

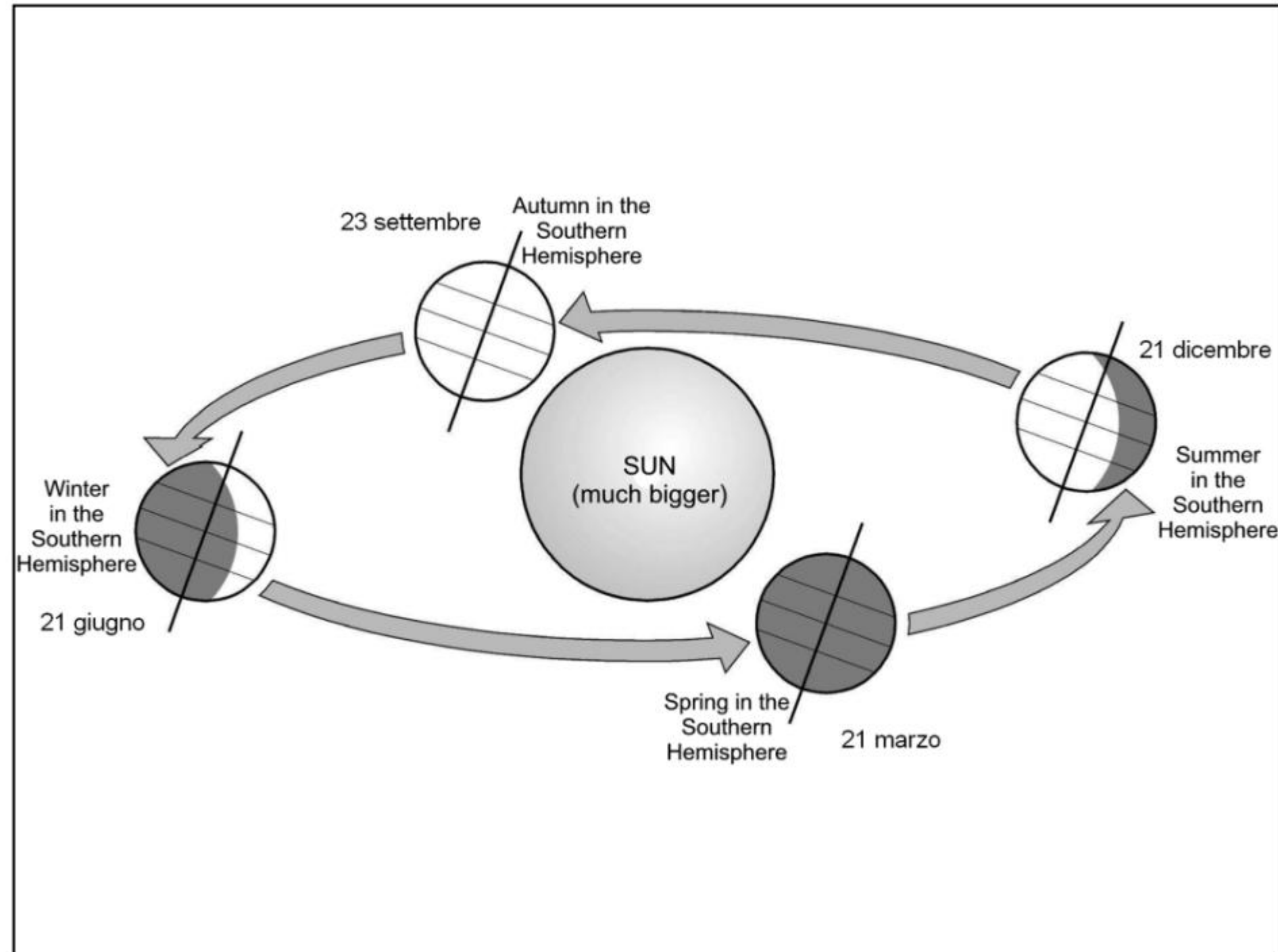


BUILDINGS HVAC SYSTEM

Solar Angles

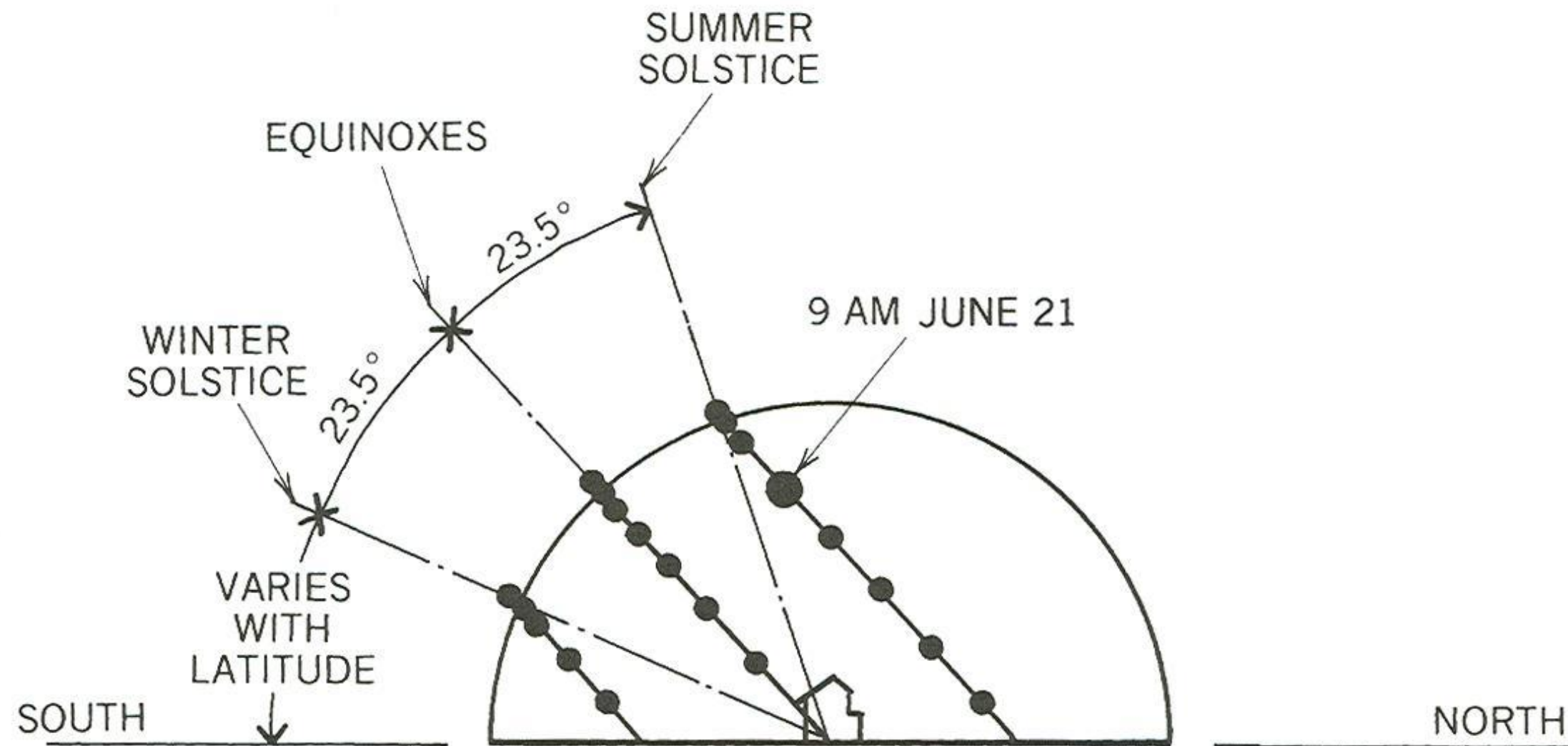


Earth orbit



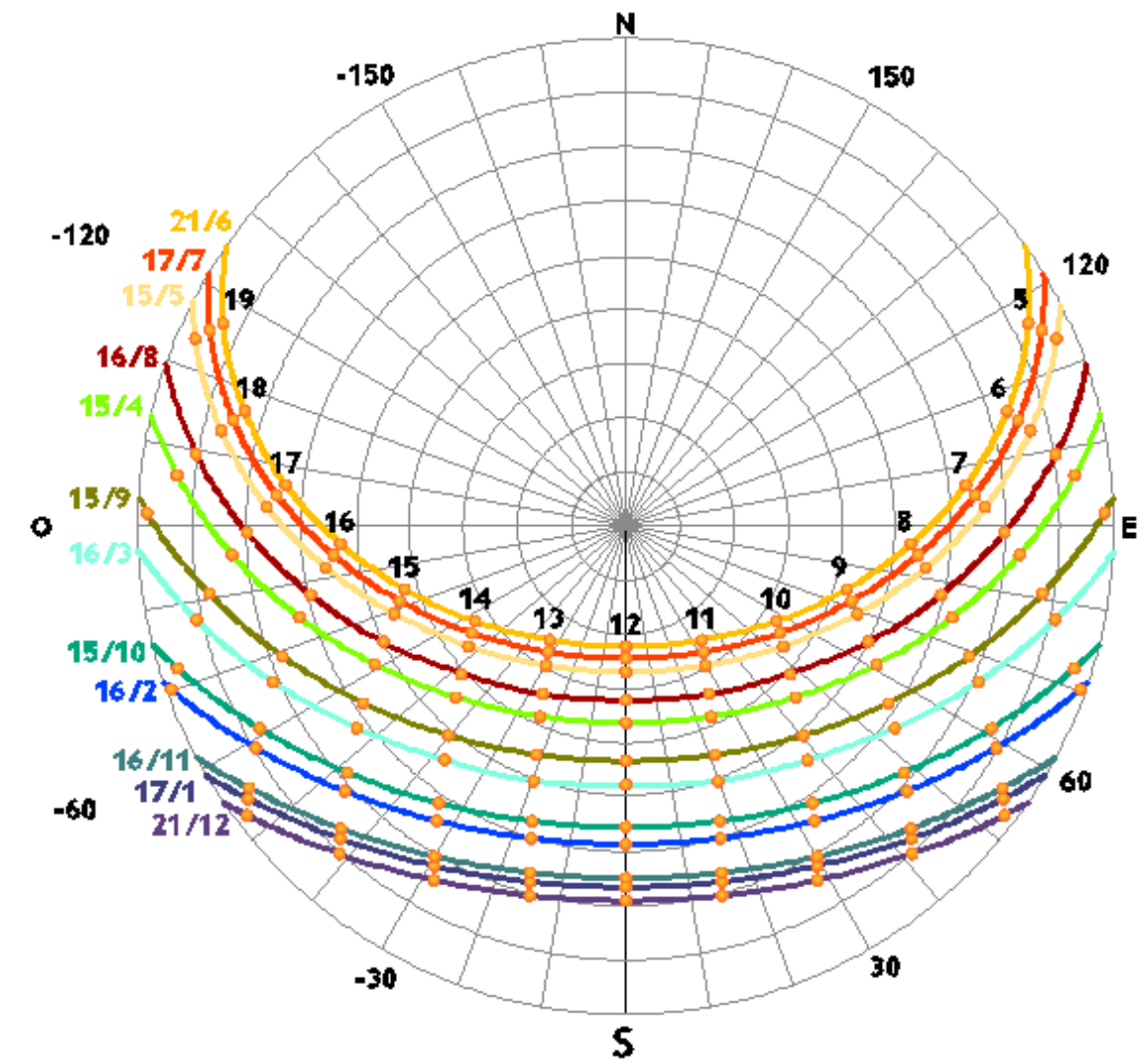
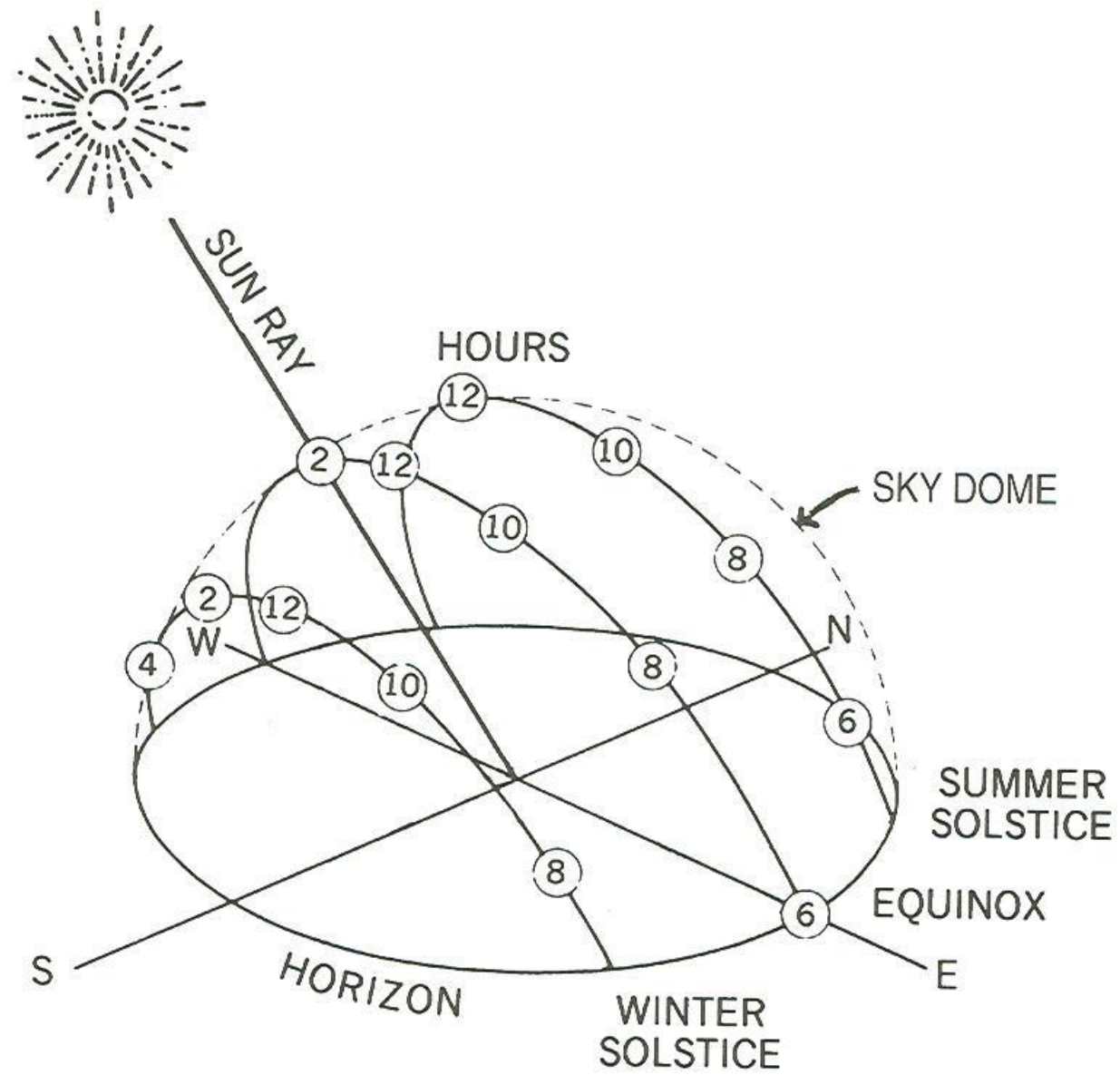


