Nyquist Diagrams: Analysis of Some Characteristic Features

Example 1 - Analyse $G(j\omega_\pi)$ when Varying the Open-Loop Transfer Function Gain

Consider the LTI system described by the strictly proper transfer function

$$G(s) = k\,\frac{\left(1-\frac{s}{2}\right)\left(1+\frac{s}{3}\right)}{(1+s)^2\,(1+10\,s)\left(1+\frac{s}{10}\right)}\,, \quad k>0$$

Let's define ω_π : ω such that $\arg G(j\omega_\pi) = -180^\circ$, i.e. the frequency response is negative real for $\omega = \omega_\pi$.

Let us identify the points where the frequency response is negative and a real number(i.e. the imaginary part is null) on the following graphs. What happens if we vary the gain k, for example if $k = 0.25, 0.5, 1, 2, \bar{k}, 5, 10$, where $\bar{k} = 10^{-11.1/20} \approx 3.59$? Do these characteristic points of the frequency response Nyquist diagram move around when we vary the gain k ?

```
clear

s=tf('s');
Gs = 4*(1-s/2)*(1+s/3)/((1+s)^2*(1+10*s)*(1+s/10));


k_lim = 10^(11.1/20); % the "magic" gain

kSET = [0.25, 0.5, 1, 2, k_lim, 5, 10];
Gs_SET = cell(numel(kSET),1);

% let's plot the Nyquist diagrams, varying the gain k
hf = figure('Units','centimeters','Position',[0.01, 0.01, 24, 28]);
ax = axes('Parent',hf); % the axes used to plot each Nyquist diagram

zoom on;

omVALS = logspace(-8,6,1e4); % a set of angular frequencies

k = kSET(1);
Gs_SET{1} = k*Gs;
get(Gs_SET{1})
```

```
      Numerator: {[0 0 -10 -10 60]}
    Denominator: {[60 726 1332 726 60]}
       Variable: 's'
        IODelay: 0
     InputDelay: 0
    OutputDelay: 0
      InputName: {''}
      InputUnit: {''}
     InputGroup: [1×1 struct]
     OutputName: {''}
     OutputUnit: {''}
    OutputGroup: [1×1 struct]
          Notes: [0×1 string]
       UserData: []
           Name: ''
             Ts: 0
       TimeUnit: 'seconds'
   SamplingGrid: [1×1 struct]
```

```
Gs_SET{1}.Name = ['k = ', num2str(kSET(1)),' '];

np = nyquistplot(ax, Gs_SET{1}, omVALS);

get(np)
```

