

# Unit 4b

## The growth theory

Prof. Stefania Rossi  
DEAMS, University of Trieste



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The material presented in these slides is primarily based on Mankiw's textbook: *Macroeconomics*, 2025 Macmillan / Worth Publishers

## Why growth matters (2 of 2)

Anything that affects the long-run rate of economic growth—even by a tiny amount—will have huge effects on living standards in the long run.

<b>Annual growth rate of income per capita</b>	<b>Increase in standard of living after 25 years</b>	<b>Increase in standard of living after 50 years</b>	<b>Increase in standard of living after 100 years</b>
2.0%	64.0%	169.2%	624.5%
2.5%	85.4%	243.7%	1,081.4%

# The lessons of growth theory

...can make a positive difference in the lives of hundreds of millions of people.

These lessons help us

- understand why poor countries are poor
- design policies that can help them grow
- learn how our own growth rate is affected by shocks and our government's policies

# The Solow model

- due to Robert Solow, won Nobel Prize for contributions to the study of economic growth
- a major paradigm:
  - widely used in policymaking
  - benchmark against which most recent growth theories are compared
- looks at the determinants of economic growth and the standard of living in the long run

## How the Solow model is different from Chapter 3's model, part 1

1.  $K$  is no longer fixed: investment causes it to grow, depreciation causes it to shrink
2.  $L$  is no longer fixed: population growth causes it to grow
3. the consumption function is simpler

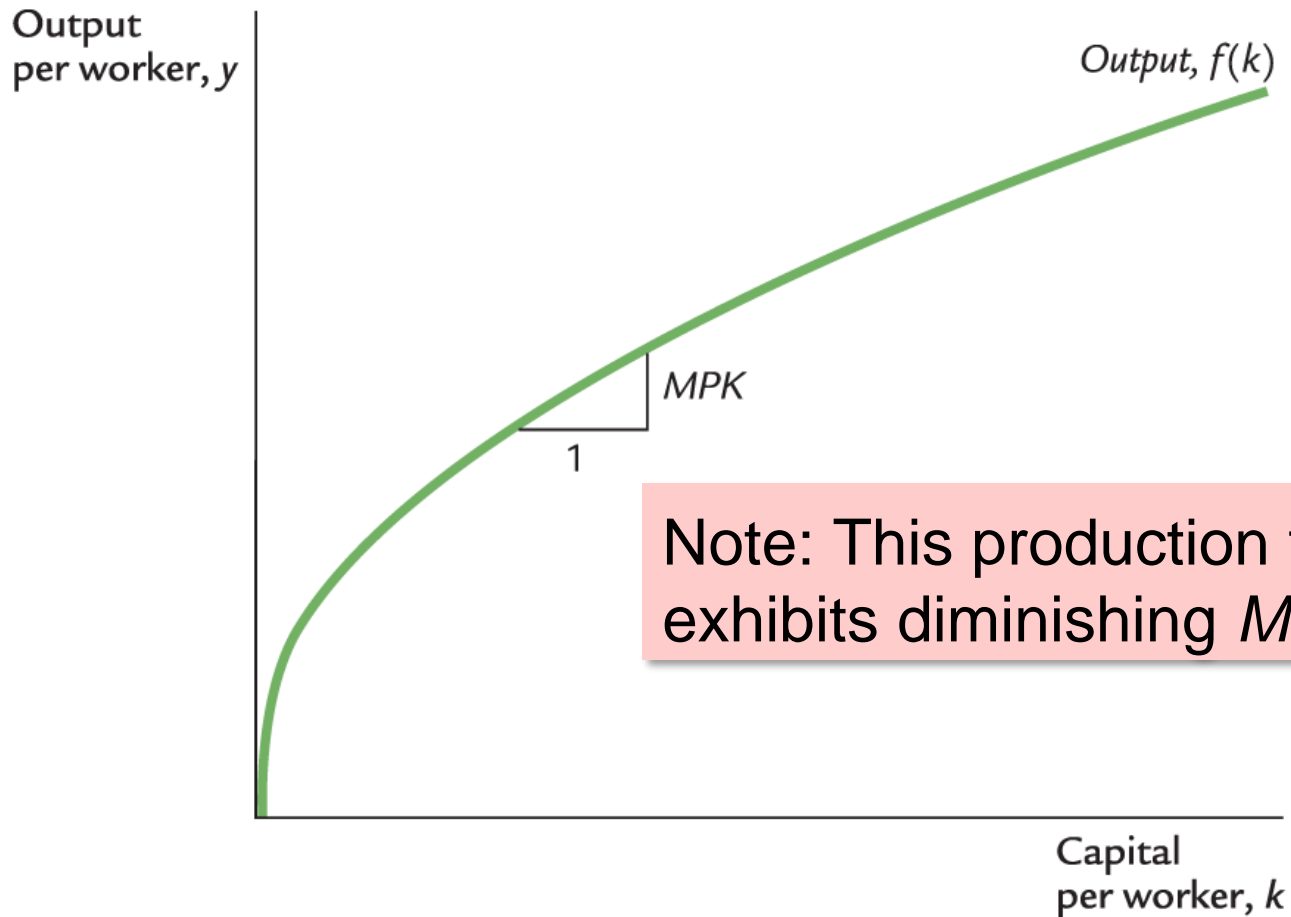
## How the Solow model is different from Chapter 3's model, part 2

4. no **G** or **T** (only to simplify presentation; we can still do fiscal policy experiments)
5. cosmetic differences

## The production function (1 of 2)

- In aggregate terms:  $Y = F(K, L)$
- Define:  $y = Y/L =$  output per worker  
 $k = K/L =$  capital per worker
- Assume constant returns to scale:  
 $zY = F(zK, zL)$  for any  $z > 0$
- Pick  $z = 1/L$ . Then
  - $Y/L = F(K/L, 1)$
  - $y = F(k, 1)$
  - $y = f(k)$  where  $f(k) = F(k, 1)$

## The production function (2 of 2)



## The national income identity

- $Y = C + I$  (remember, no  $G$ )
- In “per worker” terms:

$$y = c + i$$

where  $c = C/L$  and  $i = I/L$

# The consumption function

- $s$  = the saving rate, the fraction of income that is saved ( $s$  is an exogenous parameter)

Note:  $s$  is the *only* lowercase variable that is *not equal to* its uppercase version divided by  $L$

- Consumption function:  $c = (1-s)y$  (*per worker*)

## Saving and investment

- Saving (per worker) =  $y - c$   
=  $y - (1-s)y$   
=  $sy$
- National income identity is  $y = c + i$

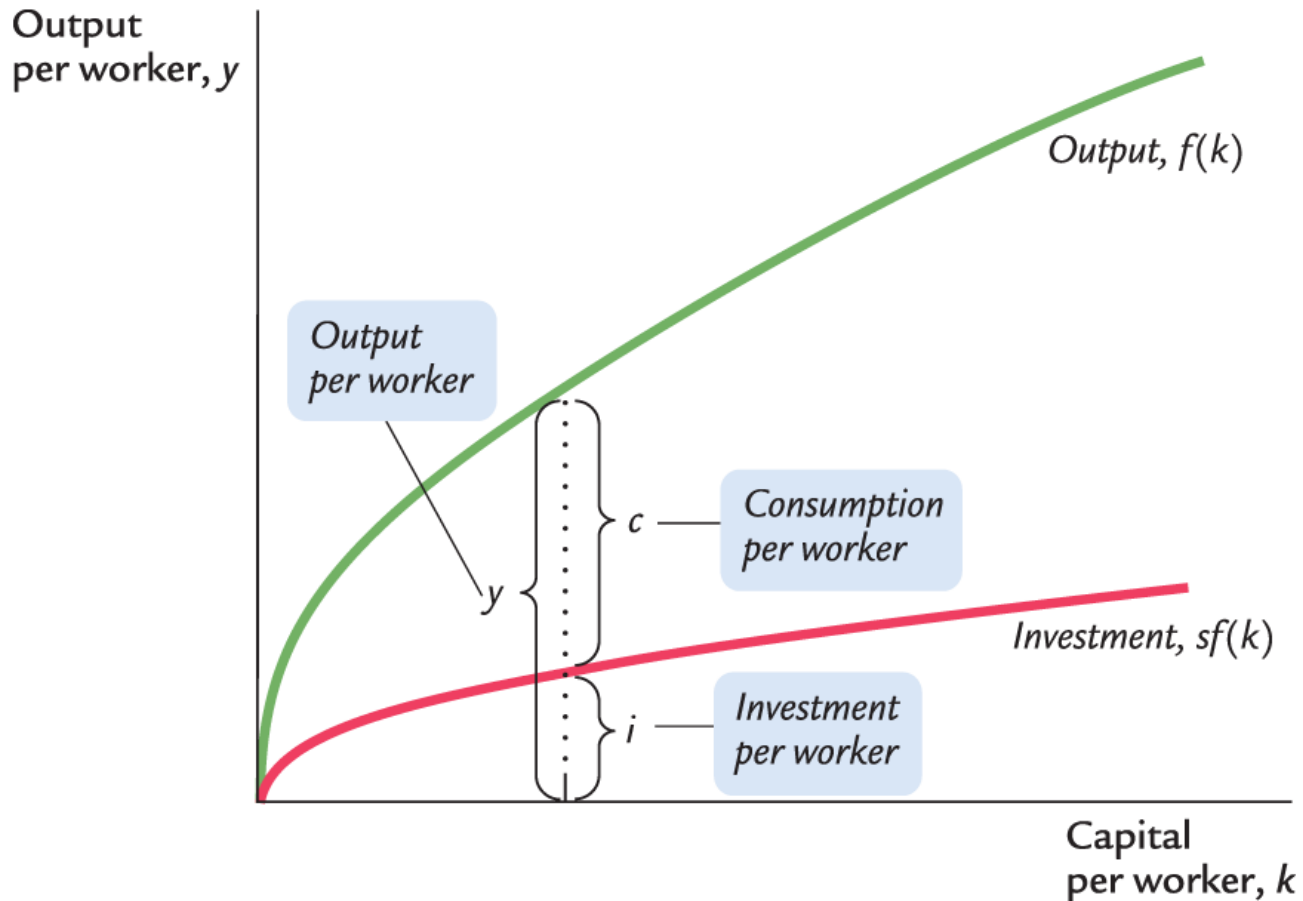
Rearrange to get  $i = y - c = sy$

*(investment = saving, as in Chapter 3!)*

- Using the results above,

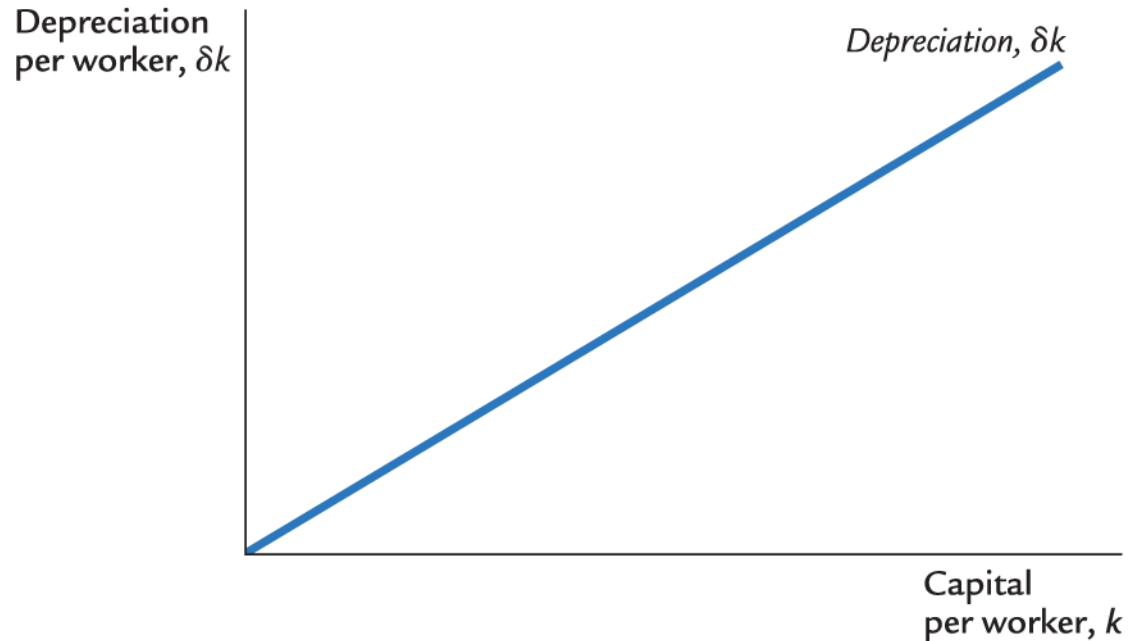
$$i = sy = sf(k)$$

# Output, consumption, and investment



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# Depreciation



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$\delta$  = the rate of depreciation  
= the fraction of the capital stock that wears out each period