Sun and seasons





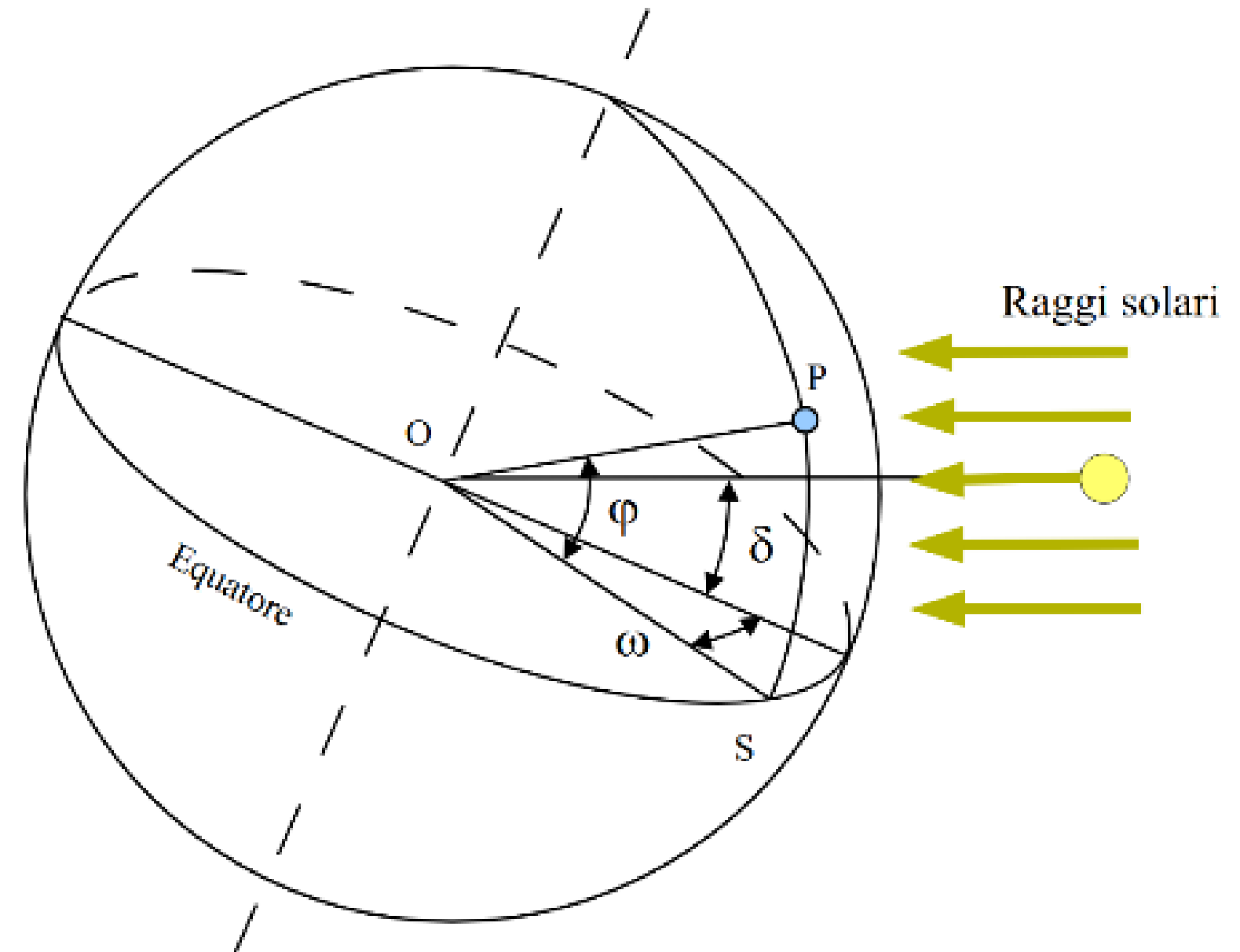
Solar path





Geographic coordinates

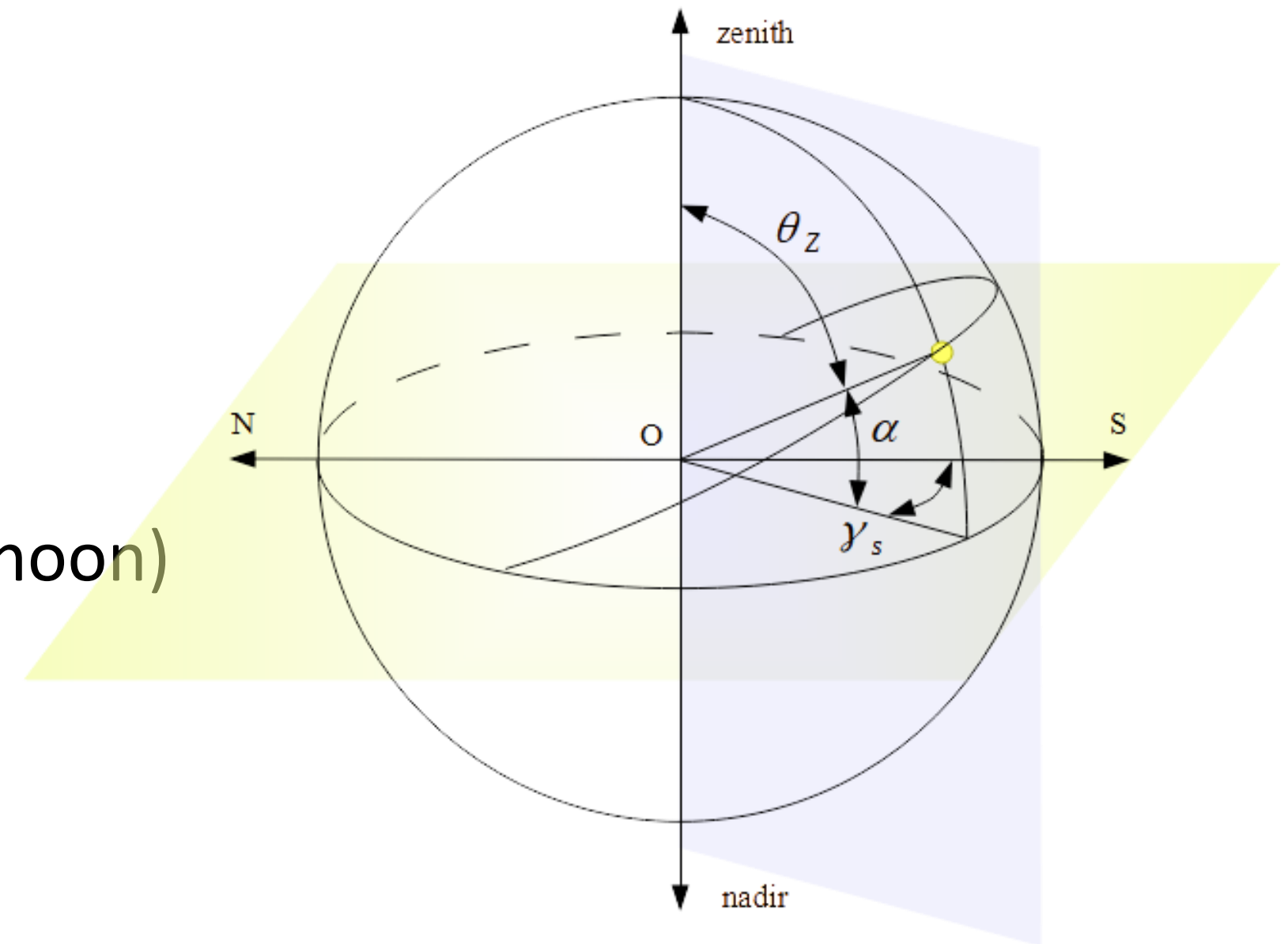
- φ : latitude
- ψ : longitude
- δ : declination
- ω : hour angle
- n : day of the year





Solar angles, horizontal coordinates

- The reference is an observer on the horizontal
- We identify the solar angles
- θ_z : zenith angle
- α : solar height
- γ_s : solar azimuth (west positive, afternoon)





Solar time

- When computing solar position and solar related quantities all must be referenced using the true solar time
- Solar time differs from the one measured by a clock
- Solar time must be recovered in function of the solar angles and the coordinates of the location
- We define the orbit angle Ω the position of earth along the path around the sun

$$\Omega = 2 \cdot \pi \cdot \frac{n - 1}{365}$$



Earth orbit declination

- The declination is the angle between the earth-sun line and the equatorial plane

$$\delta = \frac{360^\circ}{2\pi} \times [a_0 + a_1 \times \cos(\Omega) + a_2 \times \sin(\Omega) + a_3 \times \cos(2 \cdot \Omega) + a_4 \times \sin(2 \cdot \Omega) + a_5 \times \cos(3 \cdot \Omega) + a_6 \times \sin(3 \cdot \Omega)]$$

$$a_0 = 0,006918$$

$$a_1 = -0,399912$$

$$a_2 = 0,070257$$

$$a_3 = -0,006758$$

$$a_4 = 0,000907$$

$$a_5 = -0,002697$$

$$a_6 = 0,00148$$

$$a_0 \div a_6 \text{ radians}$$

