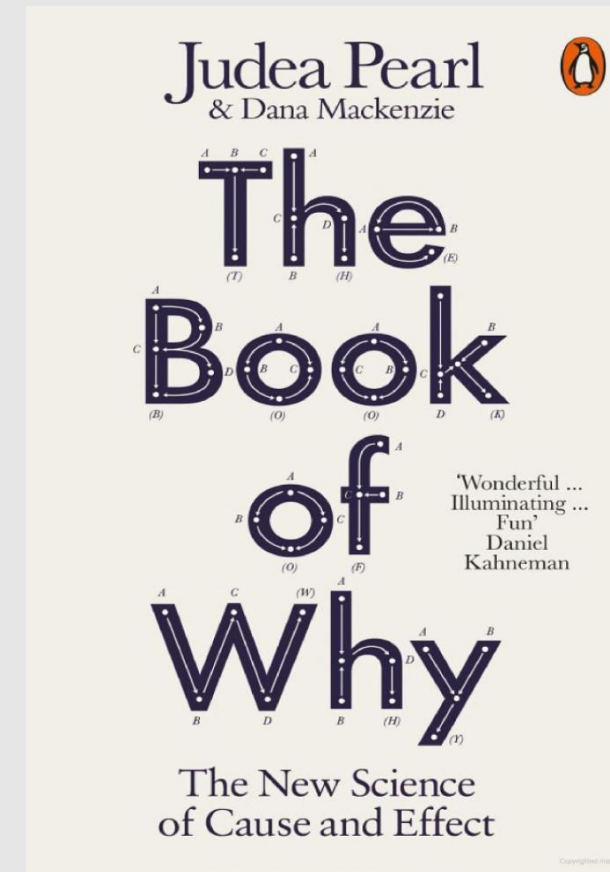
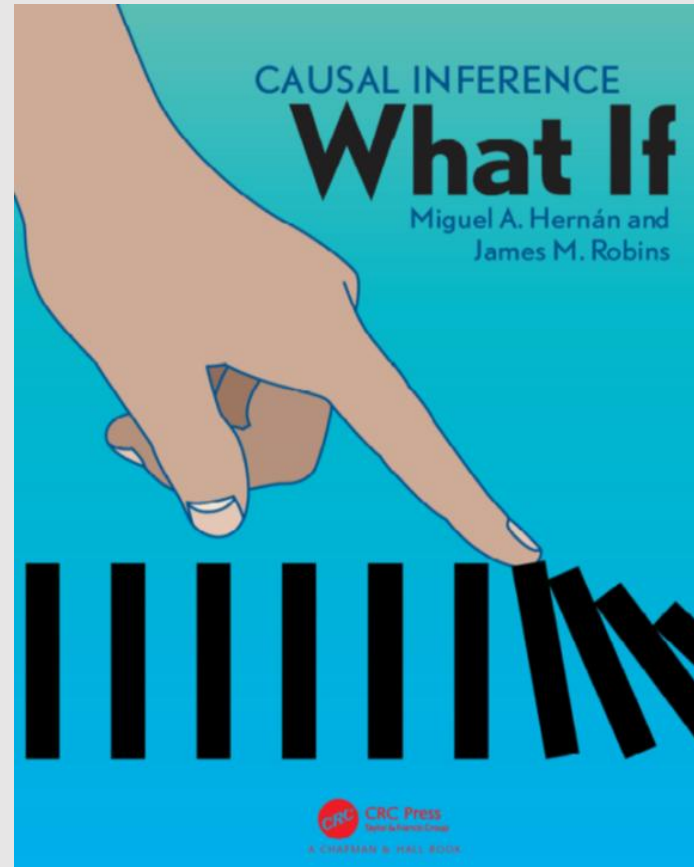
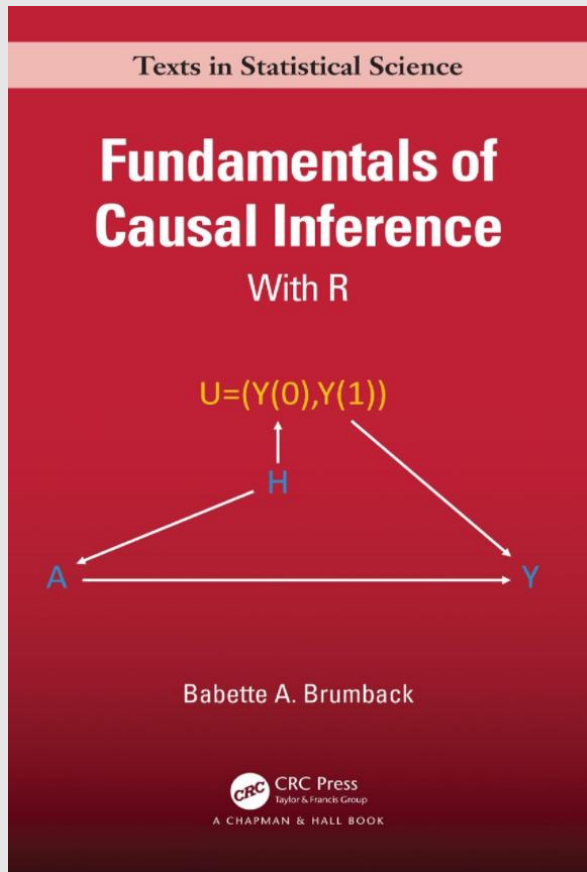


Causal Models in Epidemiological & Clinical Research (basic ideas...)



Learning without making assumptions is a myth

Different scientific aims



Descriptive modelling

Describe the outcome of interest:
which factors *affect* it and how?

Estimate a prevalence in function of age and sex



Predictive modelling

Accurate predictions of future observations.
No concern about causality and confounding (association)

Risk of developing CVD in the next x years



Explanatory modelling

Testing and comparing existing causal theories.

Effect of LDL on CVD risk

Policy recommendations (statins)

Independently from the primary scientific aim, a **basic statistical tool** is the REGRESSION model approach.

The key difference is in **the scope of** the REGRESSION model according to the main aim...

The **Descriptive** Aim (The "What")

Regression as a tool for **parsimonious** summarization.

Describing the Population: How regression describes the *average* individual.

Trend Identification: Using splines or polynomials to describe non-linear patterns

Data Reduction: Using regression to simplify complex datasets into understandable trends.

The Predictive Aim (The "Who")

Focus on accuracy and generalizability of predictions.

The **Bias-Variance** Tradeoff: Why "more variables" isn't always better.

Feature Selection: we don't care about the specific role of a variable (if it is a "confounder" or "independent predictor"..) only if *it adds signal*.

Validation: The necessity of Cross-Validation and External Validation.

The Causal Aim (The "Why")

Focus on unbiased estimation of an **effect** of an intervention/exposure.

Counterfactual Framework: Potential Outcomes.

Directed Acyclic Graphs (DAGs): How to choose variables based on theory, not (just) p-values...

Identifiability: assumptions of **exchangeability, positivity, and consistency**.

Explanatory/Causal Models (Observational Studies)

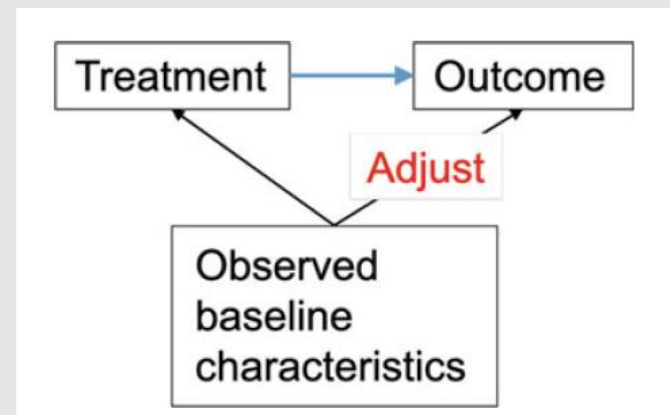
Confounding is one of the major concern in epidemiological analyses of observational studies, where we aim to estimate **causal** effects.

When treatments/exposures are compared, groups are often quite different **because of a lack** of randomization.

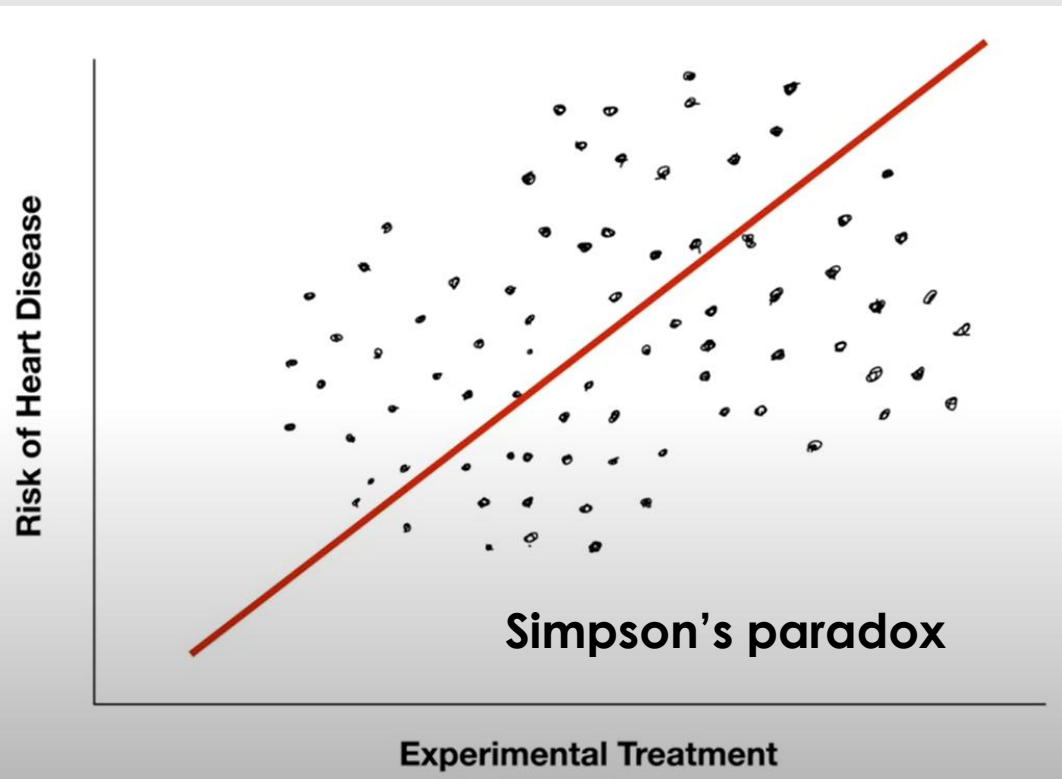
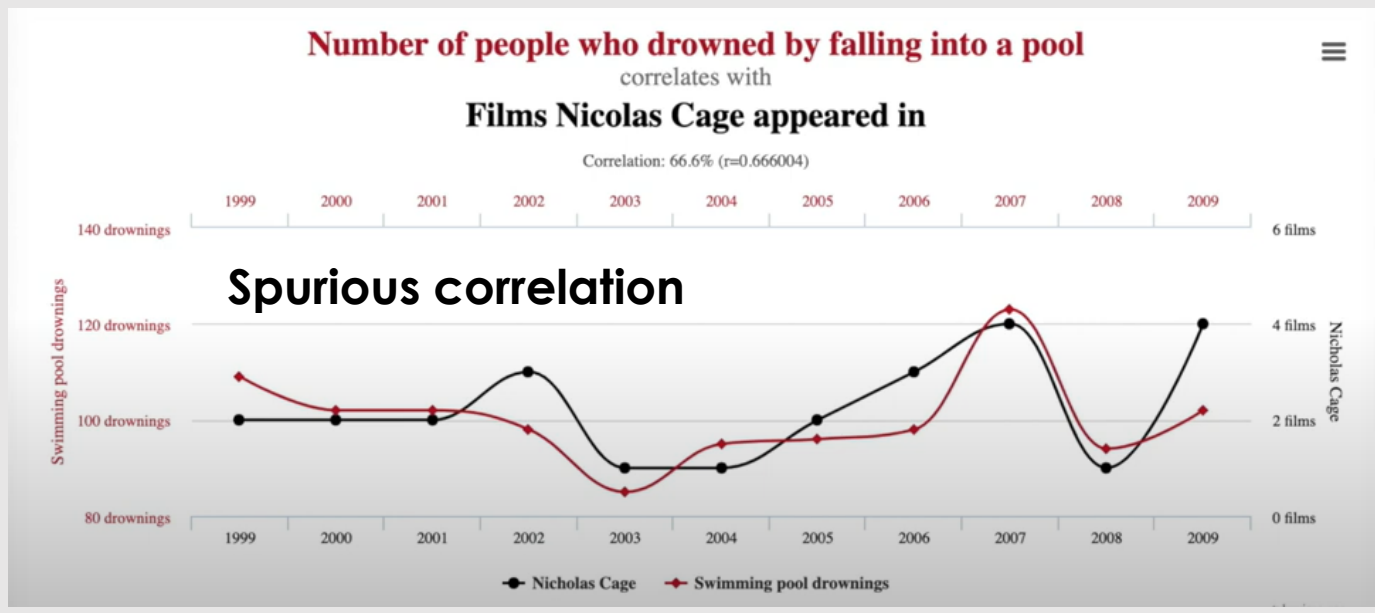
For example, subjects with specific characteristics **are more likely** to receive a certain treatment than other subjects (*confounding by indication*).

