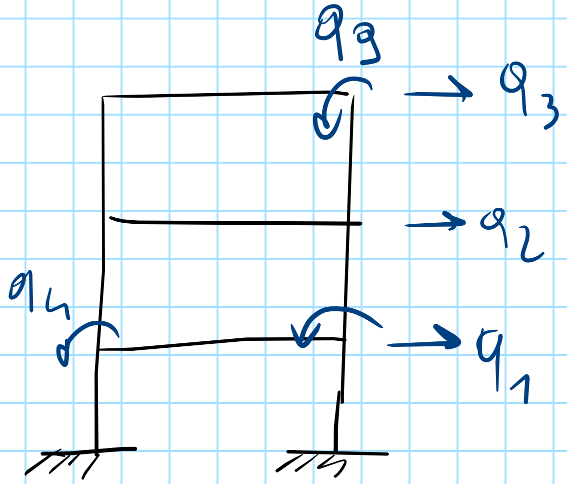
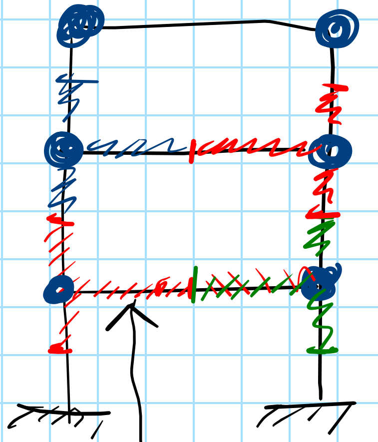
MATRIX CONDENSATION IN DYNAMICS

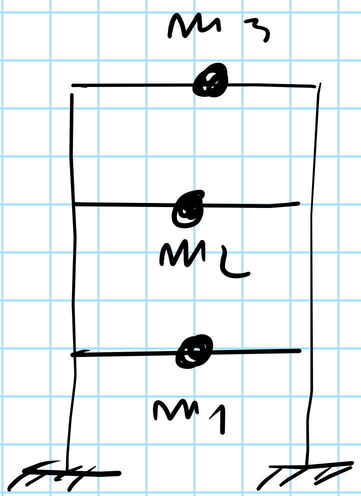
29/4/26





CONCENTRATED MASS

INFLUENCE AREA TO CALCULATE THE MASS PERTAINING TO THE NODE



MASSES AT EACH FLOOR

$$\underline{q} = \begin{bmatrix} \underline{q}_A \\ \underline{q}_B \end{bmatrix}$$

$$\underline{q}_A = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$ DISPL. DOFS

$$\underline{q}_B = \begin{bmatrix} q_4 \\ \vdots \\ q_9 \end{bmatrix}$$ ROTATIONS

$\underset{\sim}{K} \Rightarrow 9 \times 9$ MATRIX

<u>EQ OF MOTIONS</u>: $\underset{\sim}{M}\ddot{\underline{q}} + \underset{\sim}{K}\underline{q} = \underline{F}(t)$

$$E_{KIN} = \frac{1}{2}\left(m_1 \dot{q}_1^2 + m_2 \dot{q}_2^2 + m_3 \dot{q}_3^2\right)$$

ROTATIONS ARE NOT INVOLVED IN KINETIC ENERGY

$$\underset{\sim}{M}_A = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}$$

$$\left[\begin{array}{c|c} \underset{\sim}{M}_A & \underset{\sim}{0} \\ \hline \underset{\sim}{0} & \underset{\sim}{0} \end{array}\right] \begin{bmatrix} \ddot{\underline{q}}_A \\ \ddot{\underline{q}}_B \end{bmatrix} + \left[\begin{array}{c|c} \underset{\sim}{K}_{AA} & \underset{\sim}{K}_{AB} \\ \hline \underset{\sim}{K}_{BA} & \underset{\sim}{K}_{BB} \end{array}\right] \begin{bmatrix} \underline{q}_A \\ \hline \underline{q}_B \end{bmatrix} = \begin{bmatrix} \underline{F}_A(t) \\ \hline \underline{0} \end{bmatrix}$$

MATRIX CONDENSATION ALLOWS THE PROBLEM TO BE FORMULATED ONLY IN TERMS OF $\underline{q}_A$.