## Capital accumulation

*The basic idea: Investment increases the capital stock; depreciation reduces it.*

change in capital stock = investment – depreciation

$$\Delta k = i - \delta k$$

Since  $i = sf(k)$ , this becomes:

$$\Delta k = sf(k) - \delta k$$

## The equation of motion for $k$

$$\Delta \mathbf{k} = s\mathbf{f}(\mathbf{k}) - \delta \mathbf{k}$$

- The Solow model's central equation
- Determines behavior of capital over time . . .
- . . . which, in turn, determines behavior of all the other endogenous variables because they all depend on  $\mathbf{k}$ .
- Example:

income per person:  $\mathbf{y} = \mathbf{f}(\mathbf{k})$

consumption per person:  $\mathbf{c} = (1 - s) \mathbf{f}(\mathbf{k})$

## The steady state, part 1

$$\Delta \mathbf{k} = \mathbf{s}f(\mathbf{k}) - \delta \mathbf{k}$$

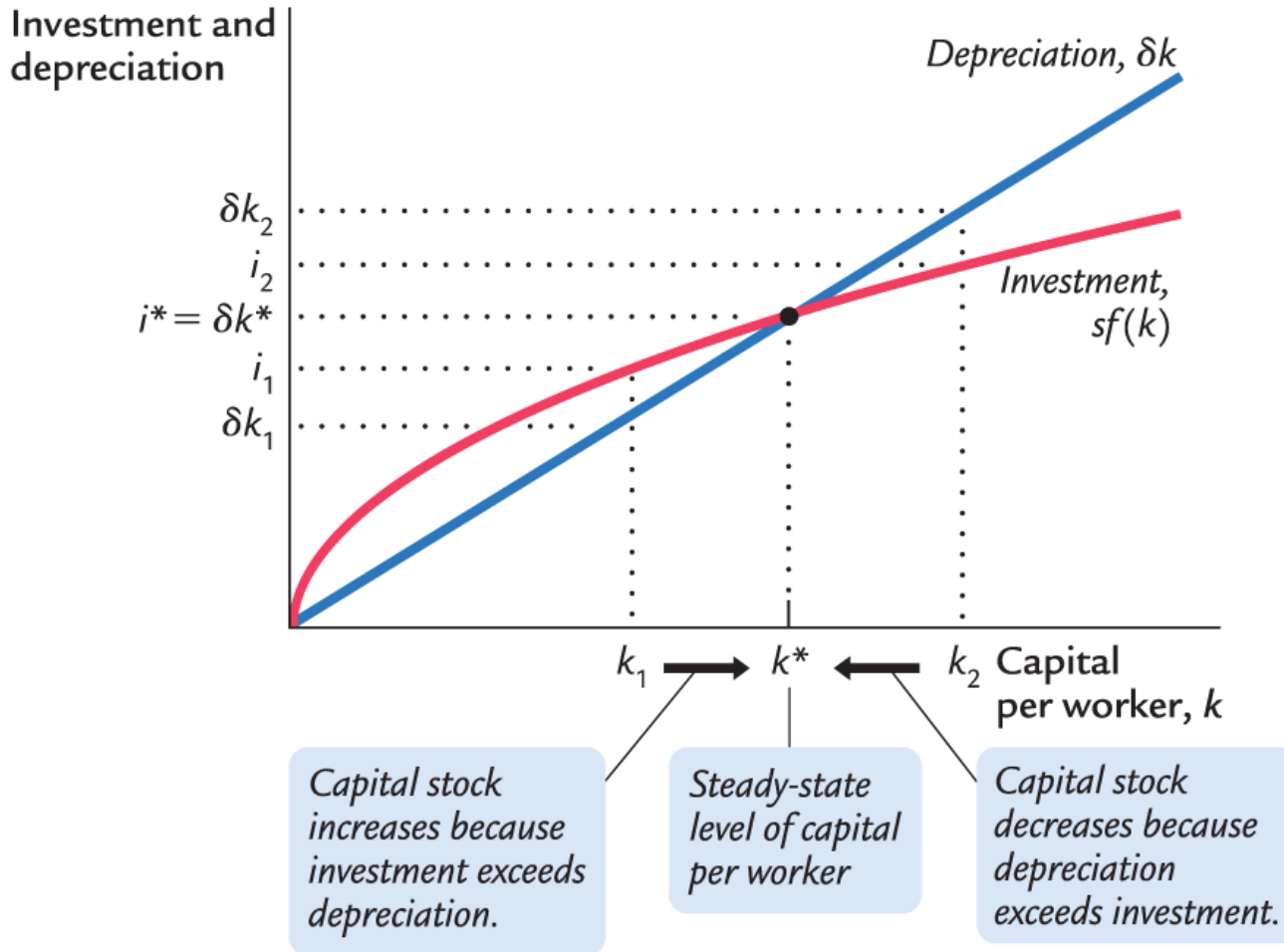
If investment is just enough to cover depreciation  
[ $\mathbf{s}f(\mathbf{k}) = \delta \mathbf{k}$ ],

then capital per worker will remain constant:

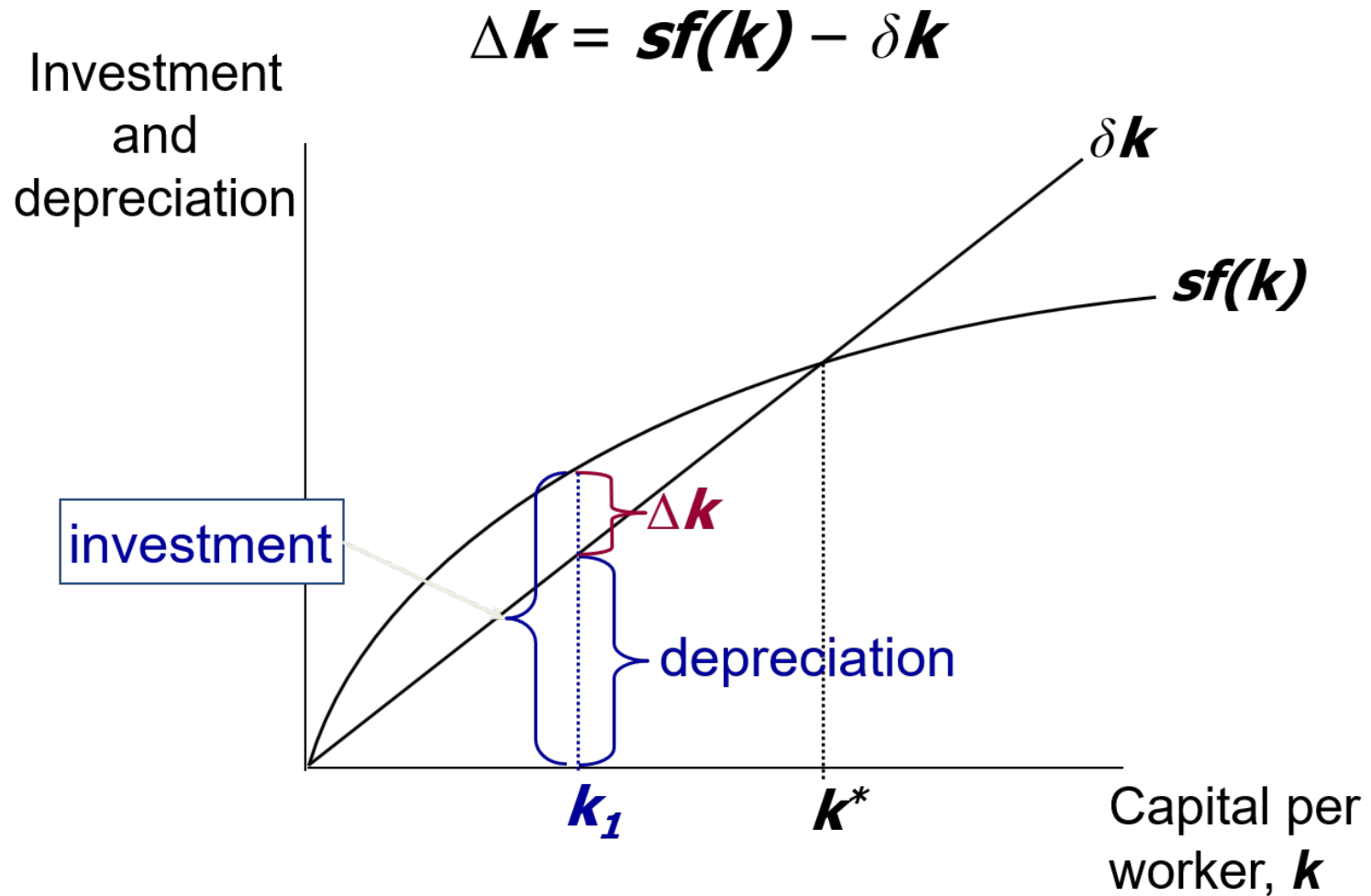
$$\Delta \mathbf{k} = 0.$$

This occurs at one value of  $\mathbf{k}$ , denoted  $\mathbf{k}^*$ , called the ***steady-state capital stock***.

# The steady state, part 2

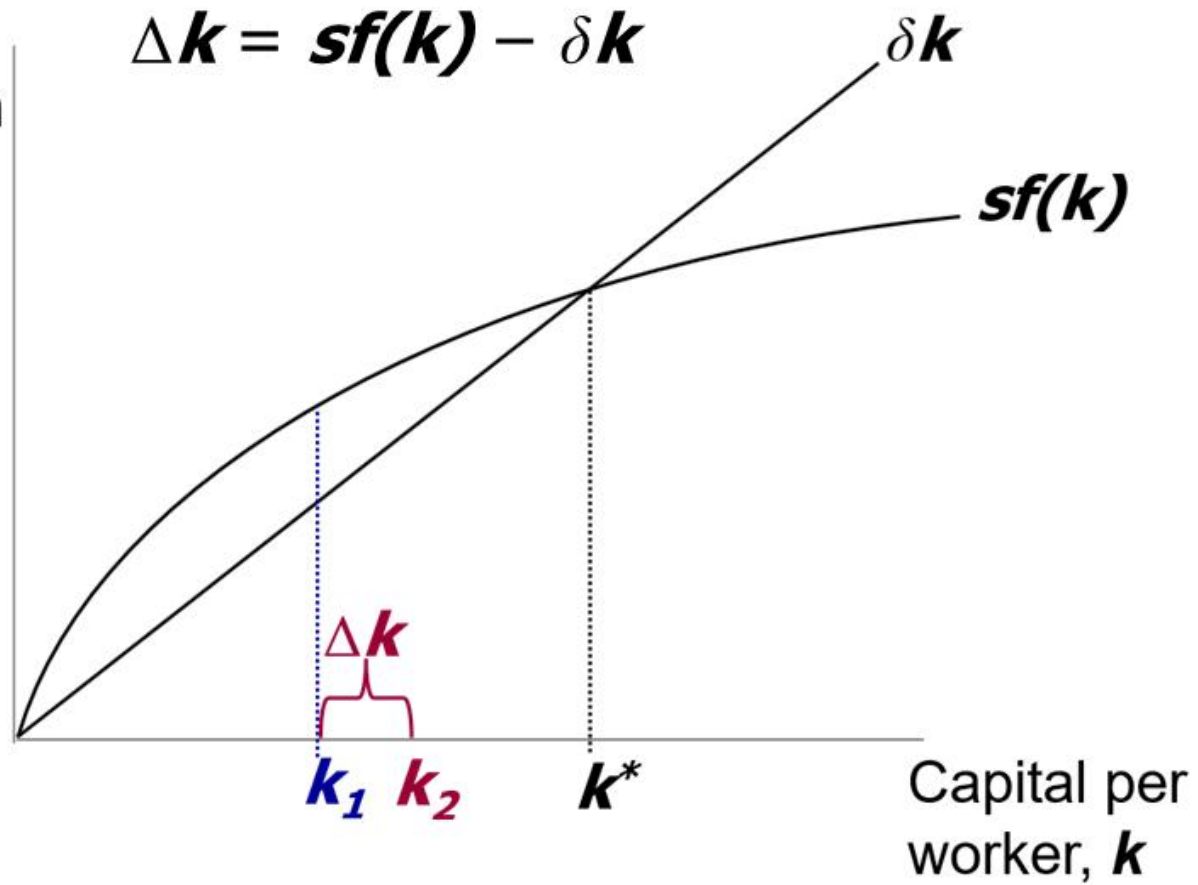


# Moving toward the steady state, part 1

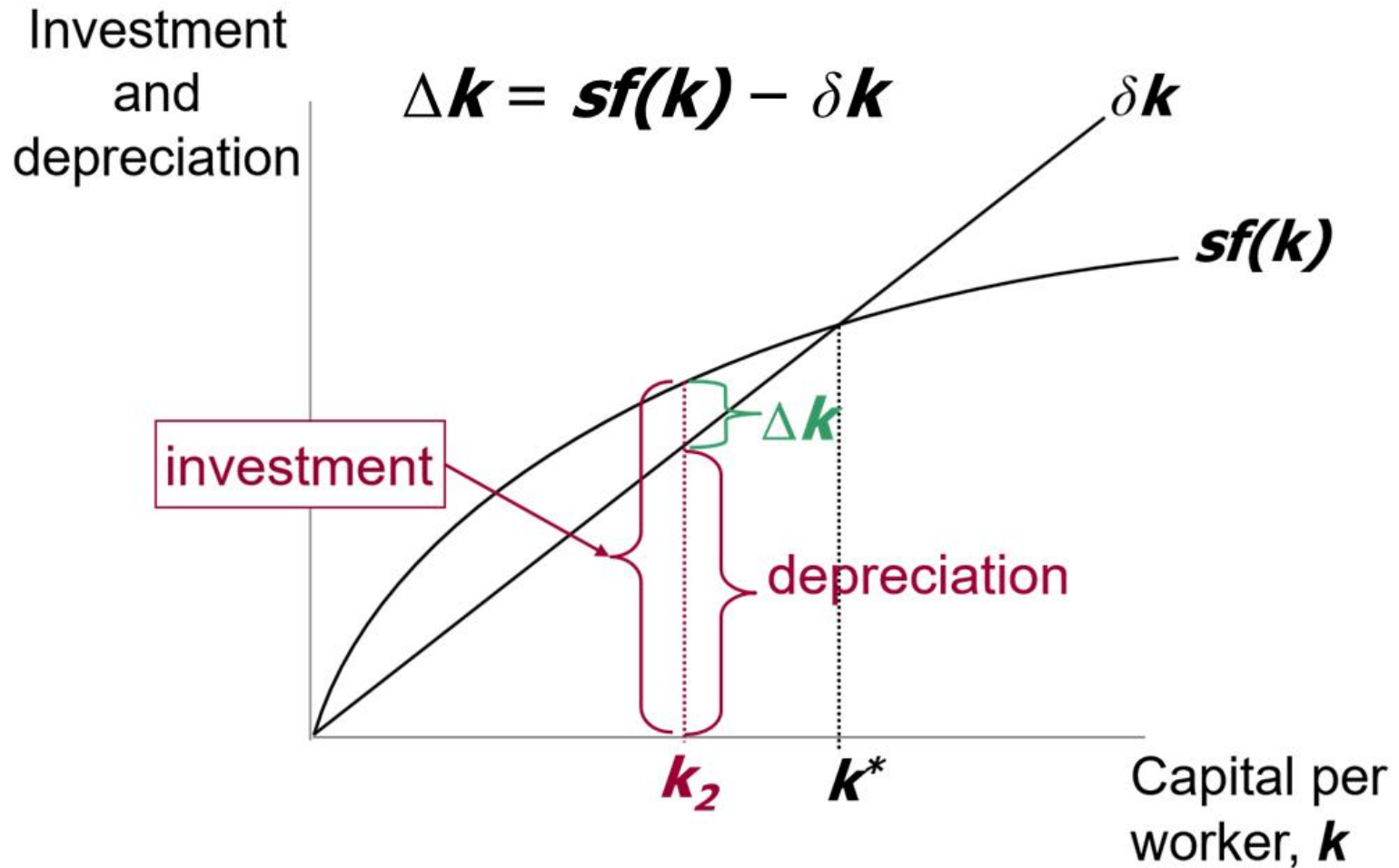


## Moving toward the steady state, part 2

Investment  
and  
depreciation

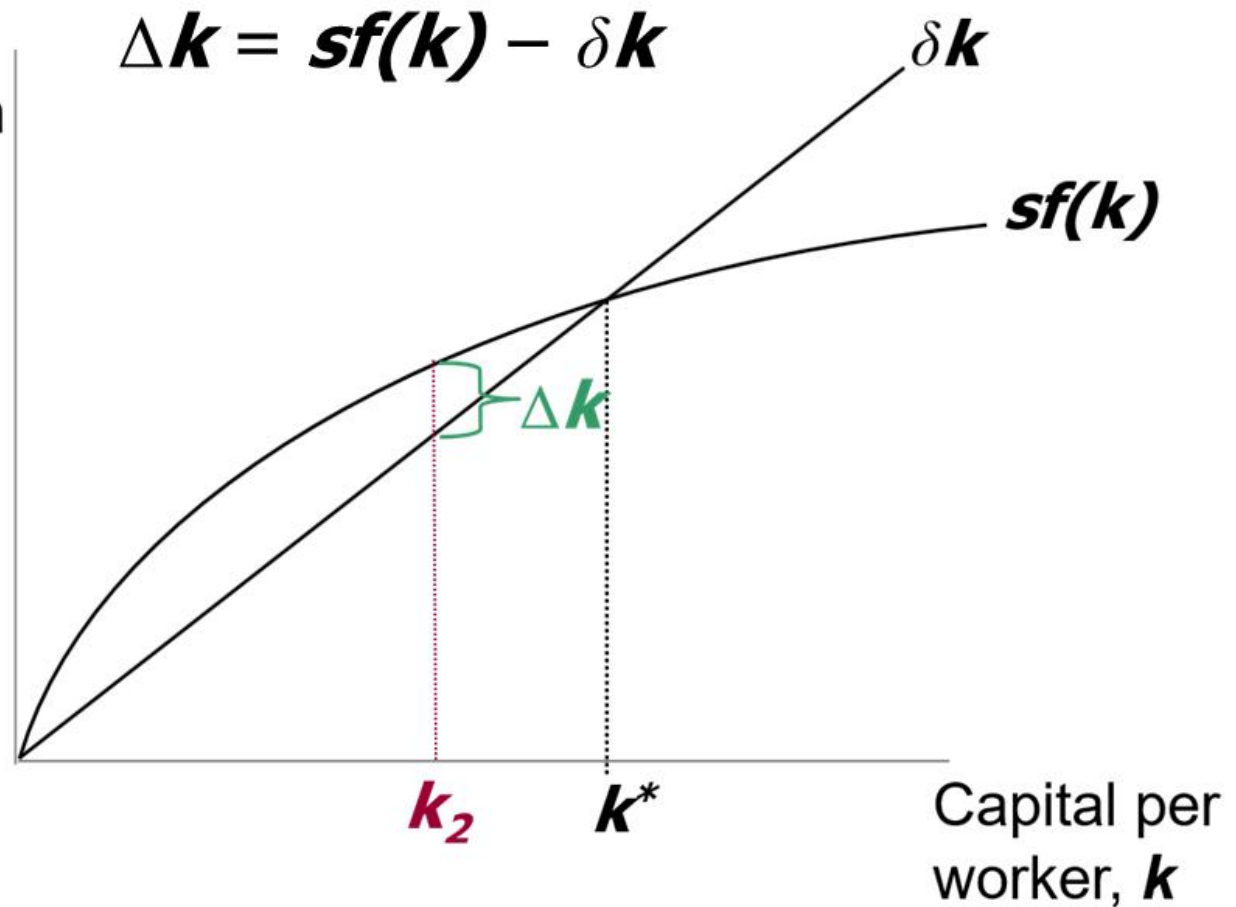


# Moving toward the steady state, part 3



# Moving toward the steady state, part 4

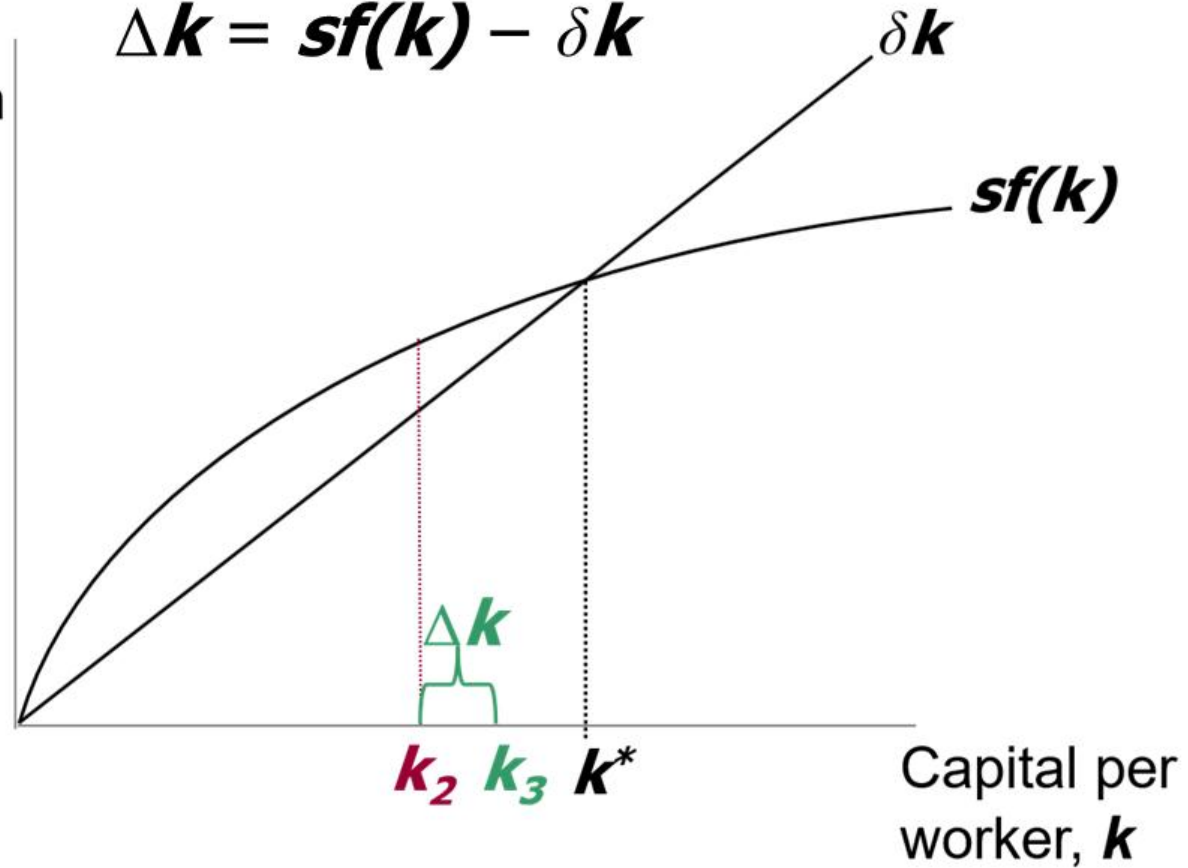
Investment  
and  
depreciation



# Moving toward the steady state, part 5

Investment  
and  
depreciation

$$\Delta k = sf(k) - \delta k$$



## Moving toward the steady state, part 6

Investment  
and  
depreciation

$$\Delta k = sf(k) - \delta k$$

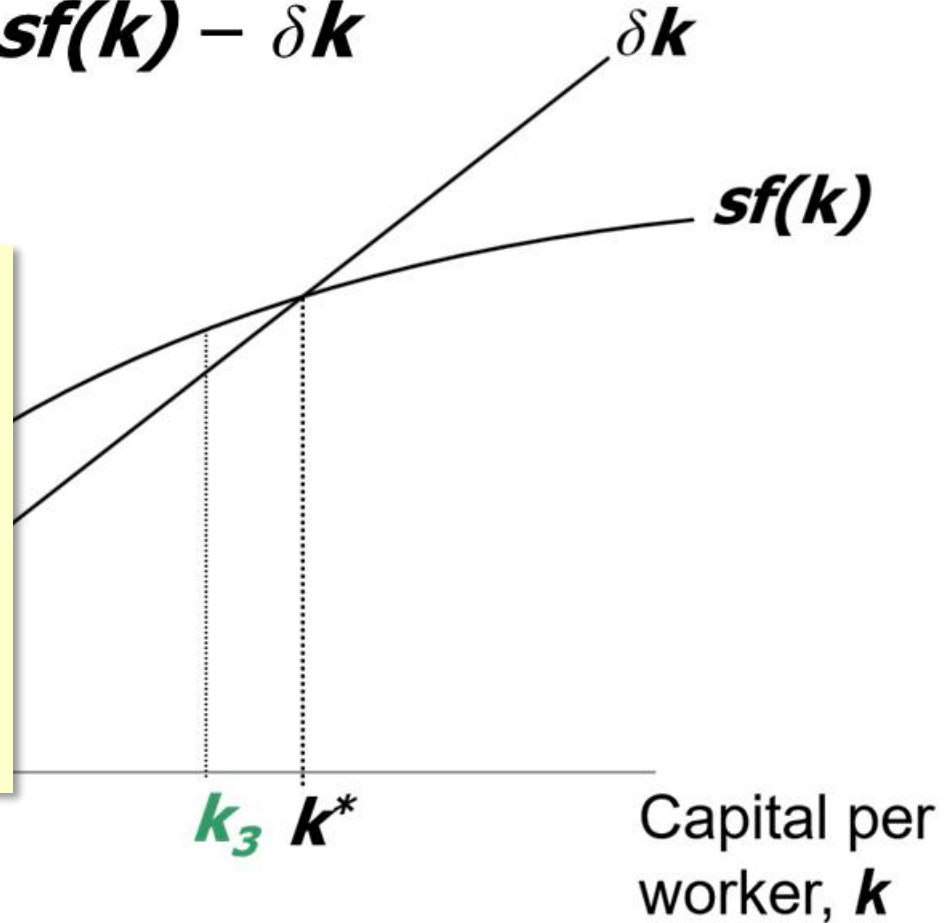
$\delta k$

$sf(k)$

*Summary:*  
As long as  $k < k^*$ ,  
investment will exceed  
depreciation,  
and  $k$  will continue to  
grow toward  $k^*$ .

$k_3$   $k^*$

Capital per  
worker,  $k$



## NOW YOU TRY

### Approaching $k^*$ from above

Draw the Solow model diagram, labeling the steady state  $k^*$ .

On the horizontal axis, pick a value greater than  $k^*$  for the economy's initial capital stock. Label it  $k_1$ .

Show what happens to  $k$  over time.

Does  $k$  move toward the steady state or away from it?

## A numerical example, part 1

Production function (aggregate):

$$Y = F(K, L) = \sqrt{K \times L} = K^{1/2} L^{1/2}$$

To derive the per-worker production function, divide through by  $L$ :

$$\frac{Y}{L} = \frac{K^{1/2} L^{1/2}}{L} = \left( \frac{K}{L} \right)^{1/2}$$

Then substitute  $y = Y/L$  and  $k = K/L$  to get

$$y = f(k) = k^{1/2}$$

## A numerical example, part 2

Assume:

- $s = 0.3$
- $\delta = 0.1$
- initial value of  $k = 4.0$

## Approaching the steady state: A numerical example

Assumptions :  $y = \sqrt{k}$ ;  $s = 0.3$ ;  $\delta = 0.1$ ; initial  $k = 4.0$

Year	$k$	$y$	$c$	$i$	$\delta k$	$\Delta k$
1	4.000	2.000	1.400	0.600	0.400	0.200
2	4.200	2.049	1.435	0.615	0.420	0.195
3	4.395	2.096	1.467	0.629	0.440	0.189
4	4.584	2.141	1.499	0.642	0.458	0.184
5	4.768	2.184	1.529	0.655	0.477	0.178
10	5.602	2.367	1.657	0.710	0.560	0.150
25	7.321	2.706	1.894	0.812	0.732	0.080
100	8.962	2.994	2.096	0.898	0.896	0.002
$\infty$	9.000	3.000	2.100	0.900	0.900	0.000

## NOW YOU TRY

Solve for the steady state

Continue to assume

$$\mathbf{s} = 0.3, \delta = 0.1, \text{ and } \mathbf{y} = \mathbf{k}^{1/2}$$

Use the equation of motion

$$\Delta \mathbf{k} = \mathbf{s} \mathbf{f}(\mathbf{k}) - \delta \mathbf{k}$$

to solve for the steady-state values of  $\mathbf{k}$ ,  $\mathbf{y}$ , and  $\mathbf{c}$ .

## NOW YOU TRY

Solve for the steady state, answers

$$\Delta \mathbf{k} = 0 \quad \text{definition of steady state}$$

$$\mathbf{s} f(\mathbf{k}^*) = \delta \mathbf{k}^* \quad \text{eq'n of motion with } \Delta \mathbf{k} = 0$$

$$0.3\sqrt{\mathbf{k}^*} = 0.1\mathbf{k}^* \quad \text{using assumed values}$$

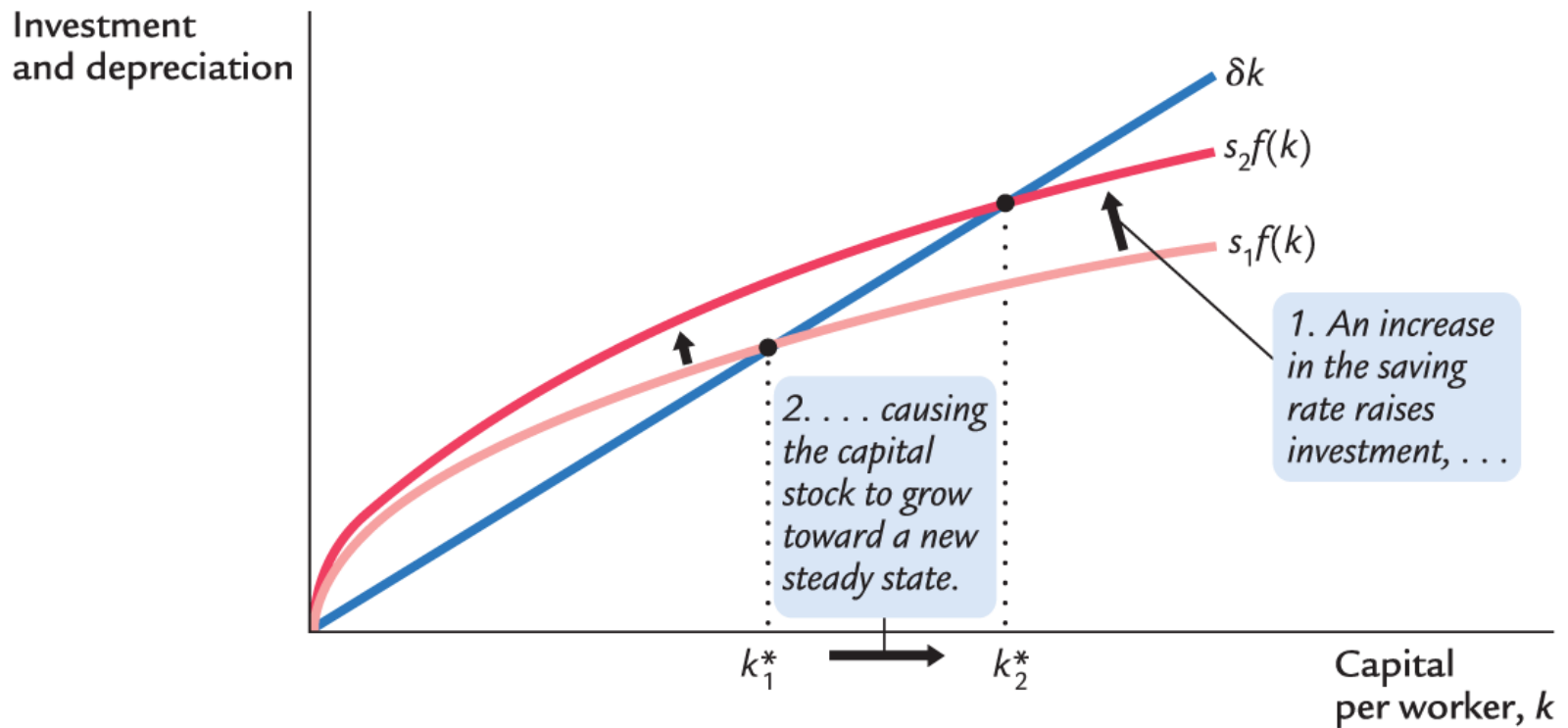
$$3 = \frac{\mathbf{k}^*}{\sqrt{\mathbf{k}^*}} = \sqrt{\mathbf{k}^*}$$

$$\text{Solve to get: } \mathbf{k}^* = 9 \quad \text{and} \quad \mathbf{y}^* = \sqrt{\mathbf{k}^*} = 3$$

$$\text{Finally, } \mathbf{c}^* = (1 - \mathbf{s})\mathbf{y}^* = 0.7 \times 3 = 2.1$$

# An increase in the saving rate

An increase in the saving rate raises investment...  
...causing  $k$  to grow toward a new steady state:



## Prediction: Countries with higher rates of saving and investment

- The Solow model predicts that countries with higher rates of saving and investment will have higher levels of capital and income per worker in the long run.
- Are the data consistent with this prediction?

## The Golden Rule: Introduction

- Different values of  $s$  lead to different steady states. How do we know which is the “best” steady state?
- The “best” steady state has the highest possible consumption per person:  $c^* = (1-s) f(k^*)$ .
- An increase in  $s$ 
  - leads to higher  $k^*$  and  $y^*$ , which raises  $c^*$
  - reduces consumption’s share of income  $(1-s)$ , which lowers  $c^*$ .
- So, how do we find the  $s$  and  $k^*$  that maximize  $c^*$ ?

## The Golden Rule capital stock, part 1

$k_{gold}^*$  = the **Golden Rule level of capital**, the steady-state value of  $k$  that maximizes consumption

To find it, first express  $c^*$  in terms of  $k^*$ :

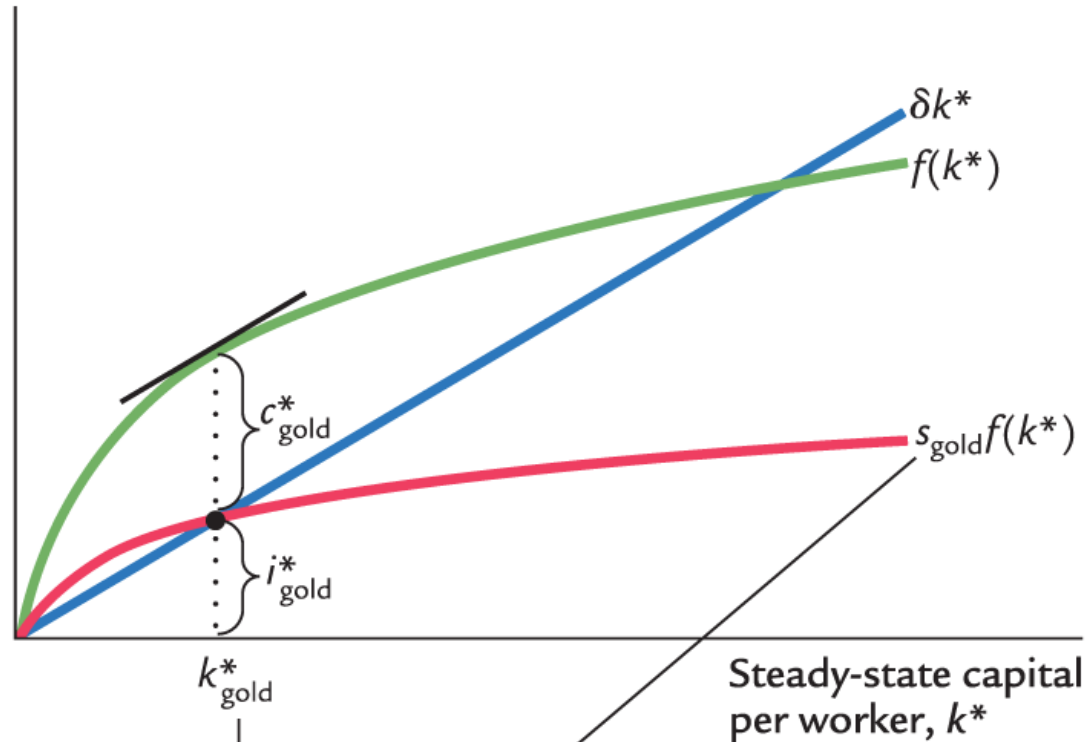
$$\begin{aligned}c^* &= y^* && - i^* \\ &= f(k^*) && - i^* \\ &= f(k^*) && - \delta k^*\end{aligned}$$

In the steady state:  
 $i^* = \delta k^*$   
because  $\Delta k = 0$ .

# The Golden Rule capital stock, part 2

Steady-state output, depreciation, and investment per worker

Then, graph  $f(k^*)$  and  $\delta k^*$ , looking for the point where the gap between them is biggest.



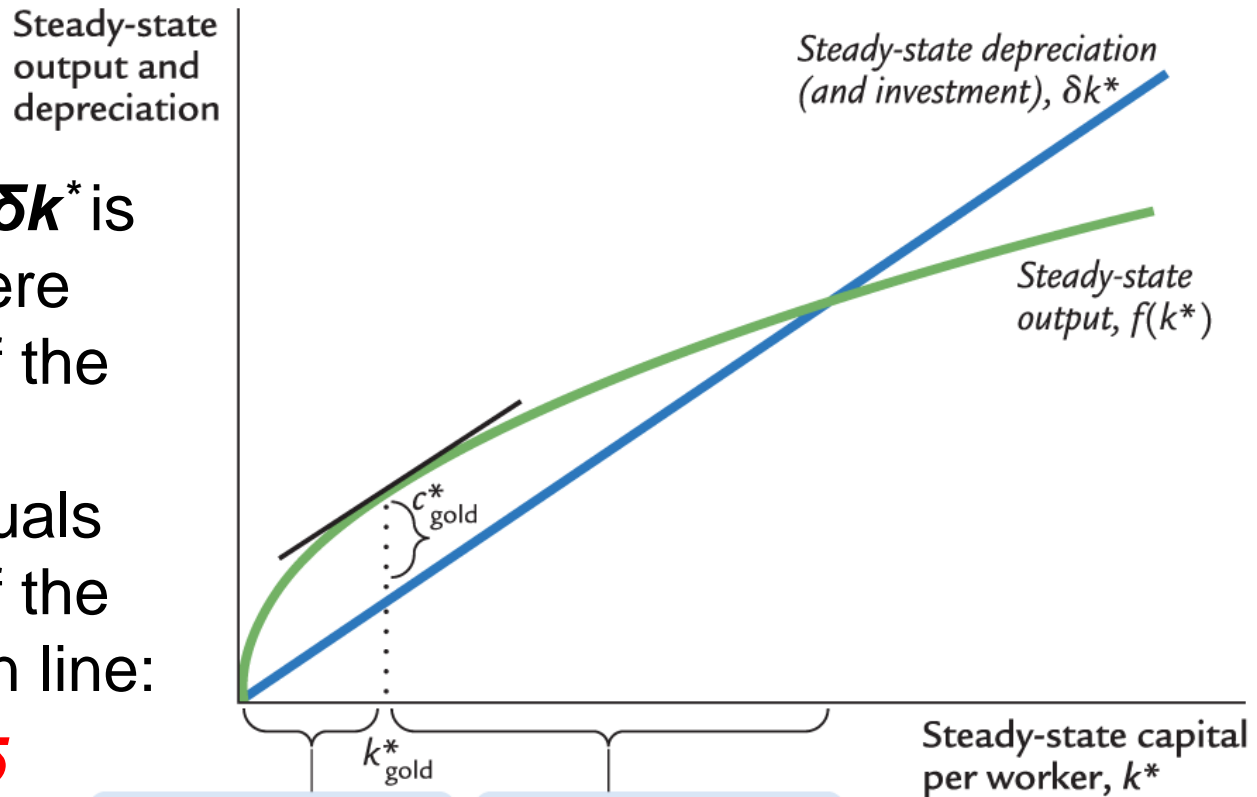
1. To reach the Golden Rule steady state . . .

2. . . . the economy needs the right saving rate.

# The Golden Rule capital stock, part 3

$c^* = f(k^*) - \delta k^*$  is biggest where the slope of the production function equals the slope of the depreciation line:

$$MPK = \delta$$



*Below the Golden Rule steady state, increases in steady-state capital raise steady-state consumption.*

*Above the Golden Rule steady state, increases in steady-state capital reduce steady-state consumption.*

## The transition to the Golden Rule steady state

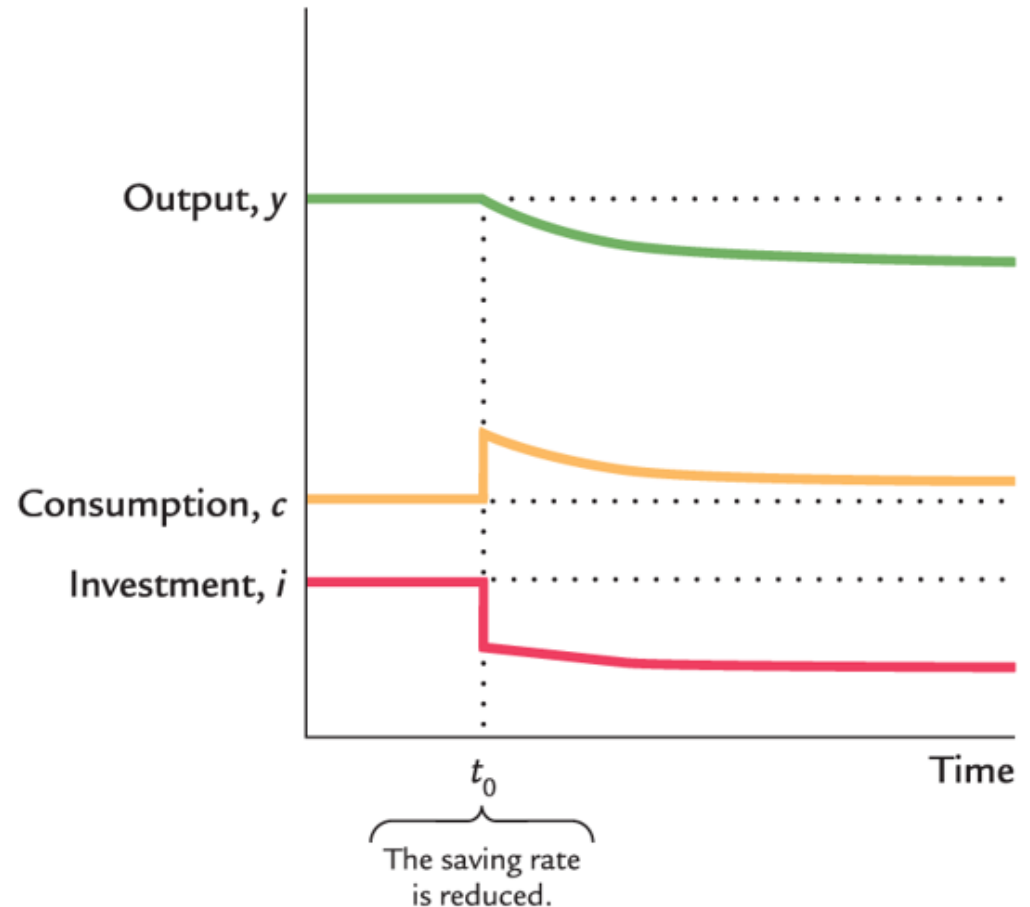
- The economy does NOT have a tendency to move toward the Golden Rule steady state.
- Achieving the Golden Rule requires that policymakers adjust  $s$ .
- This adjustment leads to a new steady state with higher consumption.
- But what happens to consumption during the transition to the Golden Rule?

# Starting with too much capital

If  $k^* > k_{gold}^*$

then increasing  $c^*$   
requires a fall in  $s$ .

In the transition to the  
Golden Rule,  
consumption is  
higher at all points in  
time.

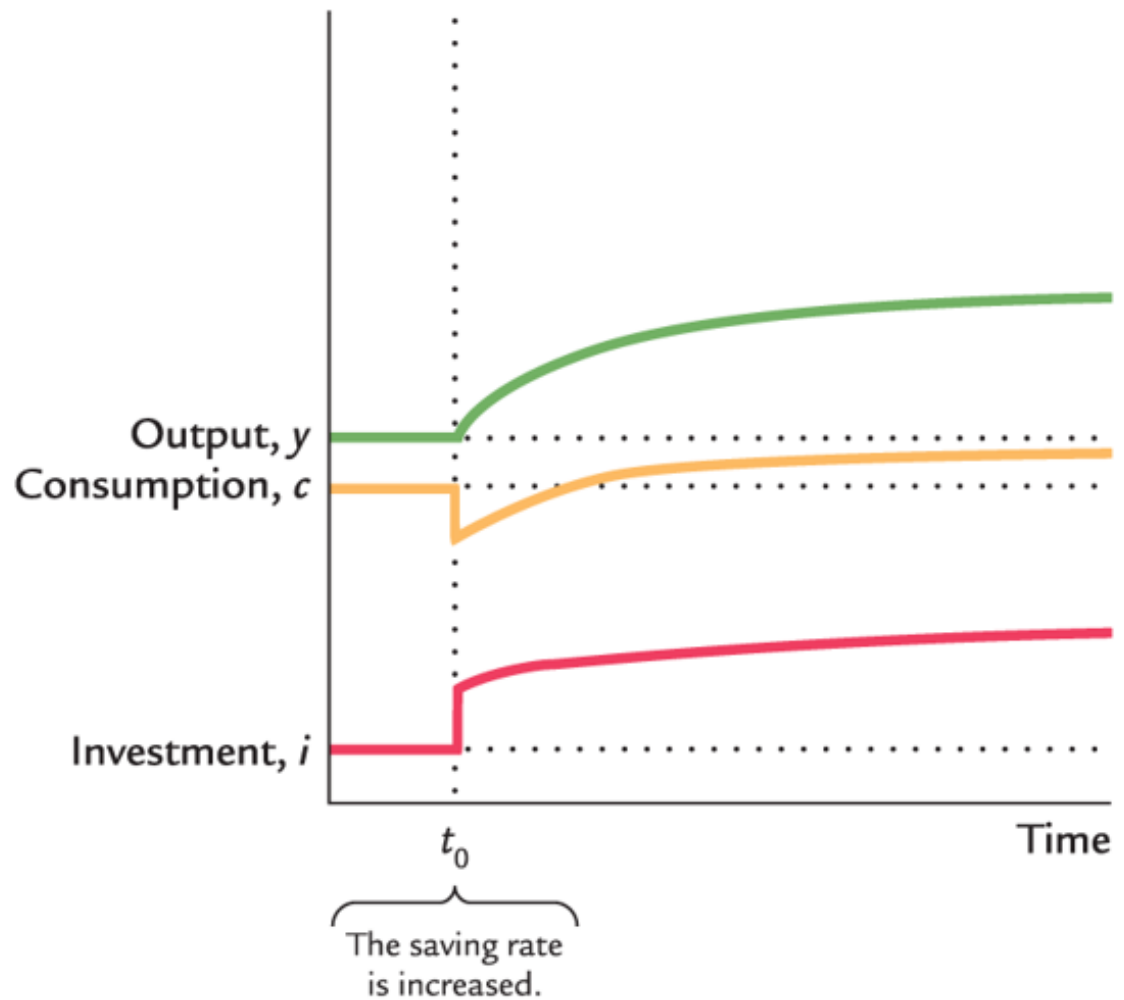


## Starting with too little capital

If  $k^* < k_{gold}^*$

then increasing  $c^*$   
requires an  
increase in  $s$ .

Future generations  
enjoy higher  
consumption, but  
the current one  
experiences an  
initial drop in  
consumption.



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## Population growth

- Assume that the population and labor force grow at rate  $n$  (exogenous):

$$\frac{\Delta L}{L} = n$$

- Example: Suppose  $L = 1,000$  in year 1 and the population is growing at 2% per year ( $n = 0.02$ ).
- Then  $\Delta L = n L = 0.02 \times 1,000 = 20$ , so  $L = 1,020$  in year 2.

## Break-even investment


- $(\delta + n)k = \text{break-even investment}$ , the amount of investment necessary to keep  $k$  constant.
- Break-even investment includes:
  - $\delta k$  to replace capital as it wears out
  - $n k$  to equip new workers with capital

(Otherwise,  $k$  would fall as the existing capital stock is spread more thinly over a larger population of workers.)

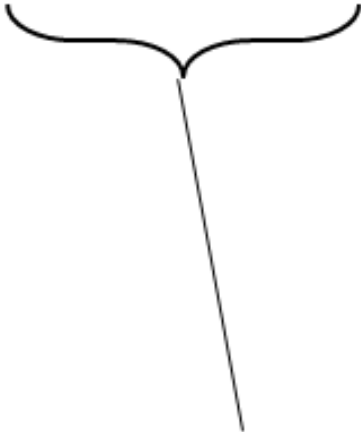
## The equation of motion for $k$ with population growth

- With population growth, the equation of motion for  $k$  is:

$$\Delta k = s f(k) - (\delta + n) k$$

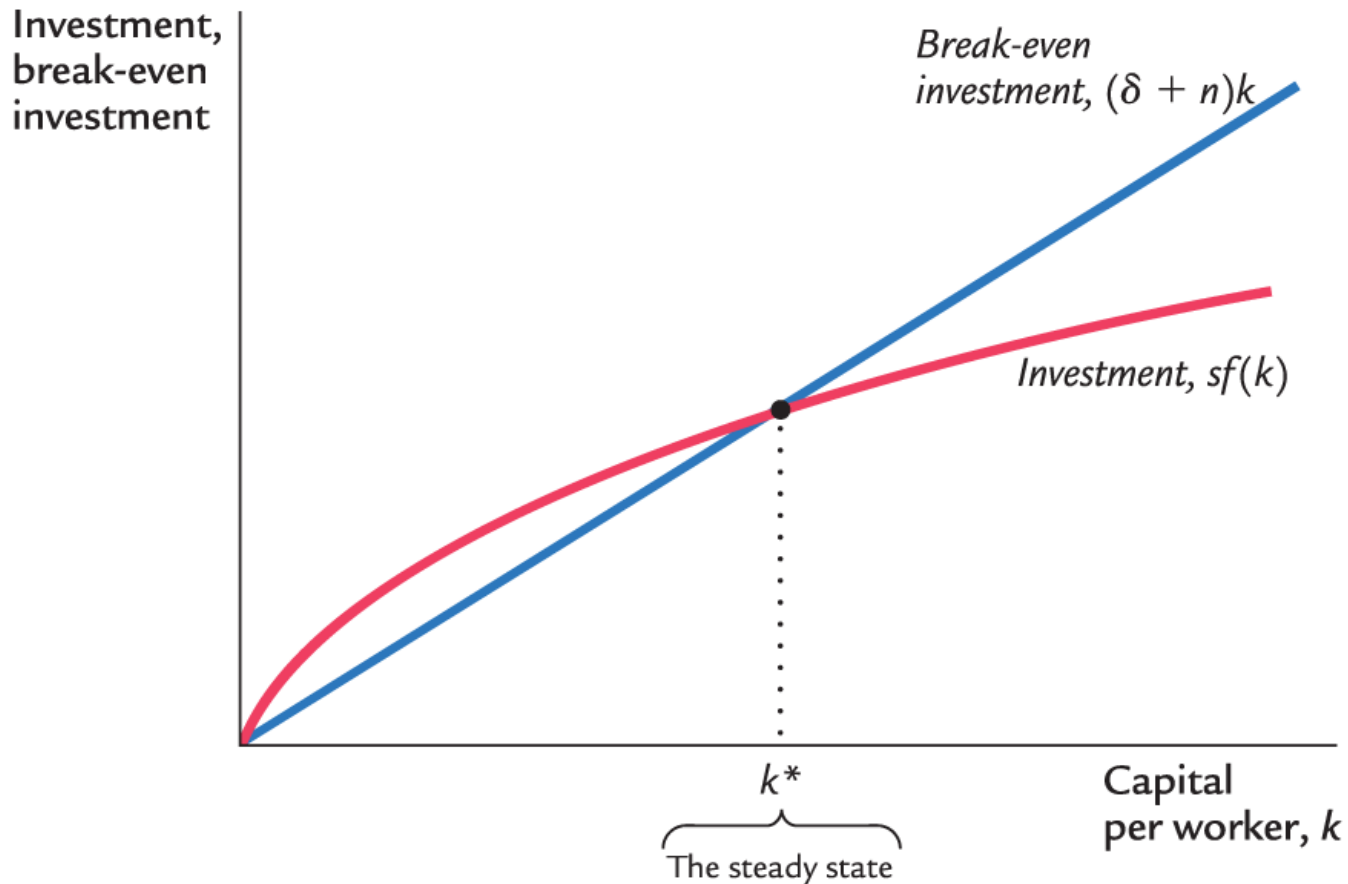


actual  
investment



break-even  
investment

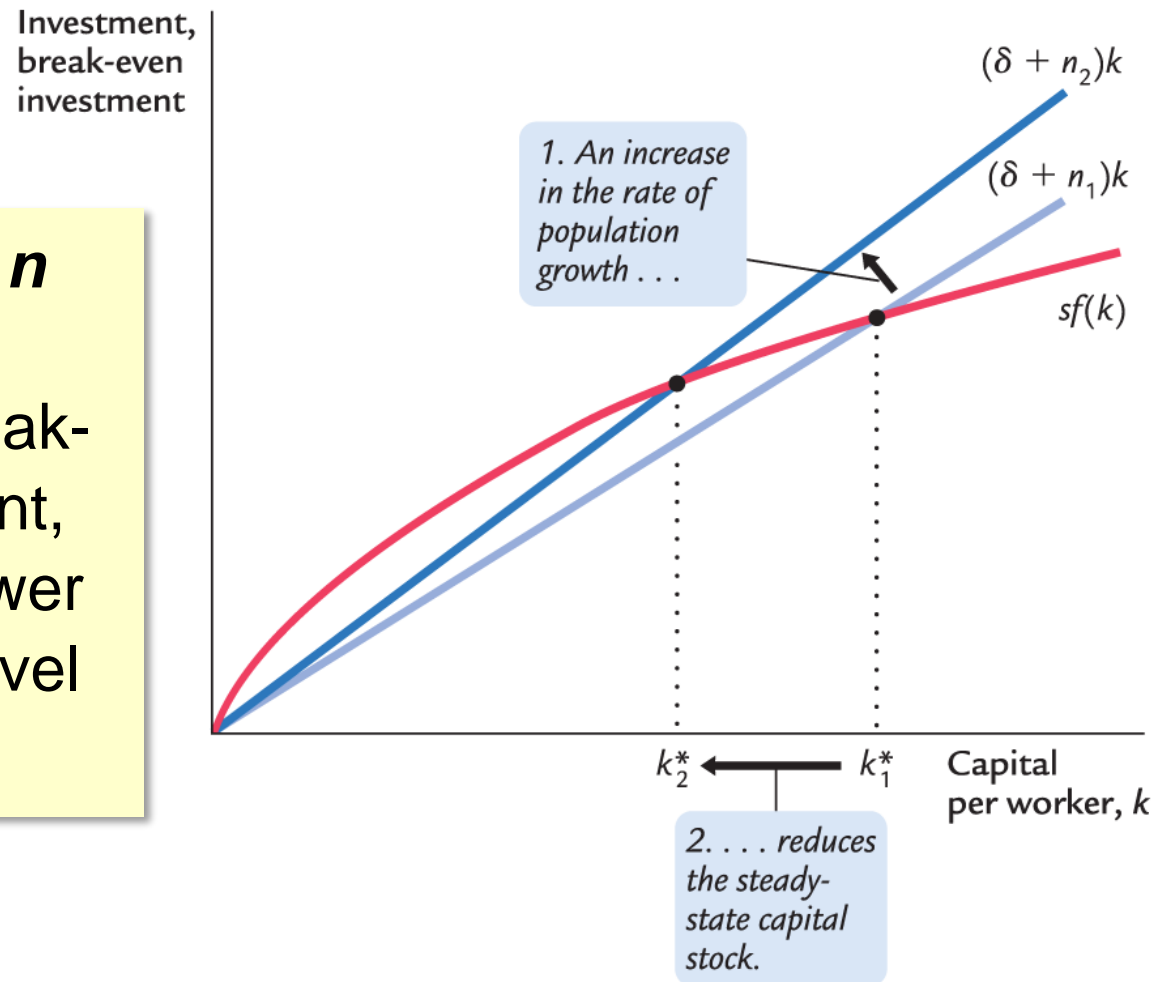
# The Solow model diagram



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# The impact of population growth

An increase in  $n$  causes an increase in break-even investment, leading to a lower steady-state level of  $k$ .



## Prediction

- The Solow model predicts that countries with higher population growth rates will have lower levels of capital and income per worker in the long run.
- Are the data consistent with this prediction?

## The Golden Rule with population growth

To find the Golden Rule capital stock, express  $c^*$  in terms of  $k^*$ :

$$\begin{aligned}c^* &= y^* - i^* \\ &= f(k^*) - (\delta + n)k^*\end{aligned}$$

$c^*$  is maximized when

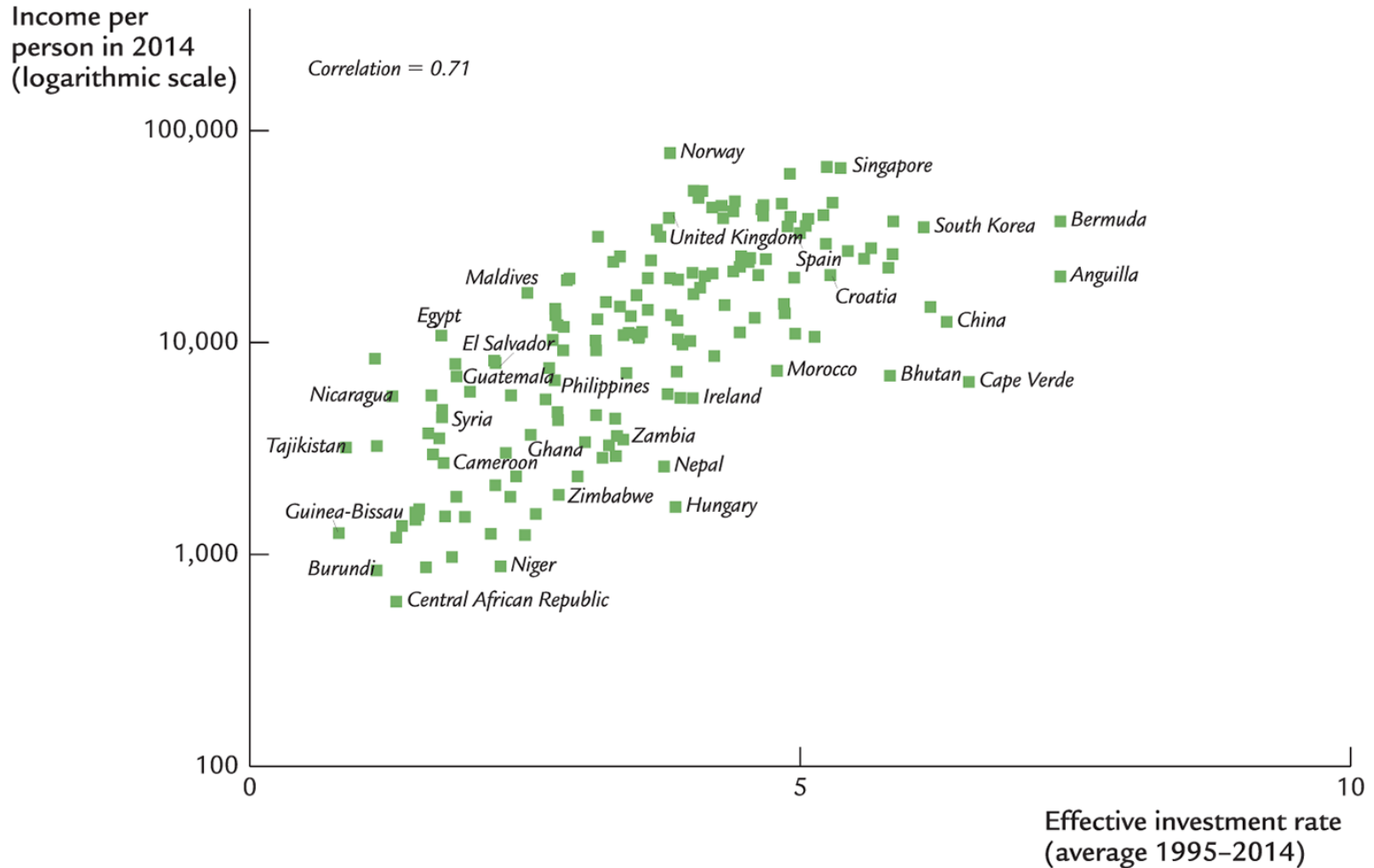
$$\text{MPK} = \delta + n$$

Or, equivalently,

$$\text{MPK} - \delta = n$$

*In the Golden Rule steady state, the marginal product of capital net of depreciation equals the population growth rate.*

# International evidence on investment rates and income per person



# Alternative perspectives on population growth, part 1

## The Malthusian model (1798)

- Predicts population growth will outstrip the Earth's ability to produce food, leading to the impoverishment of humanity.
- Since the time of Malthus, world population has increased sixfold, yet living standards are higher than ever.
- Malthus neglected the effects of technological progress.

## Alternative perspectives on population growth, part 2

### The Kremerian model (1993)

- Posits that population growth contributes to economic growth.
- More people = more geniuses, scientists, and engineers, so faster technological progress.
- Evidence, from very long historical periods:
  - As world population growth rate increased, so did the rate of growth in living standards.
  - Historically, regions with larger populations have enjoyed faster growth.

## Technological progress in the Solow model

So far, in the Solow model we have kept these assumptions:

- the production technology is held constant.
- income per capita is constant in the steady state.

Neither point is true in the real world:

- 1900–2016: U.S. real GDP per person grew by a factor of 8.58, or 1.9% per year.
- examples of technological progress abound (see the next slide).

## Examples of technological progress

- U.S. farm sector productivity nearly tripled from 1950 to 2012.
- The real price of computer power has fallen an average of 30% per year over the past three decades.
- 2000: 361 million Internet users, 740 million cell phone users  
2016: 3.4 billion Internet users, 5.0 billion cell phone users
- 2001: iPod capacity = 5GB, 1,000 songs. Not capable of playing episodes of *Game of Thrones*.  
2018: iPod touch capacity = 64GB, 30,000 songs. Can play episodes of *Game of Thrones*.

# Technological progress in the Solow model, part 1

- A new variable:  $E$  = labor efficiency
- Assume:  
Technological progress is **labor-augmenting**:  
it increases labor efficiency at the exogenous rate  $g$ :

$$g = \frac{\Delta E}{E}$$

## Technological progress in the Solow model, part 2

We now write the production function as:

$$Y = F(K, L \times E)$$

- where  $L \times E$  = number of effective workers
  - Increases in labor efficiency have the same effect on output as increases in the labor force.

## Technological progress in the Solow model, part 3

- Notation:

$y = Y / LE$  = output per effective worker

$k = K / LE$  = capital per effective worker

- Production function per effective worker:

$$y = f(k)$$

- Saving and investment per effective worker:

$$s y = s f(k)$$

## Technological progress in the Solow model, part 4

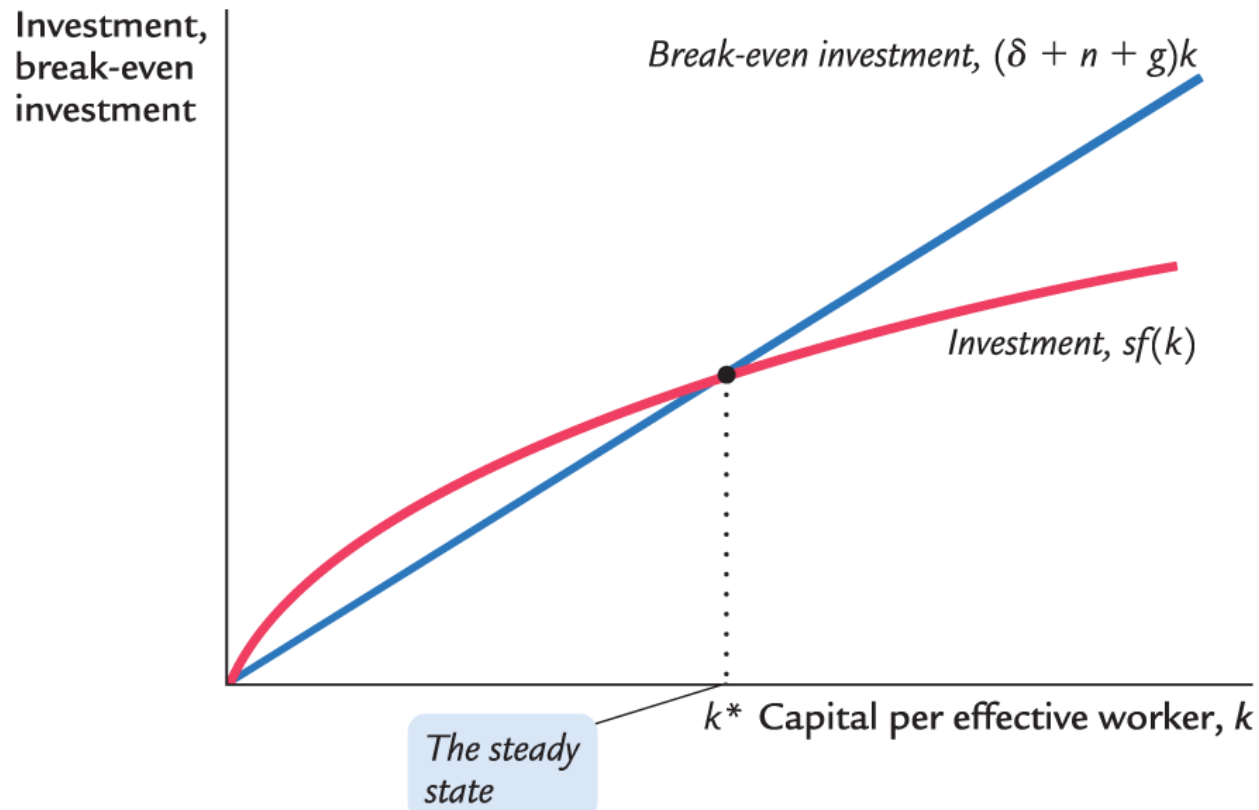
$(\delta + n + g) k$  = break-even investment:  
the amount of investment necessary to keep  $k$  constant.

Consists of:

- $\delta k$  to replace depreciating capital
- $n k$  to provide capital for new workers
- $g k$  to provide capital for the new “effective” workers created by technological progress

# Technological progress in the Solow model

$$\Delta k = s f(k) - (\delta + n + g)k$$



# Steady-state growth rates in the Solow model with tech. progress

Variable	Symbol	Steady-State Growth Rate
Capital per effective worker	$k = K/(E \times L)$	0
Output per effective worker	$y = Y/(E \times L) = f(k)$	0
Output per worker	$Y/L = y \times E$	$g$
Total output	$Y = y \times (E \times L)$	$n + g$

## The Golden Rule with technological progress

To find the Golden Rule capital stock, express  $c^*$  in terms of  $k^*$ :

$$\begin{aligned}c^* &= y^* - i^* \\ &= f(k^*) - (\delta + n + g)k^*\end{aligned}$$

$c^*$  is maximized when

$$MPK = \delta + n + g$$

Or, equivalently,

$$MPK - \delta = n + g$$

In the Golden Rule steady state, the marginal product of capital net of depreciation equals the population growth rate plus the rate of tech progress.

## Growth empirics: Balanced growth

- The Solow model's steady state exhibits **balanced growth**: many variables grow at the same rate.
  - The Solow model predicts that  $Y/L$  and  $K/L$  grow at the same rate ( $g$ ), so  $K/Y$  should be constant. This is true in the real world.
  - The Solow model predicts that real wage grows at the same rate as  $Y/L$ , while real rental price is constant. Also true in the real world.

## Growth empirics: Convergence, part 1

- Solow model predicts that, other things equal, poor countries (with lower  $Y/L$  and  $K/L$ ) should grow faster than rich ones.
- If true, then the income gap between rich and poor countries would shrink over time, causing living standards to *converge*.
- In the real world, many poor countries do NOT grow faster than rich ones. Does this mean the Solow model fails?

## Growth empirics: Convergence, part 2

- No, the Solow model does not fail because it predicts that, **other things equal**, poor countries (with lower  $Y/L$  and  $K/L$ ) should grow faster than rich ones.
  - In samples of countries with similar savings and population growth rates, income gaps shrink about 2% per year.
  - In larger samples, after controlling for differences in saving, population growth, and human capital, incomes converge by about 2% per year.

## Growth empirics: Convergence, part 3

- What the Solow model really predicts is **conditional convergence**: countries converge to their own steady states, which are determined by saving, population growth, and education.
- Each country's steady state depends on its structural characteristics:
  - 💰 **Saving rate**
  - 👶 **Population growth**
  - 🎓 **Human capital (education, skills)**

Therefore countries with **similar characteristics** will converge to **similar income levels**.

Countries with **different characteristics** will converge to **different levels**

## Growth empirics: Convergence, part 3

Example of conditional convergence:

- Let's assume 2 countries **A** and **B**: they have the same saving rate, same population growth, same education level
  - If A starts poorer than B, it will grow faster
  - Over time, A will catch up with B

## Growth empirics: Convergence, part 3

Example of conditional convergence:

If country **A** has: lower saving, higher population growth, and lower human capital than B

- It may remain poorer in the long run because its steady state is lower
- This prediction comes true in the real world.

# Growth empirics: Factor accumulation vs. production efficiency, part 1

- Differences in income per capita among countries can be due to differences in:
  1. capital—physical or human—per worker
  2. the efficiency of production (the height of the production function)
- Studies:
  - Both factors are important.
  - The two factors are correlated: countries with higher physical or human capital per worker also tend to have higher production efficiency.

## Growth empirics: Factor accumulation vs. production efficiency, part 2

- Possible explanations for the correlation between capital per worker and production efficiency:
  - Production efficiency encourages capital accumulation.
  - Capital accumulation has externalities that raise efficiency.
  - A third, unknown variable causes capital accumulation and efficiency to be higher in some countries than others.

## Policy issues

- Are we saving enough? Too much?
- What policies might change the saving rate?
- How should we allocate our investment between privately owned physical capital, public infrastructure, and human capital?
- How do a country's institutions affect production efficiency and capital accumulation?
- What policies might encourage faster technological progress?

## Policy issues: Evaluating the rate of saving, part 1

- Use the Golden Rule to determine whether the U.S. saving rate and capital stock are too high, too low, or about right.
  - If  $(MPK - \delta) > (n + g)$ , the U.S. economy is below the Golden Rule steady state and should increase  $s$ .
  - If  $(MPK - \delta) < (n + g)$ , the U.S. economy is above the Golden Rule steady state and should reduce  $s$ .

## Policy issues: Evaluating the rate of saving, part 2

To estimate  $(MPK - \delta)$ , use three facts about the U.S. economy:

1.  $k = 2.5 y$

The capital stock is about 2.5 times one year's GDP.

2.  $\delta k = 0.1 y$

About 10% of GDP is used to replace depreciating capital.

3.  $MPK \times k = 0.3 y$

Capital income is about 30% of GDP.

## Policy issues: Evaluating the rate of saving, part 3

1.  $k = 2.5 y$
2.  $\delta k = 0.1 y$
3.  $MPK \times k = 0.3 y$

To determine  $\delta$ , divide **2** by **1**:

$$\frac{\delta k}{k} = \frac{0.1 y}{2.5 y} \Rightarrow \delta = \frac{0.1}{2.5} = 0.04$$

## Policy issues: Evaluating the rate of saving, part 4

1.  $k = 2.5 y$
2.  $\delta k = 0.1 y$
3.  $MPK \times k = 0.3 y$

To determine  $MPK$ , divide **3** by **1**:

$$\frac{MPK \times k}{k} = \frac{0.3 y}{2.5 y} \Rightarrow MPK = \frac{0.3}{2.5} = 0.12$$

Hence,  $MPK - \delta = 0.12 - 0.04 = \underline{0.08}$

## Policy issues: Evaluating the rate of saving, part 5

- From the last slide:  $MPK - \delta = 0.08$
- U.S. real GDP grows an average of 3% per year (in the long-run GDP growth =  $n + g$ ) so  $n + g = 0.03$
- Thus,  
$$MPK - \delta = 0.08 > 0.03 = n + g$$
- Conclusion:

*The U.S. is below the Golden Rule steady state: Increasing the U.S. saving rate would increase consumption per capita in the long run.*

## Policy issues: How to increase the saving rate

- Reduce the **government budget deficit** (or increase the budget surplus).
- Increase incentives for **private saving**:
  - Reduce capital gains tax, corporate income tax, and estate tax, as they discourage saving.
  - Replace federal income tax with a consumption tax.
  - Expand tax incentives for IRAs (individual retirement accounts) and other retirement savings accounts.

