

**CORSO CHIMICA ANALITICA I CON LABORATORIO**  
**A.A. 2025-26**

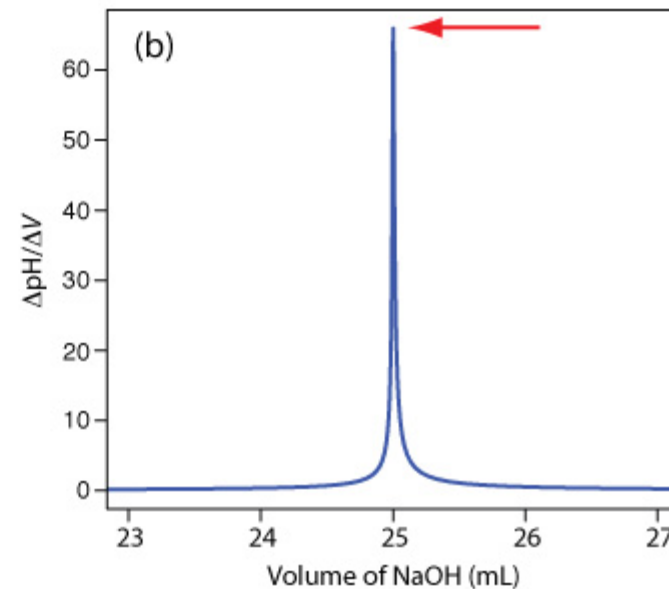
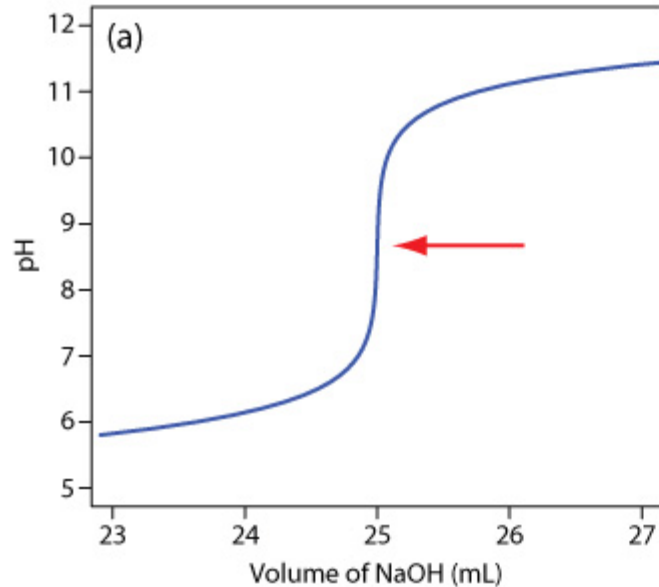
**Lezione per**  
**laboratorio informatico**

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# Determinazione del punto di equivalenza con derivata prima



Per ottenere il valore del volume al punto equivalente bisogna individuare il punto di flesso, un modo conveniente è usare la derivata prima, approssimabile a:

$$\frac{\Delta\text{pH}}{\Delta\text{V}}$$

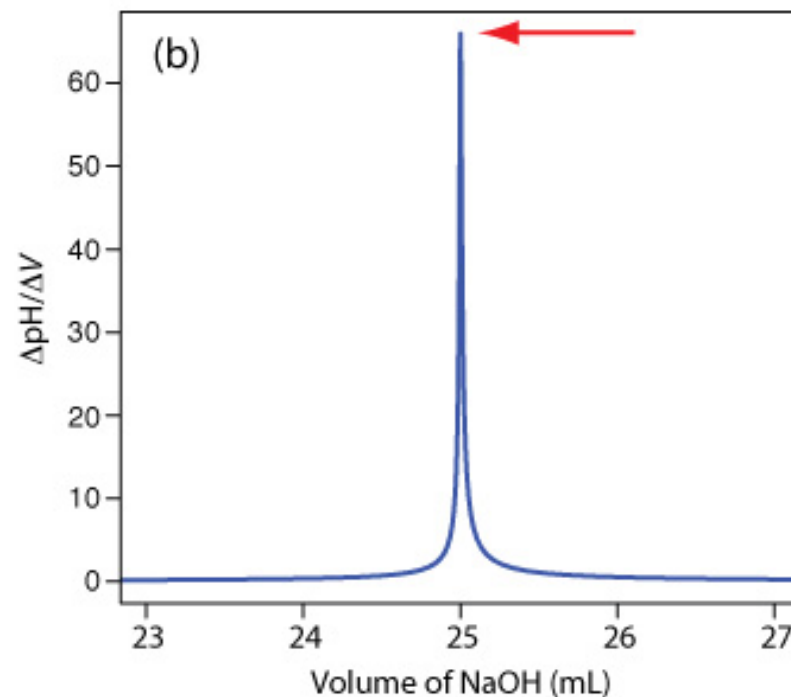
con  $\Delta$  si intende la variazione tra due misurazioni successive

Il volume corrispondente al valore più alto della derivata prima è il volume equivalente.

# Determinazione del punto di equivalenza con derivata prima

Costruzione del grafico:

X	Y
media ( $V_{n+1}$ , $V_n$ )	$\frac{\text{pH}_{n+1} - \text{pH}_n}{V_{n+1} - V_n}$



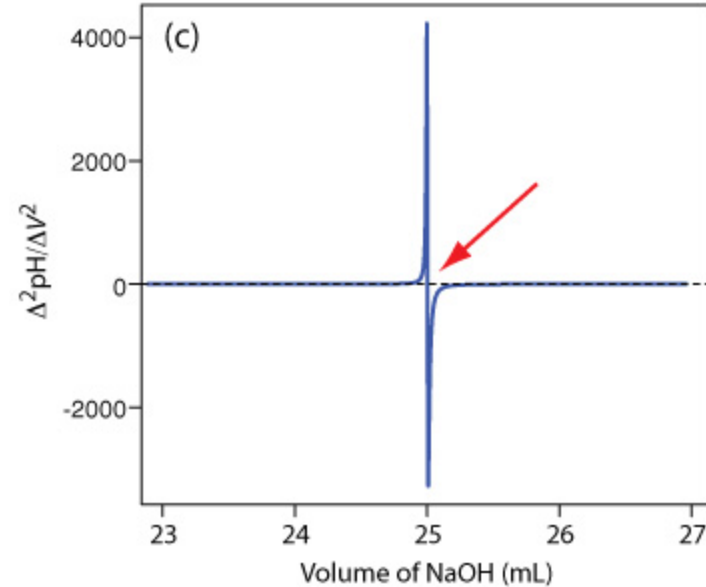
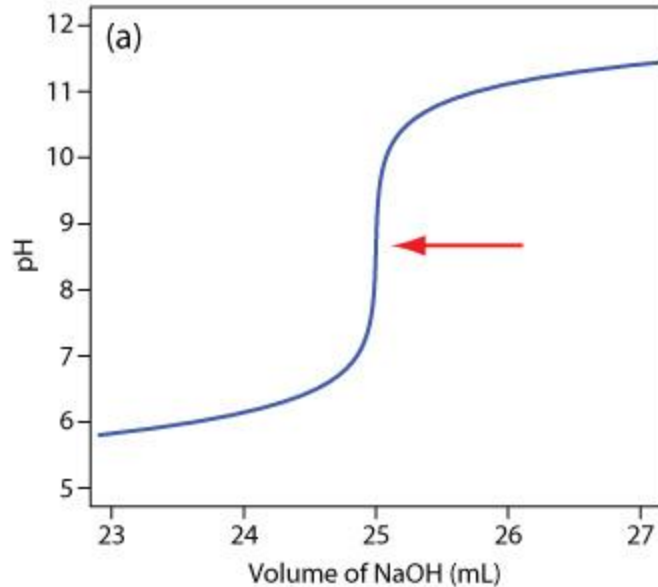
volume (mL)	pH
23.65	6.00
23.91	6.10
24.13	6.20