FROM THE "BOTTOM" PART OF THE SYSTEM:

$$\underset{\sim}{K}_{BA}\,\underline{q}_A + \underset{\sim}{K}_{BB}\,\underline{q}_B = \underline{0} \implies \boxed{\underline{q}_B = -\underset{\sim}{K}_{BB}^{-1}\,\underset{\sim}{K}_{BA}\,\underline{q}_A} \quad (*)$$

THE TOP PART BECOMES:

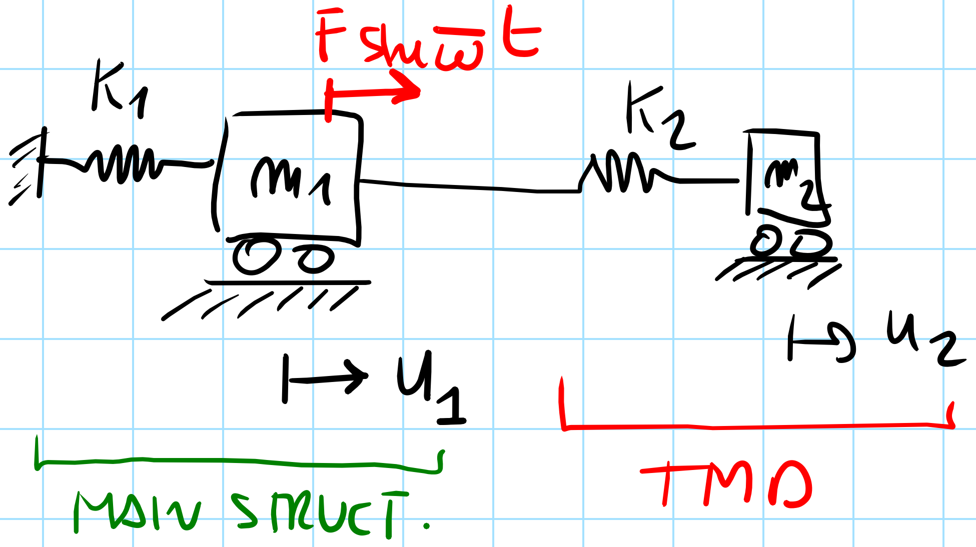
$$\underset{\sim}{M}_A\,\underline{\ddot{q}}_A + \underset{\sim}{K}_{AA}\,\underline{q}_A + \underset{\sim}{K}_{AB}\,\underline{q}_B = \underline{F}_A(t) \implies \underset{\sim}{M}_A\,\underline{\ddot{q}}_A + \underbrace{\left[\underset{\sim}{K}_{AA} - \underset{\sim}{K}_{AB}\,\underset{\sim}{K}_{BB}^{-1}\,\underset{\sim}{K}_{BA}\right]}_{}\underline{q}_A = \underline{F}_A(t)$$

CONDENSED STIFFNESS MATRIX OF DOFS $\underline{q}_A$. ($\underset{\sim}{K}_A$)

$$\underset{\sim}{M}_A\,\underline{\ddot{q}}_A + \underset{\sim}{K}_A\,\underline{q}_A = \underline{F}_A(t) \implies \boxed{\underline{q}_A(t)}$$

AFTER FROM (*) WE CAN OBTAIN $q_B(t)$

FUNCTIONING PRINCIPLE OF "TUNED-MASS DAMPERS" (TMD)



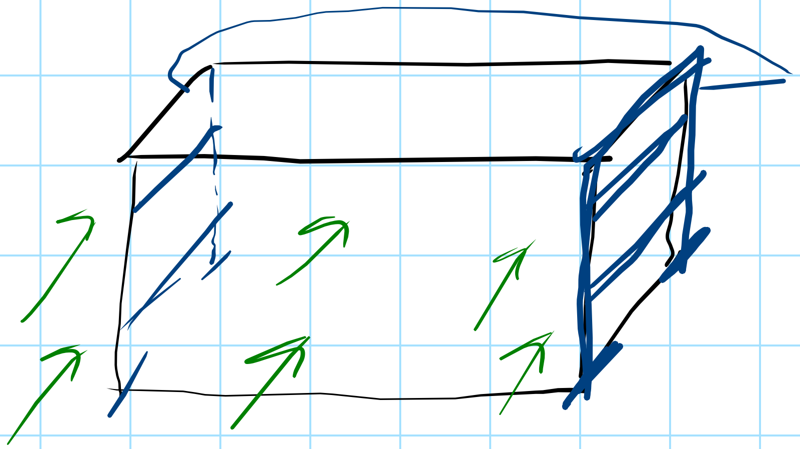
TMD : DESIGN TO LIMIT DISPL u_1 WITHIN SOME FREQUENCY RANGE

$$\mu = \frac{m_2}{m_1} \; ; \; \omega_1 = \sqrt{\frac{k_1}{m_1}} \; , \; \omega_2 = \sqrt{\frac{k_2}{m_2}}$$

