^*,q)$ .



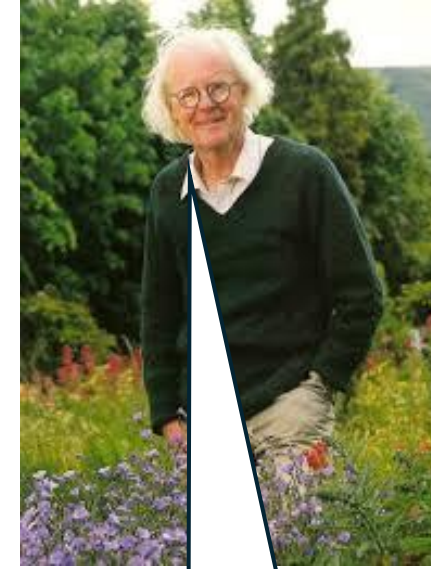
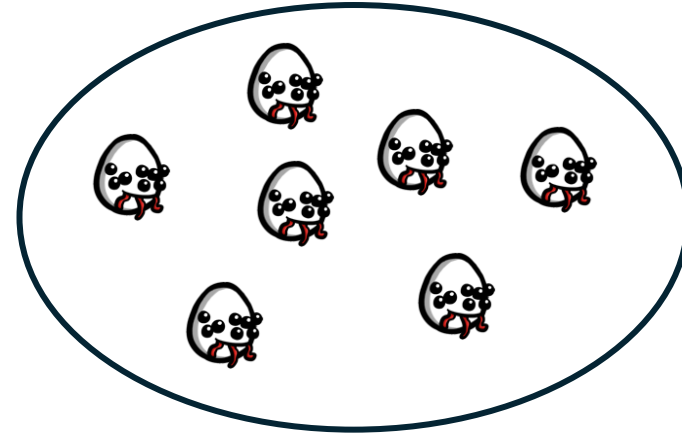
John Maynard Smith  
(1920 - 2004)

Let's check that the mutant cannot become common!

		
	Hawk	Dove
 Hawk	-1	4
 Dove	0	2

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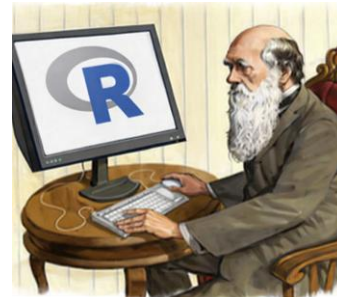
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John Maynard Smith  
(1920-2004)

A population of mutant is always invaded by  $p^*$

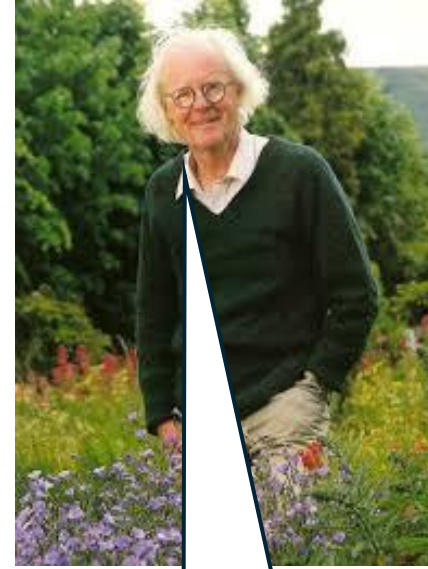
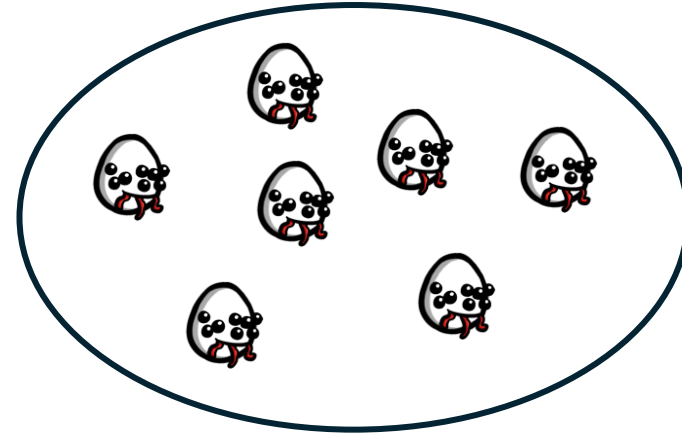
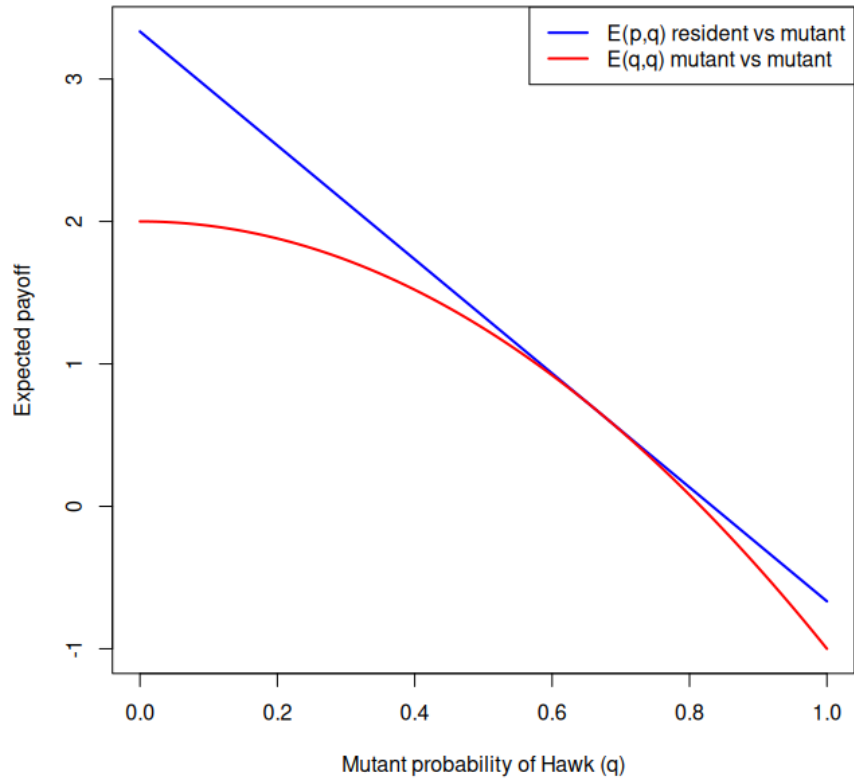
```
# payoffs of pure strategies against q
EHq <- -1*q + 4*(1-q)
EDq <- 2*(1-q)
# resident payoff vs mutant
E_pq <- p*EHq + (1-p)*EDq
# mutant payoff vs itself
E_qq <- q*EHq + (1-q)*EDq
```



		
	-1	4
	0	2





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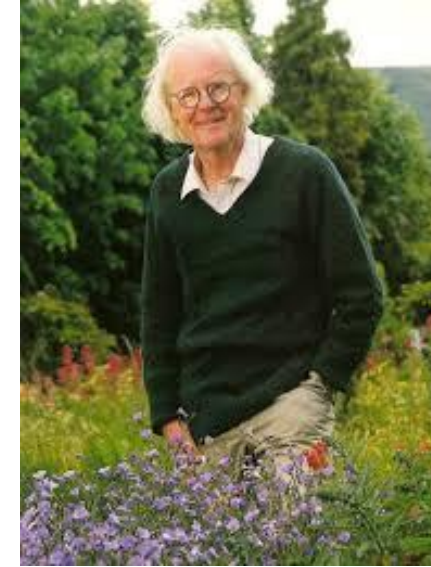


John Maynard Smith (1920-2004)

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John Maynard-Smith  
(1920-2004)

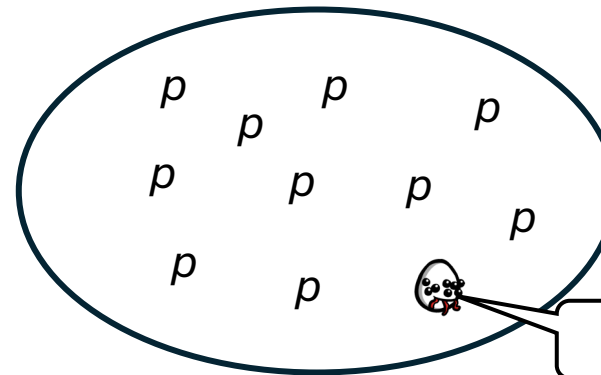
A strategy  $p$  is an ESS if for every alternative strategy  $q$  if:

First condition:

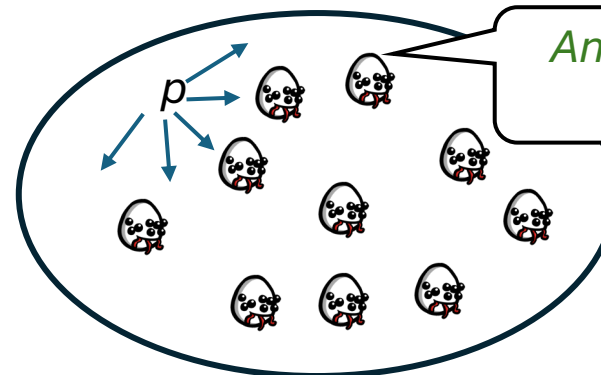
$$E(p,p) \geq E(q,p)$$

Second condition (if equality holds):

$$E(p,q) > E(q,q)$$



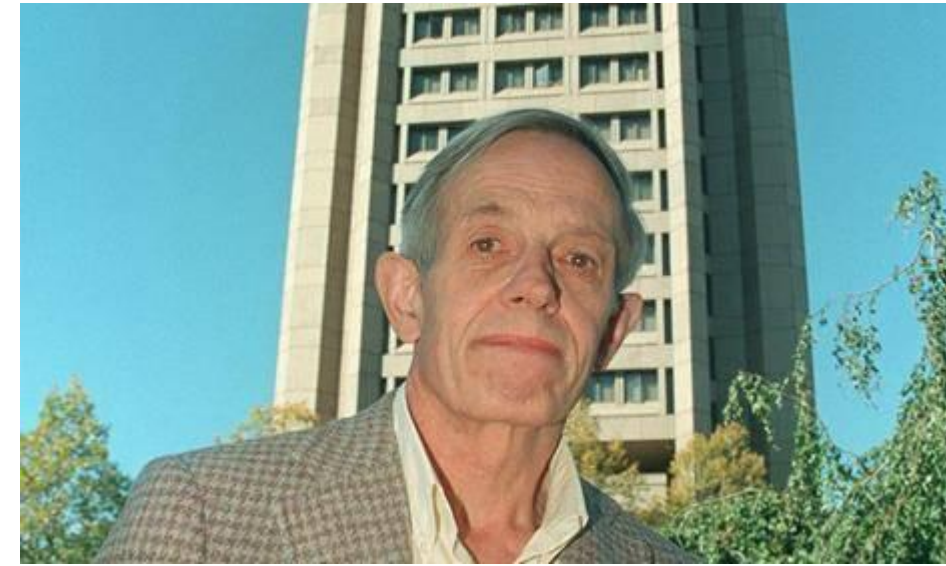
*I cannot invade easily, argh!!*



*And when I do I cannot resist **p**!  
argh!*

# Nash equilibrium

- In many strategic situations, there exists a set of strategies where **no player can improve their outcome by changing strategy alone**.
- That point is called a **Nash equilibrium**.
- Every ESS is a **Nash equilibrium**, but not every Nash equilibrium is evolutionarily stable.
- The second condition is what ensures **evolutionary stability**.

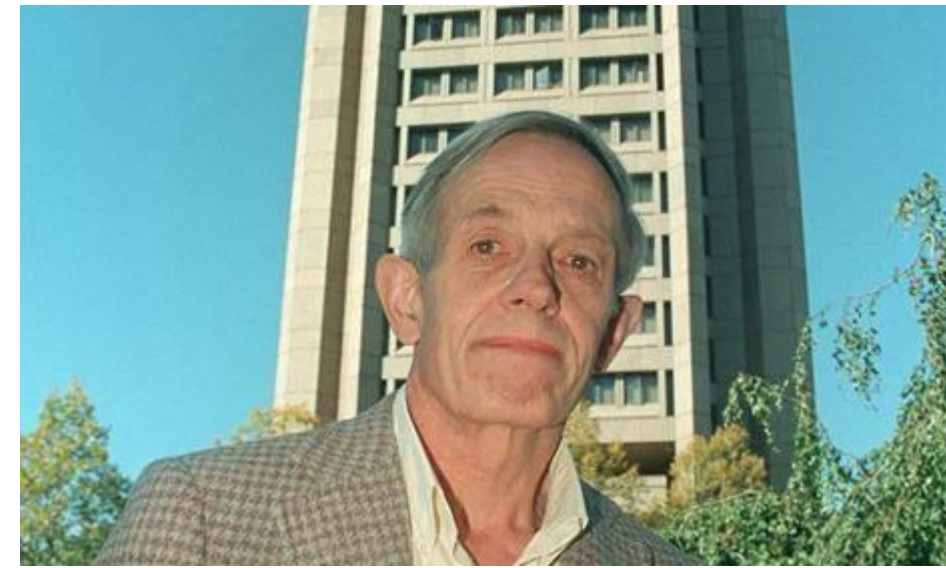


$$E(p,p) \geq E(q,p)$$

$$E(p,q) > E(q,q)$$

# Nash equilibrium

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Nash equilibrium:

$$E(p,p) \geq E(q,p)$$

Evolutionary Stable Strategy (ESS):

1° Condition:  $E(p,p) \geq E(q,p)$

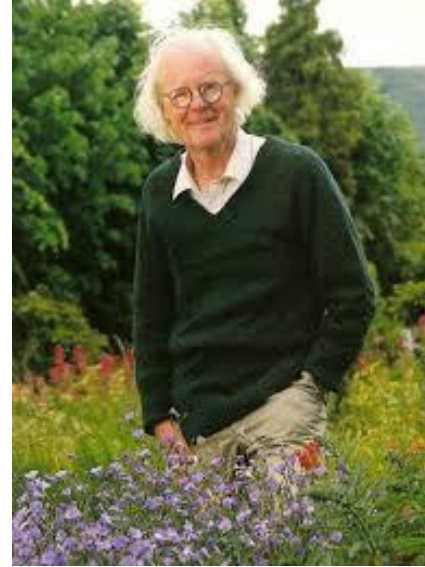
2° Condition: if  $E(p,p) = E(q,p)$ , then

$$E(p,q) > E(q,q)$$

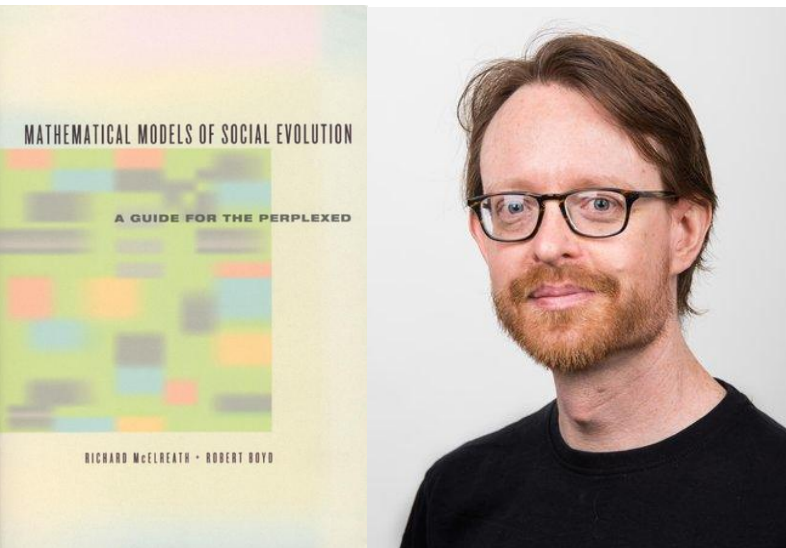
# Hawk and Doves in Maynard Smith's formulation

	Hawk	Dove
Hawk	$\frac{1}{2}v - \frac{1}{2}c$	$v$
Dove	$0$	$\frac{1}{2}v$

$v$  is the prize of winning  
 $c$  is the cost of fights/injuries



John Maynard-Smith  
(1920-2004)



