

# Autonomous thermal motors

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# Transport in Brownian motors

*P. Reimann, Phys. Rep. (2002)*

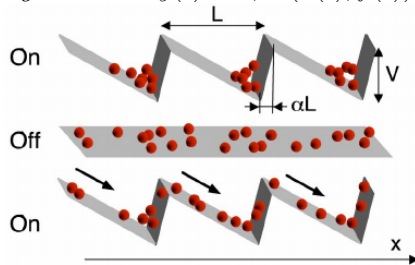
Emergence of directed transport in Brownian motors

- Spatially periodic systems
- Breaking of the spatial symmetry (ratchet potential)
- External time-dependent perturbation that drives the system out of equilibrium

# Brownian motors: Ratchets

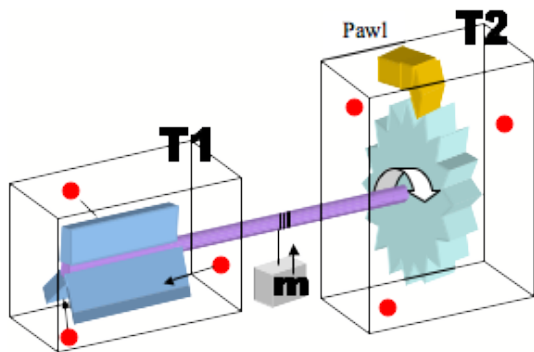
$$\dot{x}(t) = -V'(x(t), f(t)) + y(t) + \eta(t), \quad \langle \eta(t)\eta(t') \rangle = 2T\delta(t - t')$$

- $V(x)$  is a ratchet potential. Breaking of the spatial symmetry
- $f(t)$  and  $y(t)$  are either periodic or stochastic forces. Breaking of thermal equilibrium. See, e.g., [P. Reimann, Phys. Rep. \(2002\)](#)
- Example of *Tilting Ratchet*:  $f(t) = 0$  and  $y(t)$  is a correlated noise
- Example of *Pulsating Ratchet*:  $y(t) = 0$ ,  $V(x(t), f(t)) = V(x)[1 + f(t)]$



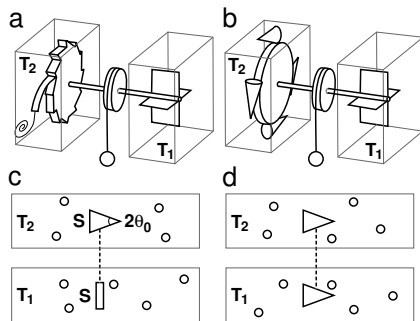
see, e.g., [Heiner Linke et al Chaos \(2005\)](#)

# Autonomous design



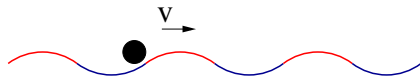
Autonomous motors: ideal design for engineering purposes  
They can operate in steady-state conditions without any external time-dependent drive

# Hard disk Brownian motor



*C. Van den Broeck, R. Kawai, and P. Meurs PRL 2004*

# Büttiker-Landauer model



*M. Büttiker, Zeit. f. Phys. B (1987); R. Landauer, J. Stat. Phys. (1988)*

- A Brownian particle moving in a  $L$ -periodic force field  $h(x)$ , with zero average  
a  $L$ -periodic temperature profile  $T(x)$

- Define

$$U(x) = - \int_0^x dy h(y)/T(y)$$

- The particle exhibits non-vanishing velocity if  $U(x) \neq U(x + L)$   
*M. Matsuo and S. ichi Sasa, Physica A (2000)*
- Ambiguity in the definition of the heat current in the overdamped regime  
see, e.g., *T. Hondou and K. Sekimoto, PRE (2000)*

# Minimal model

Minimal model for a microscopic autonomous thermal motor ?

- General features
  - breaking of the spatial inversion symmetry
  - breaking of thermal equilibrium

- Our claim:

Two interacting Brownian particles at different temperatures, diffusing in shifted periodic potentials

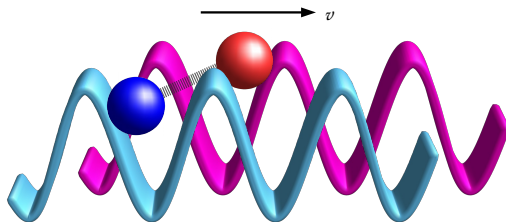
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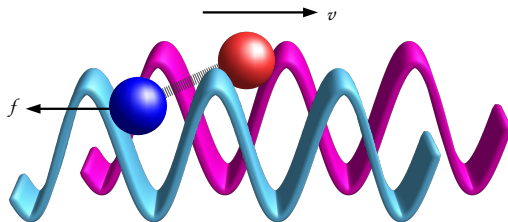
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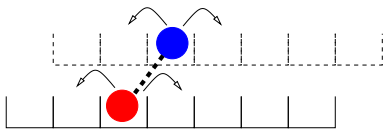
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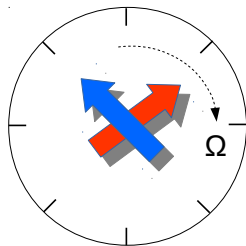
## A discrete model

System with a discrete phase space

$$U(i_1, i_2) = \frac{k}{2} \cos [(i_1 - i_2)2\pi/N_s + \varphi]$$

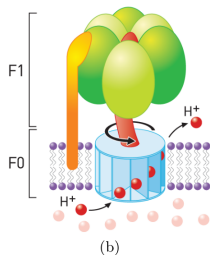
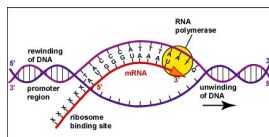
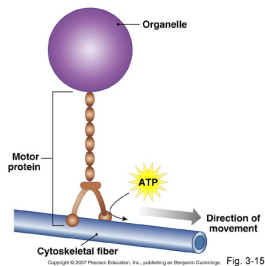


Two interacting particles moving on a lattice



Discrete xy-model  
(clock model with  $N_s$  states)

# Motors and rotors of biological interest



- Molecular motors are protein machines that convert chemical energy into useful work
- Example: Kinesin moves cargo inside cells along microtubules
- Example: RNA polymerase, transcribes DNA sequences into mRNA
- Example: ATP-synthase. The motor is driven by a proton gradient across the membrane.