## Policy issues: Allocating the economy's investment, part 1

- In the Solow model, there's one type of capital.
- In the real world, there are many types, which we can divide into three categories:
  - private capital stock
  - public infrastructure
  - **human capital:** the knowledge and skills that workers acquire through education
- How should we allocate investment among these types?

## Policy issues: Allocating the economy's investment, part 2

Two viewpoints:

1. Equalize tax treatment of all types of capital in all industries and let the market allocate investment to the type with the highest marginal product.
- 2. Industrial policy:**  
Government should actively encourage investment in capital of certain types or in certain industries because it may have positive externalities that private investors don't consider.

## Possible problems with industrial policy

- The government may not have the ability to “pick winners” (choose industries with the highest return to capital or biggest externalities).
- Politics (e.g., campaign contributions) rather than economics may influence which industries get preferential treatment.

## Policy issues: Establishing the right institutions

- Creating the right institutions is important for ensuring that resources are allocated to their best use. Examples:
  - Legal institutions, to protect property rights.
  - Capital markets, to help financial capital flow to the best investment projects.
  - A corruption-free government, to promote competition, enforce contracts, etc.

# Establishing the right institutions: North versus South Korea

After WW2, Korea split into:

- North Korea with institutions based on authoritarian communism
- South Korea with Western-style democratic capitalism

Today, GDP per capita is over 10 times higher in S. Korea than in N. Korea.



Jason Reed/Reuters/Newscom

# Policy issues: Encouraging technological progress

- **Patent laws:**  
encourage innovation by granting temporary monopolies to inventors of new products
- Tax incentives for **R&D**
- **Grants** to fund **basic research** at universities
- Industrial policy:  
encourages specific industries that are key for rapid technological progress (*subject to the preceding concerns*).

## CASE STUDY: Is free trade good for economic growth? Part 1

- Since Adam Smith, economists have argued that free trade can increase production efficiency and living standards.
- Research by Sachs & Warner:

### Average annual growth rates, 1970–89

	<b>Open</b>	<b>Closed</b>
Developed nations	2.3%	0.7%
Developing nations	4.5%	0.7%

## CASE STUDY: Is free trade good for economic growth? Part 2

- To determine causation, Frankel and Romer exploit geographic differences among countries:
  - Some nations trade less because they are farther from other nations or **landlocked**.
  - Such geographic differences are correlated with trade but not with other determinants of income.
  - Hence, they can be used to isolate the impact of trade on income.
- Findings: increasing trade/GDP by 2% causes GDP per capita to rise 1%, other things equal.

## CASE STUDY: Is free trade good for economic growth? Part 2

- A common answer is that the gains from international trade are unequal;
- although a country as a whole sees its income rise, some workers permanently lose their jobs and see a large decrease in their income.

The basic idea is:

- International trade **increases the overall size of the economy:**
  - the country becomes more efficient
  - total GDP rises

## CASE STUDY: Is free trade good for economic growth? Part 2

- ⚖️ But the benefits are not equally distributed

Not everyone gains in the same way:

- Some sectors expand (**export industries**)
- Other sectors shrink (due to import competition)

### **What happens to workers?**

Workers in expanding sectors → will **gain**

Workers in declining sectors → **lose their jobs** or earn less and these losses can be permanent

# Endogenous growth theory

- Solow model:
  - Sustained growth in living standards is due to technological progress.
  - The rate of technological progress is exogenous.

How do we incorporate the process of technological change into a growth model?

- Endogenous growth theory:
  - In this set of models, the growth rate of productivity and living standards is endogenous.

## Endogenous growth theory

In the Solow model, the long-run economic growth rate equals the rate of technological progress, which is exogenous in the model.

Hence, the Solow model is basically saying, “all I can tell you is that growth in living standards depends on technological progress. I have no idea what drives technological progress.”

**Endogenous growth theory** tries to explain the behavior of the rates of technological progress and/or productivity growth rather than merely taking these rates as given.

## The basic model, ( the AK model)

The **AK** model is a simple model of economic growth where:

- Output depends directly on capital
- $Y = A \cdot K$
- where:
- $Y$  = output (GDP)
- $K$  = capital (machines, skills, infrastructure)
- $A$  = productivity (technology) = the amount of output produced for each unit of capital (**A** is exogenous and constant)

## The basic model, ( the AK model)

- Production function:  $Y = A K$
- The **production function does not exhibit diminishing** returns to capital. This means: every extra unit of capital is just as productive as the previous one
- Key difference between this model and Solow: **MPK is constant here**, **diminishes in Solow**
- Investment:  $sY$
- Depreciation:  $\delta K$
- Equation of motion for total capital:
  - $\Delta K = sY - \delta K$

## The basic model

$$\Delta K = sY - \delta K$$

Divide through by  $K$  and use  $Y = A K$  to get:

$$\frac{\Delta Y}{Y} = \frac{\Delta K}{K} = sA - \delta$$

- If  $sA > \delta$ , then income will grow forever, and investment is the “engine of growth.”
- Note that this **does not** require the assumption of exogenous technological progress.
- Here, **the permanent growth rate depends on  $s$** . In Solow model, it does not.

## Assumption on diminishing returns of capital

$Y$  and  $K$  grow at the same rate because  $A$  is constant.

Discussion:

The **return to capital is the incentive to invest**. If capital exhibits diminishing returns, then the incentive to invest decreases as the economy grows.

Hence, investment cannot be a source of sustained growth.

However, in this model, **MPK does not fall as  $K$  rises**, so the incentive to invest never declines; people will always find it worthwhile to save and invest over and above depreciation, **so investment becomes an engine of growth**. In the AK model, capital keeps generating growth without limits, so economies **can grow indefinitely if they keep investing**.

# Endogenous model vs Solow

In the endogenous AK model

## 1. Endless growth is possible

If a country keeps investing, it can **grow forever**

Growth does **not slow down**

## 2. Saving matters a lot

Higher saving → more investment → more capital → faster growth

So policies that increase saving or investment **permanently increase growth**

## 3. No convergence (unlike Solow)

In the Solow model: poor countries catch up (convergence)

In the AK model: **countries can diverge**

countries that invest more → grow faster forever

countries that invest less → fall behind

## Assumption on diminishing returns of capital

Does capital exhibit diminishing or constant marginal returns?  
The answer is critical, for it determines whether investment explains sustained (i.e. steady-state) growth in productivity and living standards.

## Assumption on diminishing returns of capital

Does it make sense that capital does not show diminishing returns?

**No**, if capital is defined as the **stock of plants and equipment**.

**Yes**, if capital is broadly interpreted to **include the stock of knowledge**. Knowledge is a type of capital.

Some economists argue that **there are increasing returns to knowledge**.

If so, then constant returns to capital **is more plausible**, and this model may be a good description of economic growth.

## A two-sector model, part 1

The **AK model** developed above is the simplest example of an **endogenous growth model**. It is an endogenous growth model because growth is generated internally through capital accumulation rather than imposed from outside.

Economic growth arises directly from the accumulation of capital, which is a variable determined within the model.

Savings and investment decisions made by agents determine growth, not some outside technological shock.

This is the distinctive feature of endogenous growth theory.

## A two-sector model, part 1

A more sophisticated version of endogenous model incorporates two sectors:

a **manufacturing** sector that produces goods and services for either consumption or investment in physical capital  $K$  and,

a **research** sector composed of universities that produce knowledge,  $E$ , which is used in both sectors.

## A two-sector model

Two sectors:

that it is an extension of the Solow model with technological progress.

There are two differences:

First, a **fraction of the labor force** **does not** produce goods and services but rather produces “knowledge” by doing research in universities.

Second, **the rate of tech progress is not exogenous** but rather depends on how **fast the stock of knowledge grows**, which in turn depends on how much labor the economy has allocated to research.

## A two-sector model

Two sectors:

- manufacturing firms produce goods.
- research universities produce knowledge that increases labor efficiency in manufacturing.
- $u$  = fraction of labor in research ( $u$  is exogenous)

The economy can be described by three equations:

1) Manufacturing production function:

$$Y = F[K, (1 - u)EL]$$

1) Research production function:  $\Delta E = g(u)E$

2) Capital accumulation:  $\Delta K = sY - \delta K$

## A two-sector model

Manufacturing production function:

Just as in the Solow model with exogenous technological progress, output of manufacturers depends on capital and the effective labor force employed in the manufacturing sector,  $(1 - u)EL$ .

Research production function: The “output” is increases in knowledge and labor efficiency.

## A two-sector model

The “inputs” are labor and current knowledge.

The function  $g(\cdot)$  shows **how changes in the amount of labor devoted to research affect the creation of new knowledge.**

All we need is for  $g(\cdot)$  to be an increasing function. It does not matter whether a doubling of scientists causes the creation of knowledge to double, more than double, or less than double.

Capital accumulation: As in the previous model, net investment equals gross investment ( $sY$ ) minus depreciation.

## A two-sector model

- In the steady state, manufacturing output per worker and the standard of living grow at rate  $\Delta E / E = g(u)$ .
- Key variables:  $s$  and  $u$

$s$  determines the steady-state stock of physical capital,  
 $u$  determines the growth in the stock of knowledge,  
 $s$  and  $u$  determine the level of income, and  
 $u$  determines the steady-state growth rate of income.

## A two-sector model, part 1

In this model, **the steady state growth rate of the standard of living equals the growth rate of labor efficiency**, just like in the Solow model with tech progress, covered at the beginning of this chapter.

The difference here is that the rate of tech progress,  $g$ , is not exogenous: it depends on how much labor the economy has allocated to research.

## A two-sector model, part 1

This model is similar to the basic  $AK$  model in that capital exhibits constant returns to scale since capital includes both physical capital and human capital (knowledge).

## DISCUSSION QUESTION The merits of raising $u$

*Questions:*

In what ways would raising  $u$  (that is, devoting more labor to research) benefit the economy?

What are the costs of raising  $u$ ?

## DISCUSSION QUESTION The merits of raising $u$

**Increasing  $u$**  means devoting more resources to R&D.

In the short run, output of goods and services per capita will fall. However, **the pace of technological progress will rise**, so growth will speed up, and output per capita will eventually be higher than it would have been at the initial value of  $u$ .

Of course, if we increase  $u$  to its maximum possible value, 1, then no goods and services would be produced.

## DISCUSSION QUESTION The merits of raising $u$

This tradeoff suggests that there must be some kind of golden rule for  $u$  - a value of  $u$  that maximizes well-being per capita in the steady state.

At low enough values of  $u$  (values lower than this golden rule level), increases in  $u$  would, on balance, benefit the economy.

At high enough values, increases in  $u$  would likely harm the economy.

## Facts about R&D

1. Much research is done by firms seeking profits.
2. Firms profit from research:  
Patents create a stream of monopoly profits.  
There is extra profit in being first on the market with a new product.
3. Innovation produces externalities that reduce the cost of subsequent innovation.

*Much of the new endogenous growth theory attempts to incorporate these facts into models to better understand technological progress.*

## Is the private sector doing enough R&D?

- The existence of positive externalities in the creation of knowledge suggests that the private sector is not doing enough R&D.
- But there is much duplication of R&D effort among competing firms.
- Thus, many believe the government should encourage R&D.

## Economic growth as “creative destruction”

- **Schumpeter (1942)** coined term “creative destruction” to describe displacements resulting from technological progress:
  - The introduction of a new product is good for consumers but often bad for incumbent producers, who may be forced out of the market.
- Examples:
  - **Luddites (1811–1812)** destroyed machines that displaced skilled mill workers in England.
  - Walmart displaces many small retail stores.

## Economic growth as “creative destruction”

Focus:

Joseph Schumpeter proposed that economic growth occurs through a process known as “creative destruction.”

His theory viewed **new firms as continually entering the marketplace**, having monopoly power over their innovations, and reaping the profits that induced the firms to enter the market in the first place.

**Consumers benefit** from the greater choice of products, **but existing firms now face competition**. Some of these established firms cannot compete, and they go out of business.

This process continues over time, with new firms entering and established firms exiting—a process of “creative destruction.”