[rmcelreath/VLEGT](https://github.com/rmcelreath/VLEGT): Verty short course on evolutionary game theory

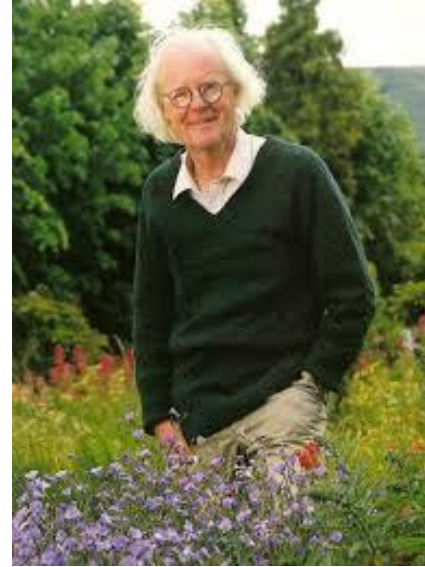


# Hawk and Doves in Maynard Smith's formulation

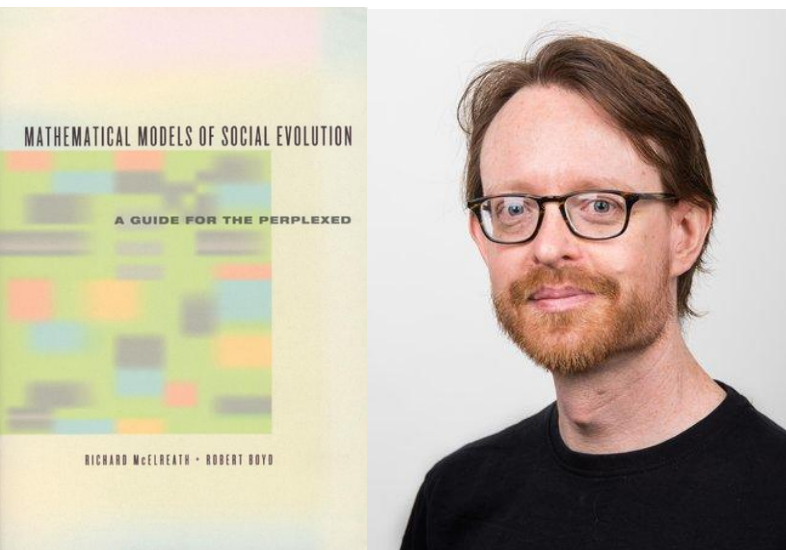
	Hawk	Dove
Hawk	$\frac{1}{2}v - \frac{1}{2}c$	$v$
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$v$  is the prize of winning  
 $c$  is the cost of fights/injuries

When are Hawks and Doves evolutionary stable?



John Maynard-Smith  
(1920-2004)

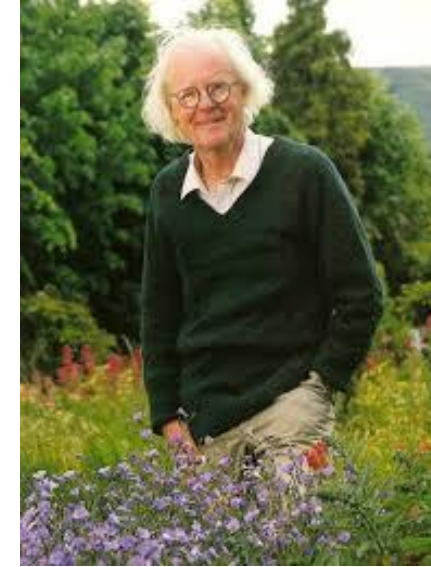


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John Maynard-Smith  
(1920-2004)

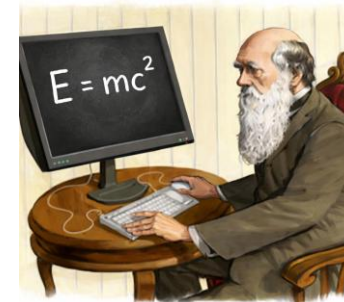
## Evolutionary stability of Pure Strategies

$$\underbrace{\frac{v - c}{2}}_{\text{Hawk-vs-Hawk}} > \underbrace{0}_{\text{Dove-vs-Hawk}}$$

Hawks are evolutionarily stable if  $v > c$   
 or "the value of the resource exceeds the cost of injury"

$$\underbrace{\frac{v}{2}}_{\text{Dove-vs-Dove}} > \underbrace{v}_{\text{Hawk-vs-Dove}}$$

Doves are evolutionarily stable if  $v < 0$   
 which is never



# Hawk and Doves in Maynard Smith's formulation

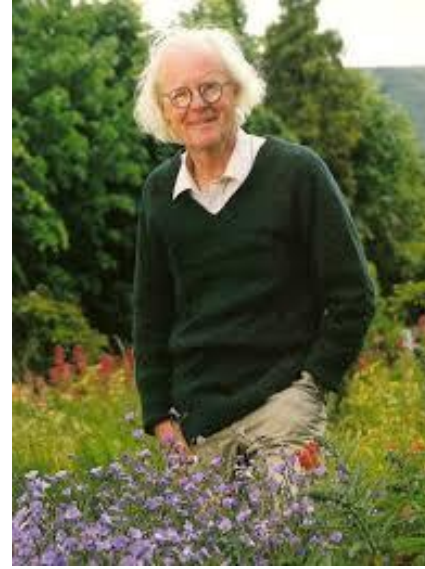
	Hawk	Dove
Hawk	$\frac{1}{2}v - \frac{1}{2}c$	$v$
Dove	$0$	$\frac{1}{2}v$

$v$  is the prize of winning  
 $c$  is the cost of fights/injuries

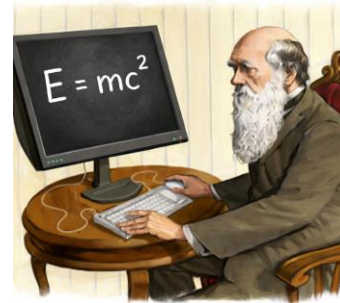
## Evolutionary stability of Pure Strategies

$$\underbrace{\frac{v - c}{2}}_{\text{Hawk-vs-Hawk}} > \underbrace{0}_{\text{Dove-vs-Hawk}}$$

Hawks are evolutionarily stable if  $v > c$   
 or "the value of the resource exceeds the cost of injury"



John Maynard-Smith  
(1920-2004)



# Hawk and Doves in Maynard Smith's formulation

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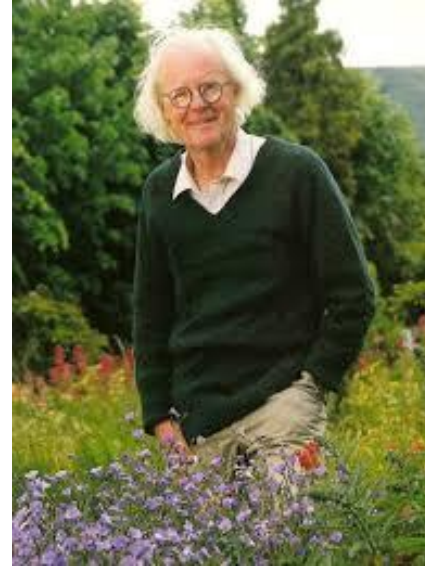
## Evolutionary stability of mixed strategies

$$p \frac{v - c}{2} + (1 - p)v$$

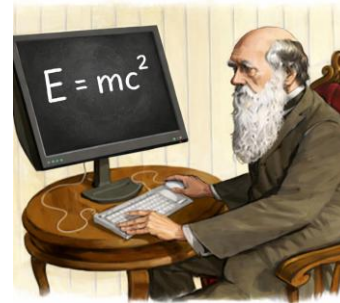
Average fitness of Hawks

$$p(0) + (1 - p)\frac{v}{2}$$

Average fitness of Doves



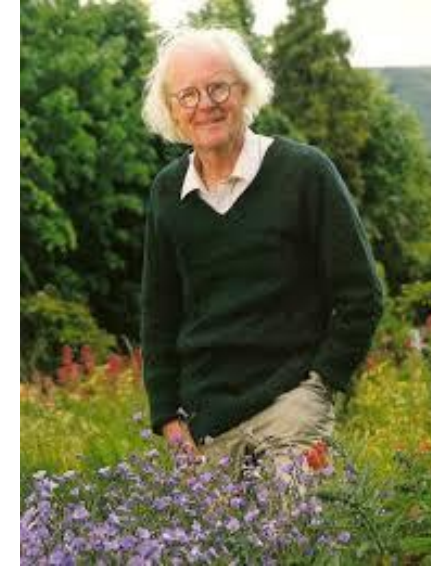
John Maynard-Smith  
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(1920-2004)

