Lezione 5 maggio 2026

martedì 5 maggio 2026 11:13

- CONICHE EUCLIDEE
- CON. AFFINI
- CONICHE PROIETTIVE

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Prove scritte : $\boxed{28/5/26}$ 11:30 – 13:30

diesonero allo scritto

orali: 4/6/26 9:30 orali

10/6 1° scritto

12/6 orali

Prove scritte: vale fino 30/9/26

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CONICHE EUCLIDEE :

Come luoghi geometrici :

in $\mathbb{E}^2_{\mathbb{R}}$ SE standard, PS standard

Def. : ① $\underbrace{s}_{\text{DIRETTRICE}}$ retta $\subseteq \mathbb{E}^2_{\mathbb{R}}$, $\underbrace{F}_{\text{FUOCO}} \in \mathbb{E}^2_{\mathbb{R}}$

punto

$$C := \{ P \in \mathbb{E}^2_{\mathbb{R}} : d(P,F) = d(P,s) \}$$

PARABOLA

② $\underbrace{F_1}, \underbrace{F_2} \in \mathbb{E}^2_{\mathbb{R}}$

FUOCHI

$$C := \{ P \in \mathbb{E}^2_{\mathbb{R}} : d(P,F_1) + d(P,F_2) = \underset{k>0}{\overset{\text{costante}}{k}} \in \mathbb{R} \}$$

ELLISSE

③ $\underbrace{F_1}, \underbrace{F_2} \in \mathbb{E}^2_{\mathbb{R}}$

FUOCHI

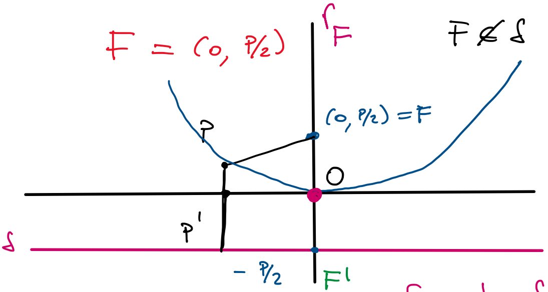
$$C := \{ P \in E^2_{\mathbb{R}} : d(P,F_1) - d(P,F_2) = k \in \mathbb{R} \underset{\text{costante}}{} \}$$

IPERBOLE

FISSO SR CARTESIANO
IN MODO OPPORTUNO
E SCRIVIAMO EQUAZIONI

① $\delta: \; y = -p/2$, $p \in \mathbb{R}$, $p > 0$

$F = (0, p/2)$



Se $P = (x, y)$

$P' = (x, -p/2)$

$r_F \perp \delta$, $r_F \ni F$: ASSE

Esercizio: se abbiamo SR cartesiano arbitrario (O' origine) e $F \notin \delta$, con una traslazione posso portare O' in O e con rotazione porto i 2 assi cartesiani negli assi fissati. (uno parallelo a δ, l'altro ortogonale)

O: punto medio sul segmento FF': VERTICE

$$C : \{ (x,y) : d(P,F) = d(P,\delta) = d(P,P') \}$$

$$\Longleftrightarrow \; \|\overrightarrow{P'P}\|^2 = \|\overrightarrow{FP}\|^2 \qquad \overrightarrow{P'P} = \begin{pmatrix} x - x \\ y - (-p/2) \end{pmatrix}$$

$$\boxed{0^2 + (y + p/2)^2 = x^2 + (y - p/2)^2} \Longleftrightarrow \boxed{x^2 = 2py}$$

$$\vec{FP} = \begin{pmatrix} x - 0 \\ y - p/2 \end{pmatrix}$$

$$y = \boxed{\frac{1}{2p}} x^2$$

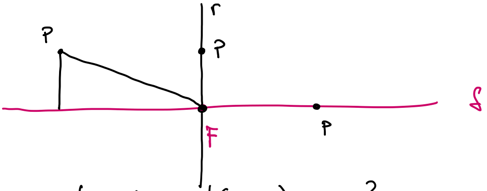
$$F = (0, p/2)$$

Esercizio: posso anche δ: $x = -\frac{p}{2}$

$F = (p/2, 0)$

trovo $y^2 = 2px$

Esercizio: Cosa succede se $F \in \delta$?



$d(P,F) = d(P,\delta)$?

è $r \perp \delta$, e $r \ni F$

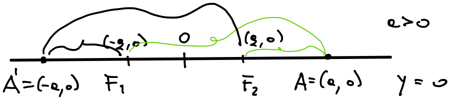
② Fisso SR cartesiano in cui

$F_1 = (-c, 0)$ $F_2 = (c, 0)$

$c \geq 0$; $k = d(P,F_1) + d(P,F_2) > 2c$

$P = (x,y)$

Conviene studiare i punti di int. con essi $x = 0$ e $y = 0$

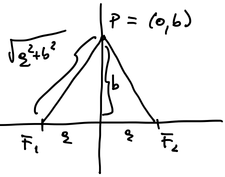


$d(A,F_1) + d(A,F_2) = +c + a + a - c$

$= 2a = k$

Cons. $(-a,0) = a - c + a + c = 2a = k$

Int. con $x = 0$



$2\sqrt{c^2+b^2} = k$, $k = 2a$

$4(c^2+b^2) = k^2 = 4a^2$

$\Rightarrow c^2 = a^2 - b^2$; $F_1 = (-c,0)$

$F_2 = (c,0)$

$$d(P,F_1) = \|\overrightarrow{F_1P}\| = \left\|\begin{pmatrix} x-(-c) \\ y-0 \end{pmatrix}\right\|$$