If these characteristics also affect the outcome, a direct comparison of treatments is **biased** and may merely reflect the lack of initial comparability.

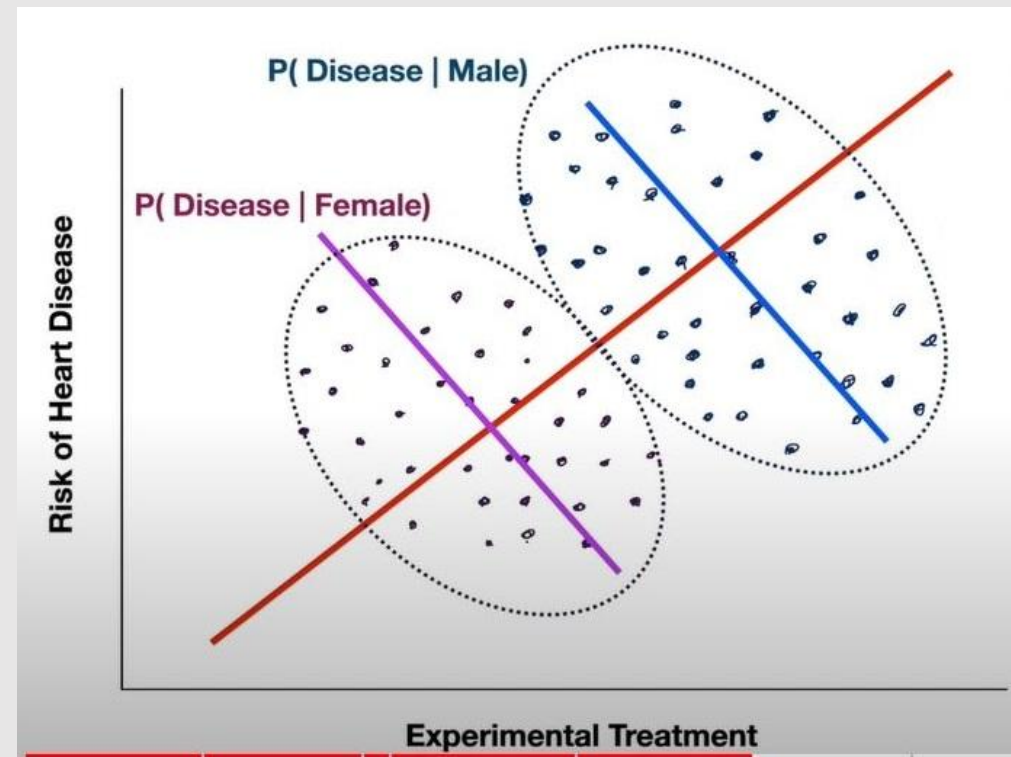
Often, randomization is not possible, and **observational studies are the only possible design**. Dealing with confounding is an essential step in such analyses.



what is the treatment effect if baseline characteristics were *similar* between treatment groups?



Treatment is **good** for a man, **good** for a woman but **bad** for a person



In this context, **causal inference** methods must bridge a gap between goals and means.

Researchers seek **causation**, but the data, on their own, only communicate **associations**.

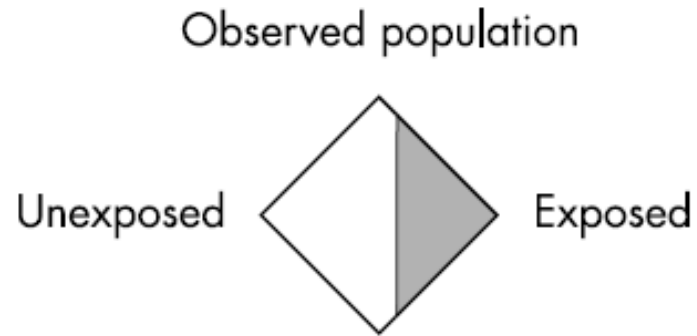
Associations usually consist of a **mixture** of causal and non-causal (spurious) components.

Therefore, a first step of **identification analysis** should determine whether, and under which conditions, it is possible to strip an observed association of all its spurious components.

Identification analysis requires causal **assumptions** about how the data were generated.

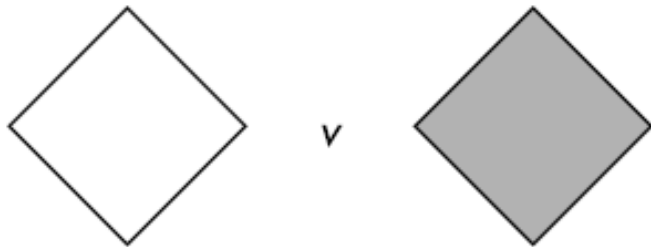
The sum of these causal assumptions is called a **causal model**, which must describe both *how the world works* (how observed and unobserved variables take their values) and *how the data were collected* (**which** variables and variable values are recorded/which **study design**).

REMIND: What is a **causal** effect?



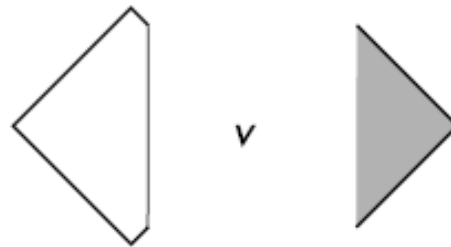
What would be?

Causation



What is ?

Association



Comparison of **potential outcomes** for **THE SAME** well defined population:

Y_1 Potential outcome if treated/exposed

Y_0 Potential outcome if control
(**not** treated/exposed)

An **association** compares some outcome in two **DIFFERENT** groups ...

Figure from Hernan: <https://pubmed.ncbi.nlm.nih.gov/15026432/>

How we **select** variables in the **causal** model ?

Depends on the scientific aim!

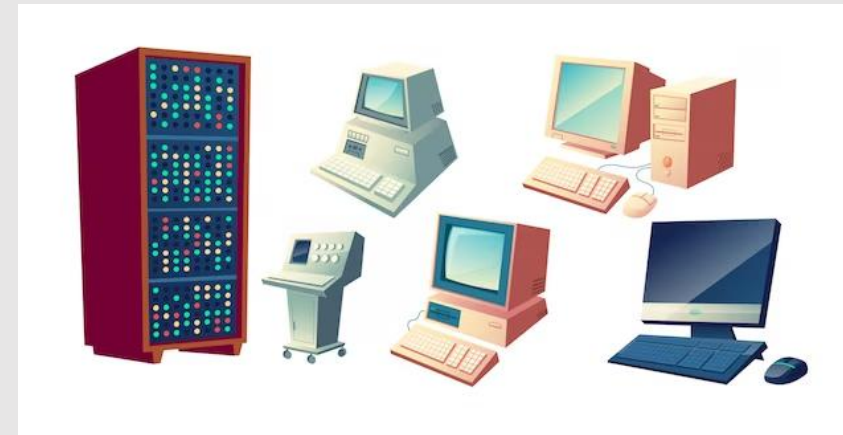
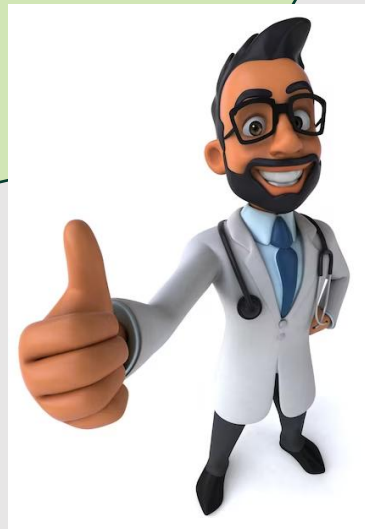
Theory-driven



Data-driven

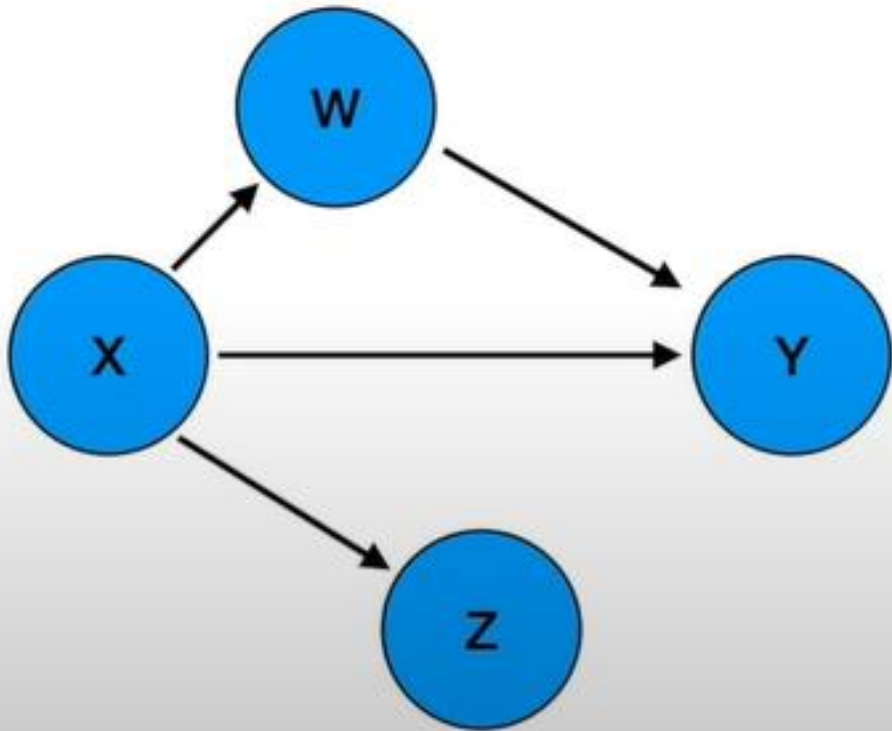


Explanatory modelling



Causality

X causes Y if when all confounders are adjusted, an intervention in X **results in a change** in Y, but intervention in Y **does not change** X



A causal model could be represented using a **DAG** (**D**irected **A**cyclic **G**raph)

Each arrow represents a causal influence

The graph is:

- **Directed** : each connection between two variables consists of an arrow
- **Acyclic** : no reverse* cycles

*A variable can't cause itself; however time varying processes can be depicted adding one *realization* of each variable per time unit

Block 3.4

Knowledge of the data-generating mechanism has to be provided by **external theory** and **understanding**