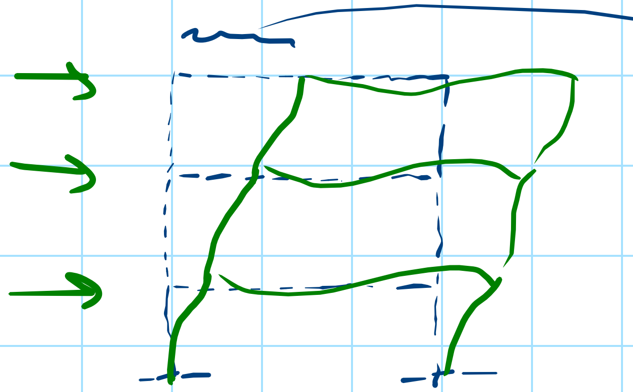
(THESE ARE NOT THE EIGEN FREQUENCIES OF THE SYSTEM)

SHEAR WALLS IN HIGH-RISE BUILDINGS

(HOW HORIZONTAL FORCES ARE DISTRIBUITED BEETWEN A FRAME AND A STIFFENING CANTILEVER) (PROBLEMA DI CONTROVENTI)

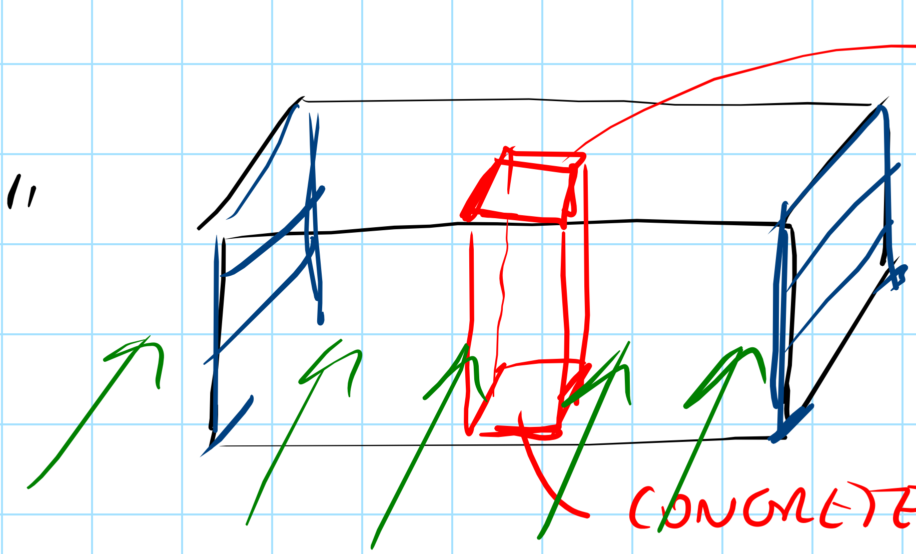


RESISTANT FRAME



QUITE LARGE DISPL. WHEN HORIZ FORCES ARE APPLIED

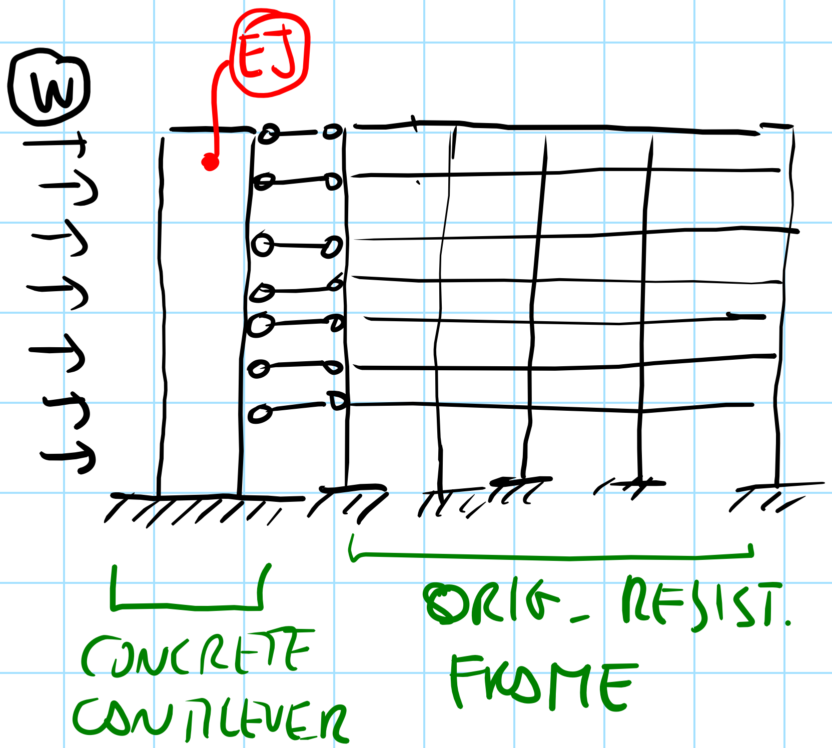
TO LIMIT HORIZ. DISPL WE USUALLY INGLOBE IN THE BUILDING "SHEAR WALLS" THAT ABSORB MOST OF THE HORIZ. FORCES, GIVING RELIEF TO THE RESISTANT FRAMES



