Lezione 11 marzo 2026 - esercizi

mercoledì 11 marzo 2026 11:20

Foglio 2, Es. 1.

$r: \begin{cases} x_1 = 1+t \\ x_2 = 2t \\ x_3 = 8-t \end{cases}$ $\qquad s: \begin{cases} x_1 = 1-z \\ x_2 = 3+z \\ x_3 = 5z \end{cases}$

$Q=(1,0,8)$ $\qquad Q'=(1,3,0)$

v_r vettore direzione $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ $\qquad v_s = \begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix}$

(base di W_r)

$r \parallel s \iff v_r$ e v_s proporzionali

$\boxed{\text{no}}$

Incidenti? $\qquad \begin{cases} 1+t = 1-z \\ 2t = 3+z \\ 8-t = 5z \end{cases}$

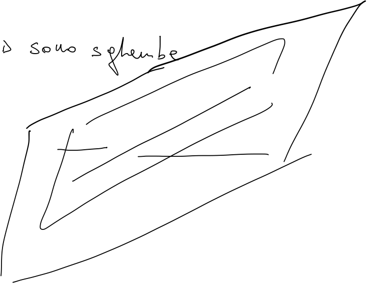
$$\begin{cases} t+z = 0 \\ 2t-z = 3 \\ -t-5z = -8 \end{cases} \qquad \left(\begin{array}{cc|c} 1 & 1 & 0 \\ 2 & -1 & 3 \\ -1 & -5 & -8 \end{array}\right)$$

$\underline{II} - 2\underline{I}$, $\quad \underline{III} + \underline{I}$

$$\left(\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & -3 & 3 \\ 0 & -4 & -8 \end{array}\right) \qquad \underline{III} - \frac{4}{3}\underline{II}$$

$$\left(\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & -3 & 3 \\ 0 & 0 & -12 \end{array}\right) \qquad \underset{=2}{rg A} \neq \underset{3}{rg(A|b)}$$

$\Rightarrow$ sono sghembe



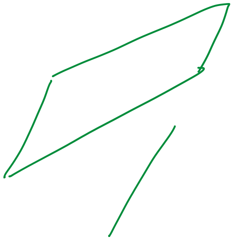
$\Rightarrow$ non sono complanari

Però $\exists$ 2 piani paralleli H e H', $H \parallel H'$

t.c. $\quad r \subseteq H, \quad s \subseteq H'$

$W_H = W_{H'} \supseteq \mathrm{Span}(v_r)$

$\supseteq \text{Span}(v_s)$



$$\Rightarrow W_H = W_{H'} = \text{Span}(v_r, v_s)$$

$$= \text{Span}\left(\underbrace{\boxed{\begin{pmatrix}1\\2\\-1\end{pmatrix}}, \boxed{\begin{pmatrix}-1\\1\\5\end{pmatrix}}}_{}\right)$$

$H \supseteq r \Rightarrow$ impongo che H passi per un punto di r, ad esempio $\underbrace{Q = (1, 0, 8)}_{}$

$$H : \begin{cases} x_1 = 1 + s_1 - s_2 \\ x_2 = 0 + 2s_1 + s_2 \\ x_3 = 8 - s_1 + 5s_2 \end{cases} \qquad r : \begin{cases} x_1 = 1 + t \\ x_2 = 2t \\ x_3 = 8 - t \end{cases}$$

eq. cart. H :

$$\det \begin{pmatrix} 1 & -1 & x_1 - 1 \\ 2 & 1 & x_2 \\ -1 & 5 & x_3 - 8 \end{pmatrix} = 0$$

$$= 1\,(x_3 - 8 - 5x_2) - 2\,(8 - x_3 + 5 - 5x_1) - 1\,(-x_2 + 1 - x_1) = 0$$

$$= \cancel{x_3} - 8 - 5\cancel{x_2} - 26 + \cancel{2x_3} + \cancel{10x_1} + \cancel{x_2} - 1 + \cancel{x_1} = 0$$

$$11x_1 - 4x_2 + 3x_3 - 35 = 0$$

$$r: \begin{cases} x_1 = 1+t \\ x_2 = 2t \\ x_3 = 8-t \end{cases}$$

$$11(1+t) - 4(2t) + 3(8-t) - 35 =$$

$$11 + 11t - 8t + 24 - 3t - 35 = 0 \quad \forall t \in \mathbb{R}$$

H':

$$\begin{cases} x_1 = 1 + s_1 - s_2 \\ x_2 = 3 + 2s_1 + s_2 \\ x_3 = 0 - s_1 + 5s_2 \end{cases}$$

eq. cartesiana

$$\det \begin{pmatrix} 1 & -1 & x_1 - 1 \\ 2 & 1 & x_2 - 3 \\ -1 & 5 & x_3 \end{pmatrix} = 0$$

$$1 \cdot (x_3 - 5x_2 + 15) - 2(-x_3 - 5x_1 + 5) - 1(-x_2 + 3 - x_1 + 1) = 0$$

$$\cancel{x_3} - \cancel{5x_2} + \cancel{15} + \cancel{2x_3} + \cancel{10x_1} - \cancel{10} + \cancel{x_2} - 4 + \cancel{x_1} = 0$$