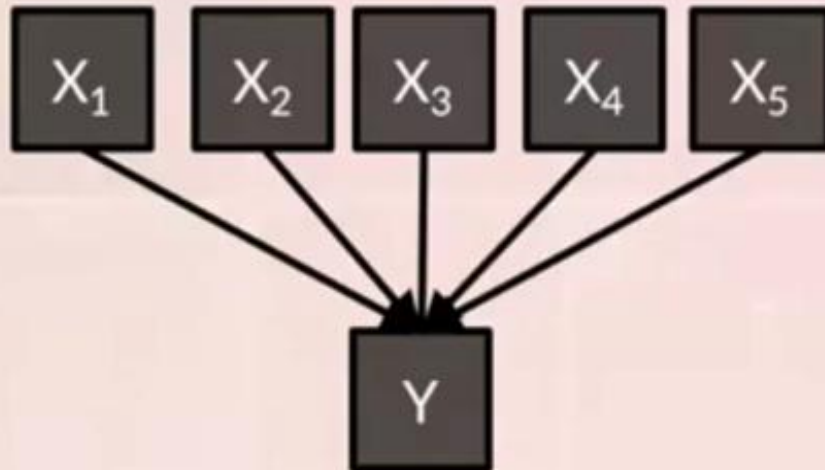


i.e. a **causal** model

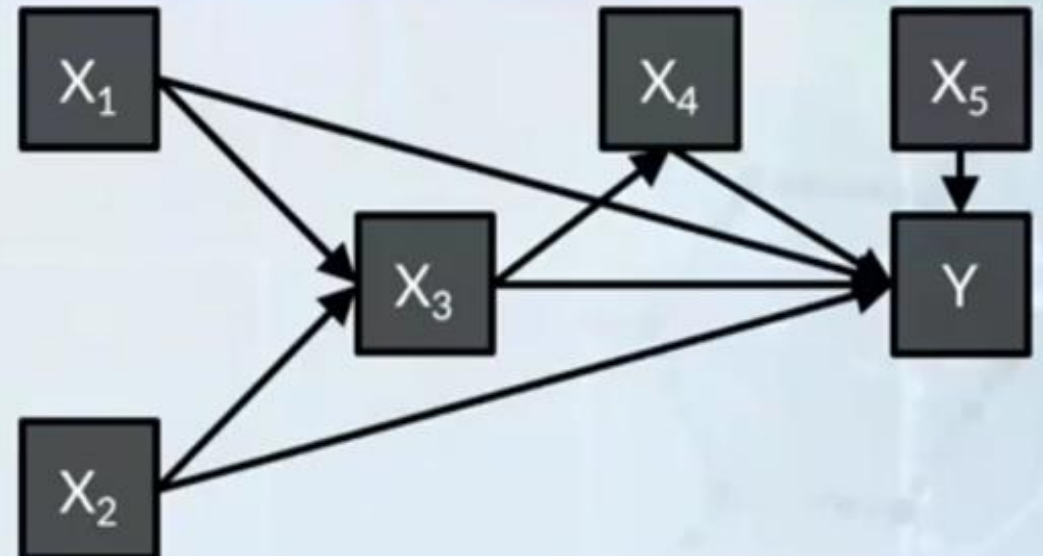
No software/algorithm can (currently) **understand** this

Prediction models can not be in general *causally interpreted* – however transparent they are ...

HOW THE ALGORITHM SEES IT



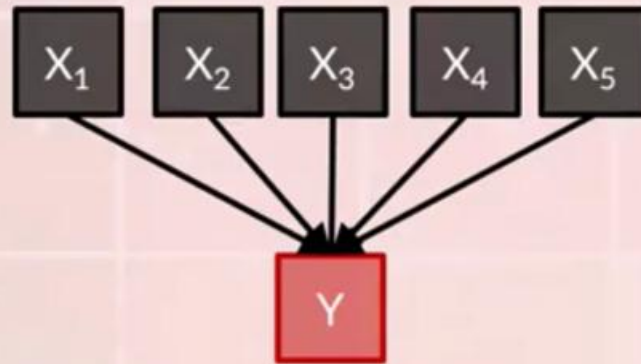
HOW NATURE CREATED IT



Predictive vs causal modelling

PREDICTIVE MODEL

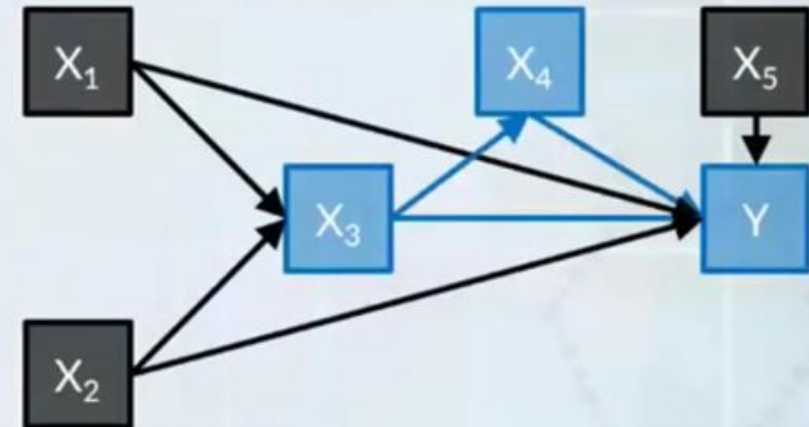
- Outcome-focused



X_i is _____ Y
'correlated with'
'a predictor of'
'associated with'

CAUSAL MODEL

- Effect-focused



The _____ of X_2 on Y is...
'total causal effect'
'direct causal effect'

Predictive vs causal modelling

Predictive Model

Aim: Predict values of outcome (p)

Maximise: performance measures (R^2 , calibration, AUC...)

Covariates selection focused on:

- Balancing precision & parsimony
- Availability of variables
- Maximising joint information

Coefficients: *associations*

Automation: Favoured

Causal Model

Aim: Estimate a causal effect

Maximise: accuracy of the effect estimate

Covariates selection focused on:

- External knowledge & judgement
- Role of variables
- Minimizing confounding

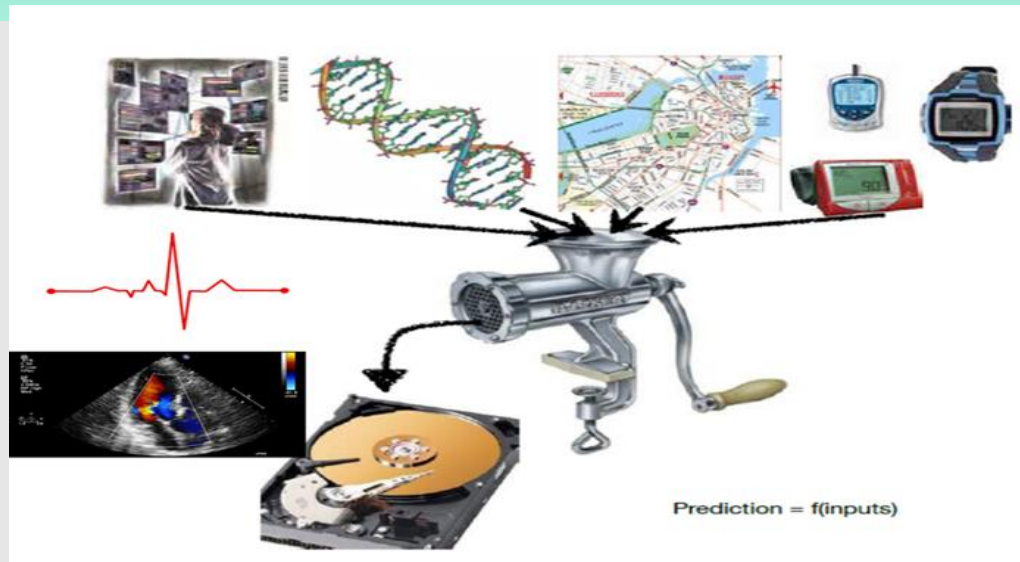
Coefficients: *Interpretable in the causal sense*

Automation: Not possible

Two extremes in regression models

Prognostic/Predictive modelling

- We are trying to find **predictors** of some outcome
- It is their **joint value** as predictors that is important
- We simply want the **most predictive** model
- We compare entire models to judge which is best



Causal/explanatory analysis

- The putative causal factor **must be** in the model
- Other factors are in the model because **help us understand** the causal factor (they are of no interest in themselves)
- We focus on the **estimation** of the putative causal effect (at a population level)



Smoking & Lung Cancer

A Tale of Two Statisticians

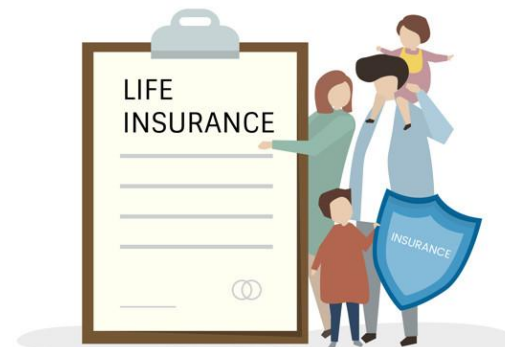
Works in public health (**explanatory**):

- I wish to establish whether it is causal
- If so I can warn smokers **to quit** and this will **benefit** their health [**intervention**]
- It is important for me to rule out possible **confounding** factors



Works in life insurance (**predictive**):

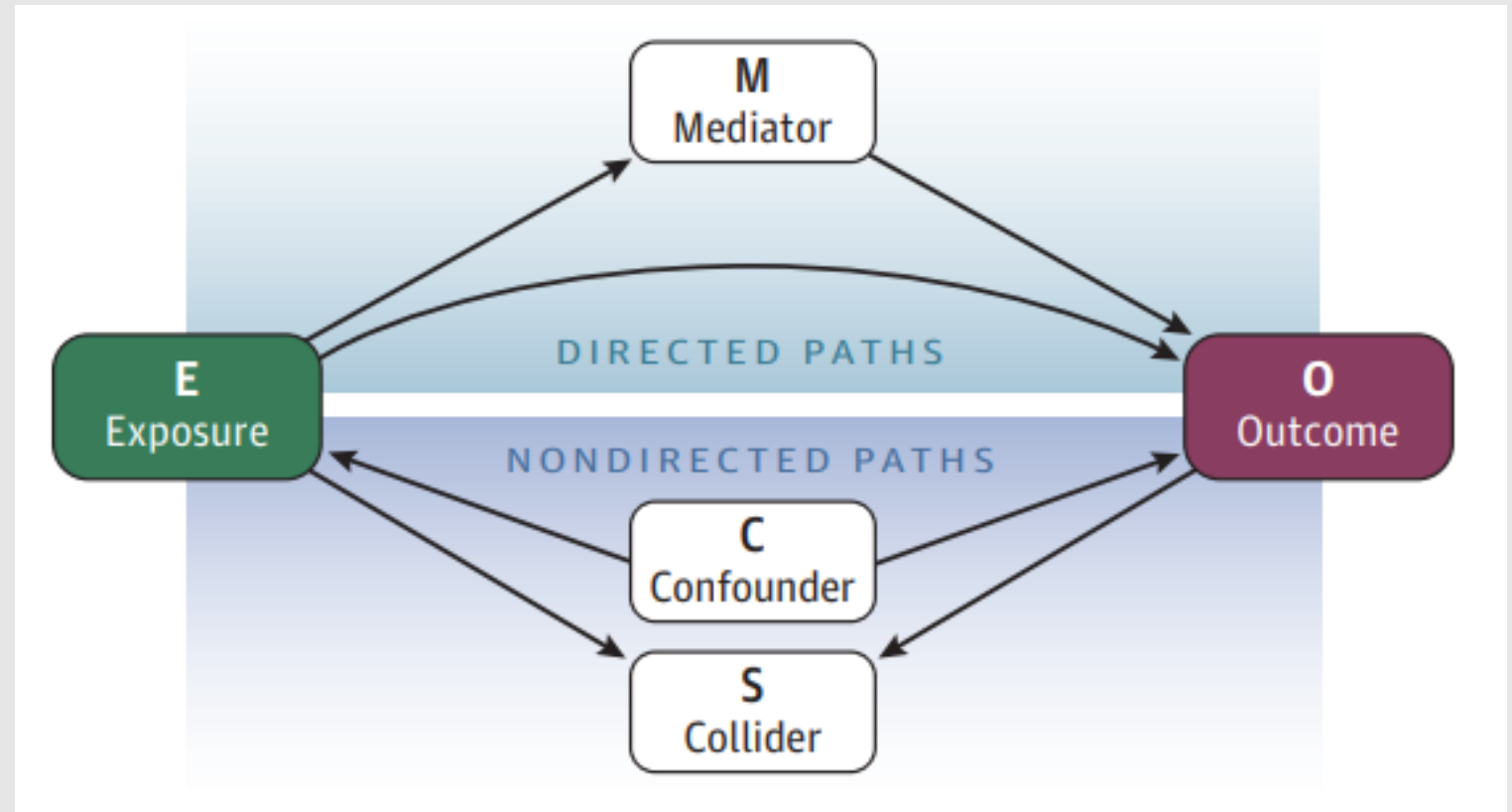
- I don't care if it is causal or not
- The data show that smokers **are much more likely** to get lung cancer
- That's enough for me to take account of it in setting the premiums



...so again: how to *represent* the role of covariates ?