- Simple relation $\delta = 23.45 \cdot \sin\left(360^\circ \cdot \frac{n+284}{365}\right)$



Equation of time

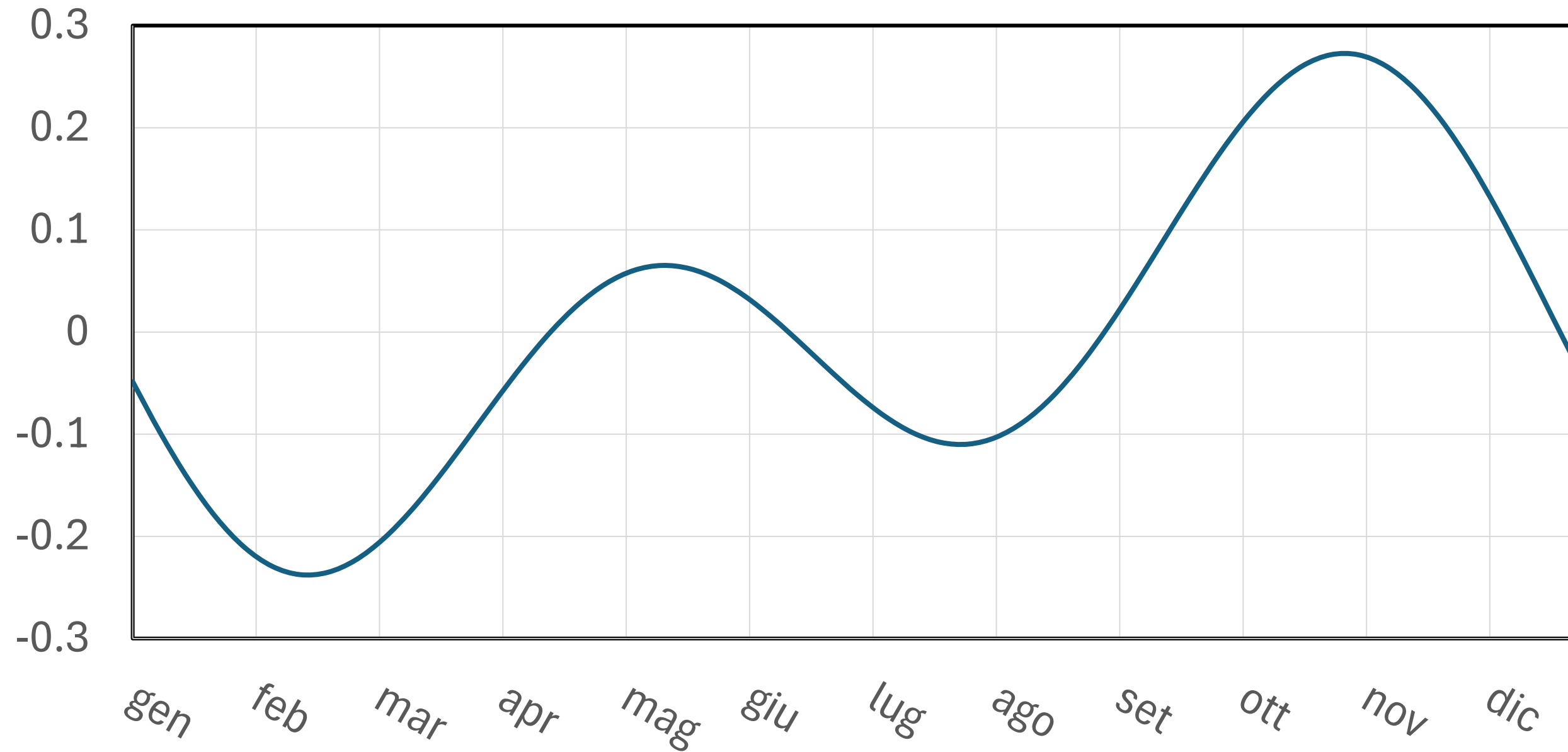
- The difference between **apparent solar time** measured by a sundial and **mean solar time** considering a uniform motion of the earth is defined as “Equation of time”
- If measured in hours can be obtained as :

$$Et = \left(\frac{24}{2\pi} \right) \times [a_1 + a_2 \times \cos(\Omega) + a_3 \times \sin(\Omega) + a_4 \times \cos(2 \cdot \Omega) + a_5 \times \sin(2 \cdot \Omega)]$$

$a_1 =$	0,0000075	$a_4 =$	-0,014615
$a_2 =$	0,001868	$a_5 =$	-0,040849
$a_3 =$	-0,032077		



Equation of Time





Solar time and clock time

- For Italy, the hour is defined with reference to the first meridian at east of Greenwich

$$t_{sa} = t_{is} + E_t + \frac{\psi - RM}{15} + \Delta t_{dst}$$

ψ longitude in degree, positive east

t_{is} local standard time, measured by a clock

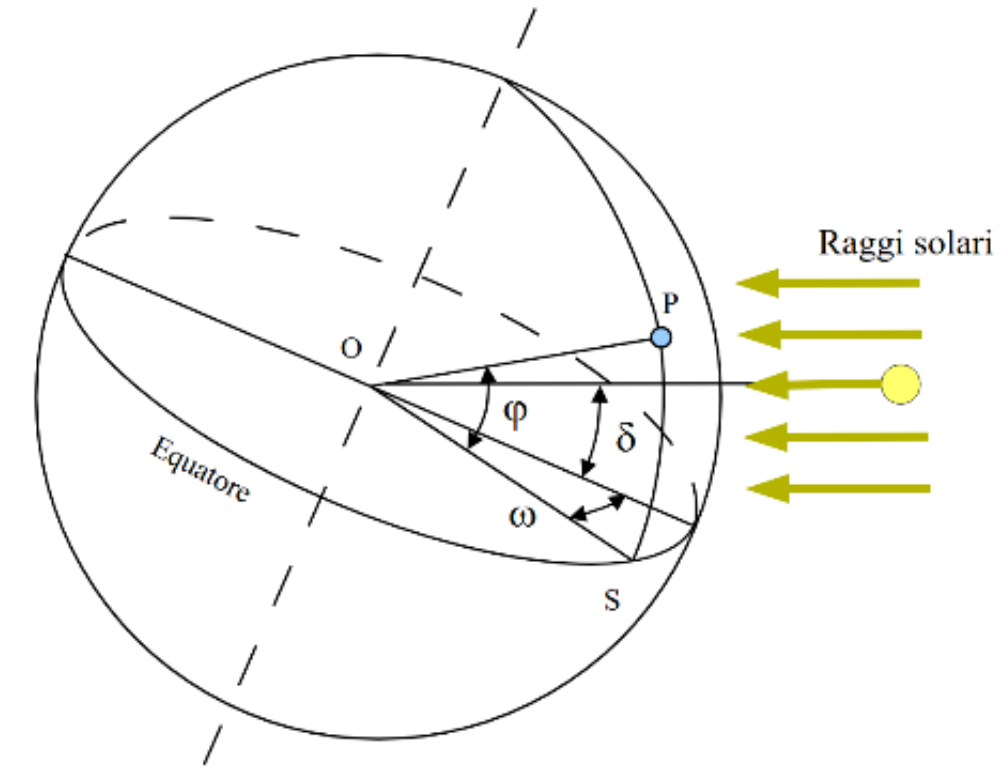
Δt_{dst} daylight saving time

RM reference meridian, Italy 15 °



Position of sun

- Hour angle
 - Positive in the afternoon
 - Negative in the morning
 - Zero at solar midday



$$\omega = 15 \times (t_{sa} - 12)$$

- Height of the sun

$$\sin(\alpha) = \cos(\varphi) \times \cos(\delta) \times \cos(\omega) + \sin(\varphi) \times \sin(\delta)$$

- Where φ is the latitude, positive at north and δ the declination.



