ADVANCED STRUCTURAL MECHANICS 2 (3 ECTS CFU) ASM2 24/3/26

STARTING FROM THE MODULE: MECCANICA AVANZATA E DINAMICA DELLE STRUTTURE (1° SEMESTER) (MADS)

MADS:
- ELASTIC LINE (IV ORDER EQUATION / E-BERNOULLI)
- WORK-ENERGY THEOREMS (CLAPEYRON, BETTI, CASTIGLIANO)
- LIMIT ANALYSIS (NOTION OF PLASTICITY / COLLAPSE OF STRUCTURES)
- BUCKLING INSTABILITY (IV ORDER EQ / GENERAL B. CONDITIONS)
- INTRO TO STRUCTURAL DYNAMICS (1 DOF + SPECTRA)

ASM 2:
- DYNAMICS OF SYSTEM OF N DOFS $\longrightarrow$ MODAL ANALYSIS
- SOME ISSUES RELATED TO STIFFNESS MATRIX IN CIVIL STRUCT. MECHANICS
- SHEAR-WALL INTERACTION WITH A FRAME
- MASONRY MODELLING / NO TENSION MATERIAL

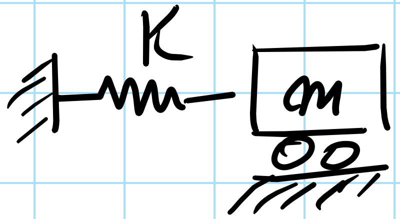
— DOMES, TANKS

— TUNED-MASS DAMPERS / BRIEF INTRO TO "METAMATERIALS"

— DESIGN PROBLEMS IN WIND TURBINE STRUCT.

LET US START WITH DYNAMICS OF MULTI-D.O.F. SYSTEMS

WHAT WE HAVE LEARNED FROM THE AUTUMN MODULE:

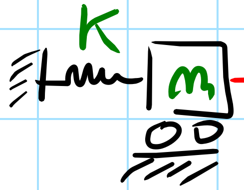


FREE OSCILLATIONS OF A RESONATOR

$$\omega = \sqrt{\frac{K}{m}}$$ CIRCULAR FREQUENCY (rad/s) (PULSAZIONE)

$$f = \frac{\omega}{2\pi}$$ FREQUENCY (FREQUENZA) (Hz)

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$ PERIOD (s)



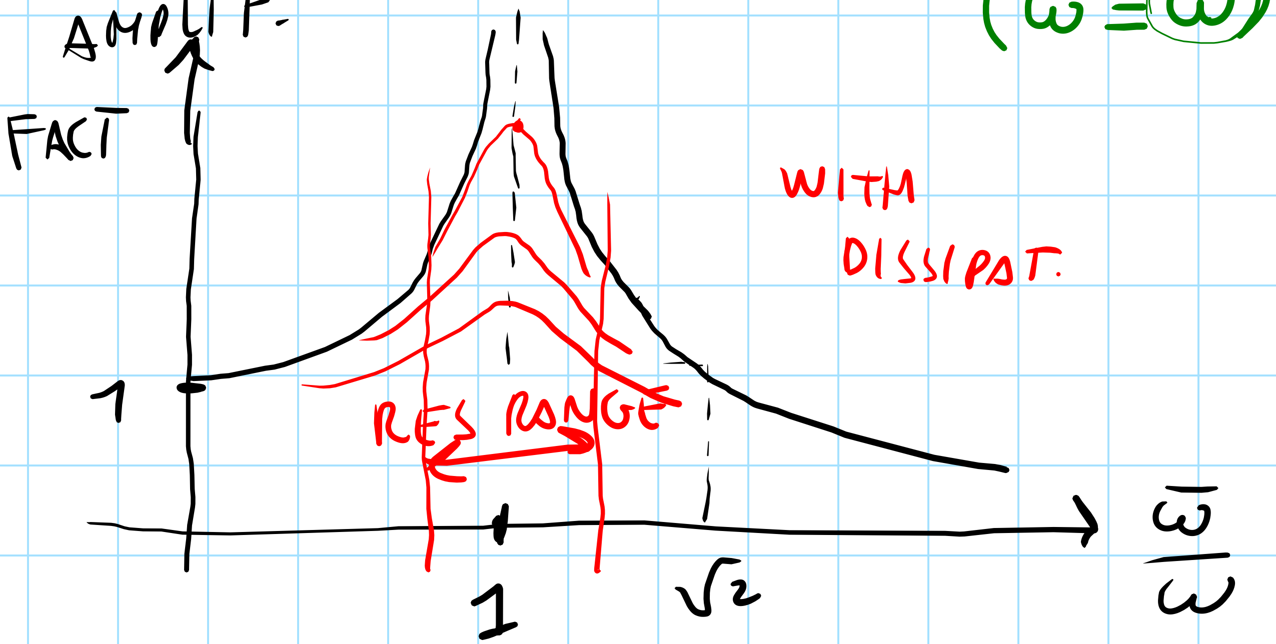
$\rightarrow F \sin \bar{\omega} t$

FORCED VIBRATIONS

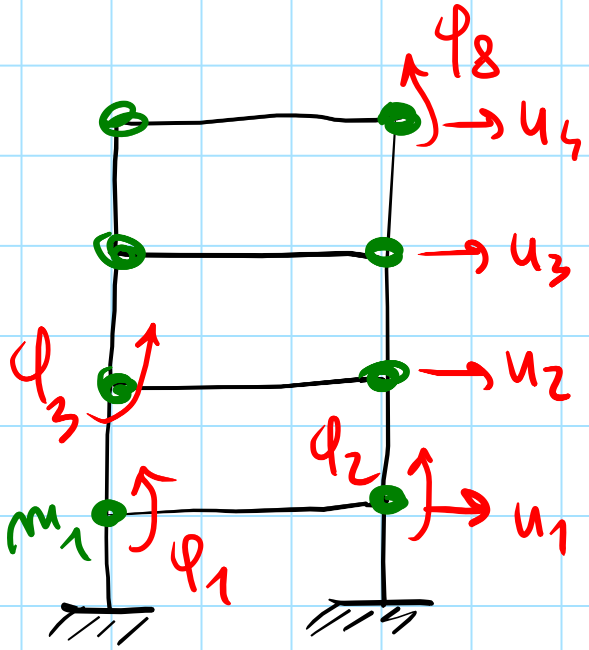
⇓

RISK OF RESONANCE

$(\bar{\omega} = \omega)$

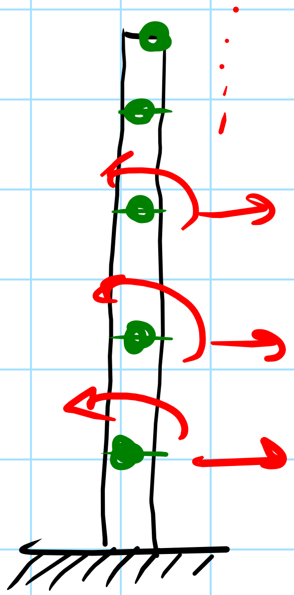


TYPICAL M-DOF STRUCTURES IN CIVIL ENGINEERING



FRAME

(DISCR WITH
12 DOFS
(4 → DISPL.
8 → ROTATIONS)



TOWER

MAINLY TWO APPROACHES:
- CONTINUOUS SYSTEMS (BEAMS AND BARS)
- MDOFS WITH SELECTED D.O.Fs (DISPL./ROTAT.)

IN THE NEXT SLIDES WE STUDY FREE OSCILLATIONS OF A GENERIC MDOF SYSTEM (NO DAMPING)

FREE OSCILLATIONS OF A MDOF-SYSTEM (NO DAMPING) (N: n° of DOFs)

$$\begin{cases} \underset{\sim}{M}\,\ddot{\underline{u}} + \underset{\sim}{K}\,\underline{u} = \underline{0} \quad (1) \\ \underline{u}(0) = \underline{u}_0 \\ \dot{\underline{u}}(0) = \dot{\underline{u}}_0 \end{cases}$$

$\underline{u}(t)$: vector of N DOFs ; $\underset{\sim}{M}$: NxN matrix of MASSES

$\underset{\sim}{K}$: NxN STIFFNESS MATRIX

$\underline{2N}$ DATA (INITIAL CONDITIONS)

(NOTE THAT THE CONSTANTS TO BE DETERMINED ARE n° = 2N)

WE SEEK SOLUTIONS IN THE FORM

$$\underline{u}(t) = \underbrace{\underline{\phi}}_{\text{VECTOR OF AMPLITUDES}}\,(A\cos\omega t + B\sin\omega t) \quad ; \quad \ddot{\underline{u}}(t) = -\omega^2\,\underline{\phi}\,\underbrace{(A\cos\omega t + B\sin\omega t)}_{*}$$

$$\text{IN EQ. (1)} \quad -\omega^2\underset{\sim}{M}\,\underline{\phi}\,(*) + \underset{\sim}{K}\,\underline{\phi}\,(*) = \underline{0} \quad (\forall t) \Longrightarrow \boxed{\left(-\omega^2\underset{\sim}{M} + \underset{\sim}{K}\right)\underline{\phi} = \underline{0}}$$

CLASSICAL EIG. PROBL.

$$(\underset{\sim}{A} - \lambda\,\underset{\sim}{I})\,\underline{\phi} = \underline{0}$$

HOMOGENEOUS SYSTEM: NON TRIVIAL SOLUTION ONLY IF

$$\det\left(-\omega^2\underset{\sim}{M} + \underset{\sim}{K}\right) = 0$$

ω^2 ARE "EIGENVALUES"

← GENERALISED EIGENVALUE PROBLEM

$\underline{\phi}$ ARE "EIGENVECTORS"

CHARACT. EQ: $\alpha_N (\omega^2)^N + \alpha_{N-1} (\omega^2)^{N-1} + \dots + \alpha_0 = 0$

UNKNOWNS

$\Rightarrow$ N SOLUTIONS : $(\omega^2)_1, (\omega^2)_2, \dots (\omega^2)_N$

WE USUALLY PUT IN THE ORDER SUCH THAT $0 \leq \omega_1 \leq \omega_2 \leq \omega_3 \leq \dots \leq \omega_N$

(LIST OF CIRCULAR FREQUENCY OF THE SYSTEM)

$\omega_1 \rightarrow$ FUNDAMENTAL OR NATURAL CIRCULAR FREQUENCY

$$\omega_i \rightarrow T_i = \frac{2\pi}{\omega_i} \rightarrow T_1 \geq T_2 \geq T_3 \dots \geq T_N > 0$$

T_1: NATURAL/FUNDAMENTAL PERIOD

$\omega_i \rightarrow \underline{\phi}_i$ (EIGENVECTOR) : i-th VIBRATION MODE

(i-simo modo di vibrare)

USUALLY $\underline{\phi}_i$ ARE NORMALISED $\rightarrow$ $\max |\phi^{(i)}_j| = 1$

$\rightarrow$ NORM $\underline{\phi}_i = 1$

THE MOTION OF THE SYSTEM MODELLED IN THE FIRST SLIDE CAN BE COMPLETELY REPRESENTED BY:

$$\underline{u}(t) = \sum_{i=1}^{N} \underline{\phi}^{(i)} \left(A_i \cos \omega_i t + B_i \sin \omega_i t \right)$$

UNKNOWNS $\rightarrow$ 2N! FROM INITIAL CONDITIONS!

MODAL ANALYSIS: STUDY OF THE SYSTEM THROUGH THE PROPERTIES OF VIBRATION MODES

MODAL MATRIX

$$\underset{\sim}{\Phi} = \begin{bmatrix} \phi_1^{(1)} & \cdots & \phi_1^{(N)} \\ \vdots & & \vdots \\ \phi_N^{(1)} & \cdots & \phi_N^{(N)} \end{bmatrix} \quad (N \times N)$$

$$\underline{\phi}^{(1)} \qquad \cdots \qquad \underline{\phi}^{(N)}$$

SPECTRAL MATRIX

$$\underset{\sim}{\Omega} = \begin{bmatrix} \omega_1^2 & & 0 \\ & \ddots & \\ 0 & & \omega_N^2 \end{bmatrix} \quad (N \times N)$$

INTRODUCE THE PRINCIPAL COORDINATES $(\underline{z}(t)) \div$ VECTOR OF N COMPONENTS

$$\underline{u}(t) = \underset{\sim}{\Phi}\, \underline{z}(t) \quad , \quad \underline{\dot{u}}(t) = \underset{\sim}{\Phi}\, \underline{\dot{z}}(t) \quad , \quad \underline{\ddot{u}}(t) = \underset{\sim}{\Phi}\, \underline{\ddot{z}}(t)$$

$$(1) \rightarrow \quad \underset{\sim}{M}\, \underset{\sim}{\Phi}\, \underline{\ddot{z}}(t) + \underset{\sim}{K}\, \underset{\sim}{\Phi}\, \underline{z}(t) = \underline{0} \quad \Rightarrow$$

$$\underbrace{\underset{\sim}{\Phi}^T \underset{\sim}{M}\, \underset{\sim}{\Phi}}_{\hat{\underset{\sim}{M}}}\, \underline{\ddot{z}}(t) + \underbrace{\underset{\sim}{\Phi}^T \underset{\sim}{K}\, \underset{\sim}{\Phi}}_{\hat{\underset{\sim}{K}}}\, \underline{z}(t) = \underline{0}$$

DIAGONAL MATRICES

$$\hat{\underset{\sim}{M}} = \begin{bmatrix} \hat{m}_1 & & & 0 \\ & \hat{m}_2 & & \\ & & \ddots & \\ 0 & & & \hat{m}_N \end{bmatrix} \quad ; \quad \hat{\underset{\sim}{K}} = \begin{bmatrix} \hat{k}_1 & & & 0 \\ & \hat{k}_2 & & \\ & & \ddots & \\ 0 & & & \hat{k}_N \end{bmatrix}$$

$$\begin{cases} \hat{m}_1 \ddot{z}_1 + \hat{k}_1 z_1 = 0 \\ \hat{m}_2 \ddot{z}_2 + \hat{k}_2 z_2 = 0 \\ \quad \vdots \qquad\qquad \vdots \\ \hat{m}_N \ddot{z}_N + \hat{k}_N z_N = 0 \end{cases}$$

N INDEPENDENT RESONATORS

$$\boxed{\begin{aligned} \hat{m}_i &= \underline{\phi}^{(i)} \cdot \underset{\sim}{M}\, \underline{\phi}^{(i)} \\ \hat{k}_i &= \underline{\phi}^{(i)} \cdot \underset{\sim}{K}\, \underline{\phi}^{(i)} \end{aligned}}$$

let us study the 1st Eq:

$$\hat{m}_1 \ddot{z}_1(t) + \hat{K}_1 z_1(t) = 0 \longrightarrow \ddot{z}_1(t) + \omega_1^2 z_1(t) = 0$$

$$\boxed{\omega_1^2 = \frac{\hat{K}_1}{\hat{m}_1}}$$