Unit 4a

The neoclassical theory of distribution: Marginal Productivity Theory

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The material presented in these slides is primarily based on Mankiw's textbook: Macroeconomics, 2025 Macmillan / Worth Publishers

In this lecture you will learn

- What determines the economy's total output/income
- How the prices of the factors of production are determined
- Marginal Productivity Theory
- How total income is distributed

Factors of production

K = capital:

tools, machines, and structures used in production

L = labor:

the physical and mental efforts of workers

The production function: $Y = F(K, L)$

- Shows how much output (***Y***) the economy can produce from ***K*** units of capital and ***L*** units of labor
- Reflects the economy's level of technology
- Exhibits constant returns to scale

Returns to scale: A review

Initially $Y_1 = F(K_1, L_1)$

Scale all inputs by the same factor z:

$K_2 = zK_1$ and $L_2 = zL_1$

(example: if $z = 1.2$, then all inputs are increased by 20%)

What happens to output, $Y_2 = F(K_2, L_2)$?

- If **constant returns to scale**, $Y_2 = zY_1$
- If **increasing returns to scale**, $Y_2 > zY_1$
- If **decreasing returns to scale**, $Y_2 < zY_1$

Returns to scale: Example 1

$$F(K,L) = \sqrt{KL}$$

$$\begin{aligned} F(zK,zL) &= \sqrt{(zK)(zL)} \\ &= \sqrt{z^2 KL} \\ &= \sqrt{z^2}\sqrt{KL} \\ &= z\sqrt{KL} \\ &= zF(K,L) \end{aligned}$$

*constant returns to scale for any **z** > 0*

Returns to scale: Example 2

$$F(K,L) = K^2 + L^2$$

$$F(zK,zL) = (zK)^2 + (zL)^2$$

$$= z^2\left(K^2 + L^2\right)$$

$$= z^2 F(K,L)$$

increasing returns to scale for any z > 1

NOW YOU TRY

Returns to scale

Determine whether each of these production functions has constant, decreasing, or increasing returns to scale:

(a) $F(K,L) = \dfrac{K^2}{L}$

(b) $F(K,L) = K + L$

NOW YOU TRY

Answers, part (a)

$$F(K,L) = \frac{K^2}{L}$$

$$F(zK,zL) = \frac{(zK)^2}{zL} = \frac{z^2K^2}{zL} = z\frac{K^2}{L}$$

$$= zF(K,L)$$

*constant returns to scale for any **z** > 0*

NOW YOU TRY

Answers, part (b)

$$F(K,L) = K + L$$

$$F(zK,zL) = zK + zL$$

$$= z(K+L)$$

$$= zF(K+L)$$

*constant returns to scale for any **z** > 0*

Assumptions

1. Technology is fixed.
2. The economy's supplies of capital and labor are fixed at:

$$K = \bar{K} \text{ and } L = \bar{L}$$

Determining GDP

Output is determined by the fixed factor supplies and the fixed state of technology:

$$\overline{Y} = F\left(\overline{K}, \overline{L}\right)$$

The distribution of national income

determined by **factor prices**, the prices per unit firms pay for the factors of production

- wage = price of ***L***
- **rental rate** = price of ***K***

Notation

W = nominal wage
R = nominal rental rate
P = price of output
W /***P*** = real wage
(measured in units of output)
R /***P*** = real rental rate

How factor prices are determined

- Factor prices are determined by supply and demand in factor markets.
- Recall that the supply of each factor is fixed.
- What about demand?

Demand for labor

- Assume that markets are competitive: each firm takes ***W***, ***R***, and ***P*** as given.
- Basic idea: A firm hires each unit of labor if the cost does not exceed the benefit.
  - cost = real wage
  - benefit = marginal product of labor

Marginal product of labor (MPL)

Definition:
The extra output the firm can produce using an additional unit of labor (holding other inputs fixed):

$$MPL = F(K, L+1) - F(K, L)$$

NOW YOU TRY

Compute and graph *MPL*

a. Determine ***MPL*** at each value of ***L***.

