Riepilogo equazioni

I PTD: $\Delta U = Q - L$

$dU = \delta Q - \delta L \quad$ con $\delta L = p dV$

$$\delta Q = \begin{cases} \left(\frac{\partial U}{\partial p}\right)_V dp + \left[\left(\frac{\partial U}{\partial V}\right)_p + p\right] dV & U = U(p,V) \\ \left[\left(\frac{\partial U}{\partial p}\right)_T + p\left(\frac{\partial V}{\partial p}\right)_T\right] dp + \left[\left(\frac{\partial U}{\partial T}\right)_p + p\left(\frac{\partial V}{\partial T}\right)_p\right] & U = U(p,T) \\ \left[\left(\frac{\partial U}{\partial V}\right)_T + p\right] dV + \left(\frac{\partial U}{\partial T}\right)_V dT & U = U(V,T) \end{cases}$$

(per sistemi idrostatici, vedi pag 3 sez. 2.1.2)

c_p e c_V:

$$c_V = \frac{1}{n}\left(\frac{\delta Q}{dT}\right)_V \overset{S.I.}{=} \frac{1}{n}\left(\frac{\partial U}{\partial T}\right)_V \xrightarrow{G.I.} \frac{1}{n}\left(\frac{dU}{dT}\right) \rightarrow dU = n c_V dT$$

$$c_p = \frac{1}{n}\left(\frac{\delta Q}{dT}\right)_p \overset{S.I.}{=} \frac{1}{n}\left[\left(\frac{\partial U}{\partial T}\right)_p + p\left(\frac{\partial V}{\partial T}\right)_p\right] = \frac{1}{n}\left(\frac{\partial H}{\partial T}\right)_p \xrightarrow{G.I.} \frac{1}{n}\left(\frac{dH}{dT}\right) \rightarrow dH = n c_p dT$$

(S.I. = sistemi idrostatici; G.I. = gas ideali)

Da cui, sempre per i G.I.:

$$\delta Q = \begin{cases} n c_V dT + p dV = dU + p dV & \xrightarrow{V\,cost} \delta Q = dU \\ n c_p dT - V dp = dH - V dp & \xrightarrow{p\,cost} \delta Q = dH \end{cases}$$

Equazioni di Clapeyron (per sistemi idrostatici):

$$\delta Q = \begin{cases} n c_p \left(\frac{\partial T}{\partial V}\right)_p dV + n c_V \left(\frac{\partial T}{\partial p}\right)_V dp & U = U(p,V) \\ n c_p dT - T\left(\frac{\partial V}{\partial T}\right)_p dp & U = U(p,T) \\ n c_V dT + T\left(\frac{\partial p}{\partial T}\right)_V dV & U = U(V,T) \end{cases}$$

Verranno dimostrate più avanti. Si possono facilmente verificare per i gas ideali. Ad esempio la prima (seconda e terza sono banali):

* $T = \frac{pV}{nR}$; $\left(\frac{\partial T}{\partial V}\right)_p = \frac{p}{nR}$; $\left(\frac{\partial T}{\partial p}\right)_V = \frac{V}{nR}$

$$\delta Q = n c_p \frac{p dV}{nR} + n c_V \frac{V dp}{nR} \qquad \text{ma } V dp = nR dT - p dV$$

$$= \cancel{n} c_p \frac{p dV}{\cancel{n}R} + \frac{\cancel{n} c_V}{\cancel{n}R}(nR dT - p dV)$$

$$= \underbrace{\frac{c_p - c_V}{R}}_{1} p dV + n c_V dT = n c_V dT + p dV \qquad \square$$