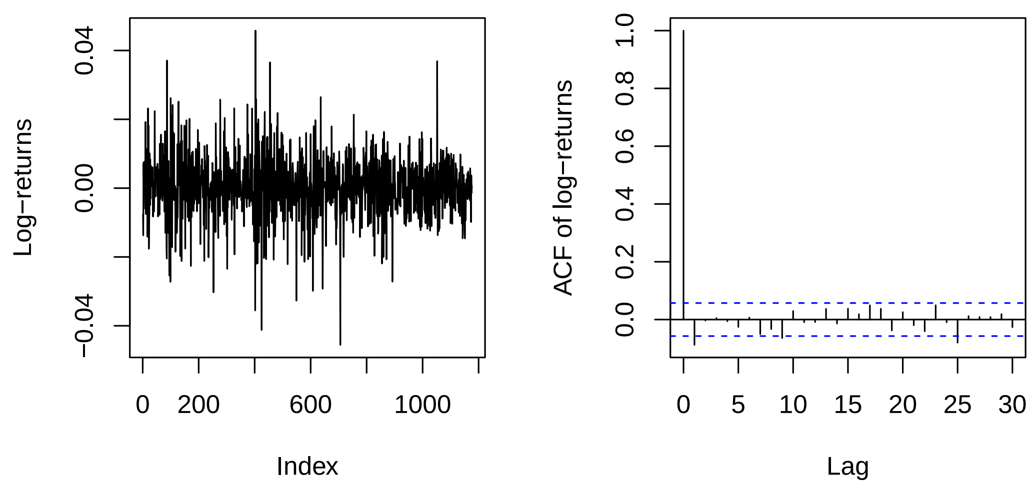
**Data Science for Insurance**
Sample exam questions (Part I)

1. The figure below shows the log-returns of McDonald's stock from January 2010 to September 2014 (left) and the corresponding correlogram or sample autocorrelation function (ACF) plot (right).



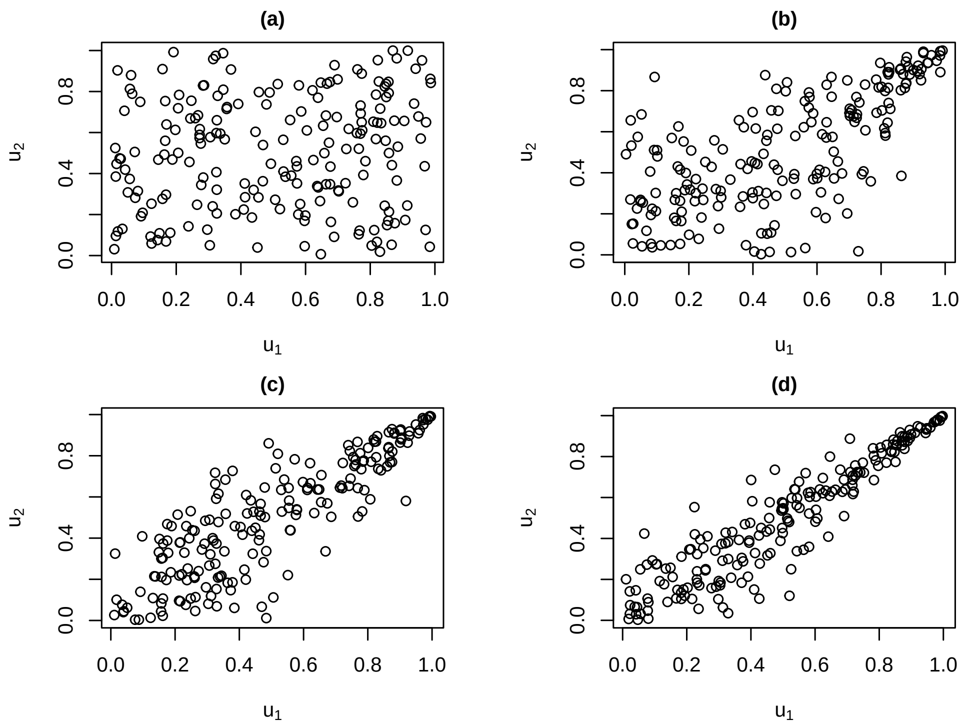

   a. Explain how the correlogram should be interpreted.
   b. Which stylized fact(s) do the two plots show?
   c. What else would you need to test whether a dataset satisfies the iid hypothesis?

2. The Joe copula is similar to the Gumbel copula, and has the form (in the bivariate case)

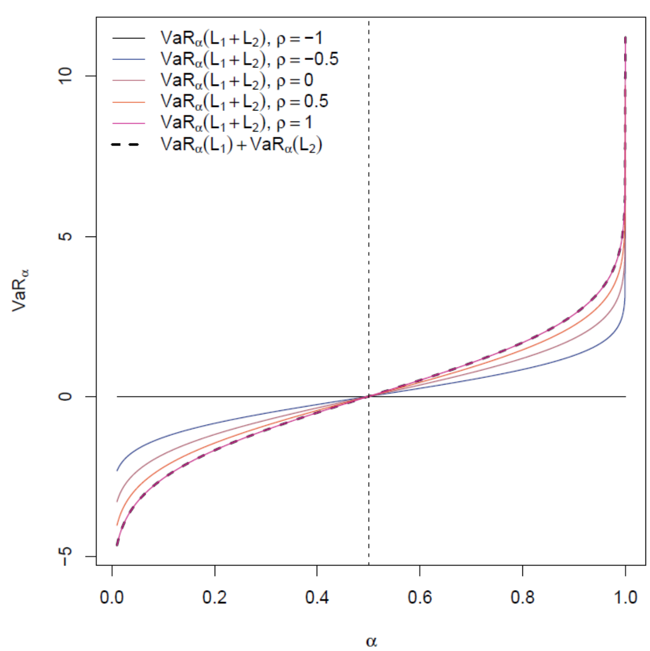
$$C(u_1, u_2) = 1 - [(1-u_1)^\theta + (1-u_2)^\theta - (1-u_1)^\theta(1-u_2)^\theta]^{1/\theta}, \quad \theta \geq 1$$

   which converges to the comonotonicity copula as $\theta \to \infty$.