$P = (x,y)$

$$d(P,F_2) = \|\overrightarrow{F_2P}\| = \left\|\begin{pmatrix} x-c \\ y-0 \end{pmatrix}\right\|$$

$$(d(P,F_1) + d(P,F_2))^2 = k^2$$

$$(d(P,F_1))^2 + 2\,d(P,F_1)\,d(P,F_2) + d(P,F_2)^2 = 4a^2$$

$$(x+c)^2 + y^2 + 2\sqrt{((x+c)^2+y^2)((x-c)^2+y^2)} + (x-c)^2 + y^2 = 4a^2$$

. . . . .

$$\underbrace{\sqrt{((x+c)^2+y^2)((x-c)^2+y^2)}}_{} = 2a^2 - (x^2+y^2+c^2)$$

svolgo e semplifico;

$$c^2x^2 = a^4 - a^2(x^2+y^2+c^2)$$

$c^2 = a^2 - b^2$; sostituisco

$$\Rightarrow b^2x^2 + a^2y^2 = a^2b^2 \quad /: a^2b^2$$

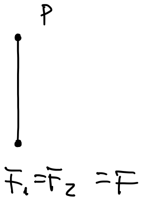
$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$$

Se $a = b$ $\Leftrightarrow$ $c^2 = a^2 - b^2 = 0$

$\Leftrightarrow$ $F_1 = (0,0) = F_2 = F$

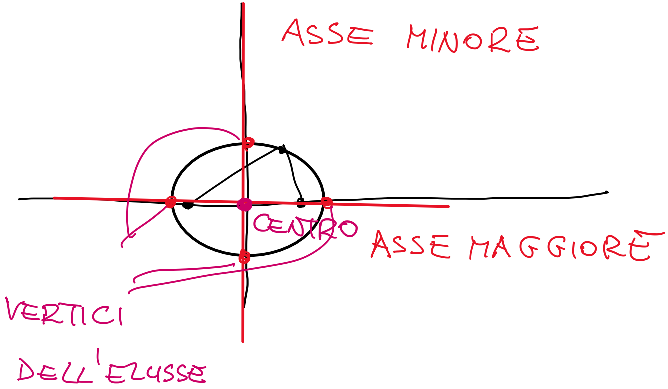
in questo caso l'ellisse si dice

CIRCONFERENZA DI CENTRO F



$d(P, F_1) + d(P, F_2) = 2d(P,F) = k$

$\Leftrightarrow$ $d(P,F) = \dfrac{k}{2}$

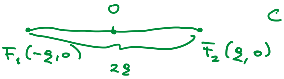


Esercizio: cosa succede se $k \leq 2g$?

$C =$ segmento

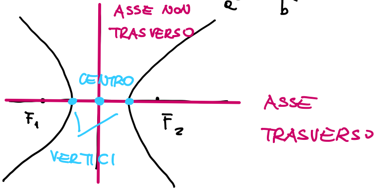
$k < 2g$?

$C = \phi$



③ $F_1 = (-g,0)$, $F_2 = (g,0)$

analog. : trovo $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$



Prop. ① C parabola

r_F è asse di simmetria

② ③ : 2 assi sono entrambi assi di simm., e il centro è centro di simmetria

Def: Ellissi e iperboli si chiamano CONICHE A CENTRO.

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Domanda: come sono caratt. le coniche in un SR cartesiano arbitrario?

Da SR Cart. speciale a SR Cart. arbitrario passo con affinità:

$$\underbrace{\begin{pmatrix}X\\Y\end{pmatrix} = A\cdot\begin{pmatrix}x\\y\end{pmatrix} + C}_{C=\begin{pmatrix}c_1\\c_2\end{pmatrix},\quad A\,\in GL(2,\mathbb{R})} \Rightarrow \begin{pmatrix}x\\y\end{pmatrix} = A^{-1}\left(\begin{pmatrix}X\\Y\end{pmatrix}-C\right) \overset{!}{=} A^{-1}\cdot\begin{pmatrix}X\\Y\end{pmatrix} - A^{-1}\cdot C$$

Come cambia eq. parabola?

$$x^2 = 2py \quad , \quad x^2 - 2py = 0$$

come scrivo questo stesso luogo in funzione di $\begin{pmatrix}X\\Y\end{pmatrix}$?

$$\begin{pmatrix}x\\y\end{pmatrix} = A^{-1}\cdot\begin{pmatrix}X\\Y\end{pmatrix} - A^{-1}\cdot C$$

$$A^{-1} = \begin{pmatrix}\alpha_{11} & \alpha_{12}\\ \alpha_{21} & \alpha_{22}\end{pmatrix} \quad , -A^{-1}\cdot C = \begin{pmatrix}\beta_1\\ \beta_2\end{pmatrix}$$

$$\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}\alpha_{11}X + \alpha_{12}Y + \beta_1\\ \alpha_{21}X + \alpha_{22}Y + \beta_2\end{pmatrix}$$

$$\Rightarrow \quad x^2 - 2py = 0 \iff (\alpha_{11}X + \alpha_{12}Y + \beta_1)^2 - 2p(\alpha_{21}X + \alpha_{22}Y + \beta_2) = 0$$

CONICHE GENERALI

Def. Conica affine è il luogo degli zeri di

$$(*) \quad \underline{a_{11}}x^2 + \underline{2a_{12}}xy + \underline{a_{22}}y^2 + \underline{2a_{13}}x + \underline{2a_{23}}y + \underline{a_{33}} = 0$$

OSS.: Se pongo

$$B := \begin{pmatrix}a_{11} & a_{12} & a_{13}\\ a_{12} & a_{22} & a_{23}\\ a_{13} & a_{23} & a_{33}\end{pmatrix}$$

$$(*) \iff (x \;\; y \;\; 1).B.\begin{pmatrix}x\\y\\1\end{pmatrix} = 0$$