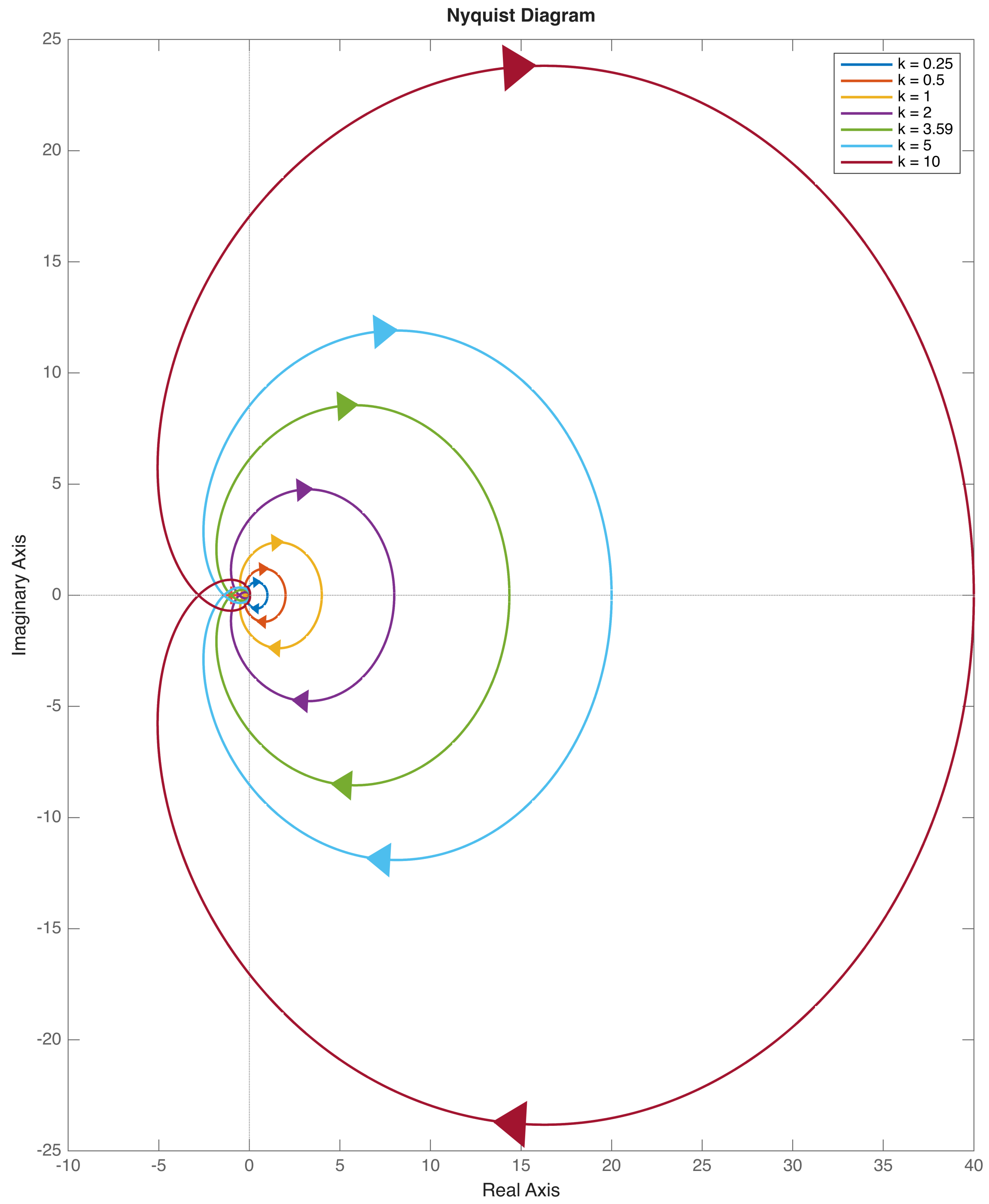
```
          AxesStyle: [1×1 controllib.chart.internal.options.AxesStyle]
    Characteristics: [1×1 controllib.chart.options.CharacteristicsManager]
      FrequencyUnit: "rad/s"
   HandleVisibility: 'on'
         IOGrouping: "none"
      InnerPosition: [0.1300 0.1100 0.7750 0.8150]
        InputLabels: [1×1 controllib.chart.internal.options.AxesLabel]
       InputVisible: on
             Layout: [0×0 matlab.ui.layout.LayoutOptions]
         LegendAxes: [1 1]
     LegendAxesMode: "auto"
     LegendLocation: "northeast"
  LegendOrientation: "vertical"
      LegendVisible: off
      MagnitudeUnit: "dB"
           NextPlot: "replace"
```

```
           OuterPosition: [0 0 1 1]
            OutputLabels: [1×1 controllib.chart.internal.options.AxesLabel]
           OutputVisible: on
                  Parent: [1×1 Figure]
               PhaseUnit: "deg"
                Position: [0.1300 0.1100 0.7750 0.8150]
      PositionConstraint: 'outerposition'
               Responses: [1×1 controllib.chart.response.NyquistResponse]
 ShowNegativeFrequencies: on
                Subtitle: [1×1 controllib.chart.internal.options.AxesLabel]
                   Title: [1×1 controllib.chart.internal.options.AxesLabel]
                   Units: 'normalized'
                 Visible: on
                  XLabel: [1×1 controllib.chart.internal.options.AxesLabel]
                 XLimits: [-1 1]
             XLimitsMode: "auto"
                  YLabel: [1×1 controllib.chart.internal.options.AxesLabel]
                 YLimits: {[-0.6000 0.6000]}
             YLimitsMode: {["auto"]}
```

```
np.LegendVisible = "on";
get(np.Responses(1))
```