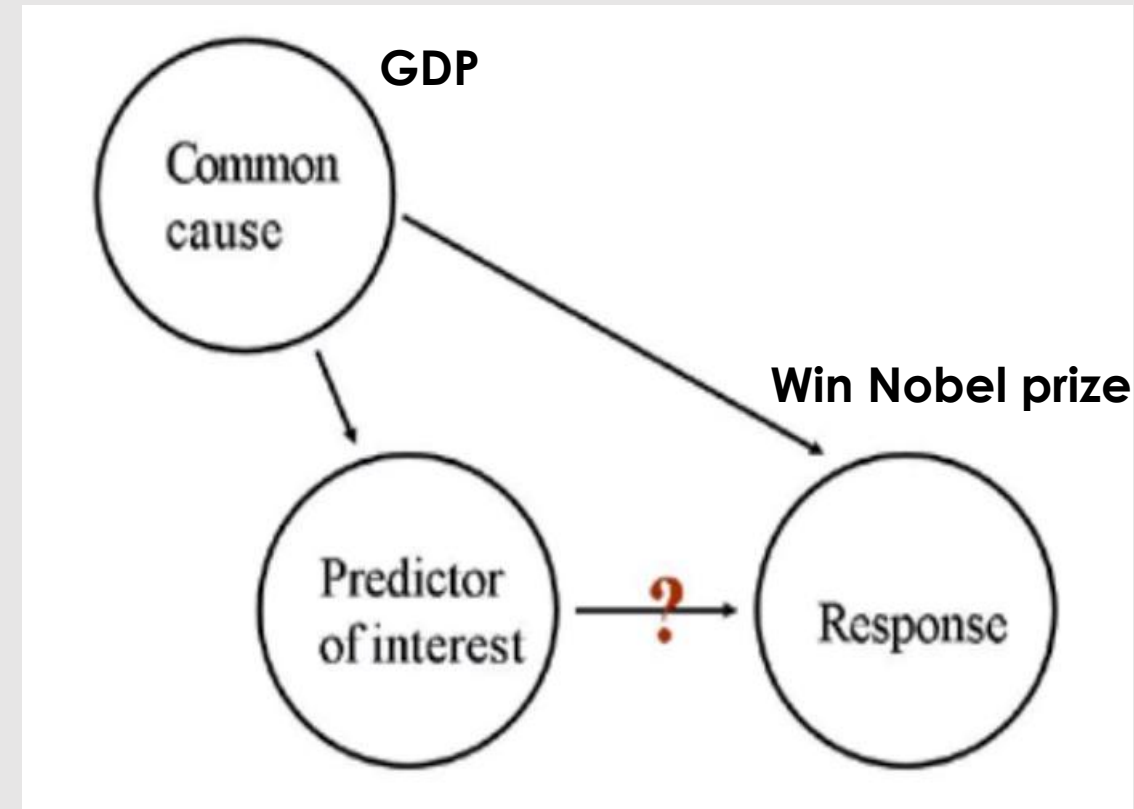
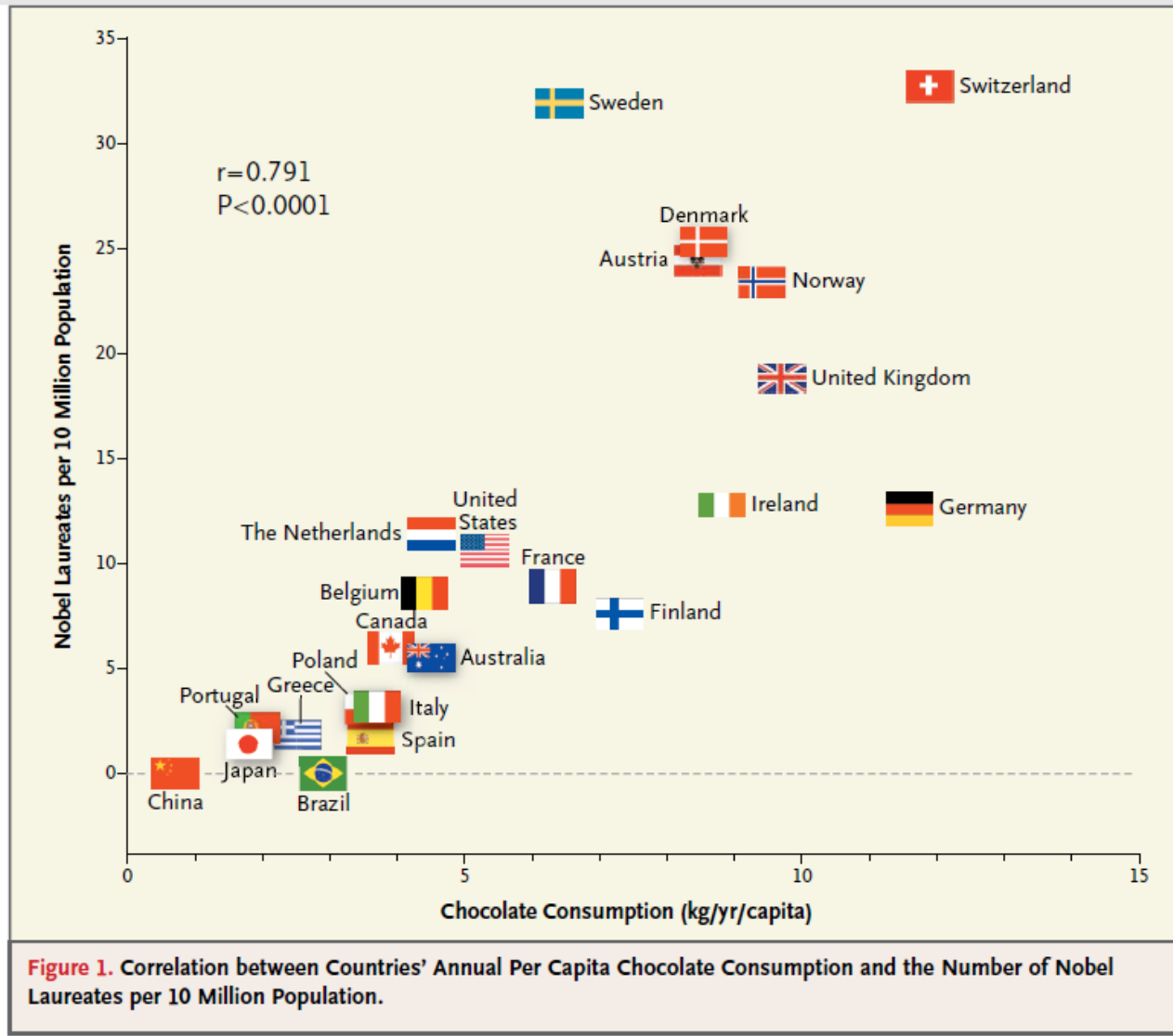
DAGs are visual representations of qualitative causal assumptions

They encode researchers' expert knowledge and beliefs about *how the world works*



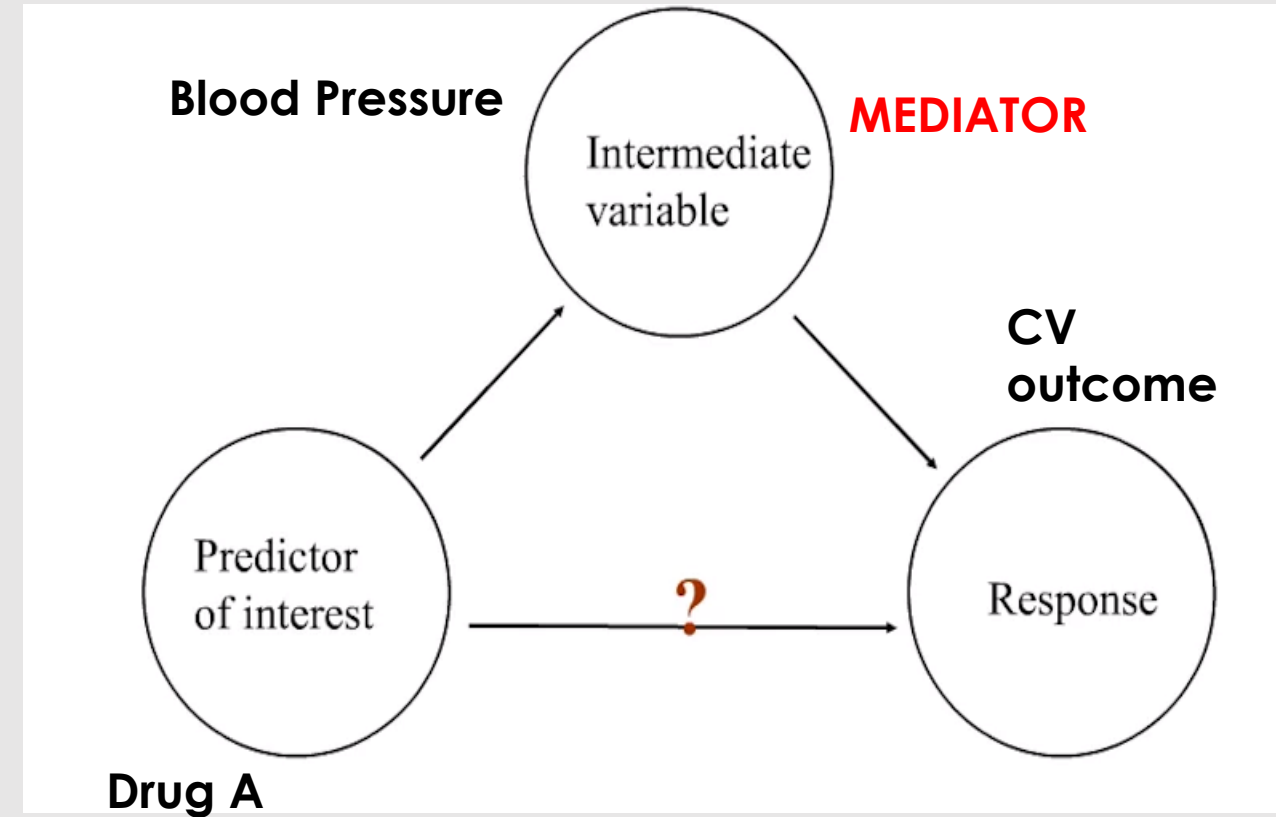
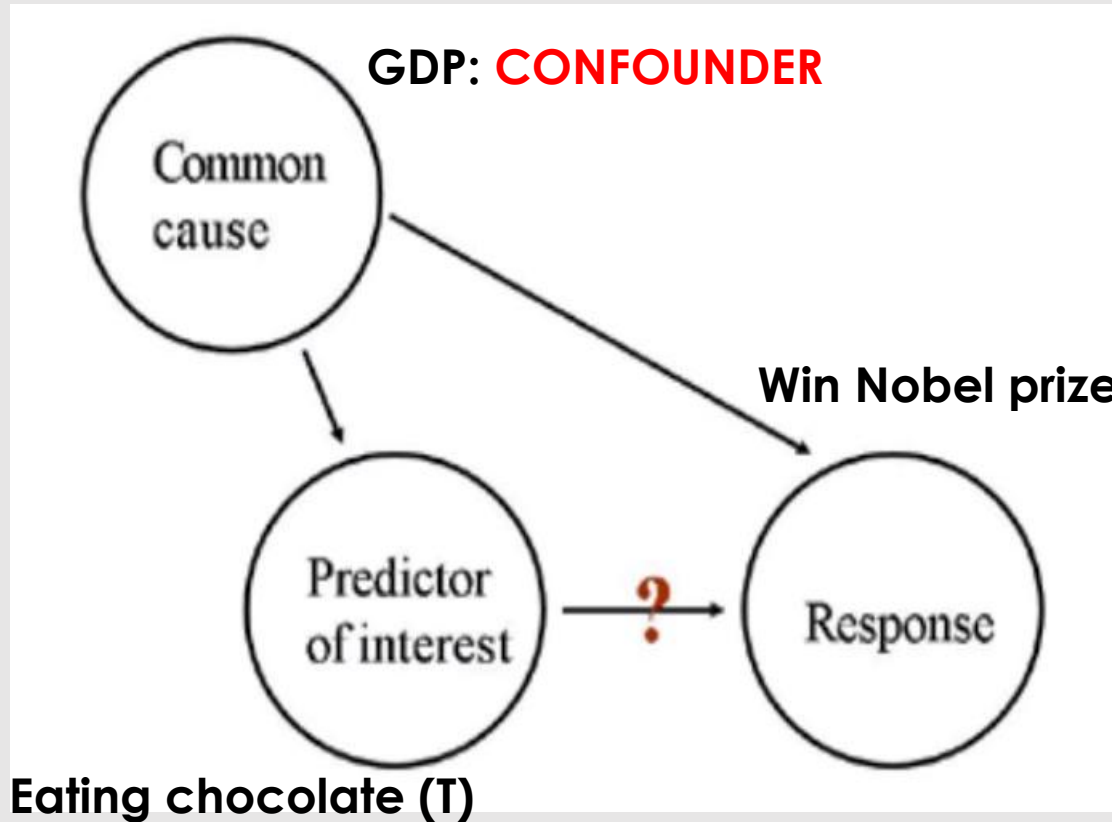
Do you remember? Chocolate Consumption, Cognitive Function, and Nobel Laureates....