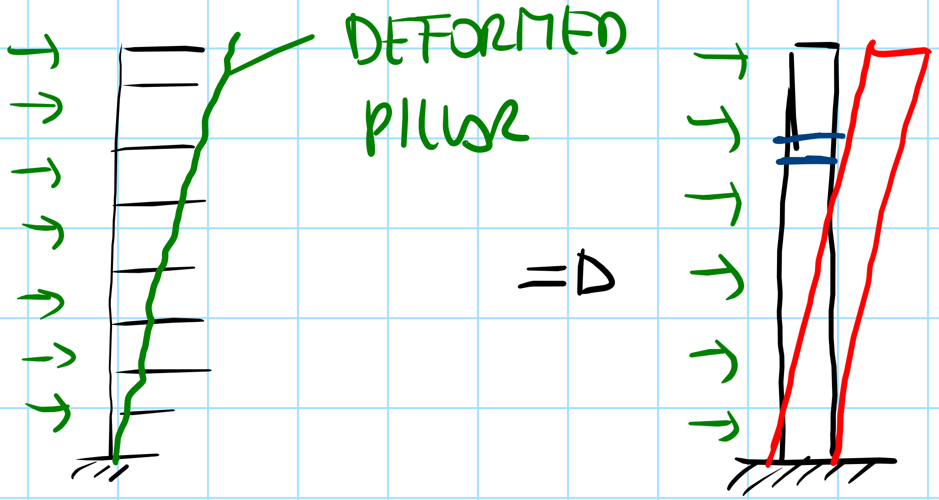
(NUCLEO DI CONTROVENTO)

CONCRETE BLOCK (CANTILEVER) (LIFTS, STAIRS) USUALLY STIFFER THAN THE FRAMES

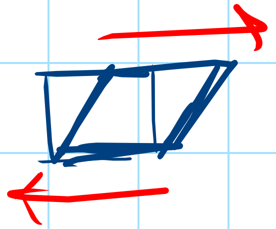
2D MODEL TO BE SOLVED ALMOST ANALYTICALLY.



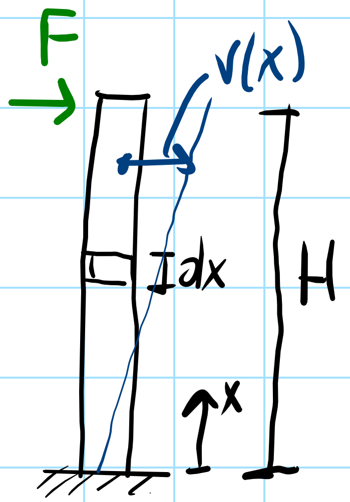
PROVIDING ALL THE FEATURES OF FRAME AND CANTILEVER, WHICH PORTION OF LOAD $\underline{W}$ IS APPLIED TO THE FRAME?

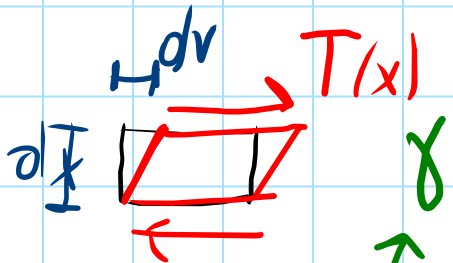


SHEAR BEAM (GA_{eq})



SHEAR BEAM BEHAVIOUR OF A FRAME SUBJECTED TO HORIZ. FORCES (VALID FOR A LARGE N° OF FLOORS)





$$\gamma = \frac{dv}{dx} = \frac{T}{GA_{eq}} = \frac{T\kappa}{GA}$$

SHEAR DEFORM.