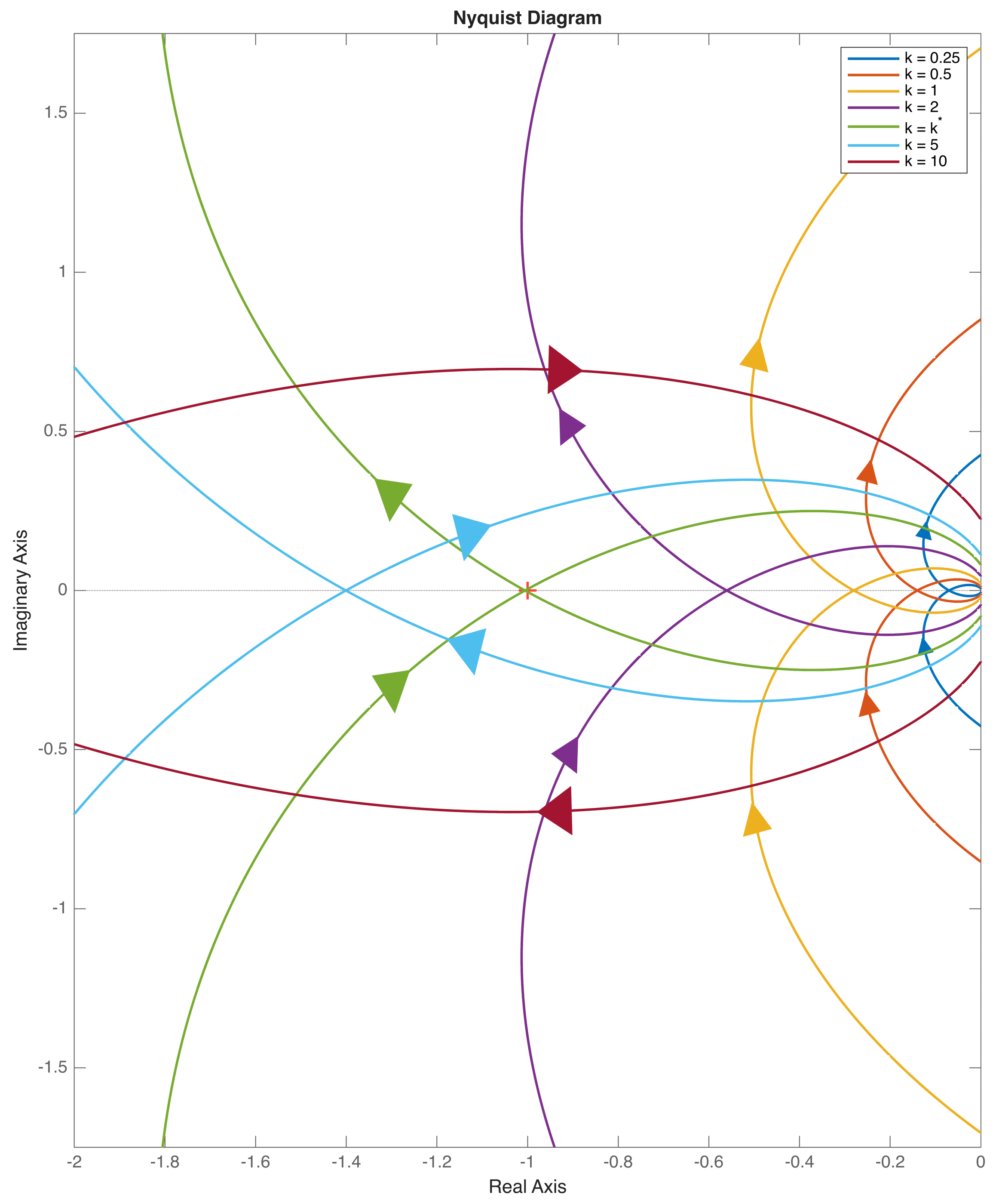
```
     SourceData: [1×1 struct]
           Name: "k = 0.25 "
        Visible: on
  LegendDisplay: on
     MarkerSize: 6
      LineWidth: 0.5000
      LineStyle: "-"
    MarkerStyle: "none"
          Color: [0 0.4470 0.7410]
```

```
np.Responses(1).LineWidth = 1.5;

for n = 2: numel(kSET)
    k = kSET(n);
    Gs_SET{n} = k*Gs;
    addResponse(np, Gs_SET{n}, omVALS);
    np.Responses(n).LegendDisplay = "on";
    np.Responses(n).Name = ['k = ', num2str(kSET(n), 3),' '];
    np.Responses(n).LineWidth = 1.5;
end % for n
```



Let's put in evidence the zone around the point $-1 + j0$:

```
% let's plot the Nyquist diagrams, varying the gain k
hf = figure('Units','centimeters','Position',[0.01, 0.01, 24, 28]);
ax = axes('Parent',hf); % the axes used to plot each Nyquist diagram
zoom on;

k = kSET(1);
Gs_SET{1} = k*Gs;
Gs_SET{1}.Name = ['k = ', num2str(kSET(1)),' '];
np = nyquistplot(ax, Gs_SET{1}, omVALS);
np.LegendVisible = "on";
np.Responses(1).LineWidth = 1.5;

for n = 2: numel(kSET)
    k = kSET(n);
    Gs_SET{n} = k*Gs;
    addResponse(np, Gs_SET{n}, omVALS);
    np.Responses(n).LegendDisplay = "on";
    if (n==5)
        np.Responses(n).Name = ['k = k^* '];
    else
        np.Responses(n).Name = ['k = ', num2str(kSET(n), 3),' '];
    end
    np.Responses(n).LineWidth = 1.5;
end % for n

np.XLimits = [-2, 0];
np.YLimits = [-1.75, +1.75];
```



