

# Statistical Methods with Application to Finance

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## Exercises (set 1)

1. Suppose  $X_1, X_2, \dots$  are iid  $N(0.06, 0.47)$ 
  - a. What is the mean of  $X_1 + X_2 + X_3$ ?
  - b. What is the standard deviation of  $X_1 + X_2 + X_3$ ?
  - c. What is the distribution of  $\exp(X_1 + X_2)$ ?
  - d. What is the probability  $P(X_1 < 1.5)$  ?
  - e. What is  $Cov(X_1, X_1 + X_2)$ ?
2. The log returns on a stock are normally distributed with mean 0.1 and standard deviation 0.13.
  - a. What is the distribution of one-period gross returns?
  - b. What is the probability that the gross return is 1.2 or more?
  - c. Assume the log-returns are also independent, what is the 0.85-quantile of the 2-period log return?
3. Suppose that the daily log returns  $r_t$  on a stock are independent and normally distributed with mean 0.001 and standard deviation 0.015.
  - a. What is  $P(r_t < 0.02)$ ?
  - b. What is the standard deviation of  $r_1 + r_2 + r_3$ ?
  - c. What is the distribution of the five-day log return?
  - d. Suppose you buy \$1000 worth of this stock, what is the probability that after five trading days your investment is worth less than \$990?
4. The prices on a stock market are assumed to follow a lognormal geometric random walk with parameters  $\mu = 0, 15$  and  $\sigma^2 = 0, 04$ .
  - a. What is the expected log return for 5 years?
  - b. What is the median log return for 10 years?
  - c. What is the median gross 10-year return?
  - d. If the stock price starts at \$100, what is the expected price after 10 years?
5. Suppose that  $X_1, X_2, \dots$  is a lognormal geometric random walk with parameters  $\mu$  and  $\sigma^2$ :

$$X_k = X_0 \exp(r_1 + \dots + r_k)$$

where  $X_0$  is a fixed constant and  $r_1, r_2, \dots$  are iid  $\mathcal{N}(\mu, \sigma^2)$ .

- a. Find  $Pr(X_2 > 1.3X_0)$  for  $\mu = 0.1$  and  $\sigma = 0.15$ .
- b. Find the first quartile of  $X_k$ , for all  $k$ .
- c. What is the expected value of  $X_k$ ?