$T(x) = +F$

G: SHEAR MODULUS

A: CROSS SECTION

K: SHEAR FACTOR >1 — $\frac{6}{5}$ (rectangular section), $\frac{32}{27}$ (circular section)

FOR OUR PROBL., $v(x)$ IS THE RESULT OF

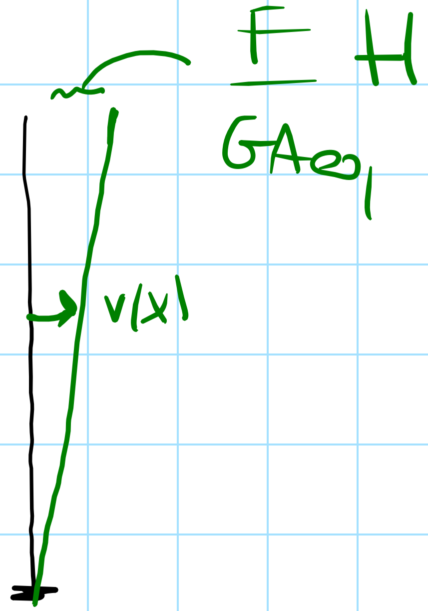
$$\begin{cases} \dfrac{dv}{dx} = \dfrac{F}{GA_{eq}} \\ v(0) = 0 \end{cases}$$

KNOWN

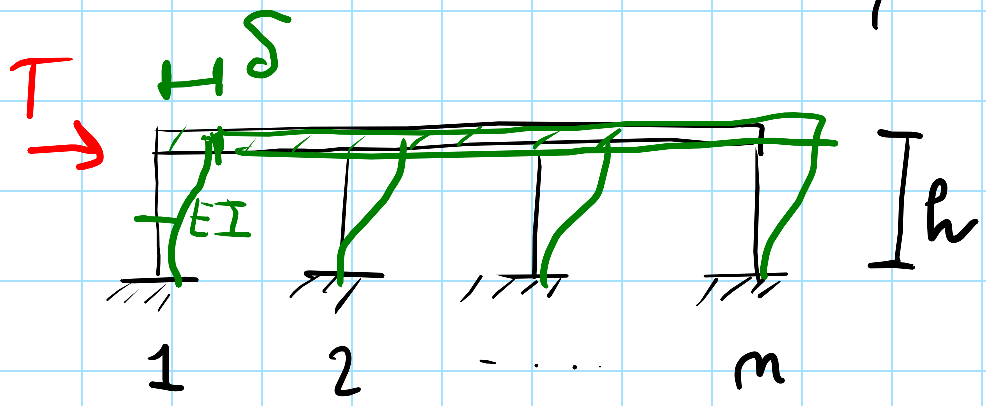
$$; \quad v(x) = \frac{F}{GA_{eq}} x + C, \quad C = 0$$

$$\boxed{v(x) = \frac{F}{GA_{eq}} x}$$

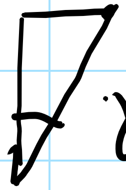
LINEAR



ESTIMATION OF "GA_{eq}" FOR A FRAME



$\delta(T)$?



$$\gamma = \frac{\delta(T)}{h}$$

$$\left(\gamma = \frac{T}{GA_{eq}} \right)$$

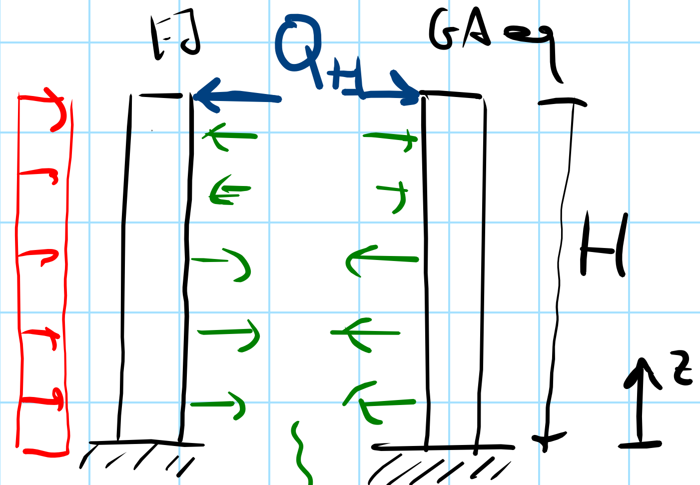
SHEAR BEAM

$$\delta = \frac{T h^3}{12 \sum_{i=1}^{m} EI_i} \longrightarrow \gamma = \frac{\delta}{h} = \frac{T h^2}{12 \sum EI_i} = \frac{1}{GA_{eq}}$$

A NOTE ON THE VALUE OF T AT A GENERIC FLOOR OF THE FRAME

DIAGR OF T

INTERACTION BETWEEN FLEXURAL CANTILEV. (EJ) AND SHEAR BEAM (GA_{eq})