Remark: As k increases from values less than 1 to larger values, the point on the Nyquist diagram where the frequency response is negative real shifts from near the origin, crossing exactly $(-1+j0)$ when $k=k^*$ and then moving away from $(-1+j0)$ as k increases.

Example 2 - Analyse $G(j\omega_c)$ when Varying the Open-Loop Transfer Function Gain

Consider the LTI system described by the strictly proper transfer function

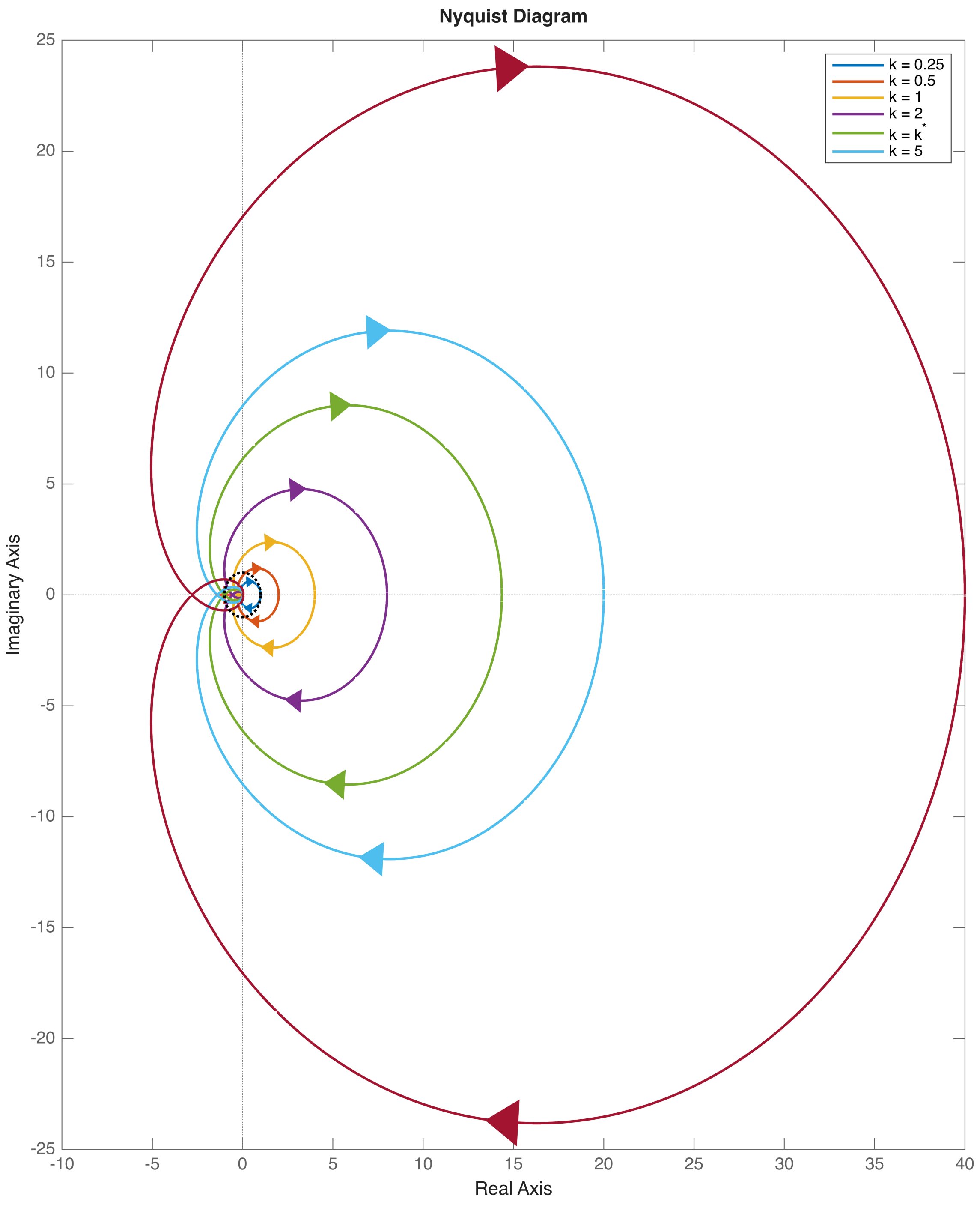
$$G(s) = k\,\frac{\left(1-\frac{s}{2}\right)\left(1+\frac{s}{3}\right)}{(1+s)^2\,(1+10\,s)\left(1+\frac{s}{10}\right)}\,, \quad k>0$$

