Interconnected Systems

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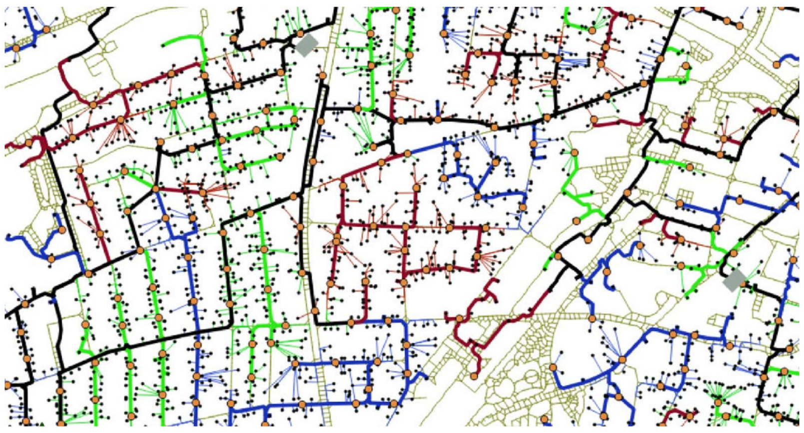


An energy distribution network model (details here).

Introduction

This live script will describe and analyse some examples of interconnected dynamic LTI systems. Please refer to Part 6 of the course material and the recommended texts for more in-depth theory.

The online course *MATLAB Onramp* can be used to provide details on simulating interconnected systems in MATLAB.

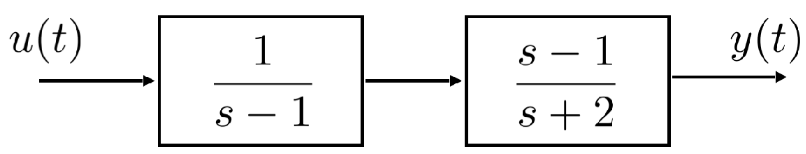
Pay Attention

We will only consider examples with factors in common between the numerator and denominator polynomial in the overall transfer function and thus present **hidden dynamics**.

Example 1: Series Interconnections

The Hidden Internal Dynamics is Unstable

Consider the system described in the following block diagram:



$$G_1(s) = \frac{1}{s-1} \quad G_2(s) = \frac{s-1}{s+2} \qquad G_{tot}(s) = G_1(s) \cdot G_2(s) = \frac{1}{s+2}$$

```
clear
s = tf('s'); % the transfer function building helper

G1s_series = 1/(s-1); G2s_series = (s-1)/(s+2);

A1 = +1; B1= +1; C1 = 1; D1 = 0;
sysG1 = ss(A1, B1, C1, D1);
```

```
G_tot_series = minreal(G1s_series*G2s_series) % let's force the pole/zero simplification -- see 'doc minreal'
```

```
G_tot_series =

    1
  -----
  s + 2

Continuous-time transfer function.
Model Properties
```

The overall model using state variables is

$$\begin{cases} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t) \\ y(t) = \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \end{cases}$$

Let's define it in MATLAB

```
A = [1 , 0; 3, -2]; B = [1; 0]; C = [1 , -1]; D = 0; % Refer to Part 6, p.6

sys_TOT_series = ss(A, B, C, D);
tf(sys_TOT_series)
```

```
ans =

     s - 1
  -----------
  s^2 + s - 2

Continuous-time transfer function.
Model Properties
```

```
% forcing the simplification
minreal(tf(sys_TOT_series))
```

```
ans =

    1
  -----
  s + 2

Continuous-time transfer function.
Model Properties
```

The Input and the Simulation Settings

Now, let's assign the simulation settings and the system input:

```
% ------------------------------------------------------------------------
Tstop = 2; % [s] - the simulation starts at the time instant 0 and ends at thee time instant Tstop

% let's define the input time function
% let's generate NT samples for the input signal
NT = 1e4;
t = linspace(0, Tstop, NT);

% the input values
U = 2 + 1.75 .* exp(-2.4.*t) .* cos(2*pi*4.*t);
```