Position of the sun, solar azimuth

- Solar azimuth
- Solar azimuth is positive facing west
- Can be obtained using the following relations

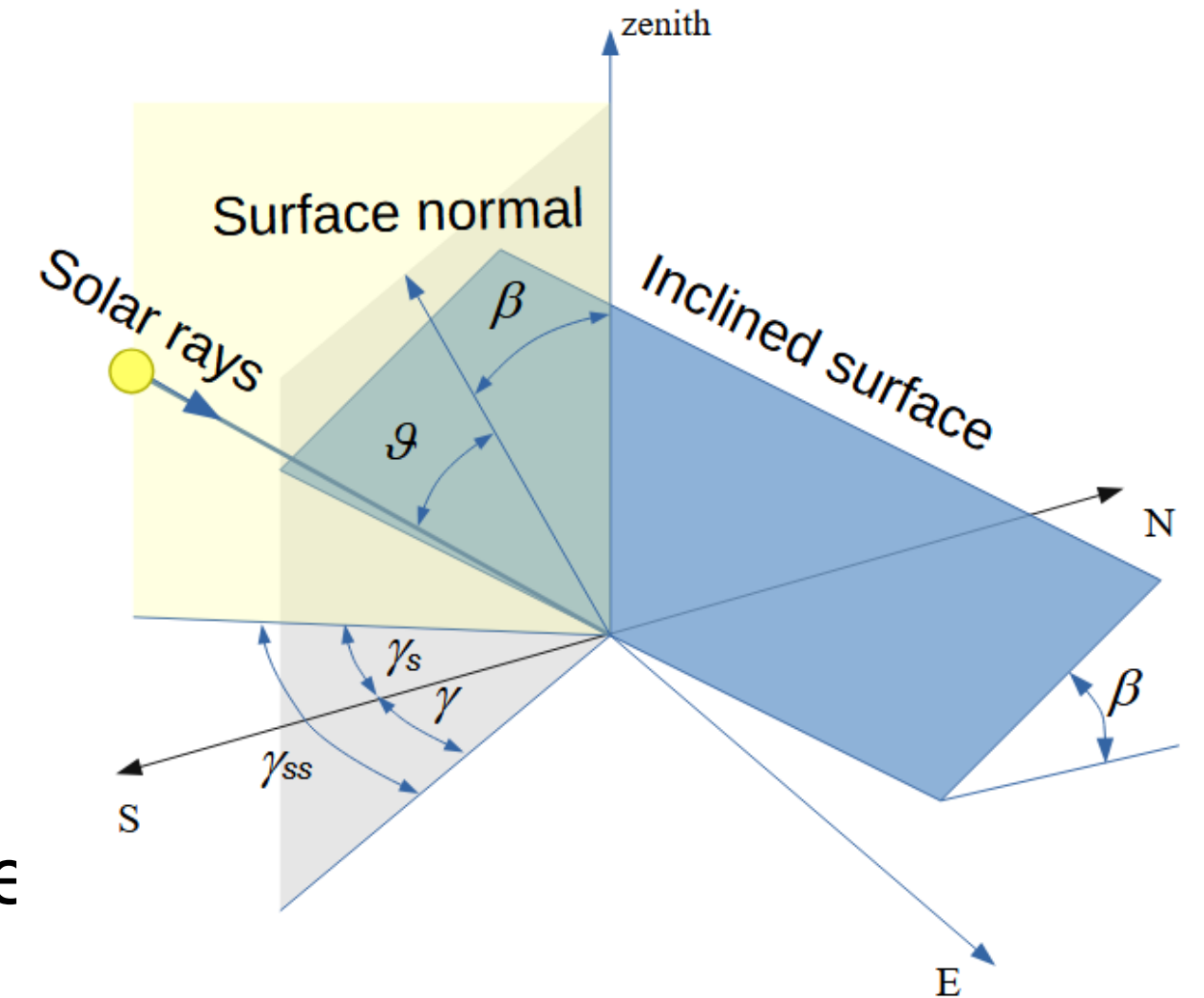
$$\cos \gamma_s = \frac{\sin \alpha \cdot \sin \varphi - \sin \delta}{\cos \alpha \cdot \cos \varphi}$$

$$\gamma_s = \text{sign}(\omega) \cdot \left| \arccos \left(\frac{\sin \alpha \cdot \sin \varphi - \sin \delta}{\cos \alpha \cdot \cos \varphi} \right) \right|$$



Surface and solar radiation

- Tilt angle β is the angle between the surface and the horizontal plane. Its value lies between 0 and 180°.
- The surface azimuth γ is defined as the displacement from south of the projection, on the horizontal plane, of the normal to the surface, negative if faces east.
- The surface-solar azimuth angle γ_{ss} is defined as the angular difference between the solar azimuth γ_s and the surface azimuth γ
- Values of γ_{ss} greater than 90° or less than -90° indicate that the surface is in the shade.



$$\gamma_{ss} = |\gamma - \gamma_s|$$



Angle of incidence

- angle between the line normal to the irradiated surface and the earth-sun line ϑ

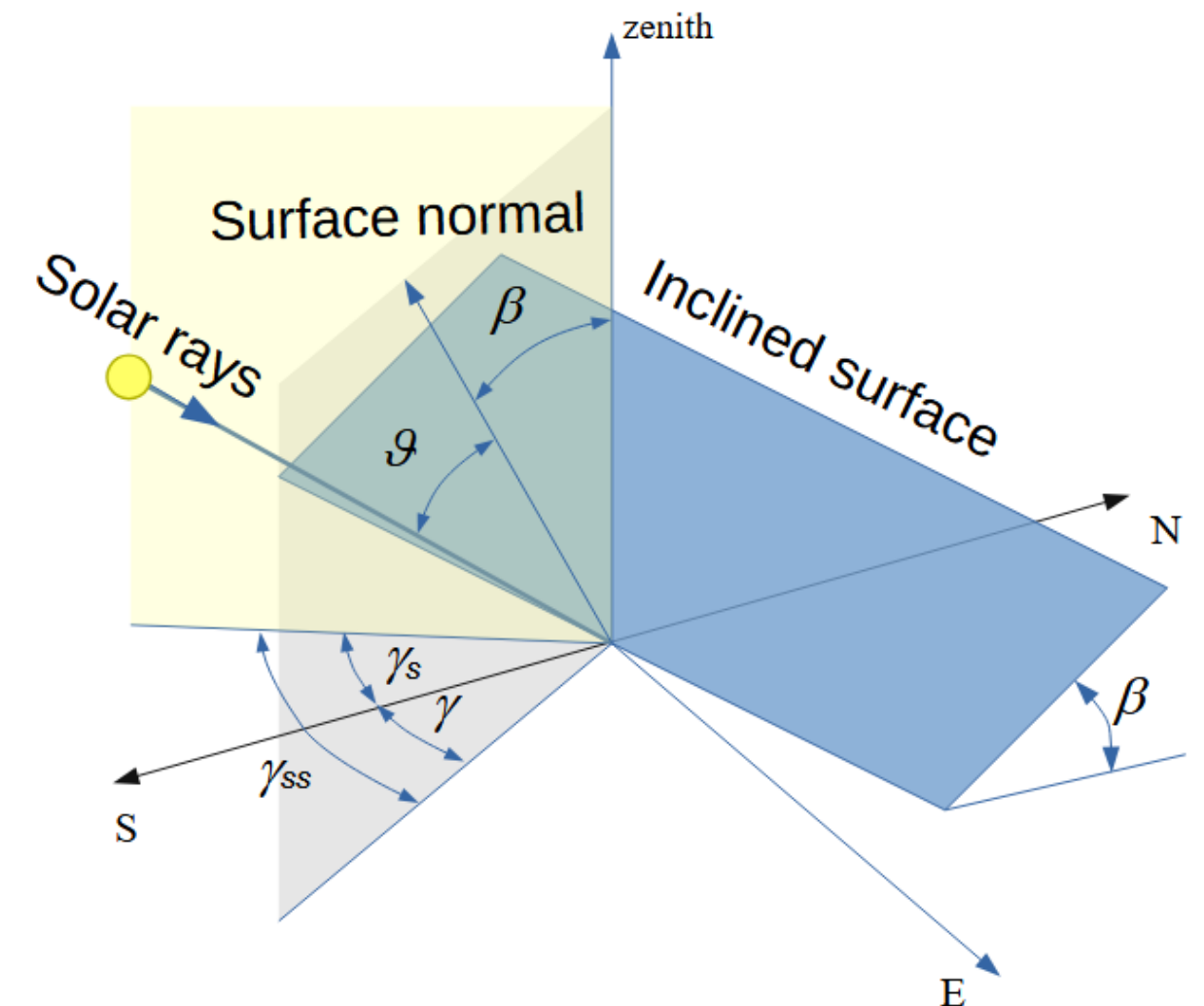
$$\cos \vartheta = \cos \alpha \cdot \cos \gamma_{ss} \cdot \sin \beta + \sin \alpha \cdot \cos \beta$$