Mathematically, we can approximate the first derivative as  $\Delta\text{pH}/\Delta V$ , where  $\Delta\text{pH}$  is the change in pH between successive additions of titrant. Using the first two points, the first derivative is

$$\frac{\Delta\text{pH}}{\Delta V} = \frac{6.10 - 6.00}{23.91 - 23.65} = 0.385$$

which we assign to the average of the two volumes, or 23.78 mL. For the second and third points, the first derivative is 0.455 and the average volume is 24.02 mL.

# Determinazione del punto di equivalenza con derivata seconda



Un altro modo conveniente è usare la derivata seconda, approssimabile a:

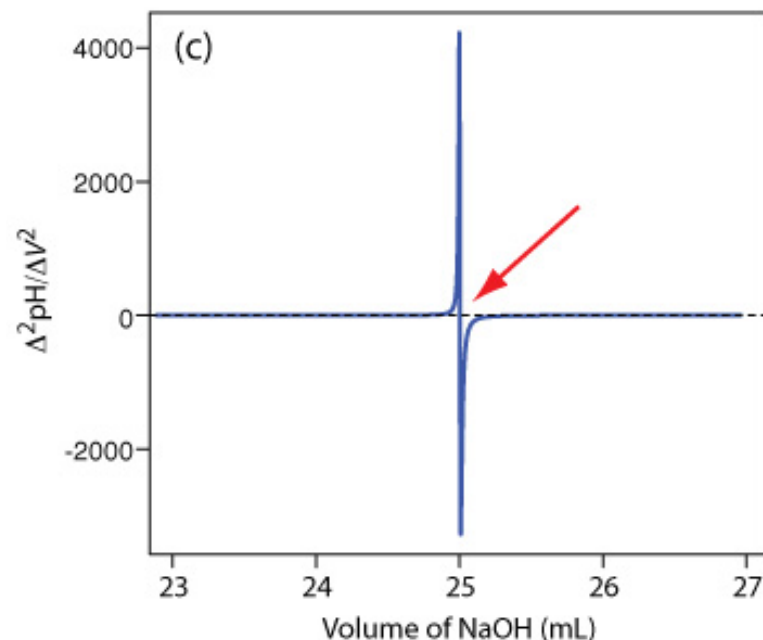
$$\frac{\Delta}{\Delta V} \left( \frac{\Delta \text{pH}}{\Delta V} \right) = \frac{\Delta^2 \text{pH}}{\Delta V^2}$$

Il volume corrispondente al valore a cui la funzione interseca l'asse x è il volume equivalente.

# Determinazione del punto di equivalenza con derivata seconda

Costruzione del grafico:

$$\begin{array}{ccc} X & & Y \\ \text{media } (\Delta V_{n+3}, \Delta V_{n+2}) & & \frac{\Delta \text{pH}_{n+3}}{\Delta V_{n+3}} - \frac{\Delta \text{pH}_{n+2}}{\Delta V_{n+2}} \end{array}$$



volume (mL)	$\Delta \text{pH} / \Delta V$
23.78	0.385
24.02	0.455

We can approximate the second derivative as  $\Delta(\Delta \text{pH} / \Delta V) / \Delta V$ , or  $\Delta^2 \text{pH} / \Delta V^2$ . Using the two points from our calculation of the first derivative, the second derivative is

$$\frac{\Delta^2 \text{pH}}{\Delta V^2} = \frac{0.455 - 0.385}{24.02 - 23.78} = 0.292$$

which we assign to the average of the two volumes, or 23.90 mL. Note that calculating the first derivative comes at the expense of losing one piece of information (three points become two points), and calculating the second derivative comes at the expense of losing two pieces of information.