$W(z)$ EXTERN. ACTION

INTERNAL DISTRIB. OF FORCES $q(z)$

UNKNOWN OF THE PROBLEM

HYPOTHESES:

THE TWO BEAMS EXCHANGE A DISTR OF FORCES $q(z)$ AND A CONCENTR FORCE Q_H ON TOP

$W(z)$ $q(z)$

$\mapsto y(z)$: COMMON DISPL. OF THE TWO BEAMS

OVERALL THE UNKNOWNS ARE:

$$y(z),\ q(z),\ Q_H$$

LET US STUDY CANTILEVER EJ CONST :

$$y''(z) = -\frac{M(z)}{EJ}$$

$$M'(z) = T(z)$$

$$y'''(z) = -\frac{T(z)}{EJ}$$

$$y^{IV}(z) = \frac{w(z) - q(z)}{EJ}$$

LET US STUDY SHEAR BEAM GA_{eq} CONST

$$y'(z) = \frac{T(z)}{GA_{eq}}$$

$$T'(z) = -q(z)$$

$$GA_{eq}\, y''(z) = -q(z)$$

GOVERNING EQ OF THE STRUCT. PROBL. + BOUNDARY CONDITIONS

EJ : $EJ\, y^{IV}(z) + q(z) = w(z)$

GA_{eq} : $-GA_{eq}\, y''(z) = q(z)$

$$\boxed{EJ\, y^{IV}(z) - GA_{eq}\, y''(z) = w(z)}$$

MODIFIED "ELASTIC LINE" (IV-ORDER PROBL)

$$y^{IV}(z) - \underbrace{\frac{GA_{eq}}{EJ}}_{\alpha^2} y''(z) = \frac{w(z)}{EJ} \quad ; \quad [\alpha] = [L^{-1}]$$

$$y^{IV}(z) - \alpha^2 y''(z) = \frac{w(z)}{EJ}$$

(NOTE THAT

αH : NON-DIMENSIONAL PARAMETER

① $y(0) = 0$, ② $y'(0) = 0$ (CANTILEVER EJ)

③ $-EJ\, y''(H) = 0$ (CANTILEVER EJ)

④ $Q_H = EJ\, y'''(H)$ (CANTL EJ) $(T(H) = -Q_H)$

$GA_{eq}\, y'(H) = Q_H$ (SHEAR BEAM GA_{eq}) $(T(H) = Q_H)$

$$\boxed{EJ\, y'''(H) = GA_{eq}\, y'(H)}$$

The general integral WHEN $w(z) = \text{CONST} = w$ IS:

$$y(z)=\underbrace{C_1+C_2 z+C_3\cosh\alpha z+C_4\sinh\alpha z}_{\text{HOMOG. PART}}-\underbrace{\frac{w z^2}{2EJ\alpha^2}}_{\text{PARTICULAR}}$$

C_1, C_2, C_3, C_4 FROM BOUNDARY CONDITIONS.

AFTER HAVING OBTAINED $y(z)$:

CANTILEVER EJ:

$M(z) = -EJ\,y''(z)$ BENDING M.

$T(z) = -EJ\,y'''(z)$ SHEAR F.

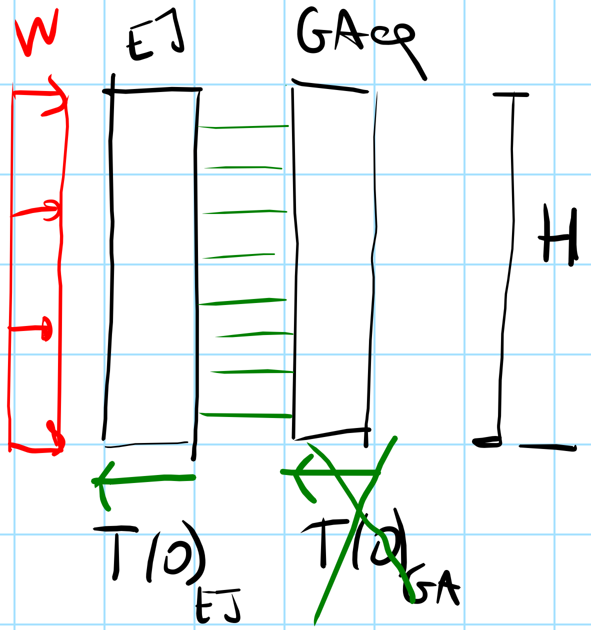
$Q_H = EJ\,y'''(H)$ SHEAR AT THE TOP

SHEAR BEAM GA_{eq}

$GA_{eq}\,y'(z) = T(z)$ SHEAR F.

