Statistical Methods with Application to Finance
a.y. 2025/2026

Exercises (set 2)

1. The prices and dividends of a stock are given in the table below.

t	P_t	D_t
1	82	0.1
2	85	0.1
3	83	0.1
4	87	0.125

   Returns and log returns are denoted by R_t and r_t, respectively.

   (a) What is R_2?

   (b) What is $R_3(2)$?

   (c) Find $r_4(3)$.

2. The monthly log return on an asset is 4.46%.

   (a) Compute the monthly net return.

   (b) If the log returns on the next two months were -7.34% and 10.77%, what is the quarterly log return on the asset?

3. The log prices on a stock are assumed to follow a random walk model

$$p_t = p_0 + \sum_{i=1}^{t} r_i$$

   where $p_0 = \log(P_0)$ is the log price at time 0 and r_1, $r_2, \ldots$ are iid $\mathcal{N}(0.1, 0.2^2)$.

   (a) What is the expected log return after 20 years?

   (b) If the stock price starts at \$100, what is the median price after 20 years?

   (c) Find the 0.05-quantile and 0.95-quantile of the 20-year gross return.

4. Assume that the simple gross return $1 + R_t$ is lognormal with log-mean zero and log-variance 0.2. Also, assume that the returns are an independent sequence.

   (a) Find the probability that a gross two-period return is less than 0.8.

   (b) What is the third quartile of the log return r_t?

   (c) Compute the probability $Pr(r_t(3) \geq 0.7)$.

   (d) What is the covariance between $r_2(1)$ and $r_2(2)$?

5. Suppose that there are two risky assets, A and B, with expected returns equal to 2% and 5%, respectively. Suppose that the standard deviations of the returns are $\sqrt{6}\%$ and $\sqrt{11}\%$ and that the returns on the assets have a correlation $\rho = 0.1$.

(a) What portfolio of A and B achieves a 3% rate of expected return?

(b) (b) What portfolios of A and B achieve an $\sqrt{5}\%$ standard deviation of return? Among these, which has the largest expected return?

(c) Is is true that

$$R_P = \omega R_A + (1-\omega) R_B$$

if R_P, R_A, R_B are log returns on the portfolio, on asset A and on asset B, respectively?

6. Consider the random walk model

$$X_t = \delta + X_{t-1} + w_t$$

with drift δ, $x_0 = 0$, and w_t a white noise series with variance σ_w^2.

(a) Show that the model can be written as $X_t = \delta t + \sum_{k=1}^{t} w_k$.

(b) Find the mean function and the autocovariance function of X_t.

(c) Suggest a transformation to make the series stationary and prove that the transformed series is stationary.

7. The series $\{w_t\}$ is white noise with zero mean and variance σ_w^2. For the following moving average models find the autocorrelation function and determine whether they are invertible.

$$(a) \quad Y_t = w_t + \frac{1}{2} w_{t-1}$$

$$(b) \quad Y_t = w_t + 2w_{t-1}$$

8. Consider the AR(1) model

$$Y_t = 5 + 0.7 Y_{t-1} + a_t$$

where $a_t \sim \text{WN}(0, \sigma_a^2)$, and assume that $\sigma_a^2 = 2$.

(a) Is this process stationary?

(b) What is the mean of this process?

(c) What is the variance of this process?

(d) What is the covariance between Y_1 and Y_3?

(e) Sketch the theoretical ACF for this model.

9. Suppose that Y_t is an AR(1) process with $\mu = 1$, $\phi_1 = 0.3$, and $\sigma_a^2 = 2$.

(a) What is the variance of Y_1?

(b) What is the covariance between Y_1 and Y_3?

(c) What is the lag-3 autocorrelation of Y_t?

10. Consider the MA(2) process

$$X_t = a_t + 0.7 a_{t-1} - 0.2 a_{t-2}$$

where a_t is a Gaussian white noise with mean 0 and variance $\sigma_a^2 = 1$.

(a) Derive the mean and the variance function of the process.

(b) Is this process invertible?

(c) Show that the ACF of the process is

$$\rho(k) = \begin{cases} 1 & k = 0 \\ 0.37 & k = \pm 1 \\ -0.13 & k = \pm 2 \\ 0 & \text{otherwise} \end{cases}$$