- For vertical surfaces

$$\cos \theta = \cos \alpha \cdot \cos \gamma_{ss}$$

- Horizontal

$$\theta = 90 - \alpha$$





External solar radiation

- External normal solar G_0 radiation can be computed as:

$$G_0 = G \times [a_0 + a_1 \times \cos(\Omega) + a_2 \times \sin(\Omega) + a_3 \times \cos(2\Omega) + a_4 \times \sin(2\Omega)]$$

$$a_0 = 1,000110$$

$$a_1 = 0,034221$$

$$a_2 = 0,001280$$

$$a_3 = 0,000719$$

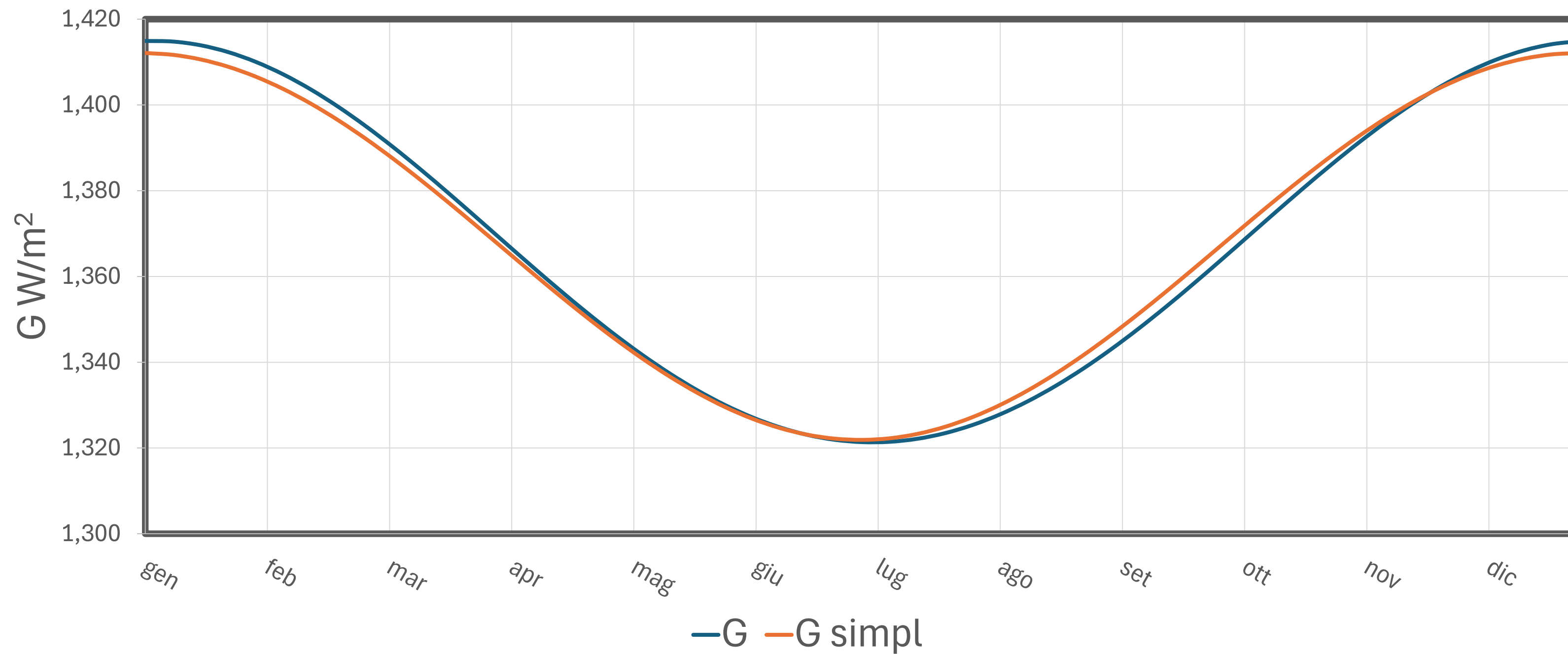
$$a_4 = 0,000077$$

- solar constant, $G = 1367 \frac{\text{W}}{\text{m}^2}$
- simplified formula $G_0 = G \cdot \left[1 + 0.033 \cdot \cos\left(\frac{2 \cdot \pi}{365}\right) \right]$
- External global radiation on a plane parallel to the horizontal is computed as

$$I_{h0} = G_0 \times \sin(\alpha)$$



External solar radiation





Solar radiation

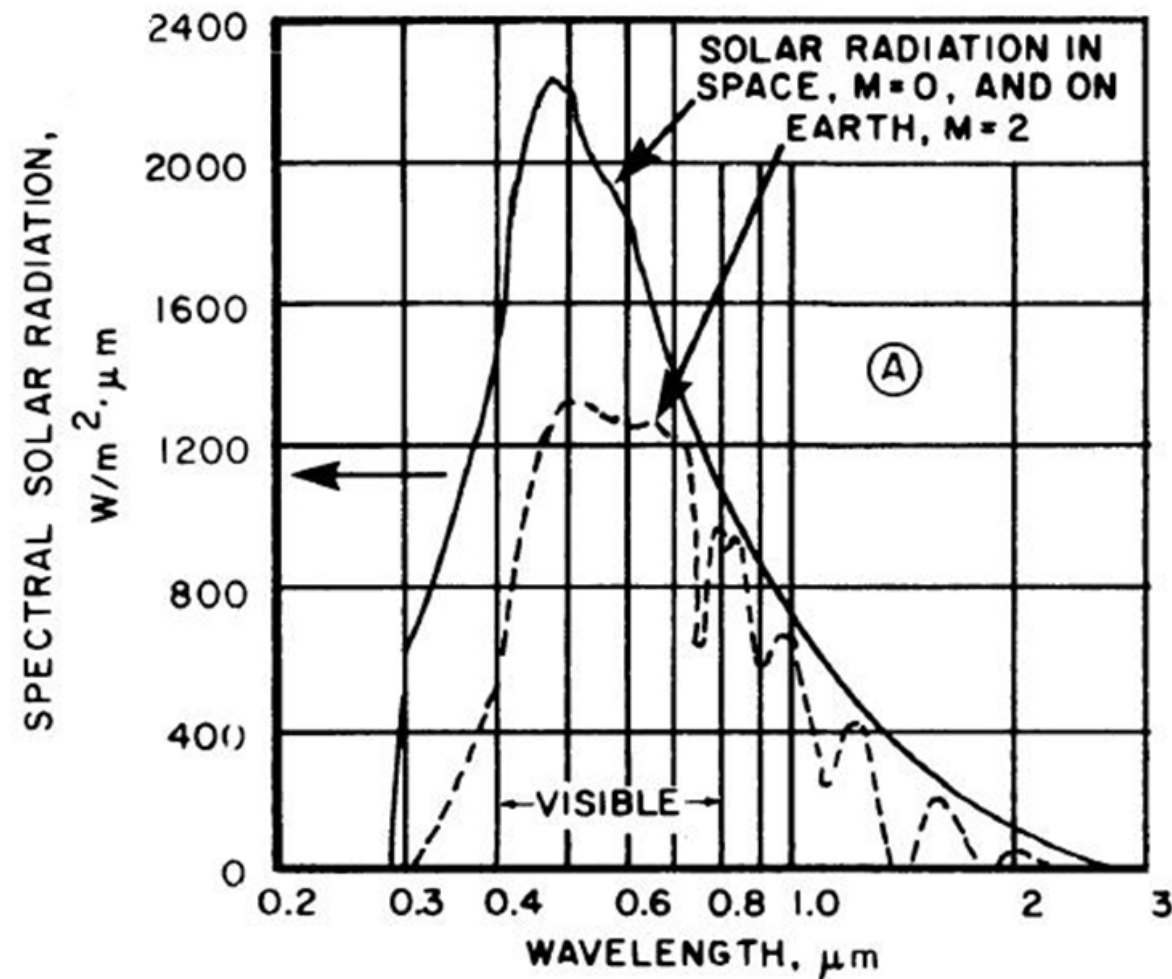


Fig. 7 Terrestrial and Extraterrestrial Solar Spectral Irradiances

- Solar radiation is absorbed
- The spectrum changes
- Solar radiation reaches the ground with two components
- Direct component
- Diffuse component



Air Mass

- The relative air mass m is the ratio of the mass of atmosphere in the actual earth/sun path to the mass that would exist if the sun were directly overhead. Air mass is solely a function of solar altitude and is obtained from

$$m = \frac{1}{[\sin \alpha + 0.50572 \cdot (6.07995 + \alpha)^{-1.6364}]}$$

α in degrees



Clear-Sky Solar Radiation

- Solar radiation on a clear day is defined with direct (beam) and diffuse components
- $E_b = E_0 \cdot \exp(-\tau_b \cdot m^{ab})$
- $E_d = E_0 \cdot \exp(-\tau_d \cdot m^{ad})$
- E_b normal irradiance measured in the direction of sun rays
- E_d diffuse radiation on a horizontal plane
- m air mass
- τ_b and τ_d beam and diffuse optical depth



Air mass exponents

$$ab = 1.219 - 0.043 \cdot \tau_b - 0.151 \cdot \tau_d - 0.204 \cdot \tau_b \cdot \tau_d$$

$$ad = 0.202 - 0.852 \cdot \tau_b - 0.007 \cdot \tau_d - 0.357 \cdot \tau_b \cdot \tau_d$$

		Annual	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Clear Sky Solar Irradiance	taub		0.344	0.369	0.417	0.461	0.465	0.476	0.465	0.456	0.44	0.419	0.376	0.34
	taud		2.401	2.32	2.196	2.101	2.14	2.159	2.198	2.238	2.247	2.27	2.336	2.43
	Ebn at noon		748	795	802	799	809	800	804	795	771	725	694	711
	Edn at noon		74	97	125	150	149	147	140	129	118	101	79	66



Fenestration

- *Clear plate or sheet glass or plastic.* Clear plate glass permits good visibility and transmits more solar radiation than other types.
- *Tinted heat-absorbing glass.* Tinted heat-absorbing glass is fabricated by adding small amounts of selenium, nickel, iron, or tin oxides. These produce colors from pink to green, including gray or bluish green, all of which absorb infrared solar heat and release a portion of this to the outside atmosphere through outer surface convection and radiation. Heat-absorbing glass also reduces visible light transmission.
- *Insulating glass.* Insulating glass consists of two panes—an outer plate and an inner plate—or three panes separated by metal, foam, or rubber spacers around the edges and hermetically sealed in a stainless-steel or aluminum-alloy structure. The dehydrated space between the glass panes usually has a thickness of 0.125 to 0.75 in. (3.2–19 mm) and is filled with air, argon, or other inert gas. Air- or gas-filled space increases the thermal resistance of the fenestration.
- *Reflective coated glass.* Reflective glass has a microscopically thin layer of metallic or ceramic coating on one surface of the glass, usually the inner surface of a single-pane glazing or the outer surface of the inner plate for an insulating glass. For a single pane, the coating is often protected by a layer of transparent polyester. The chromium and other metallic coatings give excellent reflectivity in the infrared regions but reduced transmission of visible light compared to clear plate and heat-absorbing glass. Reflections from buildings with highly reflective glass may blind drivers, or even kill grass in neighboring yards.
- *Low-emissivity (low-E) glass coatings.* Glazing coated with low-emissivity, or low-E, films has been in use since 1978. It is widely used in retrofit applications. A low-emissivity film is usually a vacuum-deposited metallic coating, usually aluminum, on a polyester film, at a thickness of about 4 10⁻⁷ in. (0.01 μm).

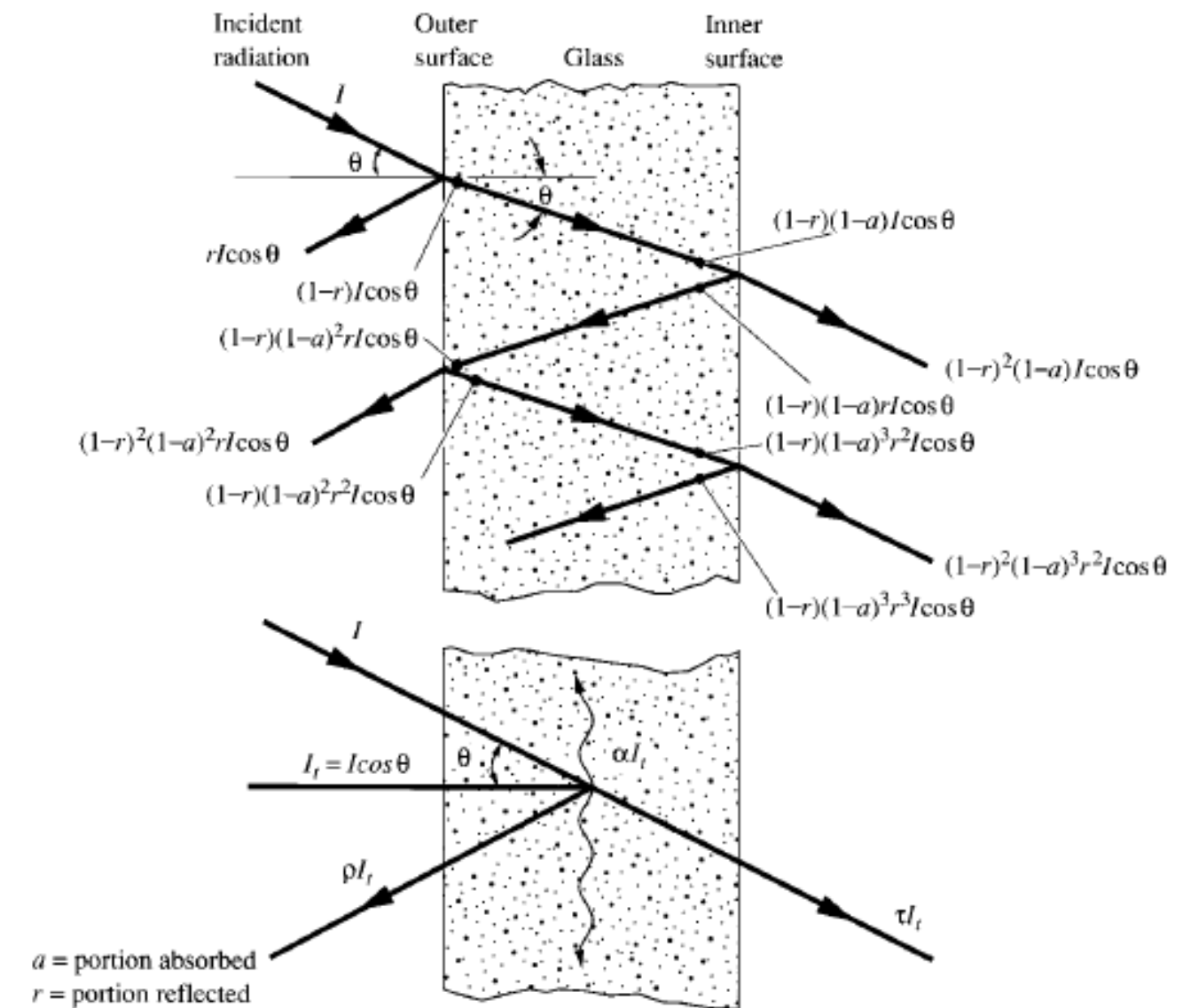


Optical properties

- Solar radiation is
 - Transmitted
 - Reflected
 - Absorbed

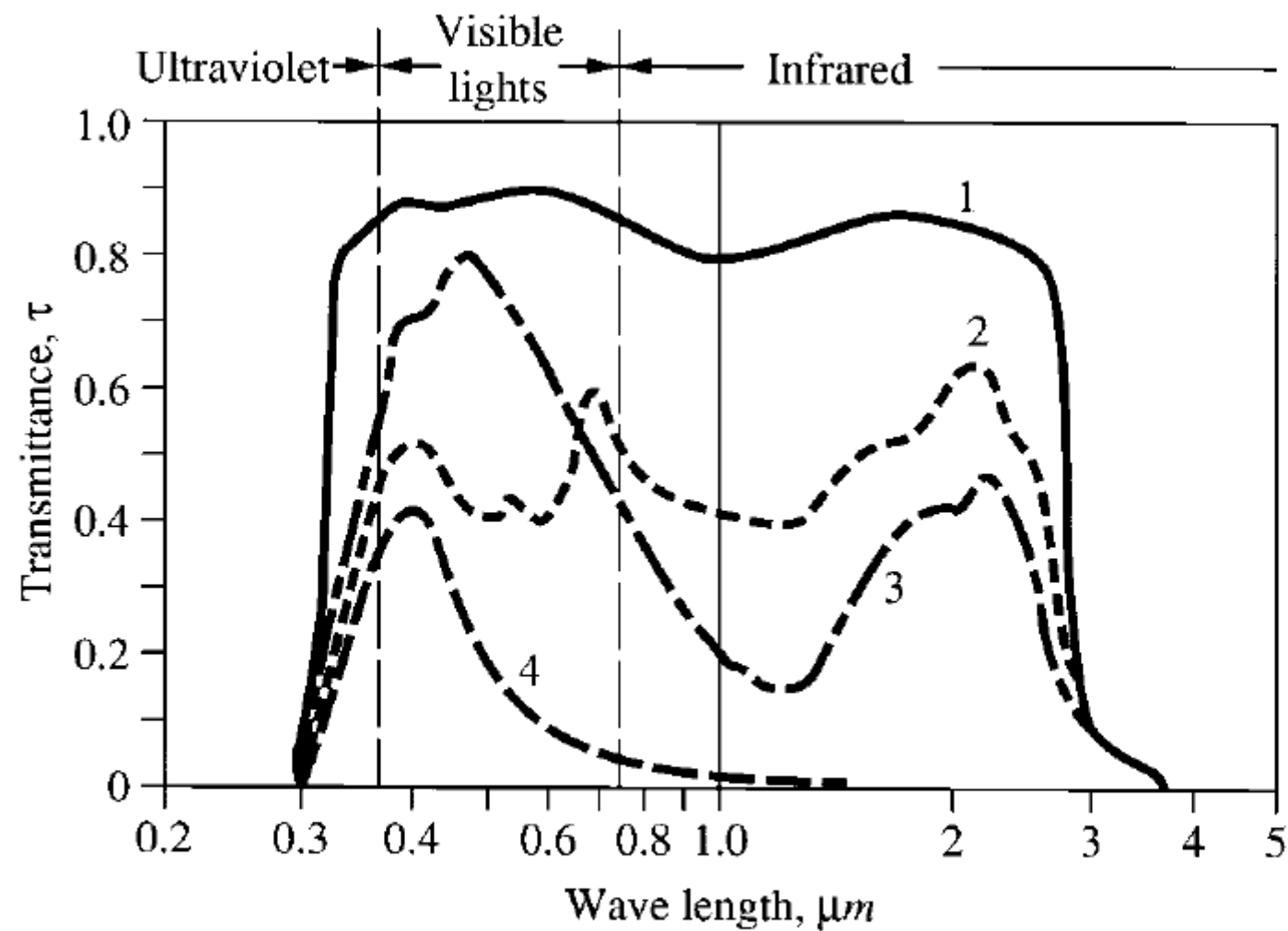
$$\tau + \alpha + \rho = 1$$

$$I \cdot \tau + I \cdot \alpha + I \cdot \rho = I$$





Spectral transmittance of window glasses



- | | |
|-----------------------|-------------------------------|
| 1 Clear plate | 3 Bluish-green heat absorbing |
| 2 Gray heat absorbing | 4 Reflective coating |

- Different glasses perform in different way
- Each glass has a spectral transmittance
- Spectral transmittance can be modified also using films



Heat through windows

- *Heat gain through window = solar radiation transmitted + inward heat flow from glass inner surface*

$$\frac{Q_{wi}}{A_s} = \frac{\tau \cdot I_t + Q_{RCi}}{A_s}$$

- Q_{RCi} inward heat flow from inner surface



Single glazing

- Q_{RCi} inward absorbed radiation + conductive heat transfer

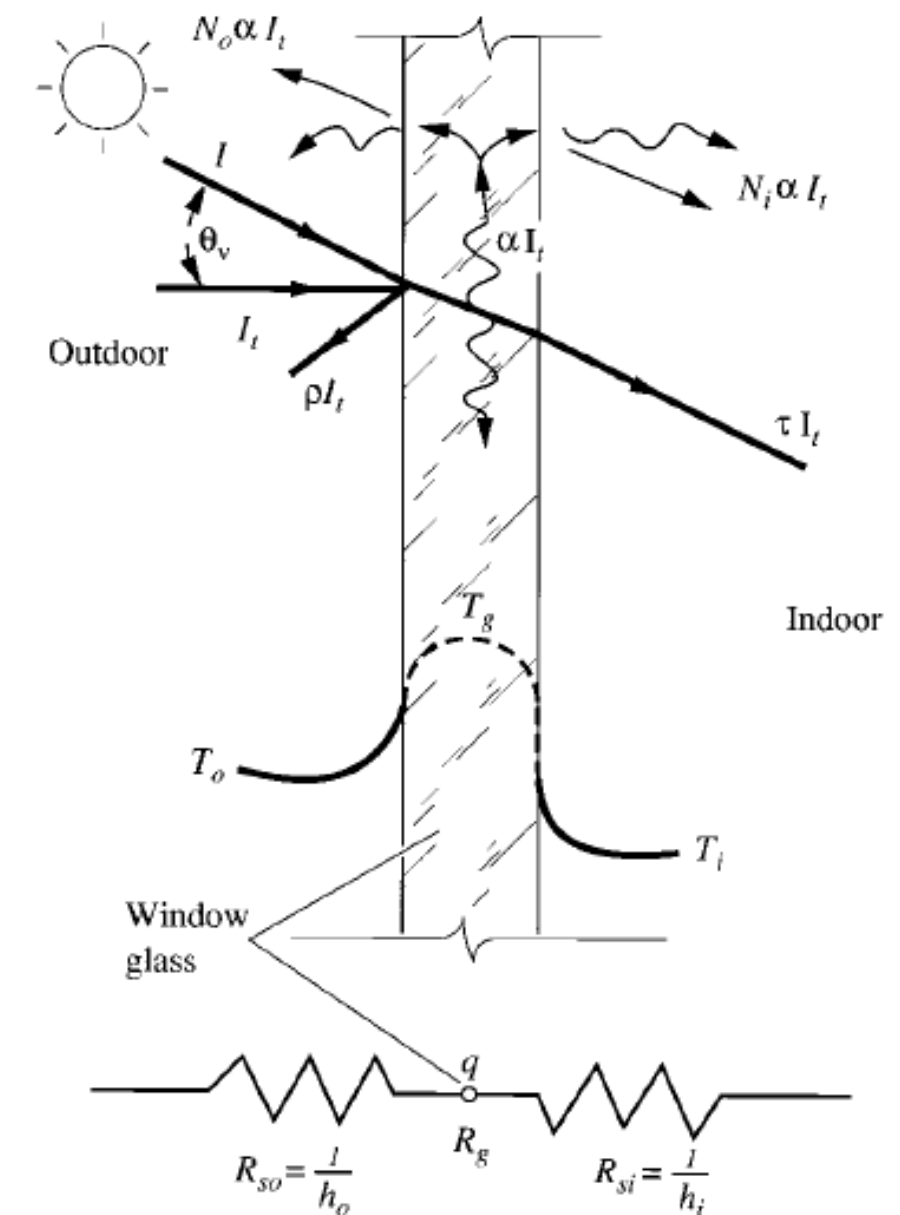
$$Q_{RCi} = U \cdot A_s \cdot \left(\frac{\alpha \cdot I_t}{h_o} + T_o - T_i \right)$$