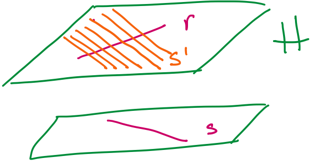
$$11x_1 - 4x_2 + 3x_3 + 1 = 0$$

$$11x_1 - 4x_2 + 3x_3 - 35 = 0$$

Il generico piano parallelo ad H (e H')

ha eq.

$$11x_1 - 4x_2 + 3x_3 + d = 0 \quad , d \in \mathbb{R}$$



Pb.: Trovare una retta $s' \parallel s$ e $s' \cap r \neq \emptyset$

basta prendere $W_s = \operatorname{Span}\left(\begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix}\right)$

e poi imporre pass. per un punto arbitrario

di r, ad es. $Q = (1, 0, 8)$

$$s' : \begin{cases} x_1 = 1 - \sigma \\ x_2 = 0 + \sigma \\ x_3 = 8 + 5\sigma \end{cases}$$

altra retta: $\begin{cases} x_1 = 2 - \delta \\ x_2 = 2 + \delta \\ x_3 = 7 + 5\delta \end{cases}$

Es. 2: $\begin{cases} x + y = 3 \\ 2y + 3z = 4 \end{cases}$ r

$v = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$

fascio di piani contenenti r:

$$\lambda(x+y-3) + \mu(2y+3z-4) = 0$$

$\lambda, \mu \in \mathbb{R}$

$$\lambda x + (\lambda + 2\mu) y + 3\mu z - 3\lambda - 4\mu = 0$$

giacitura $W_{\lambda,\mu}$ del generico piano:

$$\lambda x + (\lambda + 2\mu) y + 3\mu z = 0$$

$$\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} \in W_{\lambda,\mu} \iff 2\lambda - 2(\lambda + 2\mu) + 3\mu = 0$$

$$-\mu = 0$$

$$\Rightarrow \lambda \cdot (x + y - 3) = 0, \quad \lambda \neq 0$$

es. $\lambda = 1$: $x + y - 3 = 0$

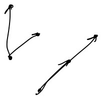
Es. 3

$P = (0,-1,1)$

$Q = (0,1,1)$

$R = (a,0,0)$

$$\vec{PQ} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}, \quad \vec{PR} = \begin{pmatrix} a \\ 1 \\ -1 \end{pmatrix}$$



$$\vec{PQ} \not\parallel \vec{PR} \quad \forall a \in \mathbb{R}$$

⇒ i 3 punti non sono mai allineati, e quindi det. 1! piano

$$L_a \quad \begin{cases} x = 0 + 0 \cdot s_1 + a\, s_2 \\ y = -1 + 2 \cdot s_1 + 1 \cdot s_2 \\ z = 1 + 0 \cdot s_1 + (-1) \cdot s_2 \end{cases}$$

$$\begin{cases} x = a\, s_2 \\ y = -1 + 2s_1 + s_2 \\ z = 1 - s_2 \end{cases}, \quad W_{L_a} = \operatorname{Span}\left(\begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} a \\ 1 \\ -1 \end{pmatrix} \right)$$

Trovare a t.c. $L_a \parallel r$

$$r: \begin{cases} y = 3 \\ x + z = 0 \end{cases}$$

giacitura W_r: $\begin{cases} y = 0 \\ x + z = 0 \end{cases}$

$$W_r = \operatorname{Span}\left(\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \right)$$

$$\Rightarrow \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \alpha \cdot \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} a \\ 1 \\ -1 \end{pmatrix}$$

$$\begin{cases} a\beta = 1 \\ 2\alpha + \beta = 0 \\ -\beta = -1 \end{cases}$$

$$\boxed{\Rightarrow a = 1}$$

$$\alpha = -\frac{1}{2}$$

$$\Rightarrow \beta = 1$$

Eq. cartesiana di L_a

$$\det\begin{pmatrix} 0 & a & x \\ 2 & 1 & y+1 \\ 0 & -1 & z-1 \end{pmatrix} = -2\left(a(z-1)+x\right)$$

$$= -2x - 2az + 2a = 0 \quad /:(-2)$$

$$x + az - a = 0$$

$$x + a(z-1) = 0$$

è una famiglia di piani contenuta nel fascio

$$\lambda x + \mu(z-1) = 0$$

Se $\lambda \neq 0 \Rightarrow$ pongo $a := \frac{\mu}{\lambda}$

$\lambda = 0$: $\mu(z-1) = 0$, $z-1=0$