Let's define $\omega_c :\ \omega$ such that $|G(j\omega_c)|=1$, i.e. the frequency response has unit modulus for $\omega=\omega_c$.

Let us identify the points where the frequency response has unitary magnitude on the following graph as intersections between the Nyquist diagram and a circle with a unitary radius and centre at the origin. What happens if we vary the gain k, for example if $k = 0.25, 0.5, 1, 2, \bar{k}, 5, 10$, where $\bar{k} = 10^{-11.1/20} \approx 3.59$? Do these characteristic points of the frequency response Nyquist diagram move around when we vary the gain k ?

```
clear
close all

s=tf('s');
Gs = 4*(1-s/2)*(1+s/3)/((1+s)^2*(1+10*s)*(1+s/10));


k_lim = 10^(11.1/20); % the "magic" gain

kSET = [0.25, 0.5, 1, 2, k_lim, 5, 10];

% let's plot the Nyquist diagrams, varying the gain k
```

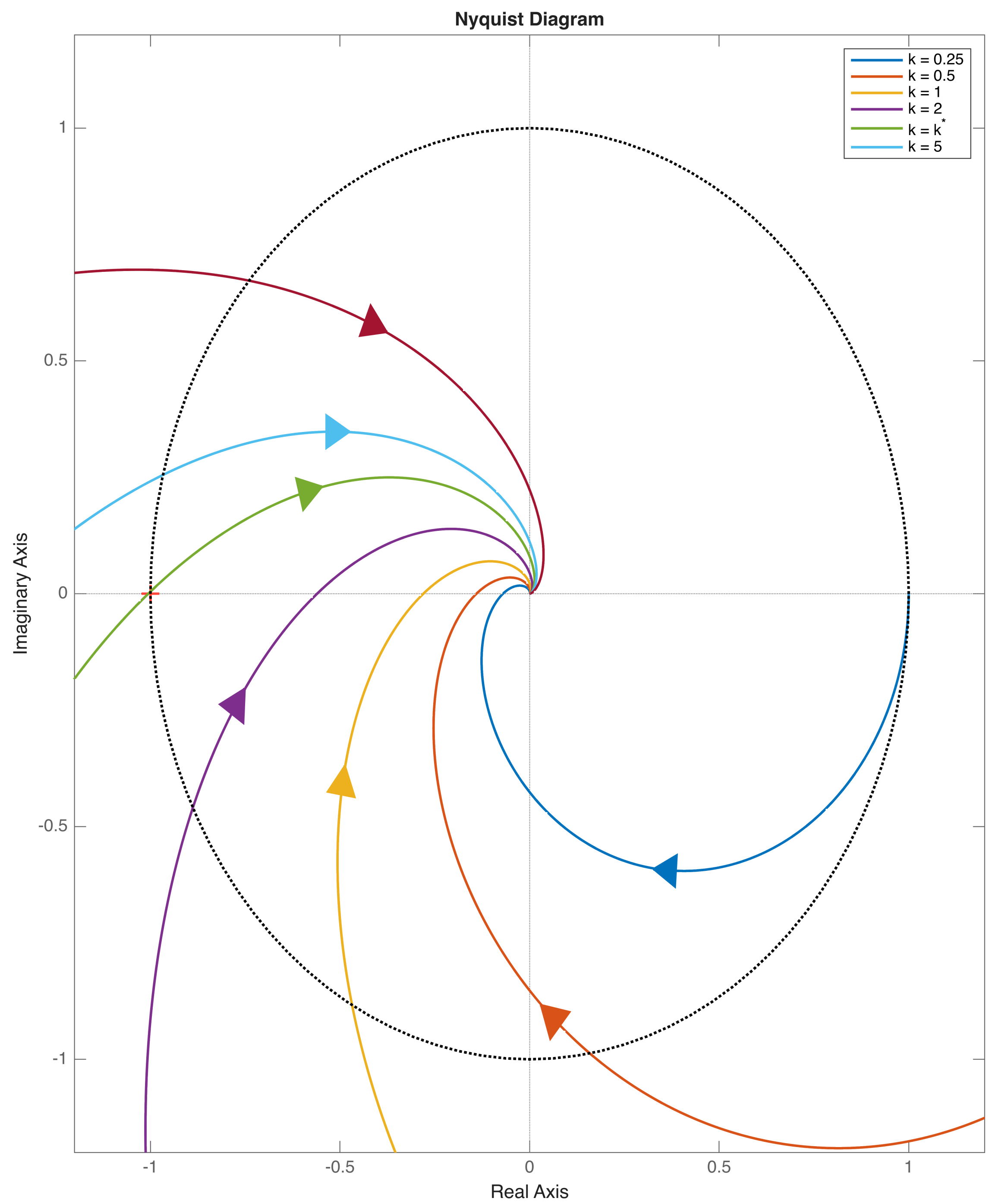
```
hf = figure('Units','centimeters','Position',[0.01, 0.01, 24, 28]);
ax = axes('Parent',hf); % the axes used to plot each Nyquist diagram
hold on;
zoom on;

plotCircle(ax, [0,0],1,[0,0,0], ':', 1.5);

omVALS = logspace(-8,6,1e4); % a set of angular frequencies

k = kSET(1);
Gs_SET{1} = k*Gs;
Gs_SET{1}.Name = ['k = ', num2str(kSET(1)),' '];
np = nyquistplot(ax, Gs_SET{1}, omVALS);
np.LegendVisible = "on";
np.Responses(1).LineWidth = 1.5;

for n = 2: numel(kSET)
    k = kSET(n);
    Gs_SET{n} = k*Gs;
    addResponse(np, Gs_SET{n}, omVALS);
    np.Responses(n).LegendDisplay = "on";
    if (n==5)
        np.Responses(n).Name = ['k = k^* '];
    else
        np.Responses(n).Name = ['k = ', num2str(kSET(n), 3),' '];
    end
    np.Responses(n).LineWidth = 1.5;
end % for n
```