N Engl J Med, 2012 Oct 18;367(16).



Eating chocolate (T)

Examples

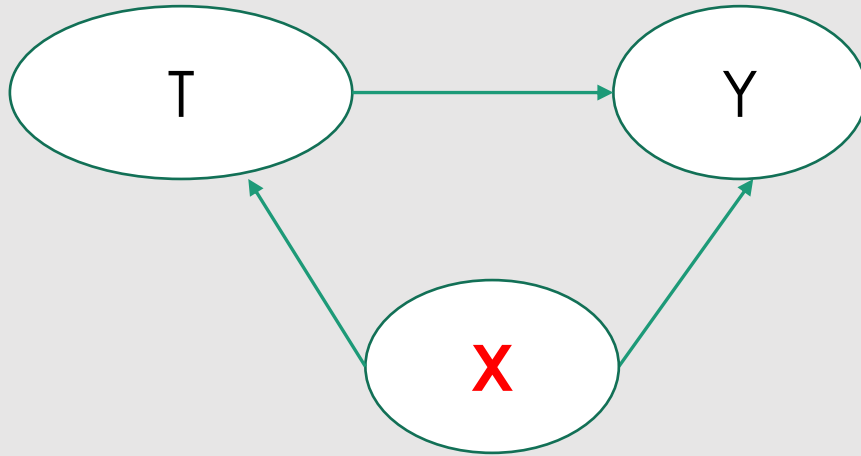


First: **Define relationships** between variables [...a priori knowledge]

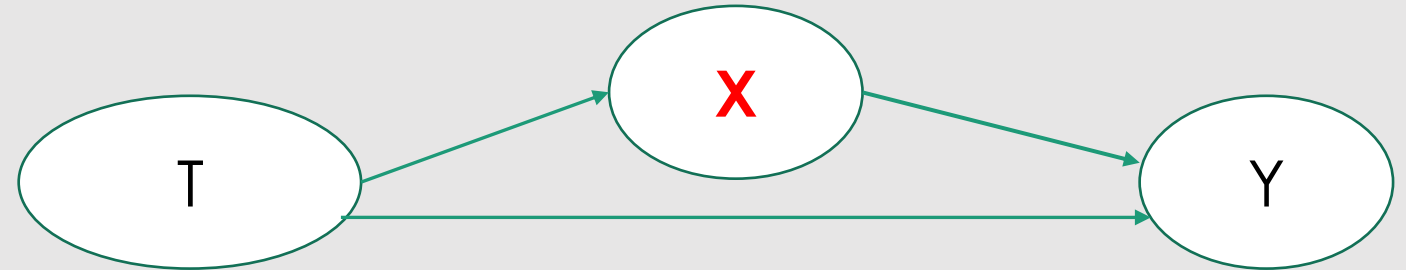
***Intermediate/Mediator vs Confounder: confounder IS NOT on the causal pathway between predictor and outcome**

Examples of different DAGs

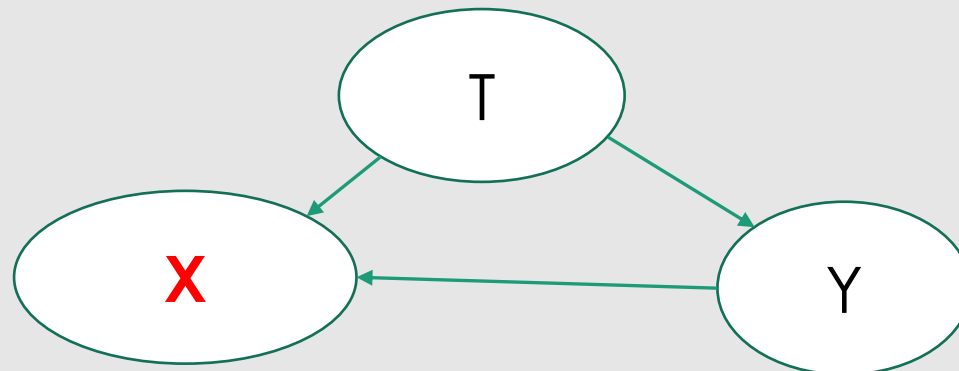
1. Confounder



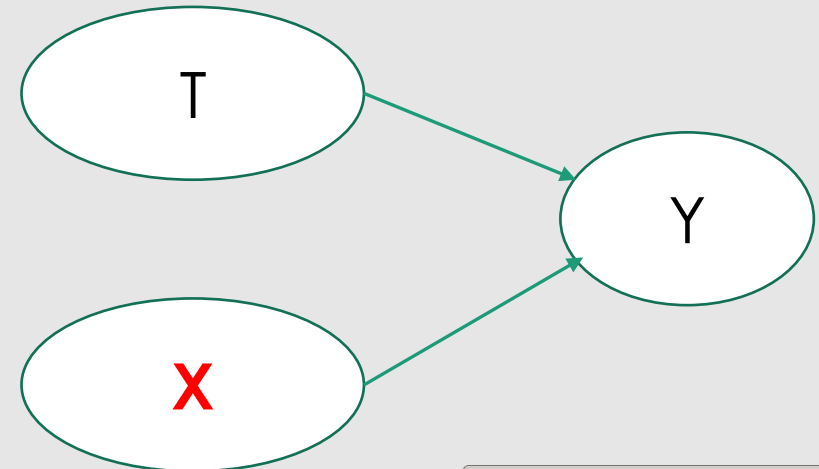
2. Mediator/Intermediate



3. Collider



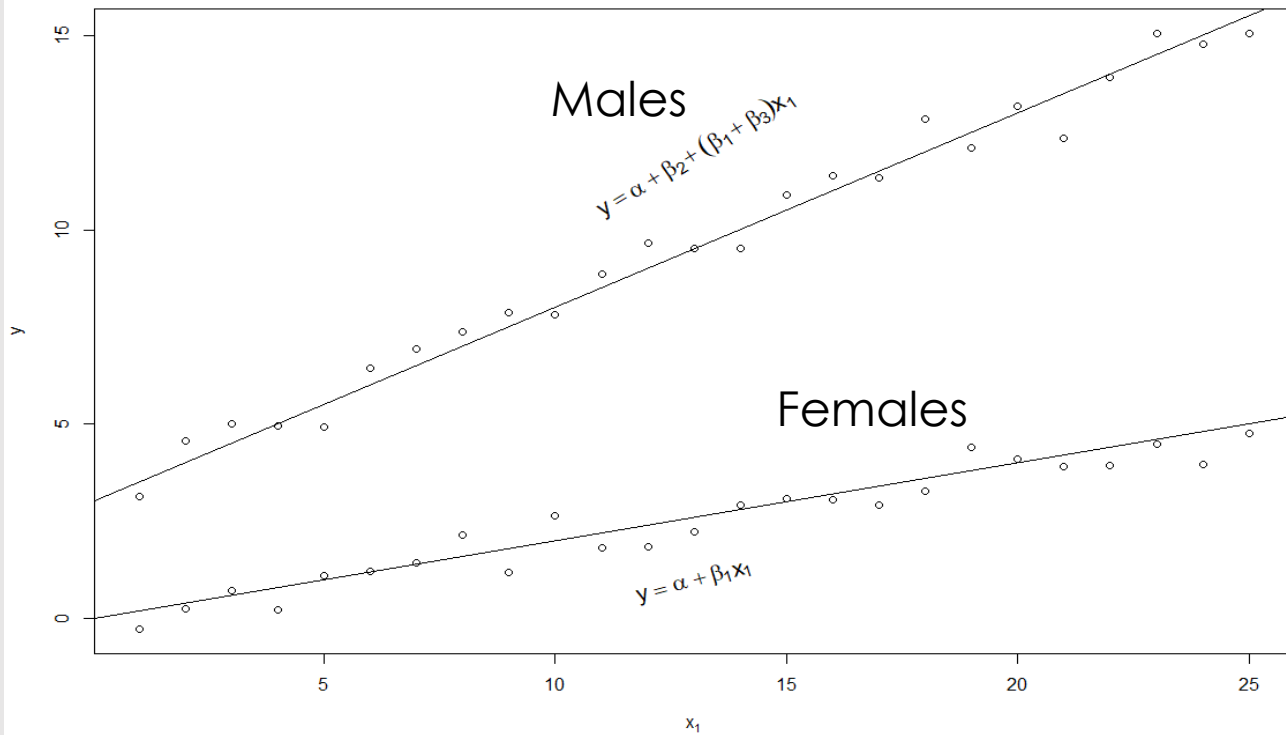
4. Independent Predictor



When a treatment has different effect in subgroups defined by X , there is **effect modification (moderators)** :

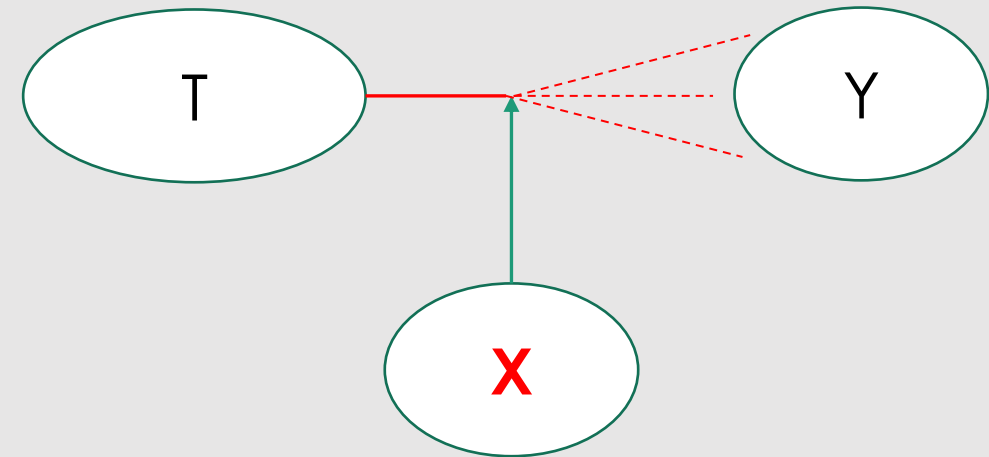
$$E(Y_1 - Y_0 | X = x_i) \neq E(Y_1 - Y_0 | X = x_j)$$

$$E(y|x) = \alpha + \beta_1 * \text{age} + \beta_2 * [\text{sex} = m] + \beta_3 * \text{age} * [\text{sex} = m]$$



! Could not be represented in a DAG !