- Heat admitted through a unit area of the single-glazing

$$\frac{Q_{wi}}{A_s} = \tau I_t + U \cdot \left(\frac{\alpha \cdot I_t}{h_o} + T_o - T_i \right)$$

- Solar heat gain coefficient (SHGC) ratio of solar heat gain entering the space to the incident solar radiation

$$SHGC = \frac{Q_{ws}}{I_t \cdot A_s} = \tau + \frac{U \cdot \alpha}{h_o}$$





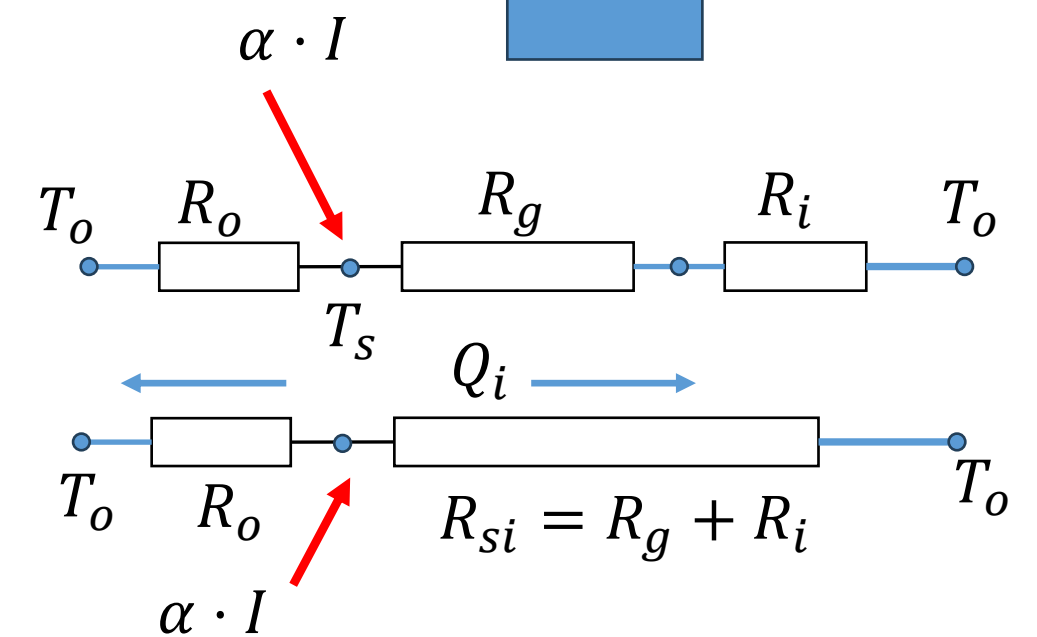
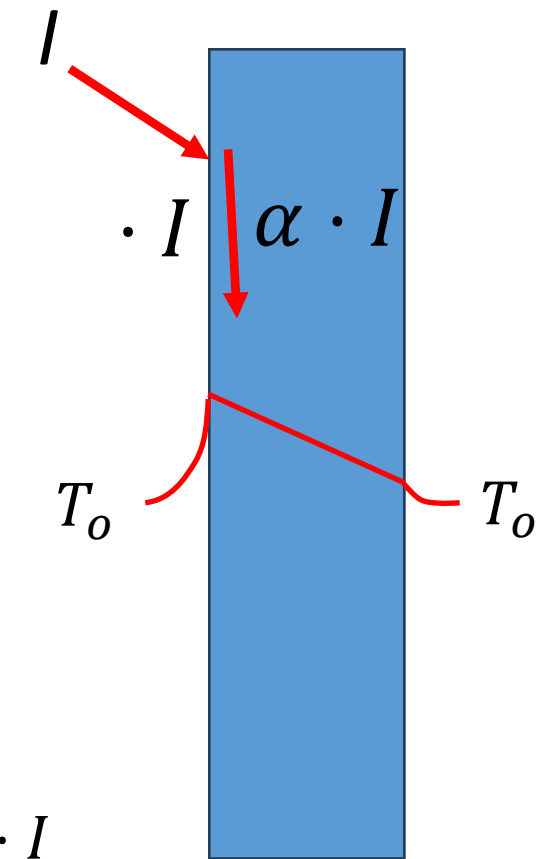
Radiation absorbed and transferred at the internal side

$$Q_o = \frac{T_s - T_o}{R_o} \quad Q_i = \frac{T_s - T_o}{R_{tot} - R_o} = \frac{T_s - T_o}{R_{si}} \quad \alpha \cdot I_t = Q_o + Q_i$$

$$\alpha \cdot I_t = \frac{T_s - T_o}{R_o} + \frac{T_s - T_o}{R_{si}} = (T_s - T_o) \cdot \left(\frac{1}{R_o} + \frac{1}{R_{si}} \right) = (T_s - T_o) \cdot \left(\frac{R_o + R_{si}}{R_o \cdot R_{si}} \right)$$

$$Q_i \cdot R_{si} = T_s - T_o \quad \alpha \cdot I_t = Q_i \cdot \cancel{R_{si}} \cdot \left(\frac{R_o + \cancel{R_{si}}}{R_o \cdot \cancel{R_{si}}} \right)$$

$$Q_i = \alpha \cdot I_t \cdot \frac{R_o}{R_o + R_{si}} \quad Q_i = \alpha \cdot I_t \cdot \frac{R_o}{R_{tot}} = \alpha \cdot I_t \cdot R_o \cdot U$$



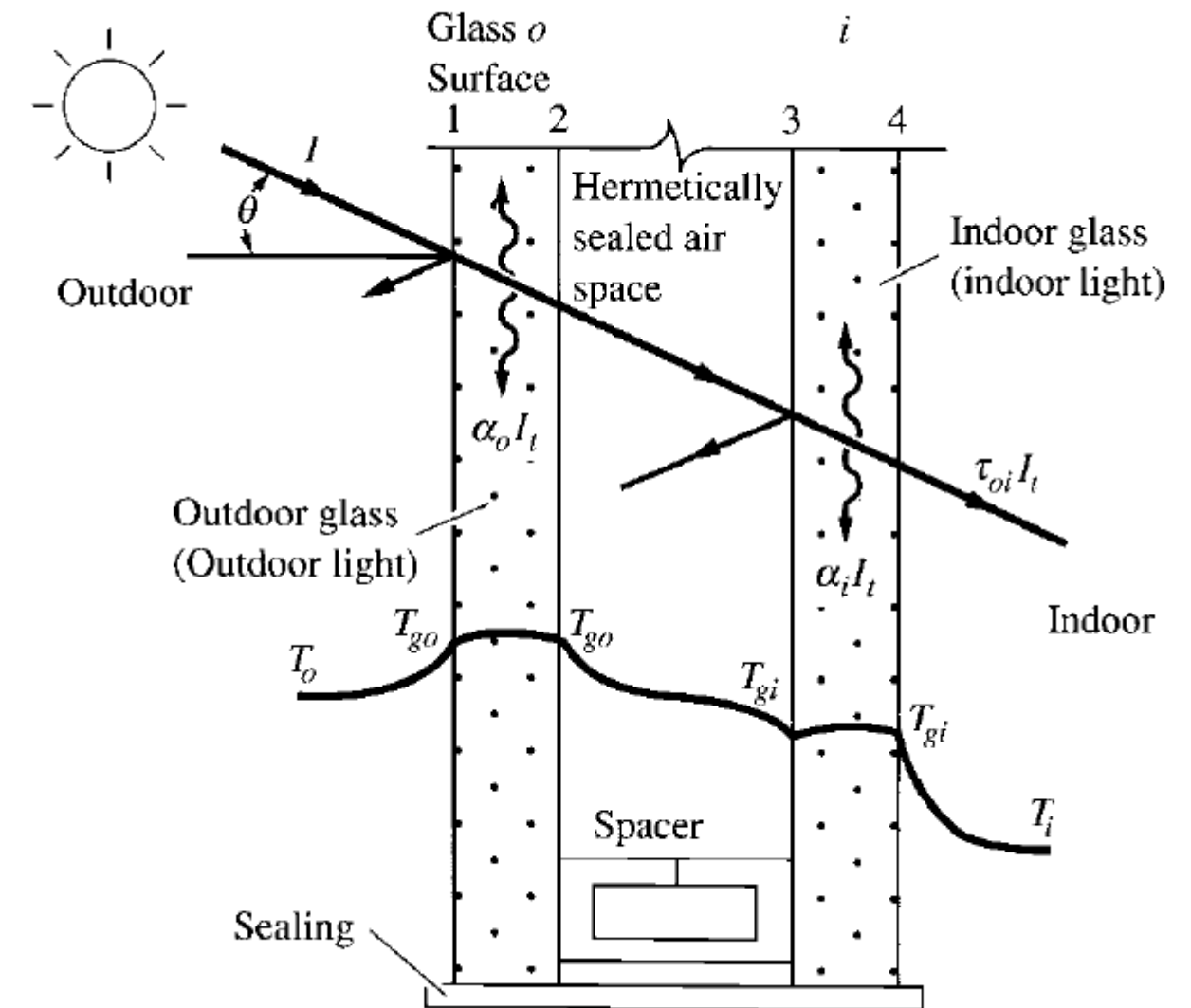


Double Glazing

$$\frac{Q_{woi}}{A_s} = \tau_{oi} \cdot I_t + U \cdot \left(\frac{\alpha_o}{h_o} + \frac{\alpha_i}{h_o} + \frac{\alpha_i}{h_o} \right) + I_t + U \cdot (T_o - T_i)$$