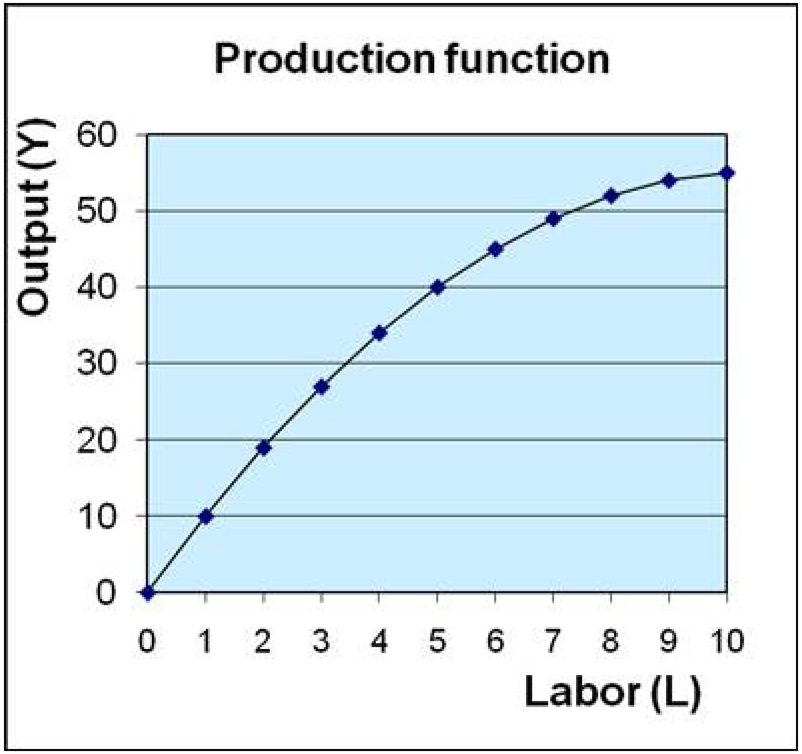
b. Graph the production function.

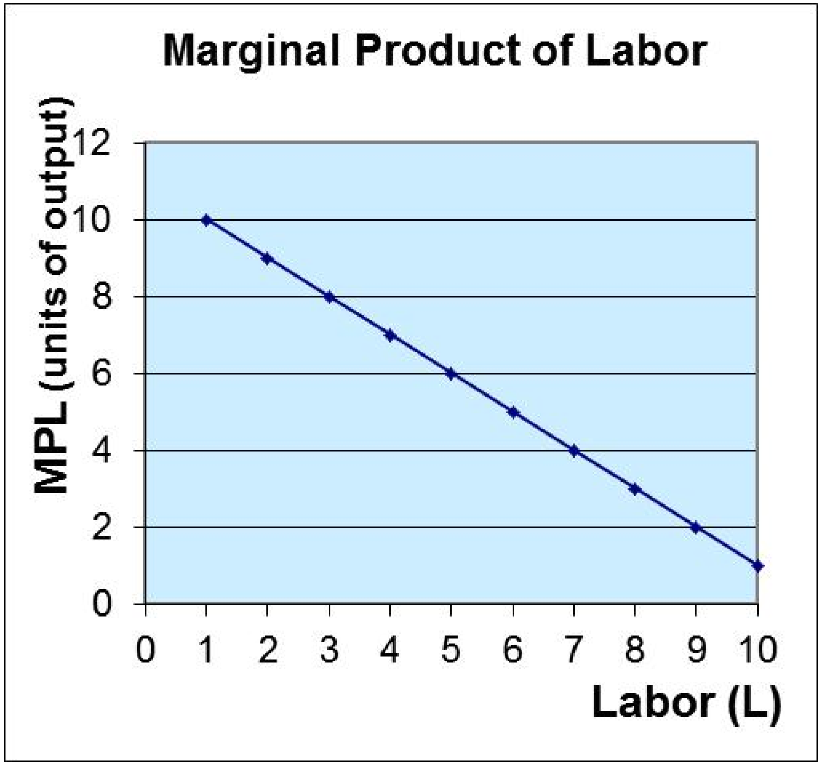
c. Graph the ***MPL*** curve with ***MPL*** on the vertical axis and ***L*** on the horizontal axis.

L	*Y*	*MPL*
0	0	n.a.
1	10	?
2	19	?
3	27	8
4	34	?
5	40	?
6	45	?
7	49	?
8	52	?
9	54	?
10	55	?

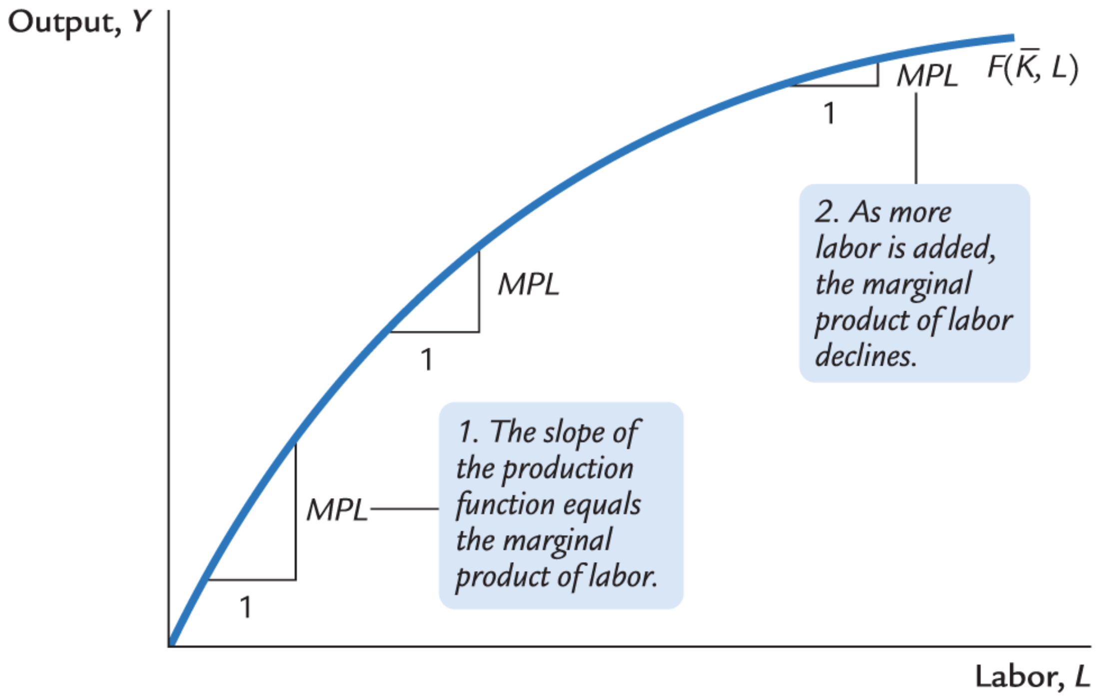
NOW YOU TRY

Compute and graph *MPL, Answers*





MPL and the production function



Diminishing marginal returns

- As one input is increased (holding other inputs constant), its marginal product falls.
- Intuition:
  If ***L*** increases while holding ***K*** fixed, machines per worker falls, worker productivity falls.

NOW YOU TRY

Identifying diminishing returns

Which of these production functions have diminishing marginal returns to labor?

a) $F(K,L) = 2K + 15L$

b) $F(K,L) = \sqrt{KL}$

c) $F(K,L) = 2\sqrt{K} + 15\sqrt{L}$

NOW YOU TRY

Identifying diminishing returns, answers

Which of these production functions have diminishing marginal returns to labor?

a) $F(K,L) = 2K + 15L$

No, ***MPL*** = 15 for all ***L***

b) $F(K,L) = \sqrt{KL}$

Yes, ***MPL*** falls as ***L*** rises

c) $F(K,L) = 2\sqrt{K} + 15\sqrt{L}$

Yes, ***MPL*** falls as ***L*** rises

NOW YOU TRY

MPL and labor demand

Suppose ***W***/***P*** = 6.

- If ***L*** = 3, should the firm hire more or less labor? Why?
- If ***L*** = 7, should the firm hire more or less labor? Why?

L	*Y*	*MPL*
0	0	n.a.
1	10	10
2	19	9
3	27	8
4	34	7
5	40	6
6	45	5
7	49	4
8	52	3
9	54	2
10	55	1

NOW YOU TRY

MPL and labor demand, answers

Suppose ***W*/*P*** = 6.

- If ***L*** = 3, should the firm hire more or less labor? Why?

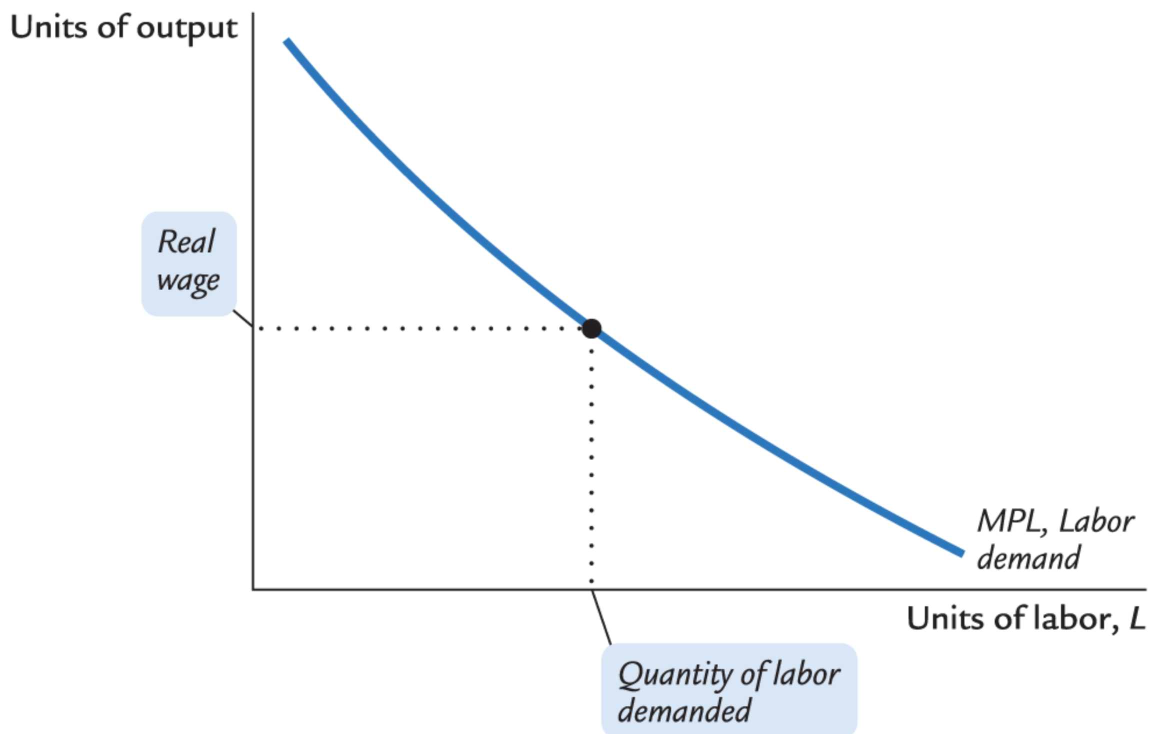
  Answer: More because the benefit of the 4th worker (*MPL* = 7) exceeds its cost (*W*/*P* = 6)

- If ***L*** = 7, should the firm hire more or less labor? Why?

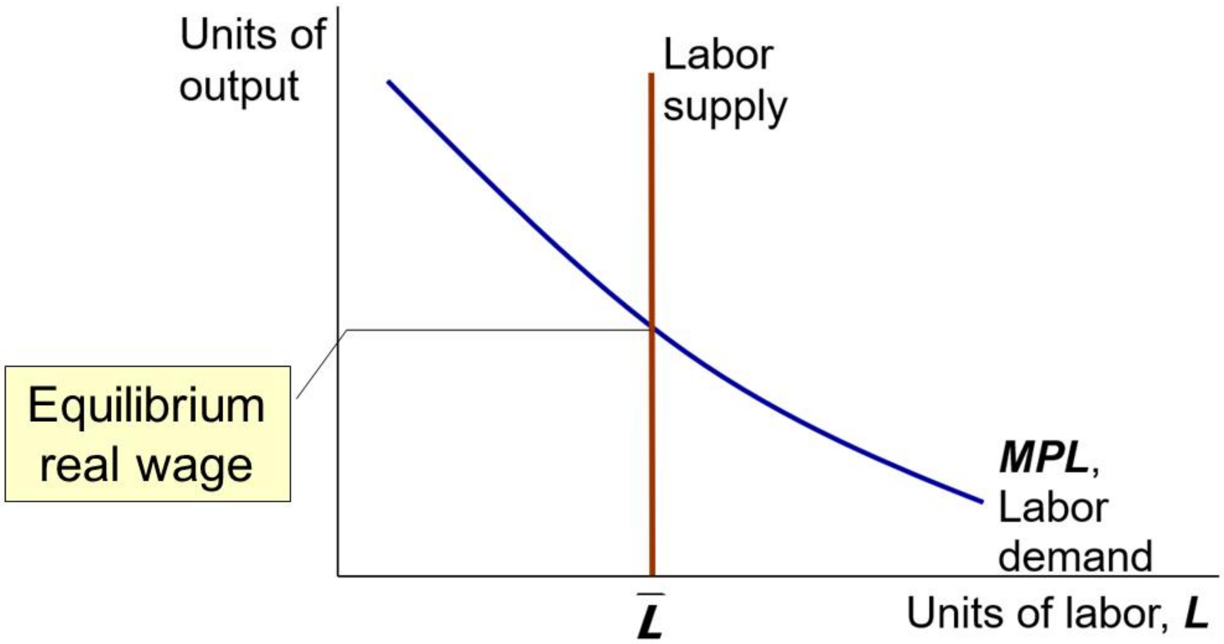
  Answer: Less because the 7th worker adds *MPL* = 4 units of output but costs the firm *W*/*P* = 6.

L	*Y*	*MPL*
0	0	n.a.
1	10	10
2	19	9
3	27	8
4	34	7
5	40	6
6	45	5
7	49	4
8	52	3
9	54	2
10	55	1

MPL and the demand for labor



The equilibrium real wage



The real wage adjusts to equate labor demand with supply.

Determining the rental rate

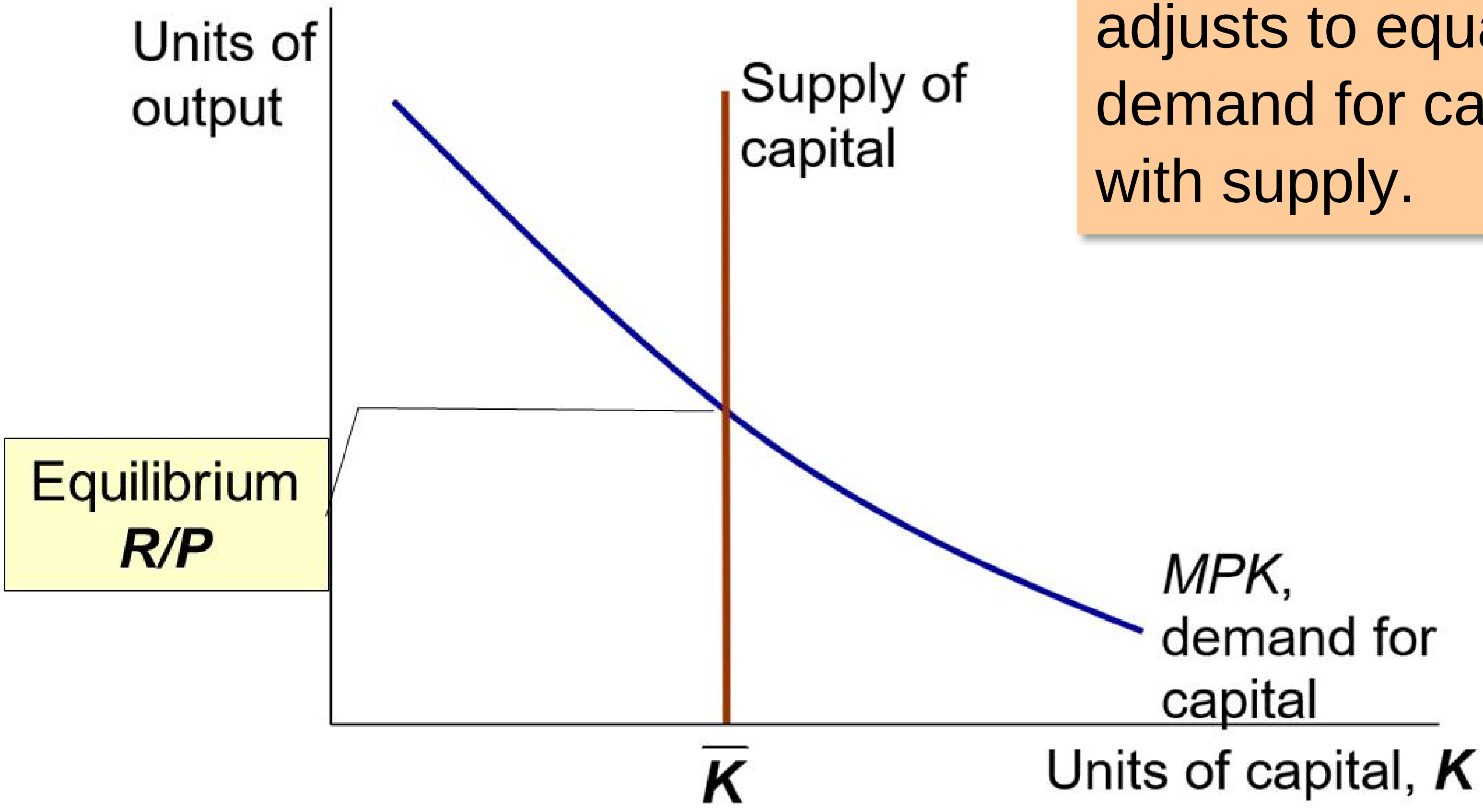
- We have just seen that ***MPL*** = ***W***/***P***.
- The same logic shows that ***MPK*** = ***R***/***P***:

  Diminishing returns to capital:
  MPK falls as ***K*** rises

  The ***MPK*** curve is the firm's demand curve for renting capital.

  Firms maximize profits by choosing ***K*** such that ***MPK*** = ***R***/***P***.

The equilibrium real rental rate



The real rental rate adjusts to equate demand for capital with supply.

The neoclassical theory of distribution

- States that each factor input is paid its marginal product
- A good starting point for thinking about income distribution

How income is distributed to L and K

$$\text{Total capital income} = \frac{W}{P}\bar{L} = MPL \times \bar{L}$$

$$\text{Total capital income} = \frac{R}{P}\bar{K} = MPK \times \bar{K}$$

If the production function has constant returns to scale, then

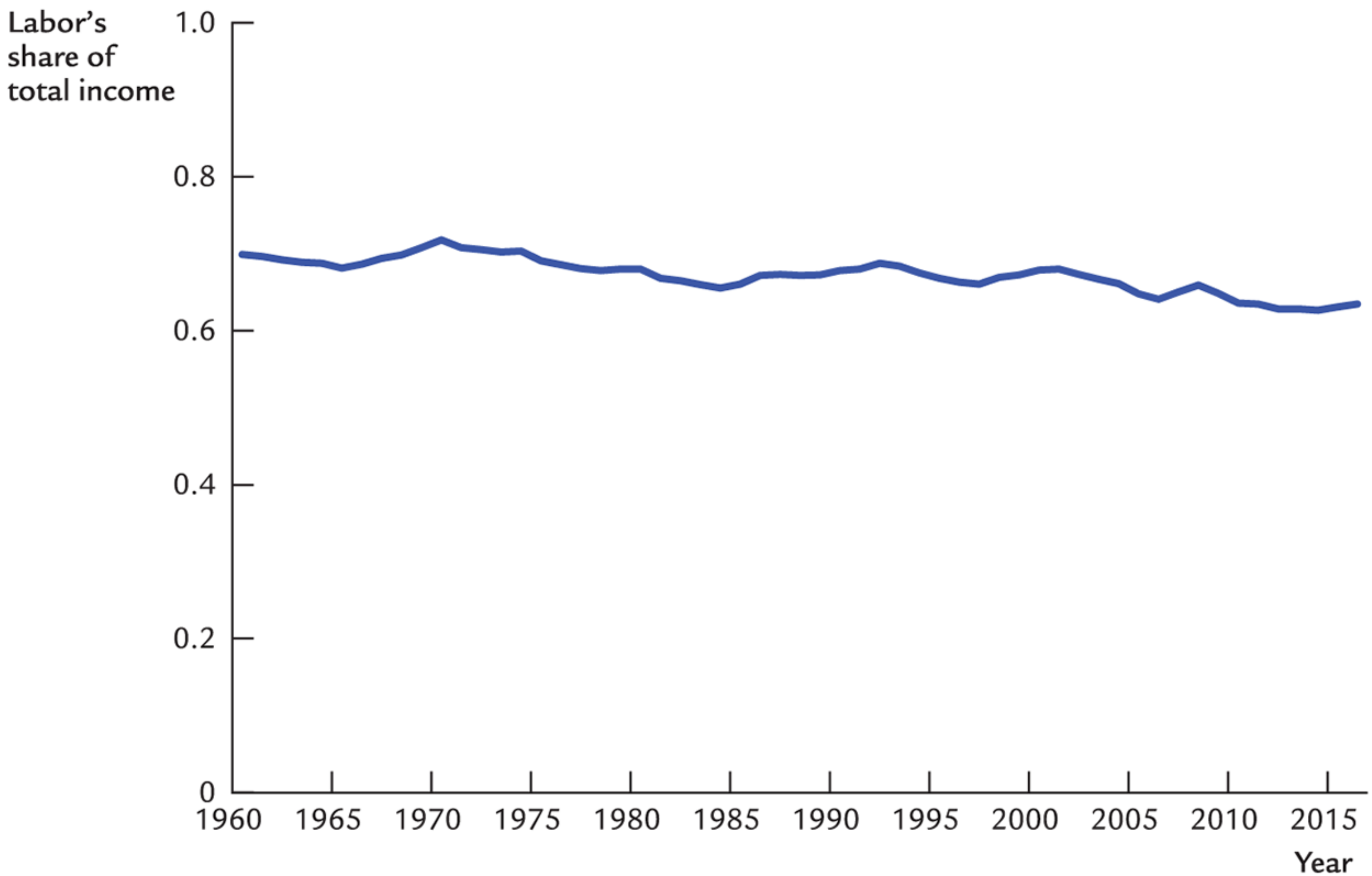
$$\bar{Y} = MPL \text{ x } \bar{L} + MPK \text{ x } \bar{K}$$

national income

labor income

capital income

The ratio of labor income to total income in the United States, 1960–2010



Mankiw, *Macroeconomics*, 10e, © 2019 Worth Publishers

The Cobb-Douglas production function (1 of 2)

- The Cobb–Douglas production function has constant factor shares:

  $\boldsymbol{\alpha}$ = capital's share of total income:

  capital income = $\boldsymbol{MPK} \times \boldsymbol{K} = \boldsymbol{\alpha Y}$

  labor income = $\boldsymbol{MPL} \times \boldsymbol{L} = (1 - \boldsymbol{\alpha})\boldsymbol{Y}$

- The Cobb–Douglas production function is:

$$Y = AK^{a}L^{1-a}$$

where ***A*** represents the level of technology

The Cobb-Douglas production function (2 of 2)

Each factor’s marginal product is proportional to its average product:

$$MPL = aAK^{a}L^{1-a} = \frac{aY}{K}$$

$$MPL = (1-a)AK^{a}L^{-a} = \frac{(1-a)Y}{L}$$

Labor productivity and wages

Theory: wages depend on labor productivity U.S. data:

Time Period	Growth Rate of Labor Productivity	Growth Rate of Real Wages
1960–2016	2.0%	1.8%
1960–1973	3.0	2.7
1973–1995	1.5	1.2
1995–2010	2.6	2.2
2010–2016	0.5	0.9

Explanations for rising inequality

From *The Race Between Education and Technology* by Goldin and Katz:

- Technological progress has increased the demand for skilled relative to unskilled workers.
- Due to a slowdown in expansion of education, the supply of skilled workers has not kept up.
- Result: Rising gap between wages of skilled and unskilled workers.