$GA_{eq}\,y'(H) = Q_H$ SHEAR AT THE TOP $\circledast$

$GA_{eq}\,y''(z) = -q(z)$ INTERACTION FORCES



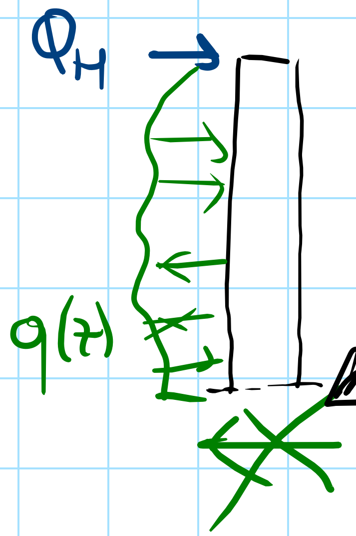
CUT AT THE FOUNDATION

$$wH = T_{EJ}(0) + \cancel{T_{GA}(0)}$$ SHOULD BE VALID!

FROM EQ PREV. PAGE $\to 0$

$$\boxed{T_{EJ}(0) = wH}$$

CONSIDER SHEAR BEAM ALONE



NO SHEAR FORCE

$$Q_H + \int_0^H q(z)\,dz = 0$$

$$Q_H + \int_0^H -GA_{eq}\, y''(z)\,dz =$$

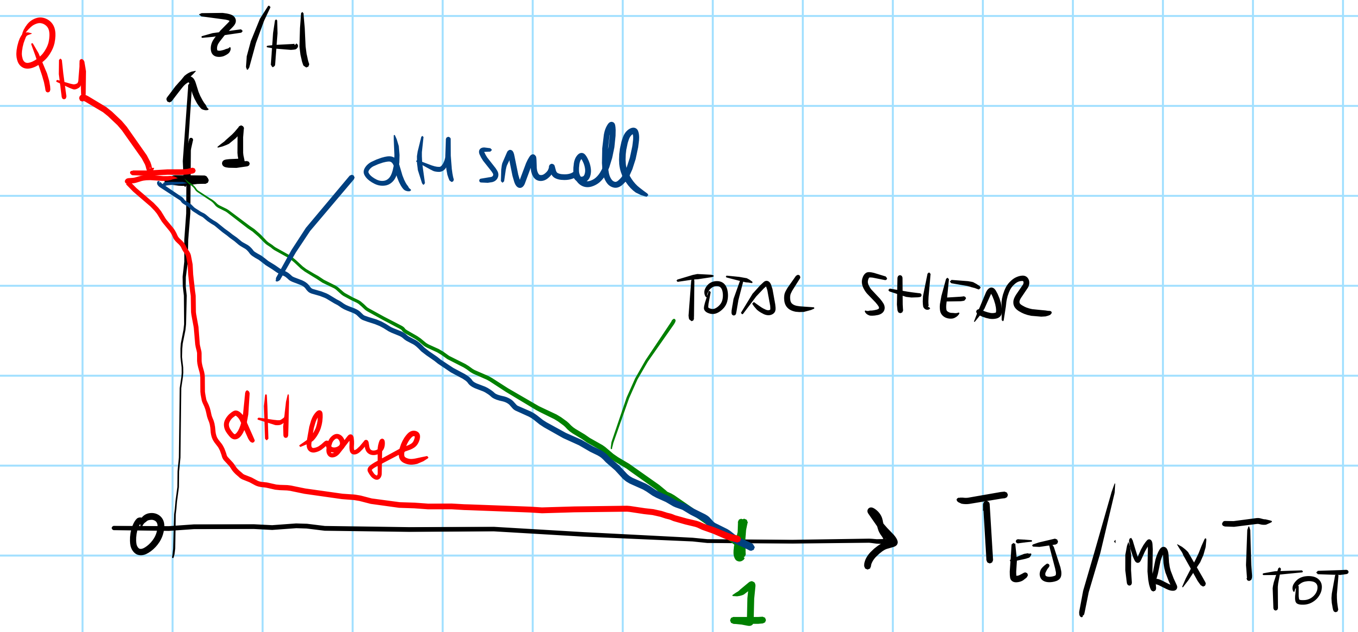
$$Q_H - GA_{eq}\left[y'(H) - y'(0)\right] =$$

$$Q_H - GA_{eq}\, y'(H) = 0$$ FOR EQ. (*)

SKETCHES OF M, T DIAGRAMS FOR THE CANTILEVER (EJ)