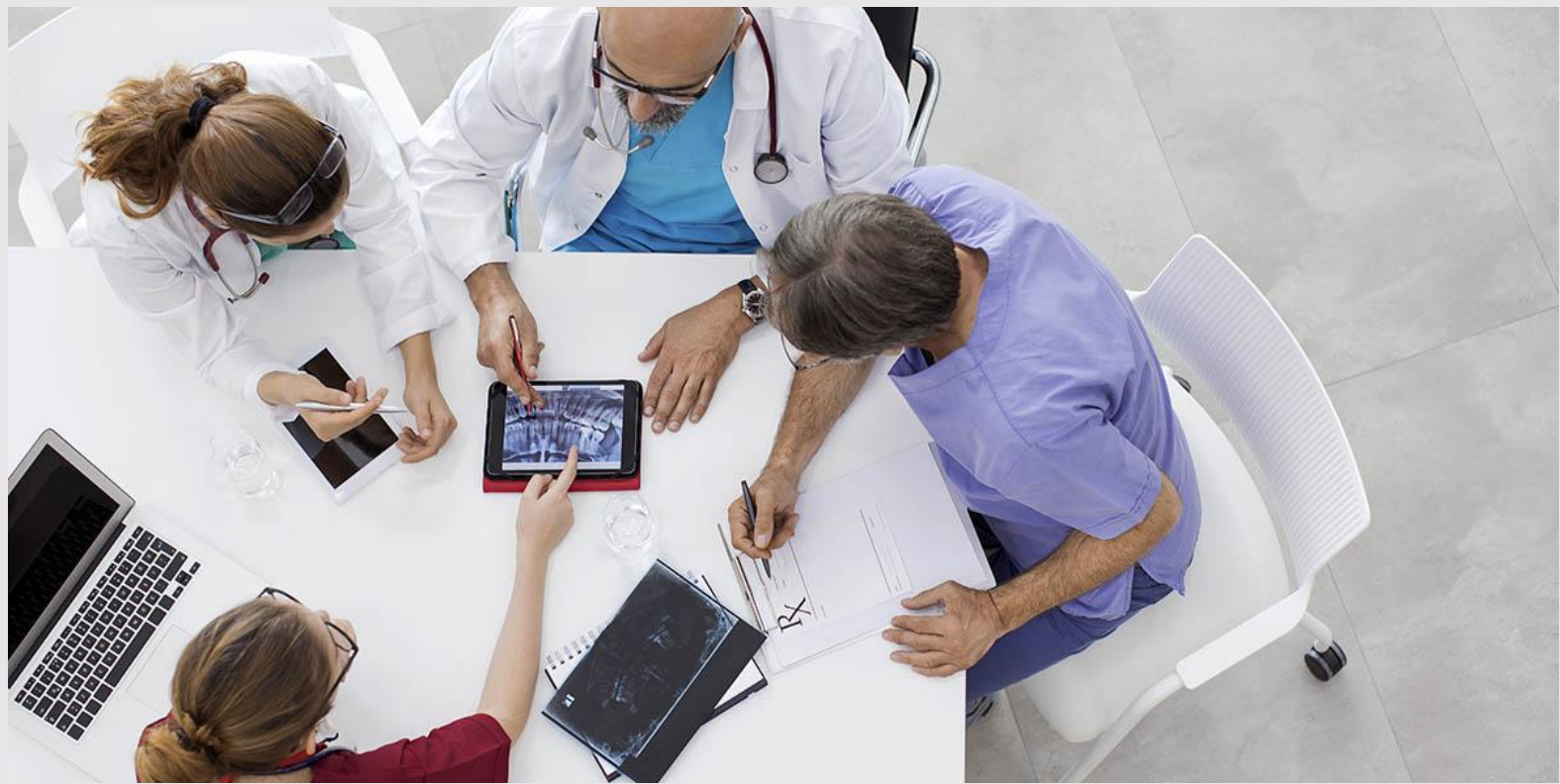
5. Effect modifier



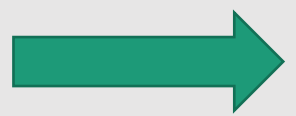
Building a DAG is a team-work...



Not this but....



This !



Basically there are **two types of effects that are of interest** in causal inference:

Average Causal Effect of a treatment or intervention on an outcome *across the entire population (or on the treated)* **ATE/ATT**

Causal effect of a treatment **within specific subgroups** or conditions defined by certain covariates (**CATE**)

MARGINAL EFFECT



- Standardization (g-formula)
- Matching
- IPTW (Inverse Probability of Treatment Weights)

CONDITIONAL EFFECT



- Stratification
- Regression coefficients

(Linear) Regression model for the outcome (no interaction)

Suppose a **true** model has a treatment T and a confounder x for outcome y :

$$y_i = \beta_0 + \beta_1 T_i + \beta_2 x_i + e_i$$

β_1 : conditional **and** marginal treatment effect (ATE = CATE)

In fact, if we want here to calculate the marginal effect we should **average (standardize)** over values of X:

$$\widehat{ATE} = \beta_0 + \beta_1 + \beta_2 E(X) - \beta_0 - \beta_2 E(X) = \beta_1$$

The conditional effect is equal to the marginal effect when there is **no interaction** between the treatment variable **and any other** covariate included in the model.

(Linear) Regression model if we ignore a confounder

Suppose a **true** model has a treatment T and a confounder x for outcome y :

$$y_i = \beta_0 + \beta_1 T_i + \beta_2 x_i + e_i$$

If x is related to the treatment, we could write:

$$x_i = \gamma_0 + \gamma_1 T_i + v_i$$

If we *ignore* the confounder x , we would fit the model:

$$y_i = \beta_0^* + \beta_1^* T_i + e_i^* \quad \beta_1^* : \text{estimated effect if } x \text{ is omitted}$$

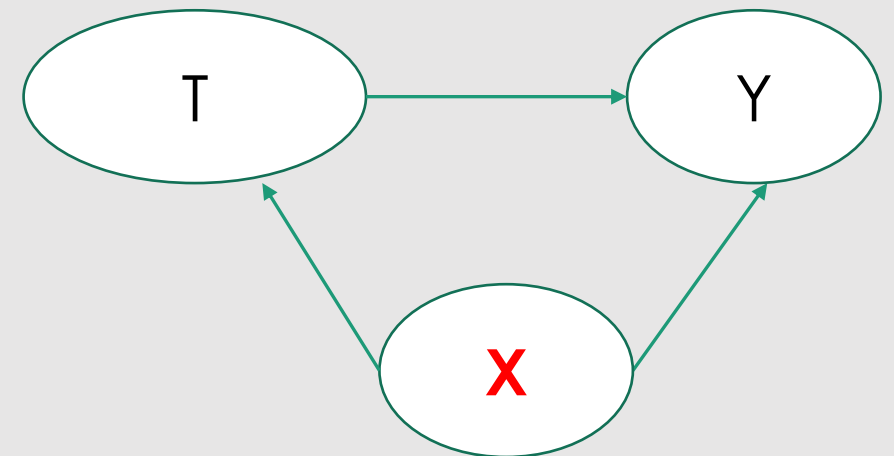
If we come back to the true model:

$$\begin{aligned} y_i &= \beta_0 + \beta_1 T_i + \beta_2 x_i + e_i = \beta_0 + \beta_1 T_i + \beta_2 (\gamma_0 + \gamma_1 T_i + v_i) + e_i \\ &= \beta_0 + \beta_2 \gamma_0 + (\beta_1 + \beta_2 \gamma_1) T_i + \beta_2 v_i + e_i \end{aligned}$$

➔ $\beta_1^* = \beta_1 + \beta_2 \gamma_1$

Estimation without x is correct only if $\beta_2 \gamma_1 = 0$

- $\beta_2 = 0$ x not associated with y
- $\gamma_1 = 0$ x not associated with T

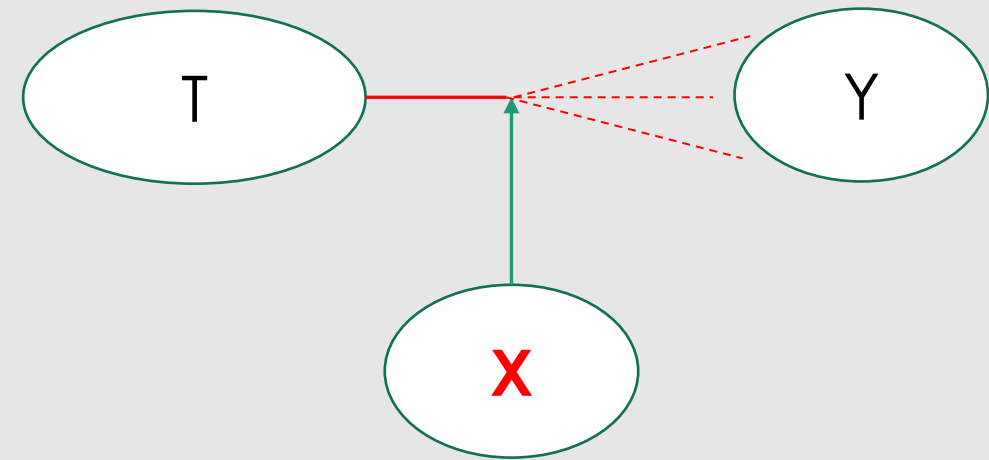


(Linear) Regression model for the outcome (with interaction)

Suppose a **true** model has a binary treatment T and a covariate (continuous) X that has an interaction with the treatment (is an **effect modifier**):

$$y_i = \beta_0 + \beta_1 T_i + \beta_2 x_i + \beta_3 x_i * T_i + e_i$$

$$\begin{aligned} \widehat{ATE} &= \frac{1}{N} \sum_{i=1}^N [\beta_0 + \beta_1 + \beta_2 x_i + \beta_3 x_i - \beta_0 - \beta_2 x_i] \\ &= \frac{1}{N} \sum_{i=1}^N [\beta_1 + \beta_3 x_i] = \beta_1 + \beta_3 E(X) \end{aligned}$$



Here the conditional effect **is not equal to** the marginal effect since there is an **interaction** between the treatment variable and a covariate included in the model.

* In **logistic regression** $\exp(\beta)$ could be only **CATE** (with/without interactions)

General basic rules

1. Confounder: include (adjust/stratify)

2. Mediator: exclude

(for *total effect* estimation, then there are more advanced topic: disentangle direct and indirect effects...)

3. Collider: **to be discussed with experts: in general exclude** (but depend on the path...)

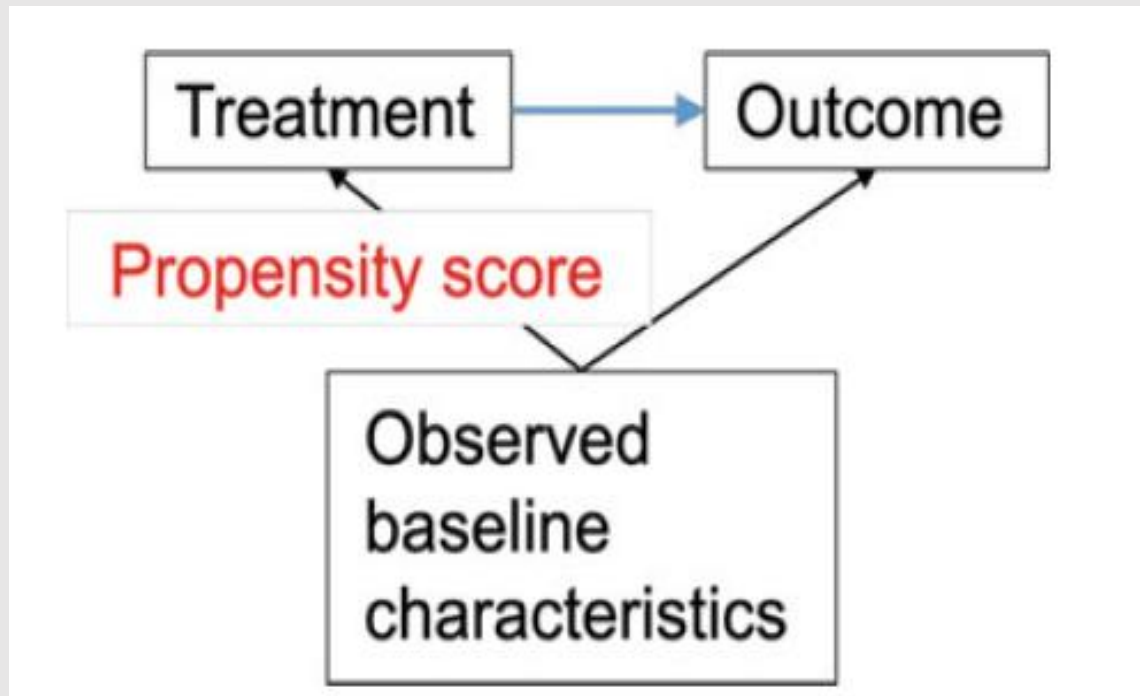
4. Independent predictor: include/exclude (in relation to sample size/precision)

5. Effect modifier: include (interaction)

Regression models are used to **estimate treatment effects** *trying to adjust* for **(measured) confounders** between treatment groups.