   a. Show that the Joe copula is the independence copula when its parameter is minimum.
   b. The figure below contains scatterplots of four bivariate random samples from various Joe copulas, with a sample size of 200 and $\theta \in \{1.1, 3, 5, 8\}$. Match the pictures from (a) to (d) to one value of $\theta$, explaining your reasoning.
   c. Do you see evidence of tail dependency in one or more of the above graphs?




3. The graph shows the behaviour of $VaR_\alpha(L_1 + L_2)$ for losses $L_j \sim N(0, 1)$ $(j = 1, 2)$ that are jointly normal with correlation $\rho$, for $\rho \in \{-1, -0.5, 0, 0.5, 1\}$. Is it possible to identify a region of superadditivity and a region of subadditivity? What happens when $\rho = 1$?




4. Consider the class of joint distributions of the pair $(X_1, X_2)$ where the marginal distributions are fixed and such that $X_1$ and $X_2$ have finite variances, and the copula is allowed to vary.

For each of the following statements, state whether they are true or false (1.5 points for each correct answer; wrong answers are weighted negatively).

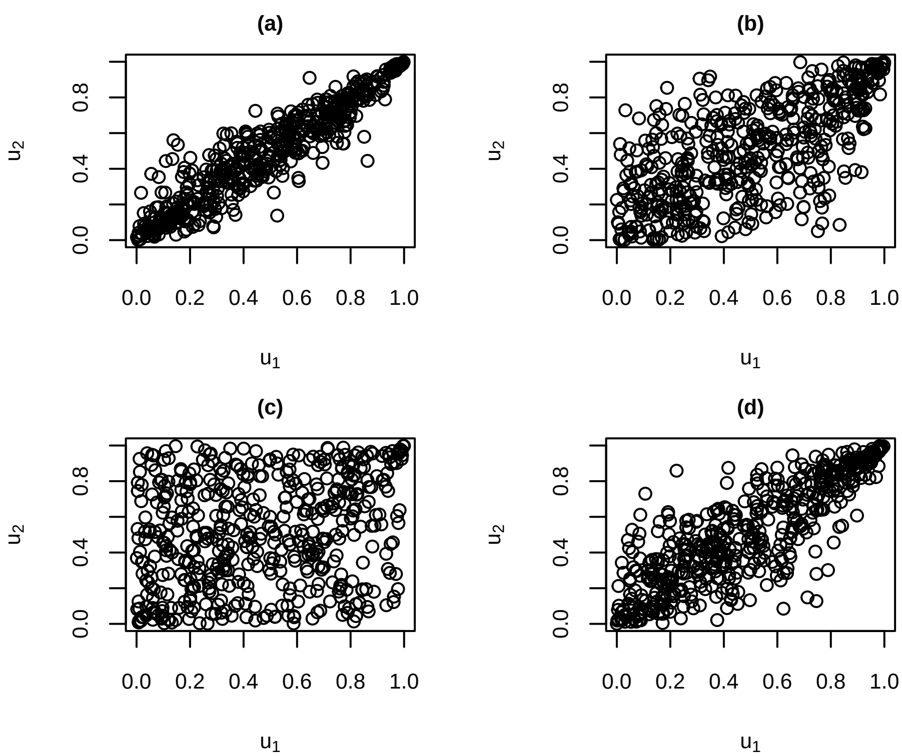
- If $(X_1, X_2)$ are jointly normal, then $\mathrm{VaR}_\alpha(\mathrm{X_1 + X_2})$ is known explicitly V F
- $\rho(X_1, X_2)$ is maximal when $(X_1, X_2)$ has the comonotonicity copula $M$ V F
- The variance of $X_1 + X_2$ is maximal when $\rho(X_1, X_2)$ is maximal V F
- For fixed $\alpha \in (0, 1)$, $\mathrm{VaR}_\alpha(\mathrm{X_1 + X_2})$ is maximal when $\rho(X_1, X_2)$ is maximal V F
- If the copula of $(X_1, X_2)$ is the normal copula $C^{Ga}_\rho$ then both marginal dfs are univariate standard normal distributions V F
- If the copula of $(X_1, X_2)$ is the normal copula $C^{Ga}_\rho$ with $\rho = -1$, then $(X_1, X_2)$ are countermonotonic V F
- If $(X_1, X_2)$ has copula $C$ then $(T(X_1), T(X_2))$ has the same copula, being $T$ a strictly increasing transformation on the ranges of $X_1$ and $X_2$ V F

5. The bivariate Gumbel-Hougaard copula has the form

$$C(u_1, u_2) = \exp\{-((-\ln u_1)^\theta + (-\ln u_2)^\theta)^{1/\theta}\}, \quad 1 \le \theta < \infty, u_1, u_2 \in [0, 1]$$

converging to the comonotone copula $M$ as $\theta \to \infty$.

a. The figures below are the scatterplots of four bivariate random samples of size $n = 500$ from Gumbel copulas with $\theta \in \{1.2, 2, 3, 5\}$.




Match the pictures (a)–(d) to one value of $\theta$, explaining your reasoning.

b. Do you see evidence of tail dependency in one or more of these graphs?

c. Can you identify some form of symmetry from the above scatter plots?