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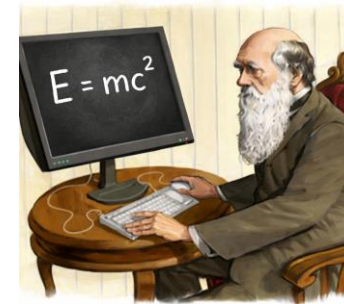
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Average fitness of Doves

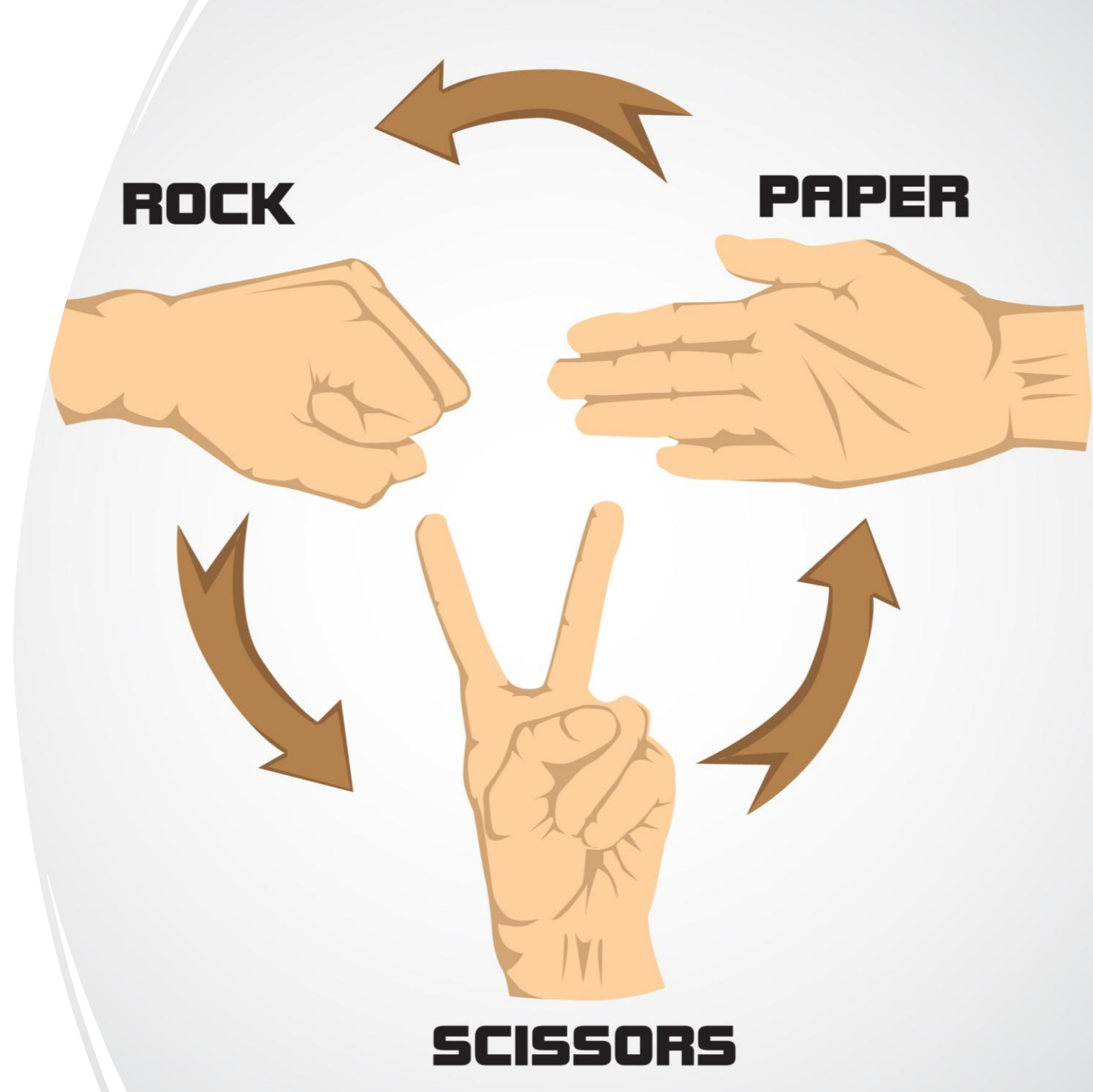
$$\underbrace{p \frac{v - c}{2} + (1 - p)v}_{\text{Hawk fitness}} > \underbrace{p(0) + (1 - p)\frac{v}{2}}_{\text{Dove fitness}} \Rightarrow p < v/c$$

Hawks increase when they are less than the ratio  $v/c$ , which is the ESS



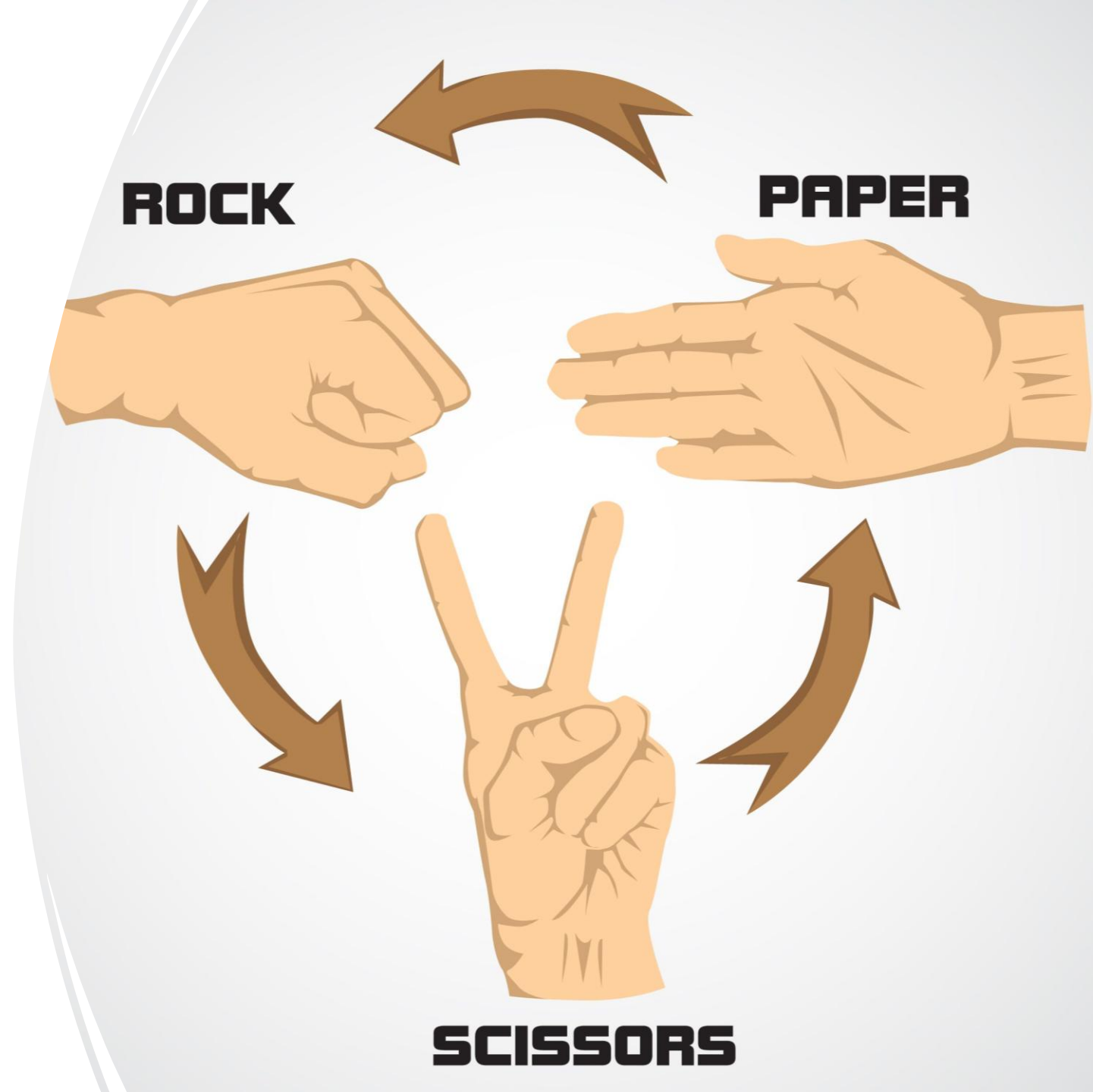
# What is the payoff matrix?

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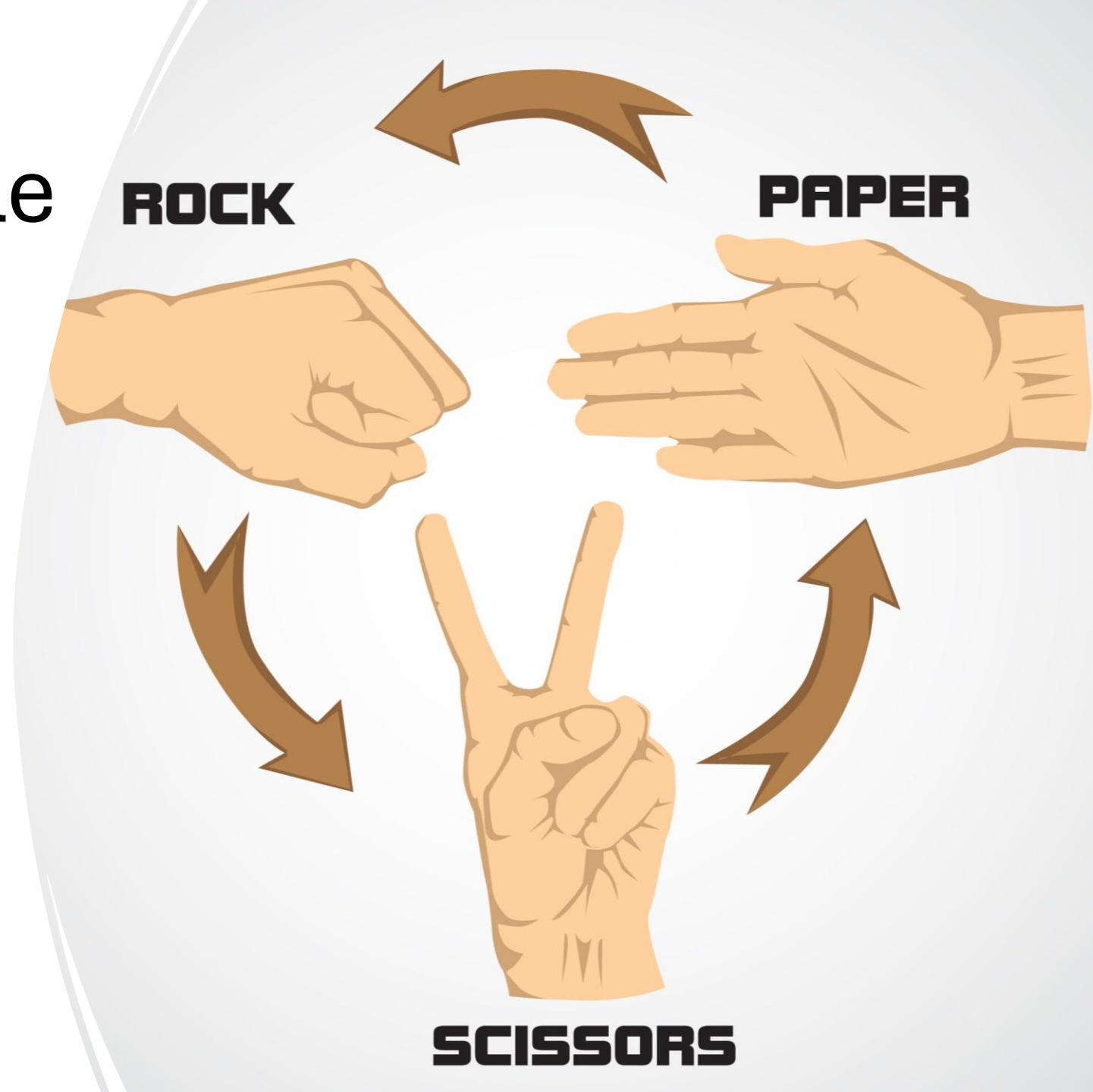
# What is the payoff matrix?

Gains of Player 1		Player 2 chooses...		
				
Player 1 chooses...		0	-1	1
		1	0	-1
		-1	1	0



# What is the Evolutionary Stable Strategy (ESS)?

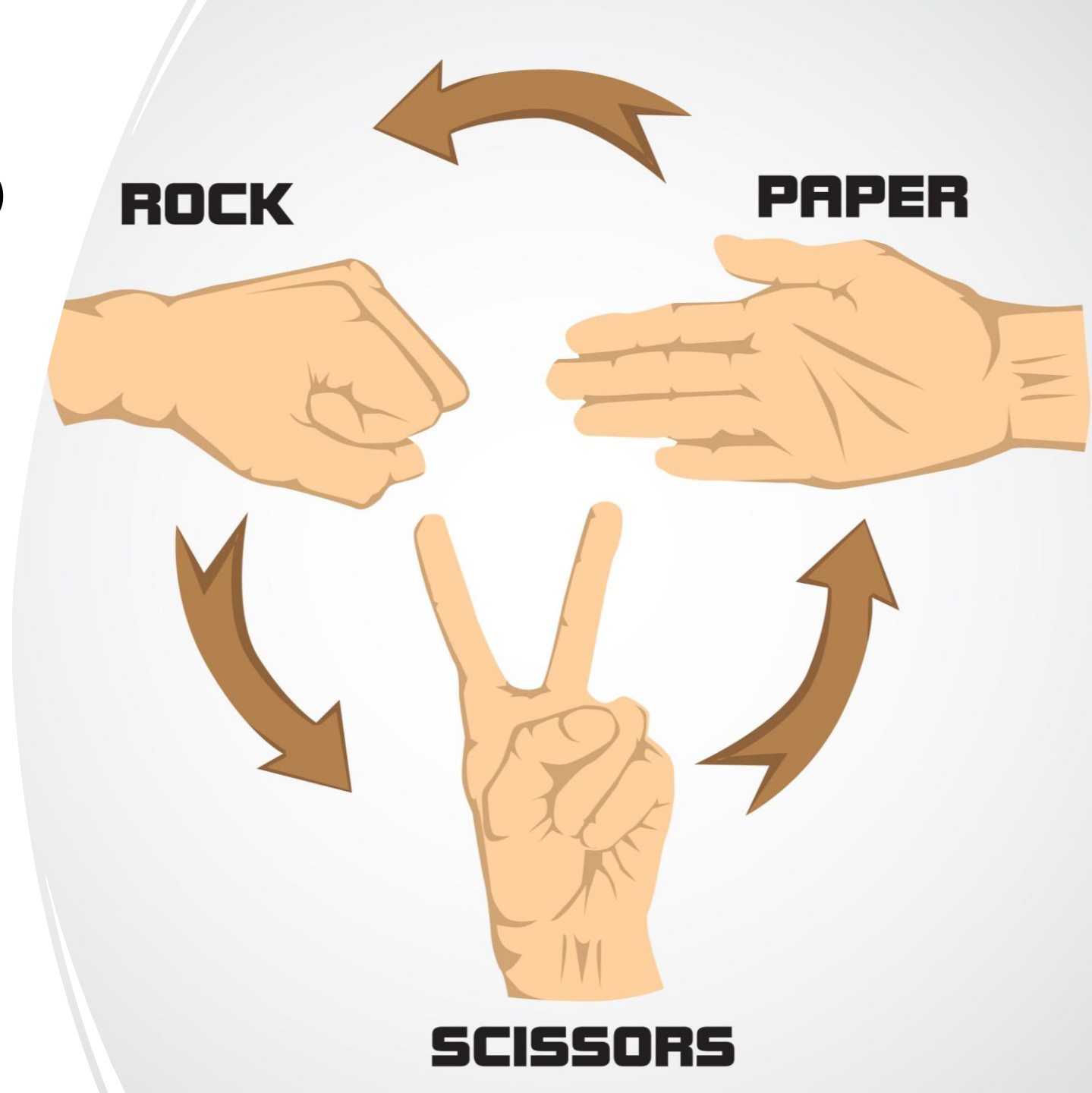
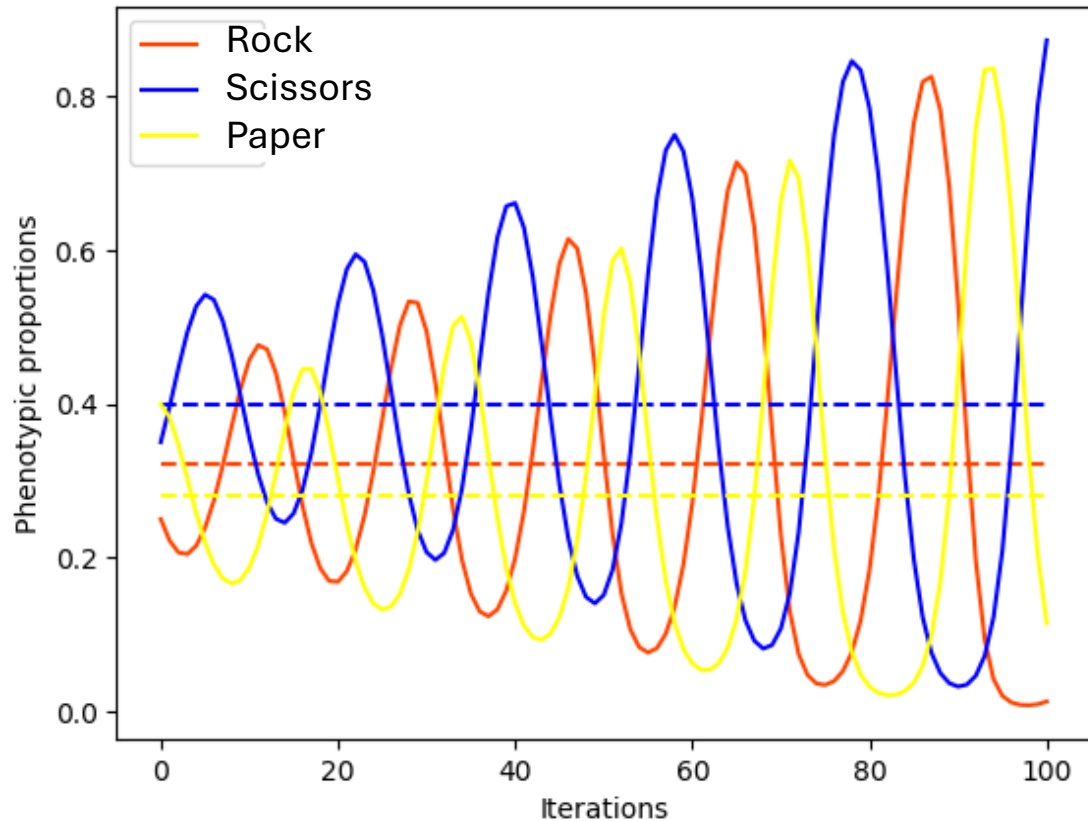
Gains of Player 1		Player 2 chooses...		
				
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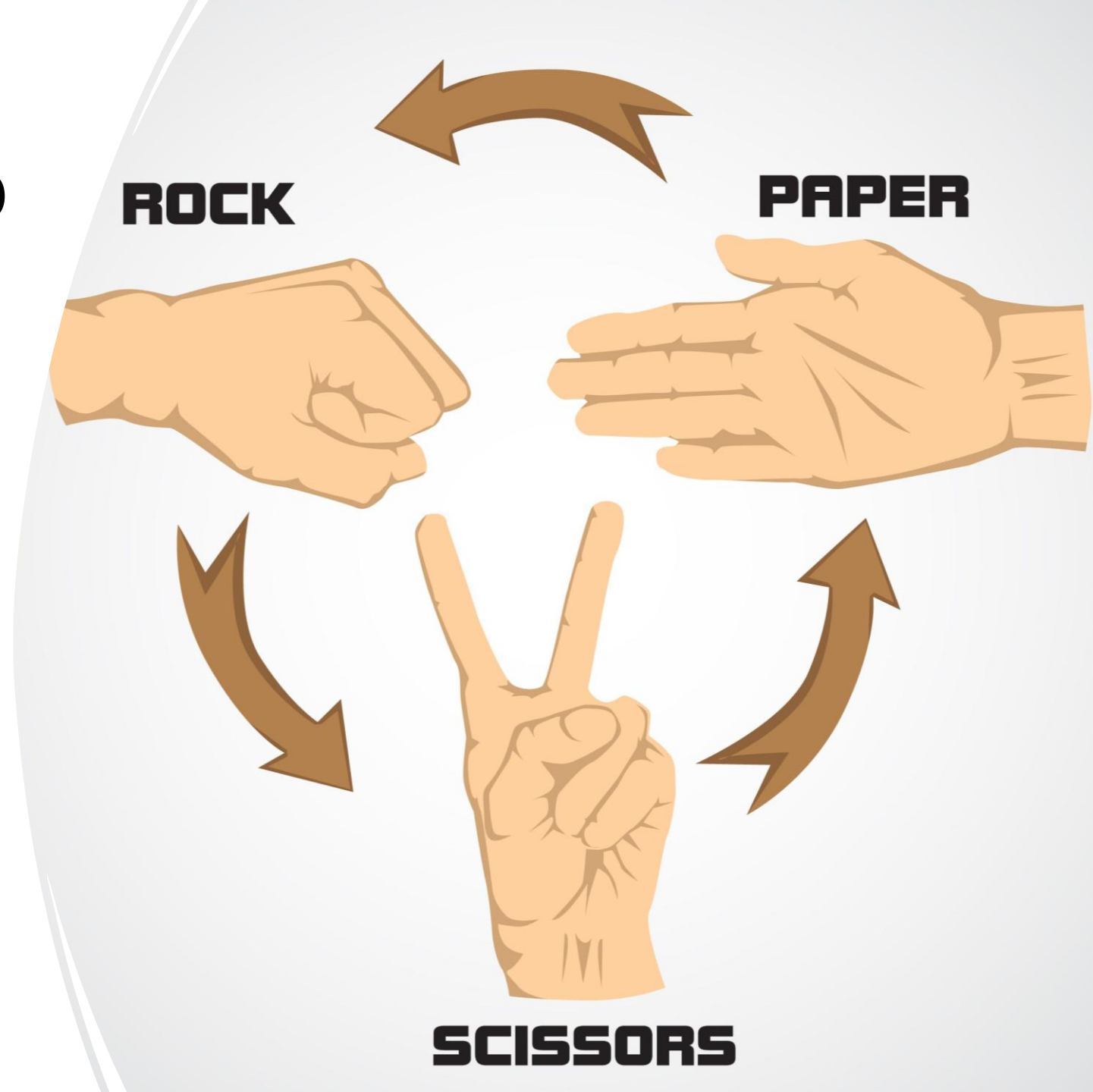
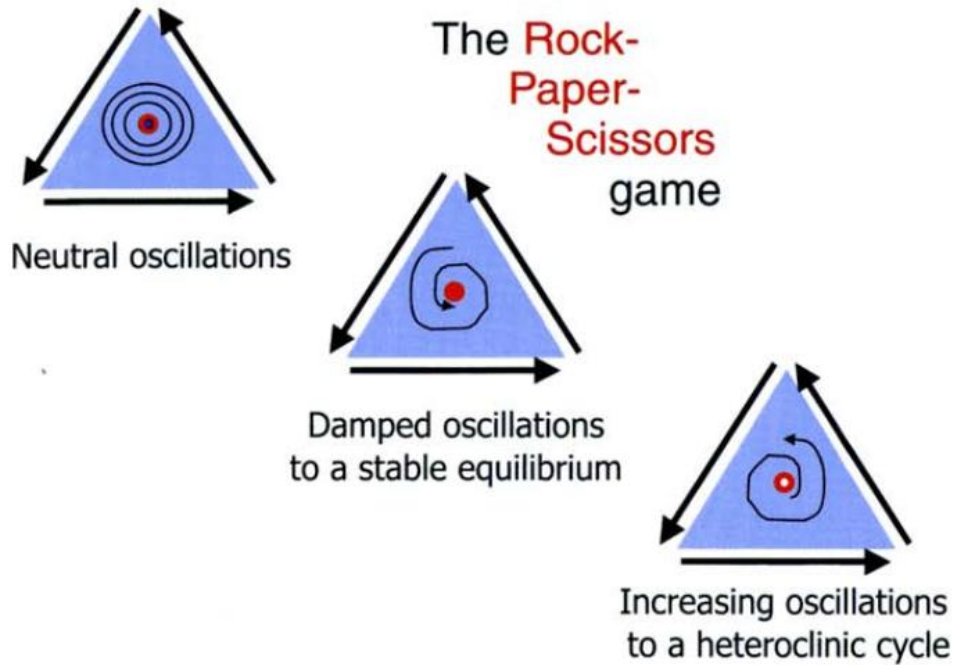


# Perturbations from the EES can lead to cyclic dynamics

Cargnellutti-Rossato et al, 2023



# Perturbations from the EES can lead to cyclic dynamics





**Orange males** are larger and more aggressive, controlling larger territories and mating with multiple females.

**Blue males** have smaller territories and focus on one female, ensuring their offspring are their own.

**Yellow males**, known as "sneakers," mimic female markings and mate without establishing a territory, increasing their chances of mating with other males.

# The Side-Blotched Lizard Game

© Ammon Corl 2025  
Origin > 5 million years ago

**Orange Morph**



**Phenotype:** Controls large territories. Has high testosterone and endurance.  
**Genotype:** OO  
**Motto:** "If you like guys very strong, with Orange you just can't go wrong!"  
**Strength:** Beats Blue Morphs. "Blues can't stand up to my might! I will beat them in a fight!"  
**Weakness:** Loses to Yellow Morphs. "Those pesky Yellows bother me! They invade my territory!"

**Orange Morph**


**Blue Morph**



**Phenotype:** Territorial. Guards females closely. Prevents extrapair matings.  
**Genotype:** BB or OB  
**Motto:** "I'm true and I'm blue, and I will watch over you."  
**Strength:** Beats Yellow Morphs. "Yellows can't sneak by me! They had better flee!"  
**Weakness:** Loses to Orange Morphs. "Orange is much too strong for me! I hate losing territory!"

**Blue Morph**

**Yellow Morph**



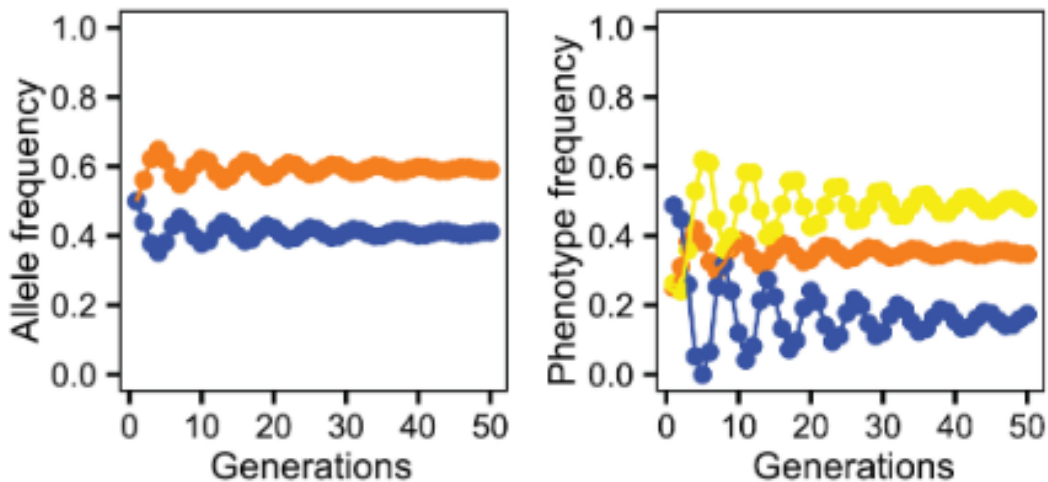
**Phenotype:** Not territorial. Sneaks. Can irreversibly transform to a blue morph.  
**Genotype:** BB or OB  
**Motto:** "While I may not win a fight, I will woo you oh so right!"  
**Strength:** Beats Orange Morphs. "Oranges can't keep me out! Their females know what I'm about!"  
**Weakness:** Loses to Blue Morphs. "While I cannot beat the Blues, I can be one if I choose!"

**Yellow Morph**



*Uta stansburiana*

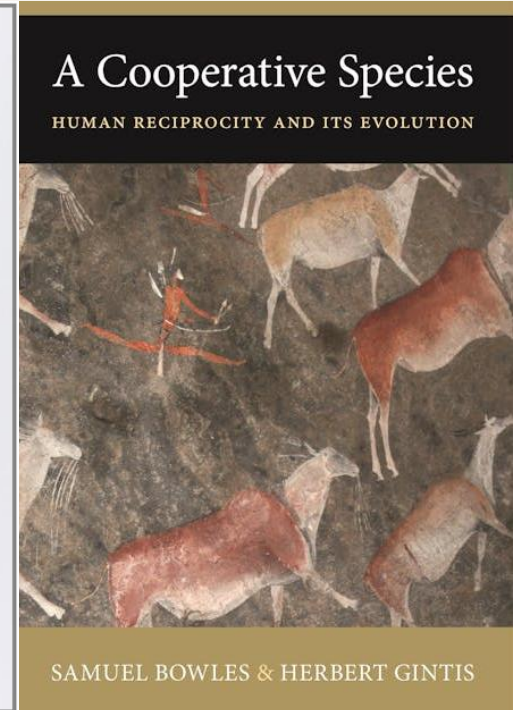
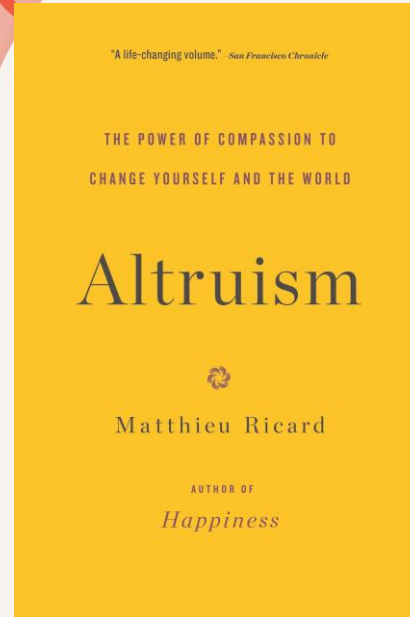
## How Lizards Play Rock-Paper-Scissors



[This Diminutive Reptile Plays Rock-Paper-Scissors - The New York Times](#)

[Corl et al., 2026, Science](#)

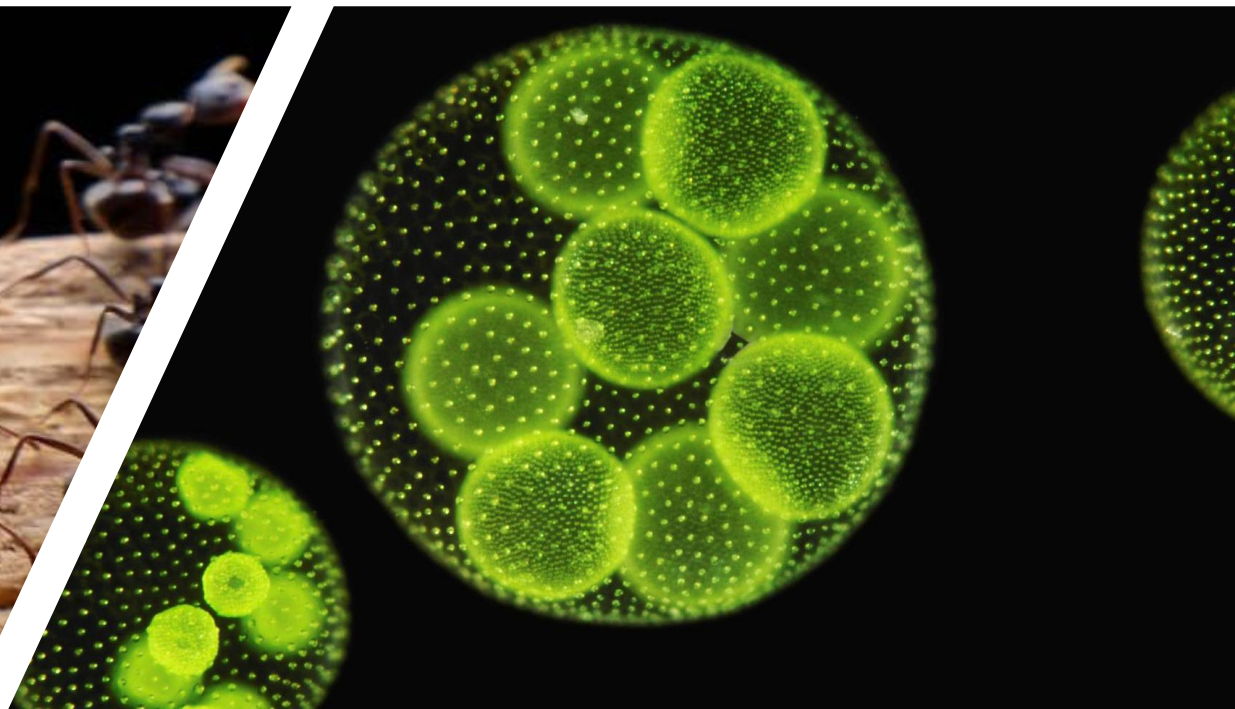
# The evolution of cooperation





## The evolution of cooperation

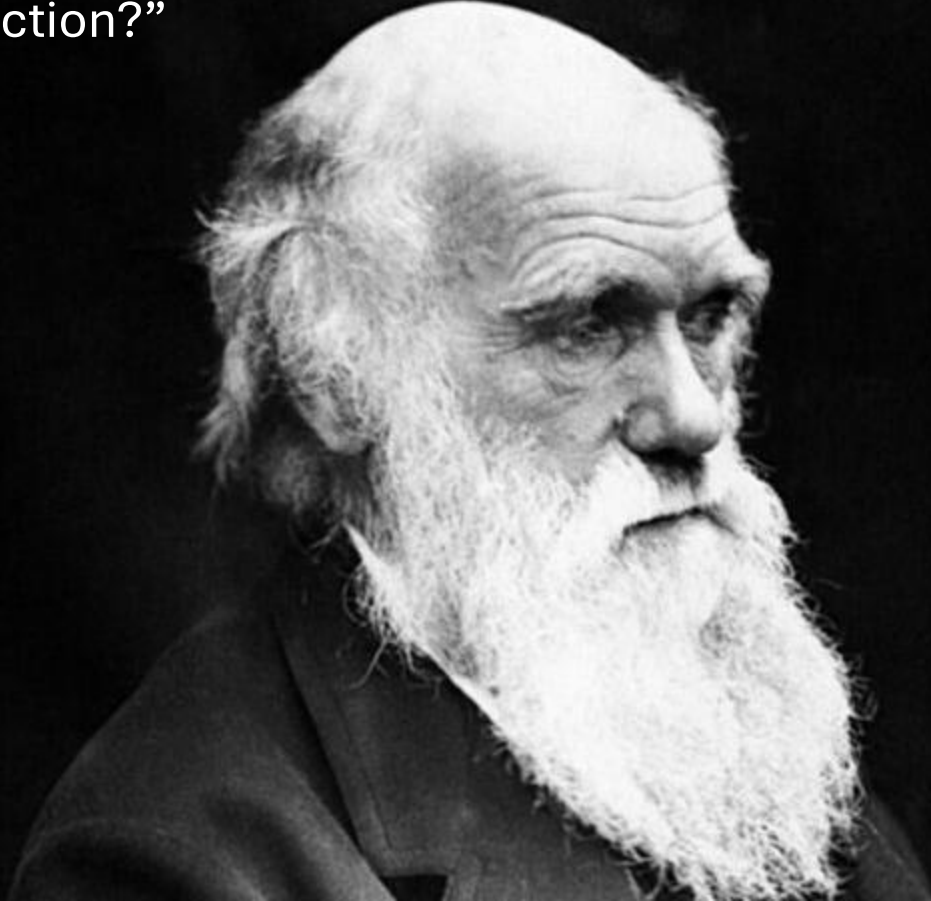




From the «On the Origin of Species»:

“It will indeed be thought that I have an overweening confidence in the principle of natural selection, when I do not admit that such wonderful and well established facts at once annihilate my theory.”

“But with the working ant we have an insect differing greatly from its parents, yet absolutely sterile; so that it could never have transmitted successively acquired modifications of structure or instinct to its progeny. It may well be asked how is it possible to reconcile his case with the theory of natural selection?”

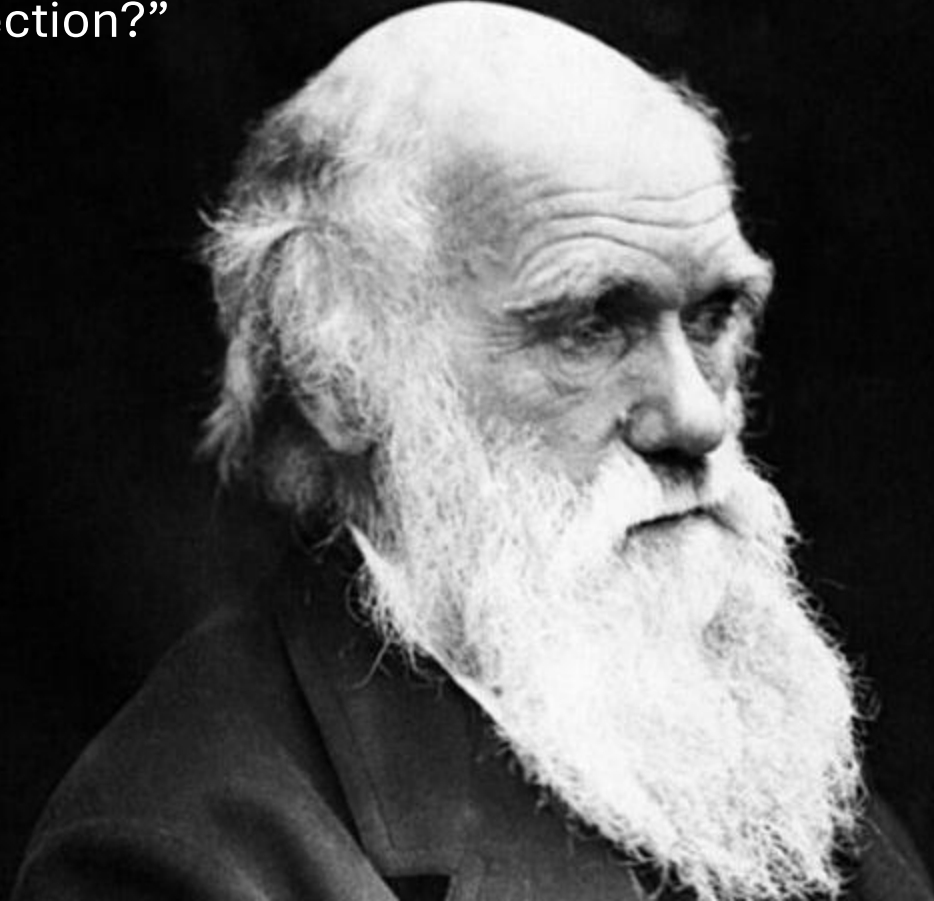




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But also he used it against Lamarck:

- “.. females,—in this case, we may safely conclude from the analogy of ordinary variations, that each successive, slight, profitable modification did not probably at first appear in all the individual neuters in the same nest, but in a few alone; and that by the long-continued selection of the fertile parents which produced most neuters with the profitable modification, all the neuters ultimately came to have the desired character.”
- «I am surprised that no one has advanced this demonstrative case of neuter insects, against the well-known doctrine of Lamarck.”