This approach relies on the underlying assumption that the **specified** model is correct. Another issue is that there is *no warning* if there is no **overlap** (**positivity violation**) between the treated and controls.

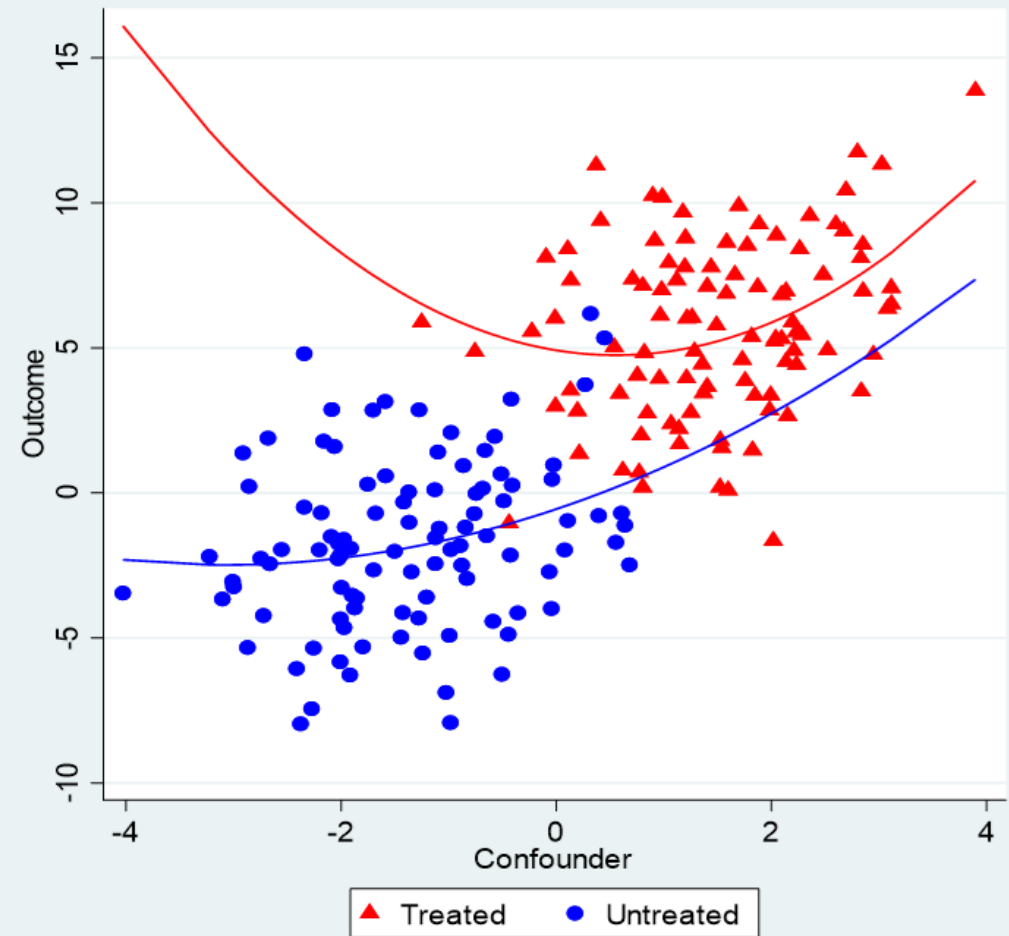
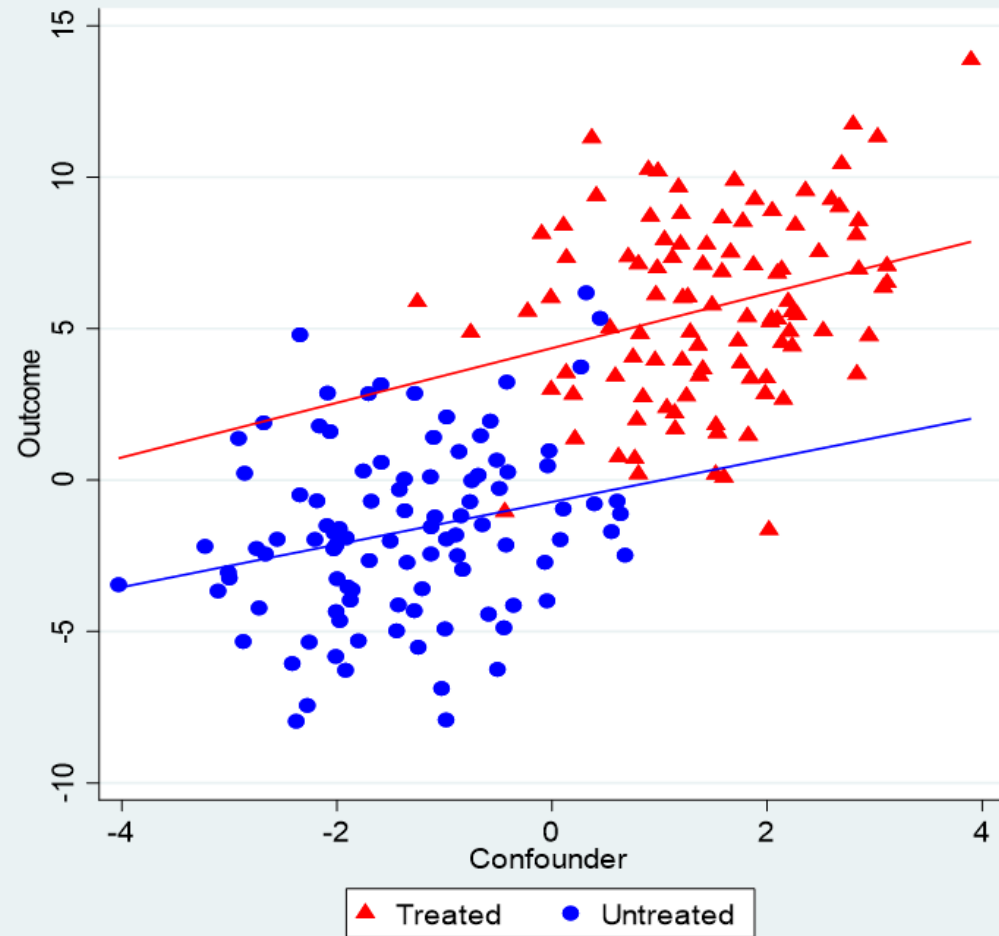
In other words, models could be estimated over regions with no or little data.



Adjustment with regression analysis is moreover **problematic** when the outcome is relatively **rare** or we have a **high-dimensional** confounders set.

An alternative is to use the **propensity score** which could be especially attractive in the setting of rare outcomes.

No overlap on the confounder range : dangerous extrapolation !



Propensity Score

The Propensity Score (**PS**) for each subject i (Rosembaum & Rubin, 1983) is defined as follows:

$$e_i = \Pr(T_i = 1 | \mathbf{X}_i)$$

The **PS** is a **balancing score**, $e = b(X)$ such that:

$$X \perp T | \mathbf{b}(X)$$

Treated ($T=1$) and control ($T=0$) subjects with the same propensity score $e(x)$ have the same distribution of the observed covariates X

T=0 | PS=p



Conditional Exchangeability

On average, within groups of individual **with the same propensity score**, we expect the subjects to be **comparable** between treatment groups

$$Y_t \perp T | X \Rightarrow Y_t \perp T | PS$$

T=1 | PS=p



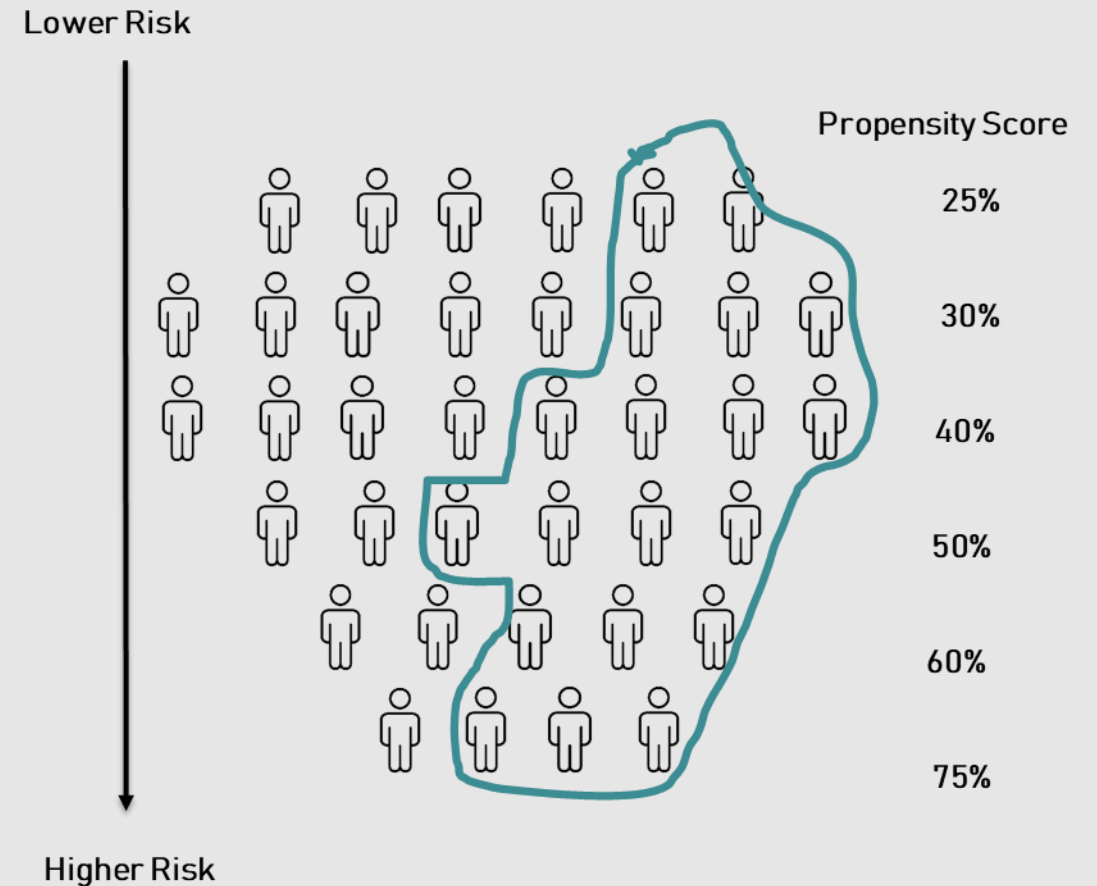
We **can estimate the propensity score** from the data:

e.g. using logistic regression:

$$\log\left(\frac{e_i}{1 - e_i}\right) = X_i\alpha$$

$$e_i = \frac{\exp(X_i\alpha)}{1 + \exp(X_i\alpha)}$$

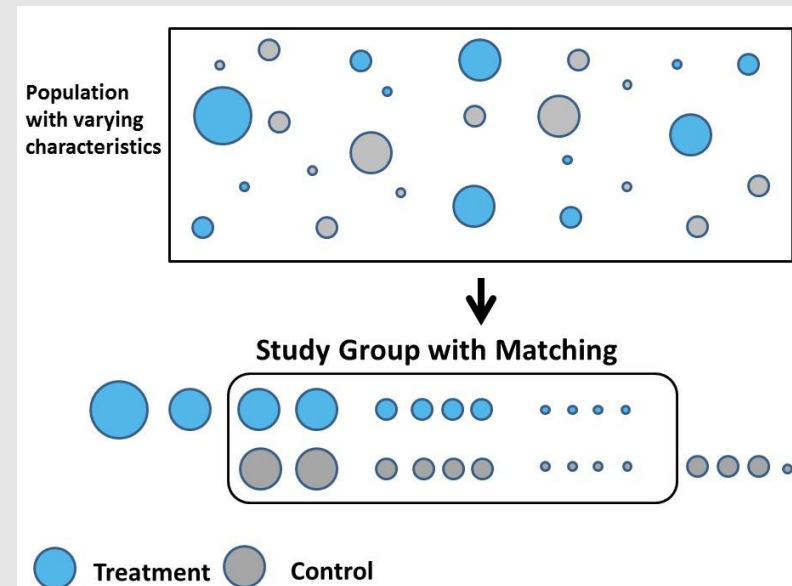
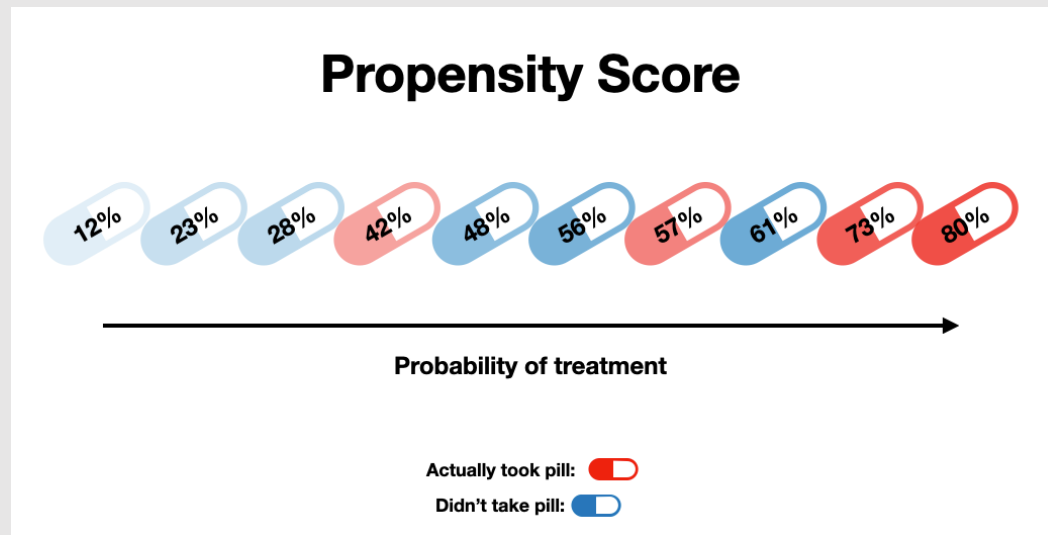
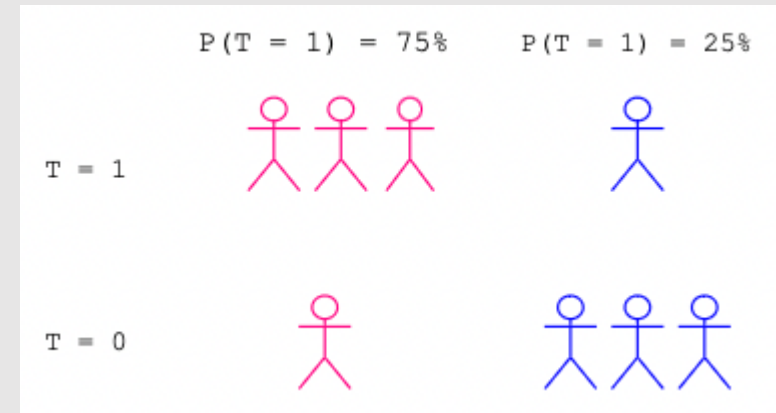
...Or using **machine learning** algorithms !
The important thing here is just to have a predicted probability to receive the treatment



Construction of the control group:

Four *main* approaches:

1. Covariate Adjustment
2. Matching
3. Stratification
4. Inverse **P**robability of **T**reatment **W**eighting (IPTW)



Covariate Adjustment: use PS as a covariate !

Method

$$Y_i = T_i\beta + f(\hat{e}_i)$$

$\hat{\beta}$ is the *average*/conditional* treatment effect (*linear regression)

- ✗ Assumption of *some* functional relationship between the outcome and the PS

✓ Simple, efficient ...

Diagnostic Tools

1. The distribution of the PS in the two groups **can help verifying the positivity assumption**

Matching

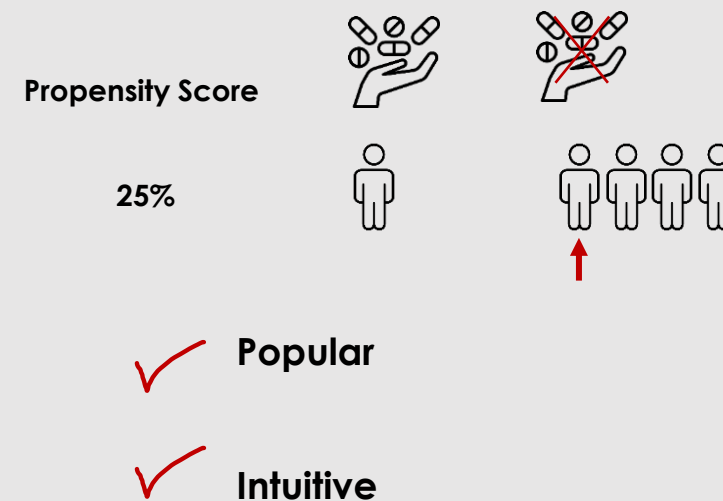
Method

Matched sets of **treated** with **untreated** subjects **who share a similar value of PS**

In the matched sample we can compare the outcome using **statistical methods** for **paired experiments**

Only the ATT effect could be estimated

- ✗ Many variants for the matching procedure
- ✗ It has been argued that appropriate SE are hard to obtain
- ✗ It discards a lot of data in a non-deterministic manner

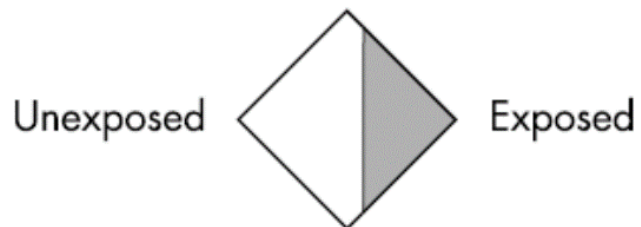


Diagnostic Tools

1. The distribution of the PS in the two groups **can help verifying the positivity assumption**
2. **Characteristics can be compared in the matched sample to assess achievement of balance**



Observed population

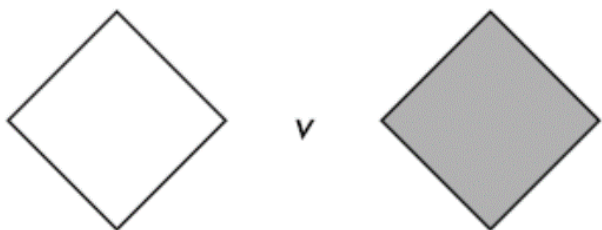


$$ITE_i = Y_1(i) - Y_0(i)$$

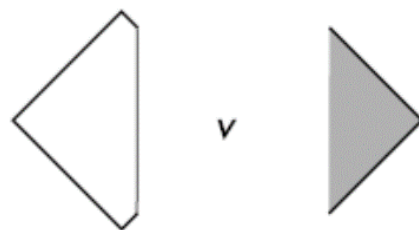
For all subjects

$$ATE = E[Y_1 - Y_0] = E[ITE_i]$$

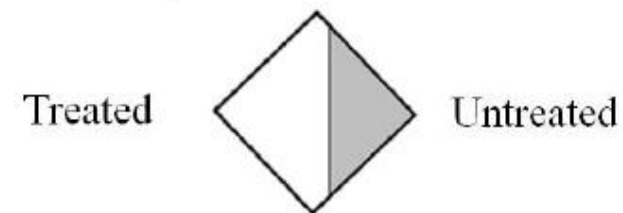
Causation



Association



Population of interest

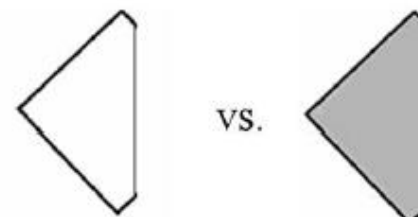


For treated subjects

$$ITE_i^{T=1} = Y_1(i) - Y_0(i)$$

$$ATT = E(Y_1 - Y_0 | T = 1)$$

Effect in the treated



$E[Y|A = 1]$

vs.



$E[Y^{a=0}|A = 1]$

Effect in the untreated



$E[Y^{a=1}|A = 0]$

vs.



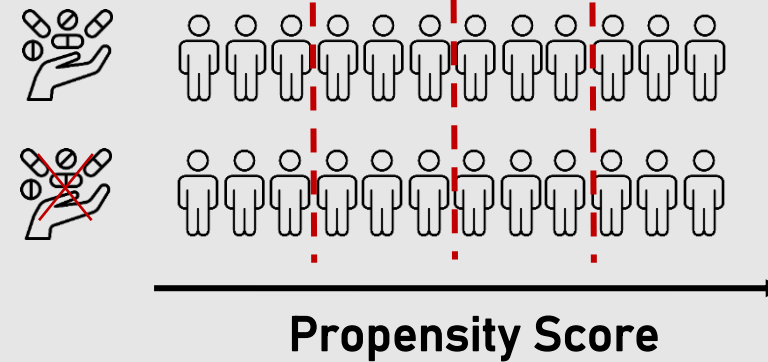
$E[Y|A = 0]$

Stratification

Method

We obtain exclusive subsets according to values of the propensity scores

In **each stratum we can** compare the outcome between the two treatment groups (**CATE**) and then eventually use a **weighted mean** to obtain the marginal treatment effect (**ATE**)

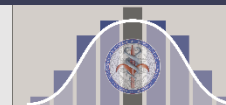


✓ Simple

- ✗ It can leave residual confounding
- ✗ We need to be careful to have a sufficient sample size per strata/outcomes

Diagnostic Tools

1. The distribution of the PS in the two groups **can help verifying the positivity assumption**
2. **Pre-treatment characteristics can be compared within strata**



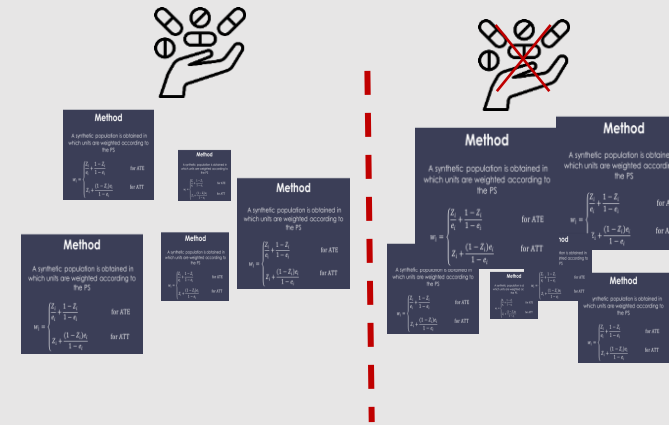
IPTW

Method

A **synthetic** population is obtained in which units are weighted according to the PS

$$w_i = \begin{cases} \frac{T_i}{e_i} + \frac{1 - T_i}{1 - e_i} & \text{for ATE} \\ T_i + \frac{(1 - T_i)e_i}{1 - e_i} & \text{for ATT} \end{cases}$$

~~X~~ **Extremes weights** → imprecise estimates



✓ **Mathematically appealing**
 ✓ **Extends to complex scenarios**

Diagnostic Tools

1. The distribution of the PS in the two groups **can help verifying the positivity assumption**
2. **Pre-treatment characteristics can be compared in the weighted dataset**