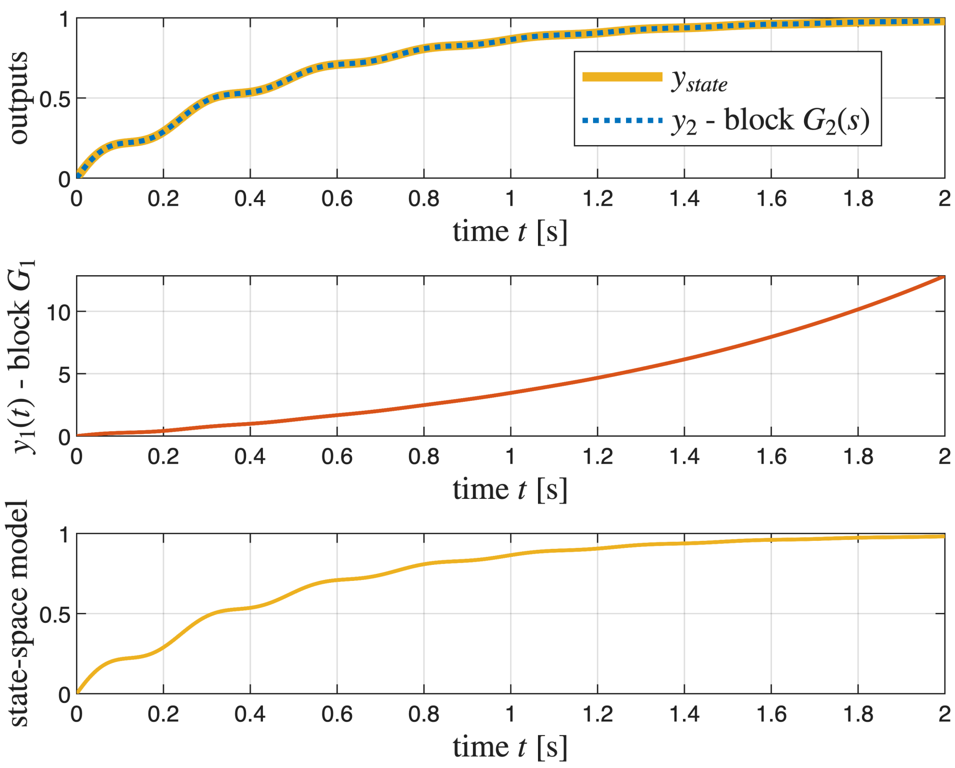
The Simulation

```
% --- simulation of G1 ONLY ---
[yG1, ~, xG1] = lsim(G1s_series, U, t);

% --- simulation of G2 ONLY ---
[yG2, ~, xG2] = lsim(G2s_series, yG1, t); % <-- PAY ATTENTION: the output of G1(s) is the input for G2(s)

% --- simulation of the full system ---
[yOut, ~, xOut] = lsim(sys_TOT_series, U, t);
% let us run the model and retrieve the results


y1 = yG1; % output of the block G1
y2 = yG2; % output of the block G2
```

```
y_state = yOut; % output of the state space model

t_stamps = t;
% time instants corresponding to the solution samples
```

The Simulation Results

```
figure('Units','pixels','Position',[0, 0, 1280, 980]);

ha1 = subplot(3,1,1);

plot(t_stamps, y_state, 'LineWidth', 3.5, 'Color', [0.9290 0.6940 0.1250]);
grid on; zoom on; hold on
plot(t_stamps, y2, 'LineWidth', 2.0, 'Color', [0 0.4470 0.7410], 'LineStyle', ':'); % output of the block G1

xlabel('time $t$ [s]', 'FontSize', 12, 'Interpreter','latex'); % labels on the plot axes
ylabel('outputs', 'FontSize', 12, 'Interpreter','latex');
legend('$y_{state}$', '$y_{2}$ - block $G_{2}(s)$',  ...
         'Interpreter', 'latex', 'FontSize', 12, 'Location', 'best' );% adding a legend


ha2 = subplot(3,1,2);
plot(t_stamps, y1, 'LineWidth', 1.5, 'Color', [0.8500 0.3250 0.0980]); % output of the block G1
grid on; zoom on;
xlabel('time $t$ [s]', 'FontSize', 12, 'Interpreter','latex'); % labels on the plot axes
ylabel('$y_1(t)$ - block $G_1$', 'FontSize', 12, 'Interpreter','latex');

ha3 = subplot(3,1,3);
plot(t_stamps, y_state, 'LineWidth', 1.5, 'Color', [0.9290 0.6940 0.1250]); % output of the state-space model
grid on; zoom on;
xlabel('time $t$ [s]', 'FontSize', 12, 'Interpreter','latex'); % labels on the plot axes
ylabel('state-space model', 'FontSize', 12, 'Interpreter','latex');
linkaxes([ha1, ha2, ha3],'x')
```



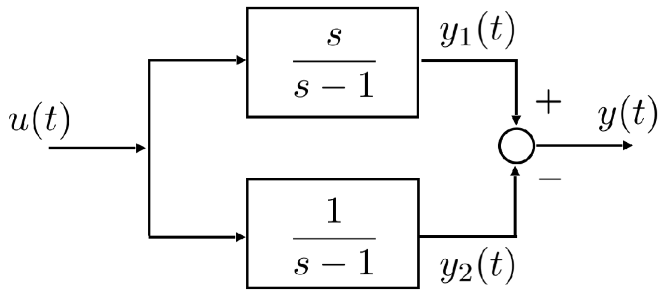
Remark

Note: as expected, the intermediate output $y_1(t)$ diverges, whereas the overall output (i.e., the second block's output $y_2(t)$, equal to the state space model output $y_{state}(t)$) does not diverge.

Example 2: Parallel Interconnections

The Hidden Internal Dynamics is Unstable

Consider the system described in the following block diagram:



$$G_1(s) = \frac{s}{s-1} \quad G_2(s) = \frac{1}{s-1} \qquad G_{tot}(s) = G_1(s) - G_2(s) = 1$$

```
clear
s = tf('s'); % the transfer function building helper

G1s_parallel = s/(s-1); G2s_parallel = 1/(s-1);
G_tot_parallel = minreal(G1s_parallel - G2s_parallel) % let's force the pole/zero simplification -- see 'doc
minreal'
```

```
G_tot_parallel =

  1

Static gain.
Model Properties
```

The overall model using state variables is

$$\begin{cases} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t) \\ y(t) = [1 \quad -1] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + [1] u(t) \end{cases}$$

Let's define it in MATLAB

```
A = [1 , 0; 0, 1]; B = [1; 1]; C = [1 , -1]; D = 1;
sys_TOT_parallel = ss(A, B, C, D);
```

The Input and the Simulation Settings

Now, let's assign the simulation settings and the system input:

```
% ---------------------------------------------------------------------
Tstop = 8; % [s] - the simulation starts at the time instant 0 and ends at thee time instant Tstop

% let's define the input time function
% let's generate NT samples for the input signal
NT = 1e4;
t = linspace(0, Tstop, NT);

% the input values
U = 2 + 1.75 .* exp(-1.4.*t) .* cos(2*pi*4.*t);
```

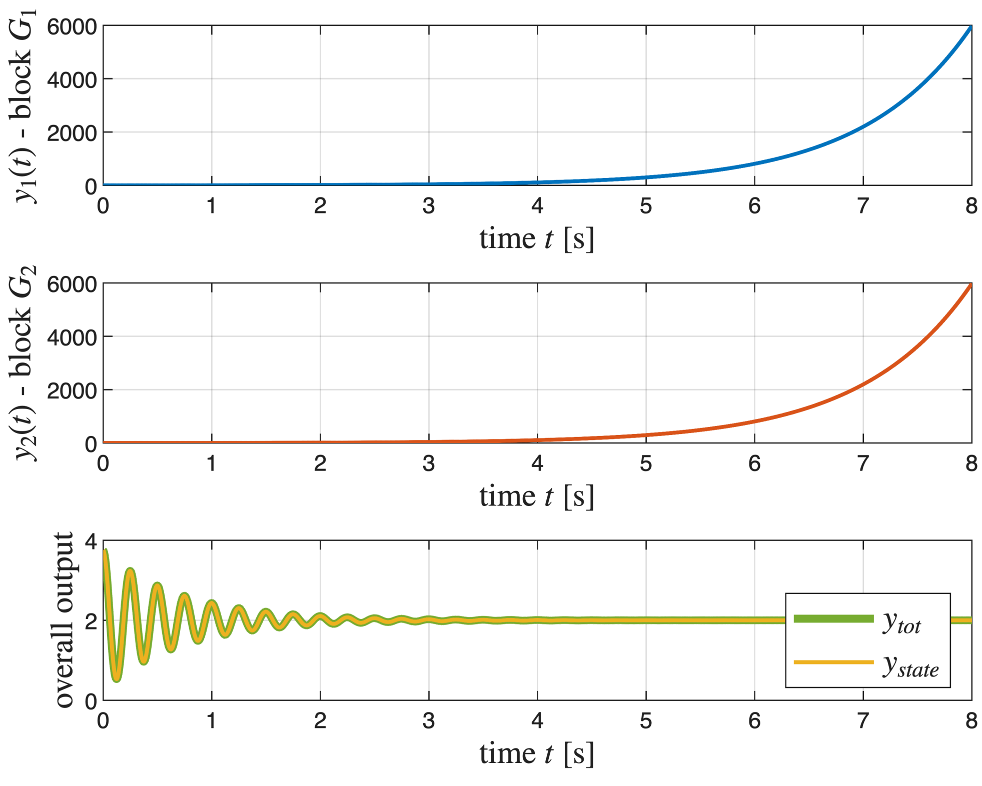
The Simulation

```
% --- simulation of G1 ONLY ---
[yG1p, ~, xG1p] = lsim(G1s_parallel, U, t);

% --- simulation of G2 ONLY ---
[yG2p, ~, xG2p] = lsim(G2s_parallel, U, t); % <-- PAY ATTENTION: the output of G1(s) is the input for G2(s)

% --- simulation of the full system ---
[yOutp, ~, xOutp] = lsim(sys_TOT_parallel, U, t);
% let us run the model and retrieve the results
```

```
y1 = yG1p; % output of the block G1
y2 = yG2p; % output of the block G2
y_tot = y1-y2;
y_state = yOutp; % output of the state space model

t_stamps = t;
% time instants corresponding to the solution samples
```

The Simulation Results

```
figure('Units','pixels','Position',[0, 0, 1280, 980]);

ha1 = subplot(3,1,1);
plot(t_stamps, y1, 'LineWidth', 1.5, 'Color', [0 0.4470 0.7410]); % output of the block G1
grid on; zoom on; hold on
xlabel('time $t$ [s]', 'FontSize', 12, 'Interpreter','latex'); % labels on the plot axes
ylabel('$y_1(t)$ - block $G_1$', 'FontSize', 12, 'Interpreter','latex');

ha2 = subplot(3,1,2);
plot(t_stamps, y2, 'LineWidth', 1.5, 'Color', [0.8500 0.3250 0.0980]); % output of the block G2
grid on; zoom on;
xlabel('time $t$ [s]', 'FontSize', 12, 'Interpreter','latex'); % labels on the plot axes
ylabel('$y_2(t)$ - block $G_2$', 'FontSize', 12, 'Interpreter','latex');

ha3 = subplot(3,1,3);
plot(t_stamps, y_tot, 'LineWidth', 3.0, 'Color', [0.4660 0.6740 0.1880]); % overall output
grid on; zoom on;
xlabel('time $t$ [s]', 'FontSize', 12, 'Interpreter','latex'); % labels on the plot axes
ylabel('overall output', 'FontSize', 12, 'Interpreter','latex');
hold on
plot(t_stamps, y_state, 'LineWidth', 1.5, 'Color', [0.9290 0.6940 0.1250]); % output of the state-space model
grid on; zoom on;
xlabel('time $t$ [s]', 'FontSize', 12, 'Interpreter','latex'); % labels on the plot axes
legend('$y_{tot}$', '$y_{state}$',...
        'Interpreter', 'latex', 'FontSize', 12, 'Location', 'southeast' );% adding a legend
linkaxes([ha1, ha2, ha3],'x')
```



Remark

Note: as expected, the intermediate outputs $y_1(t)$ and $y_2(t)$ diverge, whereas the overall output (i.e., the overall output $y_{tot}(t)$, equal to the state space model output $y_{state}(t)$) does not diverge.