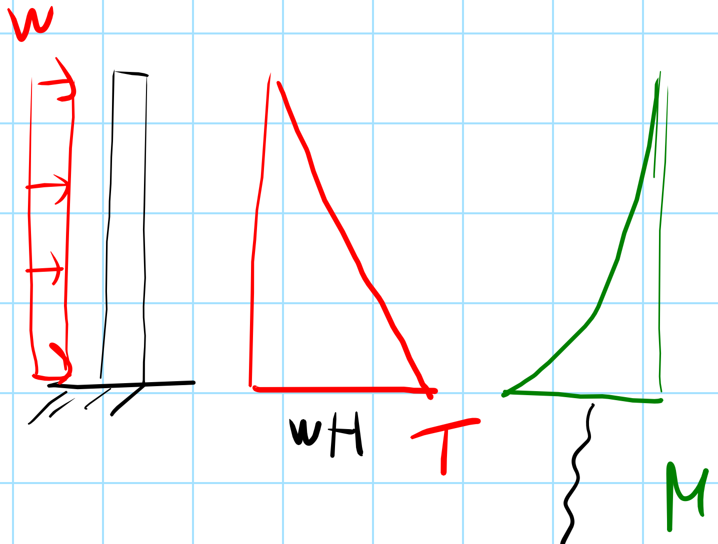
αH "large" (>10) : CANTILEVER "SOFT"

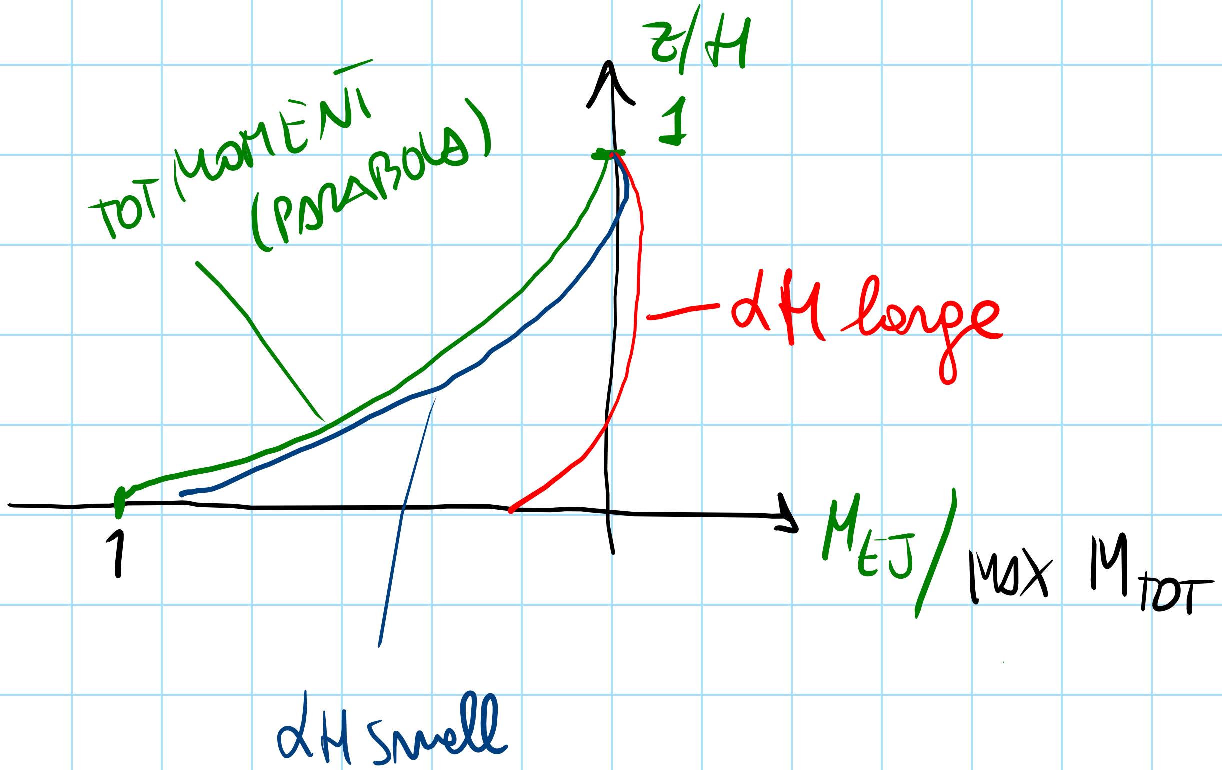
αH "small" (<1) : " "STIFF"



$\max T_{TOT} =$

$= wH$





$\max M_{TOT} = \dfrac{wH^2}{2}$

FLEXURAL CANTILEVER: