Chapter Outline

5. Wind-Generated Waves

- Wind-Wave Generation and Decay
- Wave Record Analysis for Height and Period
- Wave Spectral Characteristics
- Wave Spectral Models

- Wave Prediction-Early Methods
- Wave Prediction-Spectral Models

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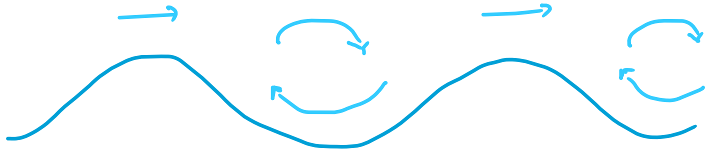
Wind-Wave Generation and Decay

Wind blowing over the surface of a water body will transfer energy to the water.
How does a horizontal wind initiate the formation of waves on an initially flat water surface?

This process is best explained by a resonance model proposed by **Phillips** (1957, 1960):
There are turbulent eddies in the wind field that exert a fluctuating pressure on the water surface. These pressure fluctuations vary in magnitude and frequency and they move forward at a range of speeds. The pressure fluctuations cause water surface undulations to develop and grow. The key to their growth is that a resonant interaction occurs between the forward moving pressure fluctuations and the free waves that propagate at the same speed as the pressure fuctuations.

Although the Phillips model explains the initiation of wave motion, it is insufficient to explain the continued growth of the waves. This growth is best explained by a shear flow model proposed by **Miles** (1957):
Below a point on the velocity profile where the wind velocity equals the wave celerity, air flow is reversed relative to the forward moving wave profile. This results in a flow circulation and a momentum transfer to the wave that selectively amplifies the steeper waves.



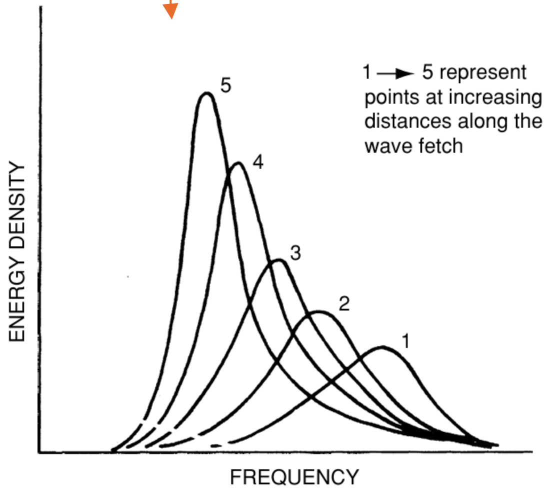
Wind-Wave Generation and Decay

- As the waves propagate through the area where the wind is acting, they grow in average height and period.

- After leaving the area of active wind generation the surface profiles become smoother and the crests become longer and more easily recognized.

  → These freely propagating waves are commonly called **swell**. As swell propagate their average height decreases some what owing to air resistance and internal friction, but more importantly because of angular speading of the wave field.

Note that crests propagate in a range of directions around the dominant wind direction.



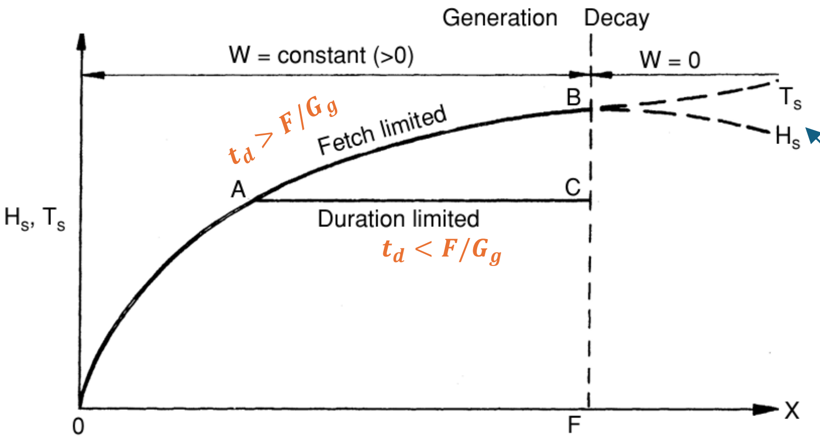
Wind-Wave Generation and Decay

If the wave heights from a wave record are ordered by size one can define **H_{33}** which is the average height of the highest one-third of the waves. This is commonly called the **significant height Hs** and it is approximately the height an experienced observer will report when visually estimating the height of waves at sea [note that H_{100} is the average wave height].

At the same way we define a **significant period Ts** as the average period of the highest one-third of the waves in the record.

The significant wave height and period as well as the resulting spectrum of wind-generated waves depend primarily on the distance over which the wind blows (known as the fetch length F), the wind velocity W (commonly measured at the 10 m elevation), and the duration of the wind td.

There is a fixed limit to which the average height and period can grow. At this limiting condition the rate of energy input to the waves from the wind is balanced by the rate of energy dissipation because of wave breaking and surface water turbulence.
This condition, which is known as a fully developed sea, is usually not reached even in large storms.



Outside of the region where the wind is blowing the **waves propagate as swell**. In this region the **significant height will decrease** and the **significant period will increase**. Energy dissipation and lateral spreading of the waves will decrease the wave height.

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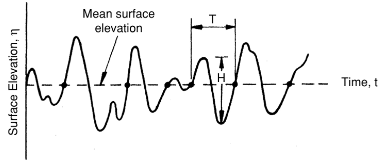
Wave Record Analysis for Height and Period

Wind-generated waves are much more complex than the simple monochromatic waves considered to this point.

It is important to be able to predict these waves for a given wind condition.

A water surface time history measured at a point in a storm would show an irregular trace →



A wave record taken at the same time at a nearby location would be significantly different but would have similar statistical properties

As the (1) wind velocity **W**, (2) distance or fetch **F** over which the wind blows, and/or (3) duration of the wind **td** increase, the average height and period of the resulting downwind waves will increase (within limits).

Our understanding of wind-generated waves at sea comes largely from the **analysis of wave records**. Most of these wave records are point measurements of the water surface time history for a time period of several minutes.

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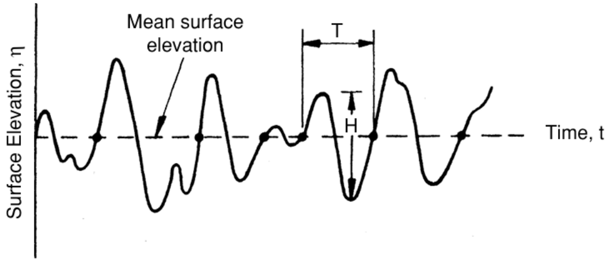
Wave Record Analysis for Height and Period

Analysis of wave records is done:
(1) by identifying individual waves in the record and statistically analyzing the heights and periods of these individual waves and
(2) by conducting a Fourier analysis of the wave record to develop the wave spectrum.

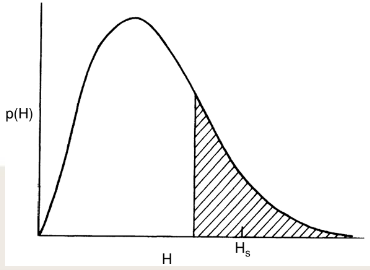
(1) statistically analyzing the heights and periods

The most commonly used analysis procedure is the zero-upcrossing method (Pierson, 1954). A mean water surface elevation is determined and each point where the water surface crosses this mean elevation in the upward direction is noted



The time elapsed between consecutive points is a *wave period* and the *maximum vertical distance between crest and trough is a wave height*. Note that some small surface undulations are not counted as waves so that some higher frequency components in the wave record are filtered out

We can build the p(H) = probability of occurrence of the height H.



Wave Record Analysis for Height and Period

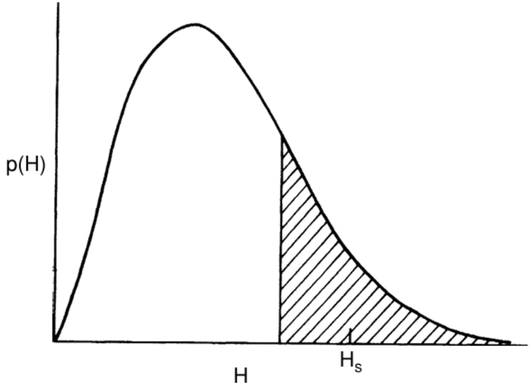
(1) statistically analyzing the heights and periods

We can build the p(H) = probability of occurrence of the height H.

Rayleight proposed the distribution $p(H) = \frac{2H}{(H_{\text{rms}})^2} e^{-(H/H_{\text{rms}})^2}$

Being $H_{\text{rms}} = \sqrt{\sum \frac{H_i^2}{N}}$

Hi are the individual wave heights in a record containing N waves.



Employing the **Rayleigh distribution** leads to the following useful relation ships: $Hs = 1.416\, H_{rms}, \quad H_{100} = 0.886\, H_{rms}$

The cumulative probability distribution P(H) (i.e., the percentage of waves having a height that is equal to or less than H) is:

$$P(H) = \int_o^H p(H)dH = 1 - e^{-(H/H_{\text{rms}})^2}$$

→ We are more interested in the percentage of waves that have a height greater than a given height: $1 - P(H) = e^{-(H/H_{\text{rms}})^2}$

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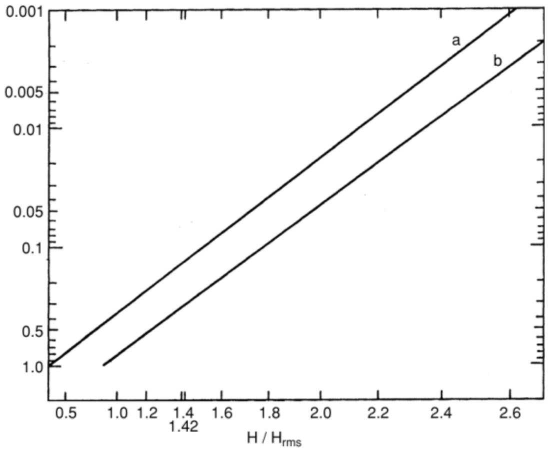
Wave Record Analysis for Height and Period

(1) statistically analyzing the heights and periods

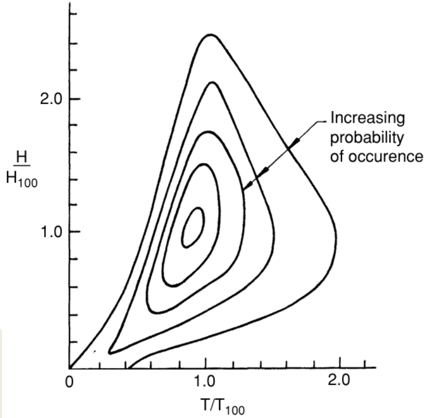
There is no upper limit to the wave heights defined by the Rayleigh distribution. In a storm, however, the highest wave that might be expected will depend on the length of the storm as well as its strength:

$$H_{\max} = 0.707\ H_s \sqrt{\ln N}$$

For example, a storm having a 6 hour duration of high waves having an average period of 8s would about 2700 waves and H_{max} would be 1.99Hs.



Line a gives the probability P that any wave height will exceed the height (H/Hrms)
Line b gives the average height of the n highest fraction of the waves.



This figure shows the distribution of the **wave height** versus **wave period** for each wave in a typical record, nondimensionalized by dividing each height and period value by the average height and average period, respectively. **The contour lines are lines of equal probability** of occurrence of a height–period combination.

Wave Spectral Characteristics

(2) Fourier analysis of the wave record

An alternate form to (1) is the period spectrum where the wave energy density S is plotted versus the wave period (f=1/T) and direction θ. From the small-amplitude wave theory, the energy density in a wave is $\rho g H^2 / 8$, this leads to the following expression for a directional wave spectrum $S(f,\theta) = \sum_\theta \sum_f \frac{H^2}{8}$

Neglecting the dependency on wave direction, the general form of the spectrum has the form $S(f) = \frac{A}{f^5} e^{-B/f^4}$

where **A and B adjust the shape and scale** of the spectrum and can be written either as a function of the generating factors (W, F) or as a function of a representative wave height and period (Hs, Ts).

The nth moment of a spectrum is defined $m_n = \int_0^\infty S(f) f^n df$

So, for example, the **zeroth moment m_0** would just be the area under the spectral curve.

It leads to a useful fact: wave height and period for the wave spectrum that can be derived from the spectrum $H_s = 4\sqrt{m_0}$

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Wave Spectral Characteristics

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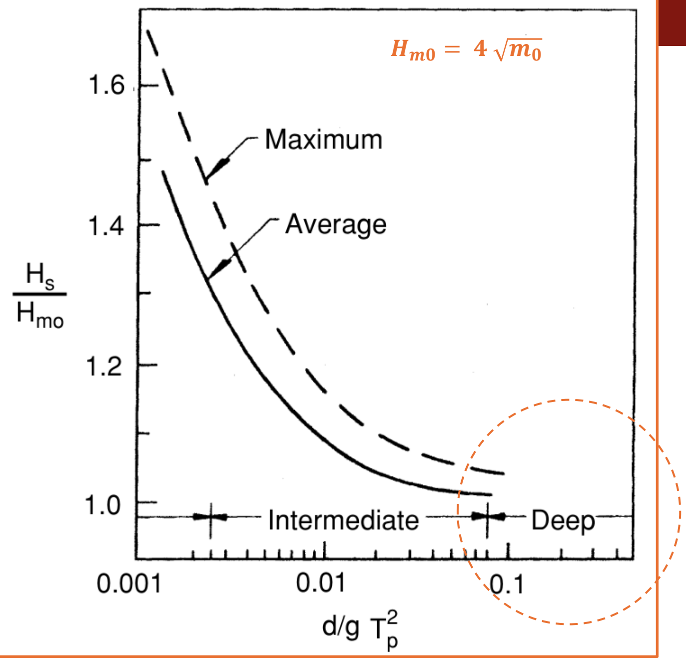

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Wave Spectral Models

(2) Fourier analysis of the wave record

Spectral Models are derived from empirical fits to selected sets of wave measurements, supported by dimensional and theoretical reasoning. Most common, are four: Bretschneider, Pierson–Moskowitz, **JONSWAP**, and TMA spectra.

$$S(T) = \frac{\alpha g^2}{(2\pi)^4} T^3 e^{-0.675(gT/2\pi W F_2)^4}$$

Bretschneider

$$S(f) = \frac{\alpha g^2}{(2\pi)^4 f^5} e^{-0.74(g/2\pi W f)^4}$$

Pierson–Moskowitz

$$S(f) = \frac{\alpha g^2}{(2\pi)^4 f^5} e^{-1.25(f_p/f)^4} \gamma^a$$

JONSWAP

$$S(f)_{\mathrm{TMA}} = S(f)_J \Phi(f, d)$$

TMA

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Wave Spectral Models

(2) Fourier analysis of the wave record

$$S(T) = \frac{\alpha g^2}{(2\pi)^4} T^3 e^{-0.675(gT/2\pi W F_2)^4}$$

Bretschneider

$$\alpha = 3.44 \frac{F_1^2}{F_2^2}$$

$$F_1 = \frac{g H_{100}}{W^2} \qquad F_2 = \frac{g T_{100}}{2\pi W}$$

Bretschneider empirically related **F1** and **F2** to the **wind speed W**, the **fetch F**, and the **wind duration td** to calculate the wave period spectrum – and to develop the forecasting relationship for the average wave height and period.

$$S(f) = \frac{\alpha g^2}{(2\pi)^4 f^5} e^{-0.74(g/2\pi W f)^4}$$

Pierson–Moskowitz

In **Pierson-Moskowitz** model he wind speed W is measured at an elevation of 19.5 m which yields a speed that is typically 5% to 10% higher than the speed measured at the standard elevation of 10 m. The coefficient α has a value of 8.1×10^{-3}.
Note that the fetch and wind duration are not included since this spectrum assumes a fully developed sea.

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Wave Spectral Models

(2) Fourier analysis of the wave record

$$S(f) = \frac{\alpha g^2}{(2\pi)^4 f^5} e^{-1.25(f_p/f)^4} \gamma^a$$

JONSWAP

$$a = e^{-[(f-f_p)^2/2\sigma^2 f_p^2]}$$

$$\sigma = 0.07 \text{ when } f < f_p$$

$$\sigma = 0.09 \text{ when } f \geq f_p$$

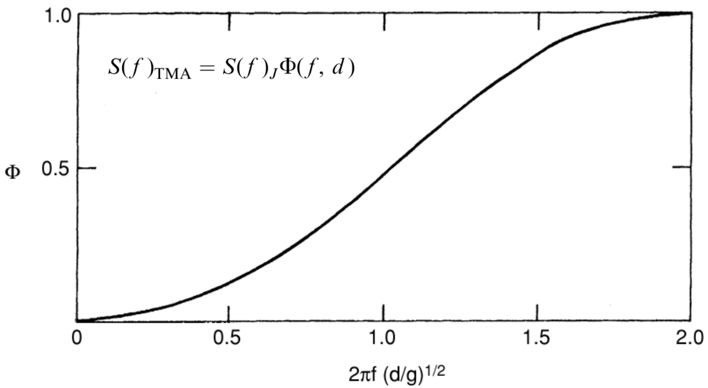
$$\alpha = 0.076\left(\frac{gF}{W^2}\right)^{-0.22}$$

$$f_p = \frac{3.5g}{W}\left(\frac{gF}{W^2}\right)^{-0.33}$$

This spectrum results from a **Jo**int **N**orth **S**ea **Wa**ve **P**roject operated by laboratories from four countries. Wave and wind measurements were taken with sufficient wind durations to produce a deep water fetch limited model spectrum.

In the JONSWAP spectrum, γ typically has values ranging from 1.6 to 6 but the value of 3.3 is recommended for general usage. The coefficient γ is simply the ratio of S(f) at the peak frequency for the JONSWAP and Pierson-Moskowitz spectra.

The previous three models were developed for deep water conditions. As wind waves propagate into intermediate and shallow depths there is a period-dependent change in the shape of the spectrum versus that for deep water.
The **TMA spectrum** is a wave spectrum based on the generation of waves in deep water that then propagate without refracting into intermediate/shallow water depths. The spectral form is a JONSWAP spectrum modified by a depth and frequency dependent factor $\phi(f, d)$.



→ Check the Matlab code Jonswap.m

Wave Spectral Models

(2) Fourier analysis of the wave record

Directional Wave Spectra

The components that make up a wave spectrum at a particular location will typically be propagating in a range of directions. A point measurement of the water surface elevation time history will not detect this directional variability, so an analysis of this time history yields a one-dimensional spectrum.
But, during recent years, wave gages that can detect the full directionality of the wave field at a given location have come into more common use. Consequently, directional spectral data are becoming available and significant development of directional spectral models has taken place.

Generally, the short period components of the wave spectrum have a wider range of directions, while the wave energy is more focused on the dominant direction for the frequencies near the spectral peak.

Simple idea: $S(f,\theta) = S(f)\, G(f,\theta)$
Note that modifying a one-dimensional spectrum to a directional spectrum does not change the total energy density, thus $\int_{-\pi}^{\pi} G(f,\theta)d\theta = 1$

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Wave Spectral Models

(2) Fourier analysis of the wave record

Directional Wave Spectra

Simple idea: $S(f,\theta) = S(f)\,G(f,\theta)$

One of the originally proposed (St. Dennis and Pierson, 1953) directional spreading functions was a simple cosine squared function that is independent of frequency:

$$G(f,\theta) = G(\theta) = \frac{2}{\pi}\cos^2\theta$$

The angle θ is usually measured clockwise starting at zero in the dominant wave direction and has a practical range of $-\pi/2$ to $+\pi/2$.

A much more complex directional spreading function (Mitsuyasu et al., 1975), which is based on extensive measurements of directional wave spectra:

$$G(f,\theta) = G(s)\cos^{2s}\left(\frac{\theta}{2}\right)$$

Γ is the gamma function which is tabulated in some mathematical handbooks.

$$G(s) = \frac{2^{2s-1}}{\pi}\frac{\Gamma^2(s+1)}{\Gamma(2s+1)}$$

The parameter **s** was originally given as a function of wave frequency, wave peak frequency, and wind speed.

$$s = S_{\max}(f/f_p)^5 \text{ when } f < f_p$$
$$= S_{\max}(f/f_p)^{-2.5} \text{ when } f > f_p$$

$S_{\max} = 10$ Wind waves
$S_{\max} = 25$ Swell with short decay distance
$S_{\max} = 75$ Swell with long decay distance

Wave Prediction-Early Methods

Early methods for wave prediction were simple empirical formulations relating the wave height and period to some representative wind speed, fetch, and later duration.

$$H_s, T_s = f(W, F, t_d, g)$$

Sverdrup and Munk (1947) developed a more rigorous wave prediction procedure. This procedure involved relatively simple wave energy growth concepts with empirical calibration, using the small amount of available data.

This procedure was improved by Bretschneider (1952, 1958) over subsequent years by improved calibration using accumulated Weld data sets. The method is now known as the SMB method after the three authors.

$$\frac{gH_s}{W^2}, \frac{gT_s}{2\pi W} = f\left(\frac{gF}{W^2}, \frac{gt_d}{W}\right)$$

Dimensionless significant wave height and period to the dimensionless fetch and duration.

Either the fetch or the duration term would control, depending on whether wave generation were fetch or duration limited.

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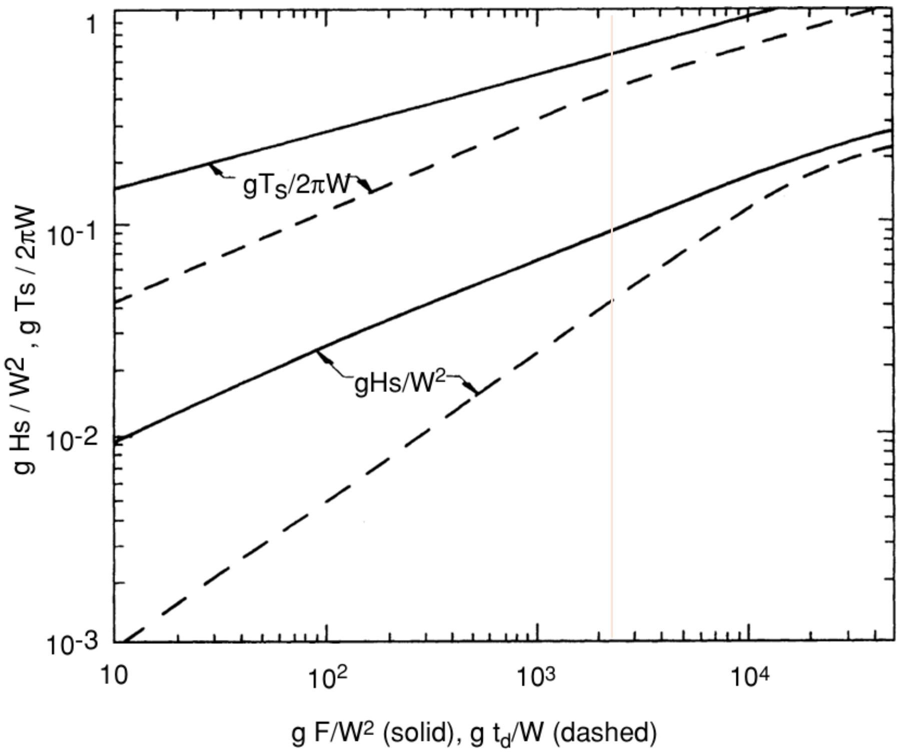
Wave Prediction-Early Methods



The figure is based on a large amount of field data that are not shown.

$$\frac{gH_s}{W^2}, \frac{gT_s}{2\pi W} = f\left(\frac{gF}{W^2}, \frac{gt_d}{W}\right)$$

Either the fetch or the duration term would control, depending on whether wave generation were fetch or duration limited.

Example

A deep lake has a wind with an average velocity of 30m/s blowing over it for a period of 2 hours. The fetch in the direction of the wind is 20km. Using the SMB method, what significant wave height and period will be generated at the down wind end of the lake after two hours?

$$\frac{gt_d}{W} = \frac{9.81(2)(3600)}{30} = 2354 \rightarrow \frac{gH_s}{W^2} = 0.043 \quad \frac{gT_s}{2\pi W} = 0.40$$

$$H_s = 3.9\ m, \quad T_s = 7.7\ m$$

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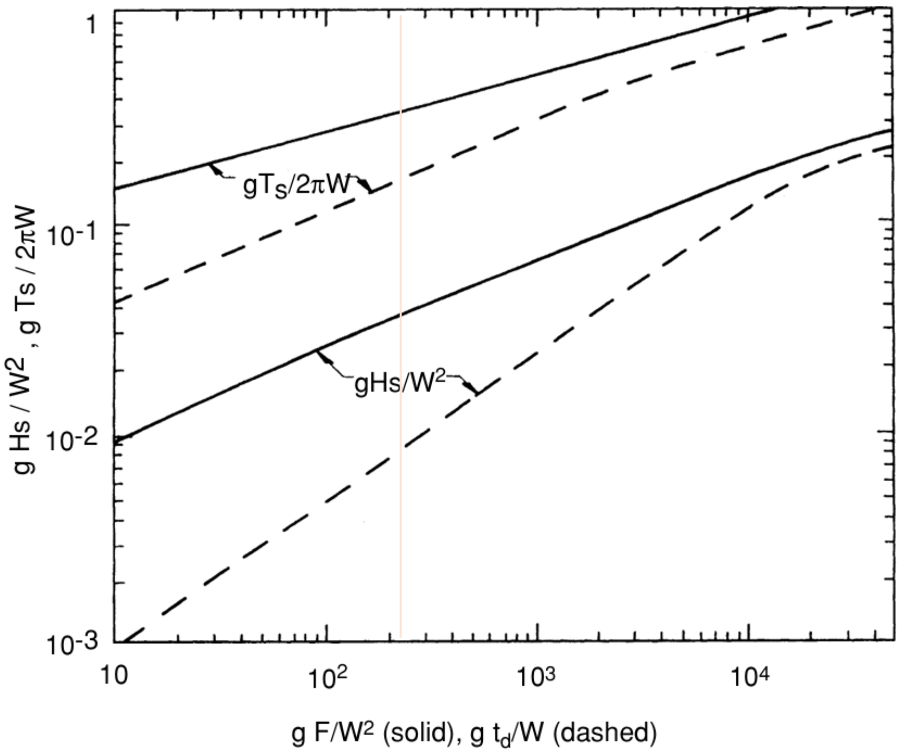
Wave Prediction-Early Methods



The figure is based on a large amount of field data that are not shown.

$$\frac{gH_s}{W^2}, \frac{gT_s}{2\pi W} = f\left(\frac{gF}{W^2}, \frac{gt_d}{W}\right)$$

Either the fetch or the duration term would control, depending on whether wave generation were fetch or duration limited.

Example

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$$\frac{gF}{W^2} = \frac{9.81(20{,}000)}{(30)^2} = 218 \quad \rightarrow \quad \frac{gH_s}{W^2} = 0.034 \quad \frac{gT_s}{2\pi W} = 0.33$$

$$H_s = 3.1\ m, \qquad T_s = 6.4\ m$$

The smaller values, Hs=3.1 m and Ts=6.4 s control, and wave generation is Fetch limited.

Wave Prediction-Spectral Models

Going back to SMB-Bretschneider model: W, F, td → Hs, Ts → H100, T100 → S(T)

$$S(T) = \frac{\alpha g^2}{(2\pi)^4} T^3 e^{-0.675(gT/2\pi W F_2)^4}$$

Bretschneider

$$\alpha = 3.44 \frac{F_1^2}{F_2^2}$$

$$F_1 = \frac{gH_{100}}{W^2} \qquad F_2 = \frac{gT_{100}}{2\pi W}$$

$$T_{100} = 0.77 T_p = \frac{0.77}{0.95} T_s = 0.81 T_s$$

$$H_{100} = 0.886 H_{\text{rms}} = \frac{0.886}{1.146} H_s = 0.63 H_s$$

SMB

W, F, td → Hs, Ts