$$\frac{Q_{woi}}{A_s} = SHGC_{oi} \cdot I_t + U \cdot (T_o - T_i)$$

$$\tau_{oi} = \frac{\tau_o \cdot \tau_i}{1 - \rho_2 \cdot \rho_3}$$





Shading Coefficient

- The Shading Coefficient SC is defined as the solar heat gain of the specified glass over the shading coefficient of a double-strength sheet glass

$$SC = \frac{SHGC_W}{SHGC_{DSA}} = \frac{SHGC_W}{0.87} = 1.15 \cdot SHGC_W$$



Shading Coefficients

Type of glass	Thickness of glass, in.	Solar transmittance		Glass	Venetian blinds		Roller shade		Draperies	
					Med.*	Light†	Opaque, white	Translucent	Med.‡	Light§
Clear	$\frac{3}{32}$	0.87 to 0.79			0.74	0.67	0.39	0.44	0.62	0.52
Heat-absorbing	$\frac{3}{16}$ or $\frac{1}{4}$ $\frac{3}{8}$	0.46			0.57	0.53	0.30	0.36	0.46	0.44
		0.34			0.54	0.52	0.28	0.32		
		0.24			0.42	0.40	0.28	0.31	0.38	0.36
Reflective-coated				0.30	0.25	0.23				
				0.40	0.33	0.29				
				0.50	0.42	0.38				
Insulating glass		Outer	Inner							
Clear out	$\frac{3}{32}$, or $\frac{1}{8}$	0.87	0.87		0.62	0.58	0.35	0.40		
Clear in										
Heat-absorbing out	$\frac{1}{4}$	0.46	0.80		0.39	0.36	0.22	0.30		
Clear in										
Reflective glass				0.20	0.19	0.18				
				0.30	0.27	0.26				
				0.40	0.34	0.33				

*Med. indicates medium color.

†Light indicates light color.

‡Draperies Med. represents draperies of medium color with a fabric openness of 0.10 to 0.25 and yarn reflectance of 0.25 to 0.50.

§Draperies Light represents draperies of light color with a fabric openness below 0.10 and yarn reflectance over 0.50.



Shading of fenestration

- Effect of a setback

$$S_W = P_V \cdot \tan \gamma$$

$$S_H = P_H \cdot \tan \Omega = P_H \cdot \tan \beta / \cos \gamma$$

- Projection factor F_{pro}

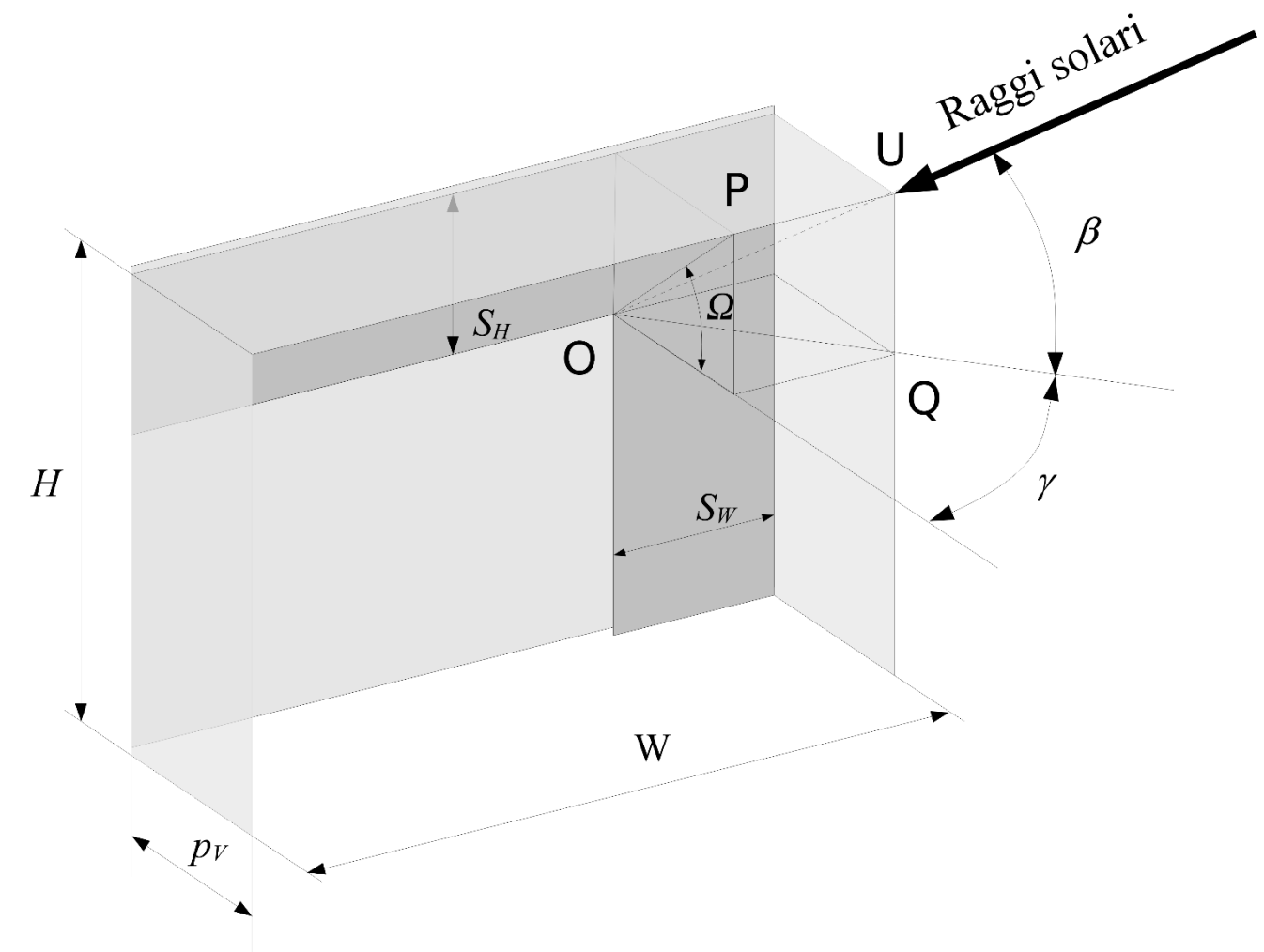
$$F_{pro} = \frac{P_H}{H}$$

- Net sunlit area

$$A_S = (W - S_W) \cdot (H - S_H)$$

- Shaded area

$$A_{Sh} = A_W - A_S$$





Split methods

- Dynamic building energy simulations require detailed environmental data such as temperature, humidity, wind velocity and direction, usually available from a number of climatic stations.
- However, direct normal radiation (DNI) and diffuse horizontal radiation are also required in order to compute solar load.
- Unfortunately, continuous records of DNI are scarce due to the cost of the equipment: the monitoring stations equipped with solar trackers are very rare.
- An intermediate solution is to record diffuse and global irradiance, but there are numerous locations around the world where only global insolation has been monitored. Therefore, a great number of climatic data report only this value.
- Therefore often the only possible way is to detect the global radiation and then to use solar radiation splitting algorithms to obtain its components.



Pyranometer with shadow ring



- Pyranometer measure the irradiance on a flat surface
- an adjustable shadow ring for the measurement of diffuse radiation only.
- The band must be adjusted for solar incidence
- The band could be automated for remote stations



Splitting Models

- In literature, more than 150 models have been developed. Nevertheless, defining a universal model able to provide the best possible result at any specific location is complex.
- Three different types of models can be considered: polynomial, exponential and logistic models. All of these categories use predictors, intended as a measurement or an evaluated variable, which is required for applying the model.
- In all the models that will be described the clearness index k_t is used as predictor. The diffuse fraction k_d instead is the model outcome.

$$\bullet \quad k_t = \frac{I_h}{I_{h0}} \qquad k_d = \frac{I_{h,d}}{I_h}$$

- Where: I_h = global solar radiation on horizontal plane
- I_{h0} = extraterrestrial solar radiation on horizontal plane
- $I_{h,d}$ = diffuse solar radiation on horizontal plane



The Oliveira Model

- The Oliveira model can be illustrated with a conditional equation where k_d varies with a fourth degree polynomial curve when k_t is lower than 0.75 and higher than 0.17, otherwise it is set to a constant value.

$$\bullet k_d = \begin{cases} k_t = 0: & 0 \\ 0 < k_t \leq 0.17: & 1 \\ 0.17 < k_t \leq 0.75: & 0.97 + 0.8 * k_t - 3 * k_t^2 - 3.11 * k_t^3 + 5.2 * k_t^4 \\ k_t > 0.75: & 0.18 \end{cases}$$



The Torres Model

- Similarly to the Oliveira model, the Torres model can be illustrated with a conditional equation where k_d varies with a fourth degree polynomial curve within a slightly different clearness index range: when k_t is lower than 0.755 and higher than 0.225. If k_t is less than 0.225, it uses a low slope linear curve, while k_d is constant when k_t is higher than 0.755.

- $k_d =$

$k_t = 0:$	0
$0 < k_t \leq 0.225:$	$0.9943 - 0.1165 * k_t$
$0.225 < k_t \leq 0.755:$	$1.4101 - 2.9918 * k_t + 6.4599 * k_t^2 - 10.329 * k_t^3 + 5.514 * k_t^4$
$k_t > 0.755:$	0.18



The Al Riahi Model

- The Oliveira, the Torres and the Al Riahi models share the same conditional form: unlike the two above mentioned, a linear equation is used when k_t is between 0.25 and 0.7, otherwise it is set constant.

$$\bullet k_d = \begin{cases} k_t = 0: & 0 \\ 0 < k_t \leq 0.25: & 0.932 \\ 0.25 < k_t \leq 0.70: & 1.293 - 1.631 * k_t \\ k_t > 0.70: & 0.151 \end{cases}$$



The Boland Model

- The Boland splitting method has been introduced in Italian standard, it uses two coefficients

$$a = -5.0033$$

$$b = 8.6025$$

- $k_d = \frac{1}{1 + e^{a + b \times k_T}}$