Let's put in evidence the zone around the point $-1 + j0$:

```
hf = figure('Units','centimeters','Position',[0.01, 0.01, 24, 28]);
ax = axes('Parent',hf); % the axes used to plot each Nyquist diagram
```

```
hold on;
zoom on;

plotCircle(ax, [0,0],1,[0,0,0], ':', 1.5);

omVALS = logspace(-8,6,1e4); % a set of angular frequencies

k = kSET(1);
Gs_SET{1} = k*Gs;
Gs_SET{1}.Name = ['k = ', num2str(kSET(1)),' '];
np = nyquistplot(ax, Gs_SET{1}, omVALS);
np.LegendVisible = "on";
np.Responses(1).LineWidth = 1.5;

for n = 2: numel(kSET)
    k = kSET(n);
    Gs_SET{n} = k*Gs;
    addResponse(np, Gs_SET{n}, omVALS);
    np.Responses(n).LegendDisplay = "on";
    if (n==5)
        np.Responses(n).Name = ['k = k^* '];
    else
        np.Responses(n).Name = ['k = ', num2str(kSET(n), 3),' '];
    end
    np.Responses(n).LineWidth = 1.5;
end % for n

% let's modify the view
p = getoptions(np);
p.ShowFullContour = 'off';
p.XLim = {[-1.2, 1.2]};
p.YLim = {[-1.2, 1.2]};
setoptions(np,p);
```



Remark: When k increases from values less than 1 to larger values, the point on the Nyquist diagram where the frequency response has unit modulus rotates from $(1 + j0)$, passing through precisely the point $(-1 + j0)$ for $k = \bar{k}$ and then moving away from $(-1 + j0)$ as k increases further.

```
function plotCircle(ax, Centre,radius,Color, Line_Style, Line_Width)
    xc = Centre(1); yc = Centre(2);
    r = radius;
    LineSty = Line_Style;
    LineW = Line_Width;
    LineCol = Color;

    th = 0:pi/50:2*pi;
    xunit = r * cos(th) + xc;
    yunit = r * sin(th) + yc;
    plot(ax, xunit, yunit, 'Color', LineCol, 'LineStyle', LineSty, ...
        'LineWidth',LineW, 'HandleVisibility','off');
    % in this way (using 'HandleVisibility') the circle doesn't have any
    % legend -- see https://stackoverflow.com/questions/13685967/how-to-show-legend-for-only-a-specific-subset-
of-curves-in-the-plotting.

end % function
```