Lezione 30 aprile 2026

mercoledì 29 aprile 2026 21:39

ALCUNE APPLIC. delle Geom. proiett.

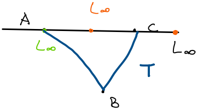
In $\mathbb{P}^2_{\mathbb{R}}$

TRIANGOLI PROIETTIVI

dati 3 punti di $\mathbb{P}^2_{\mathbb{R}}$, NON allineati: A, B, C

e i 3 segmenti

$c = AB$, $b = AC$, $a = BC$

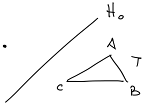


$L(A,C) \cong \mathbb{P}^1_{\mathbb{R}}$

L_∞ punto improprio

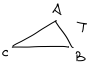
$\mathbb{P}^1_{\mathbb{R}} \setminus \{L_\infty\} \cong \mathbb{A}^1_{\mathbb{R}}$

Esaminiamo i casi di posiz. tra T e $x_0 = 0$: H_0

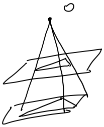


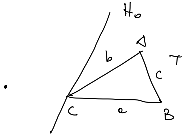
In:

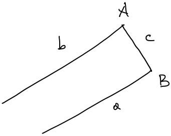
$\mathbb{P}^2_{\mathbb{R}} \setminus H_0 \cong \mathbb{A}^2_{\mathbb{R}}$

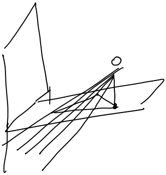


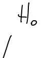
in $\mathbb{A}^2_{\mathbb{R}}$

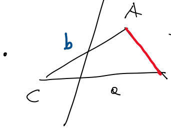


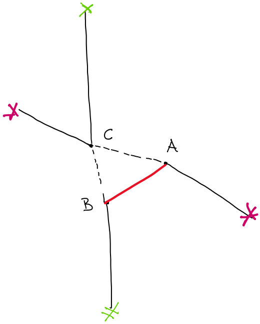




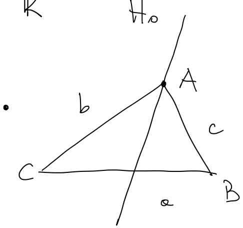




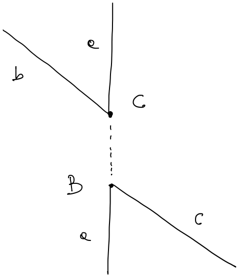




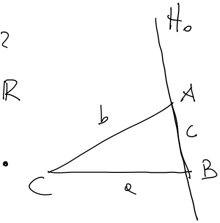
$\mathbb{P}^2_{\mathbb{R}}$



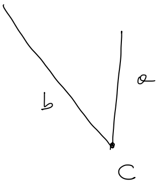
$\mathbb{A}^2_{\mathbb{R}}$

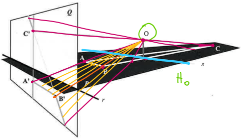


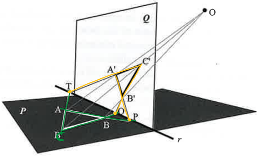
$\mathbb{P}^2_{\mathbb{R}}$



$\mathbb{A}^2_{\mathbb{R}}$







A, B, C   A', B', C'

Def.: Due triangoli piani nello spazio affine si dicono

OMOLOGHI se è possibile associare i vertici di uno a quelli dell'altro

A → A'
B → B'   in modo che $AA' \cap BB' \cap CC' = \{O\}$
C → C'

Teorema di Desargues: 2 triangoli sono omologhi se e solo se i 3 punti di intersezione delle rette corrispondenti a lati si incontrano in 3 punti allineati

Dim.: $\mathbb{A}^3_{\mathbb{R}} \subseteq \mathbb{P}^3_{\mathbb{R}}$; $O = L(A,A') \cap L(B,B') \cap L(C,C')$

A, B, C   A', B', C'

$[v_A], [v_B], [v_C]$ $[v_{A'}], [v_{B'}], [v_{C'}]$ $\Downarrow$

$O = [v_O]$

$$v_0 = \alpha_A v_A + \alpha_{A'} v_{A'}$$

$$= \alpha_B v_B + \alpha_{B'} v_{B'}$$

$$= \alpha_C v_C + \alpha_{C'} v_{C'}$$

$$[\underbrace{\alpha_A v_A - \alpha_C v_C}_{v_1}] = [-\alpha_{A'} v_{A'} + \alpha_{C'} v_{C'}] \in L(A,C) \cap L(A',C')$$

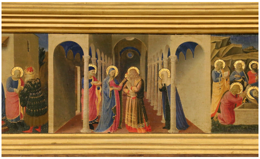
$$[\underbrace{-\alpha_B v_B + \alpha_C v_C}_{v_2}] = [\alpha_{B'} v_{B'} - \alpha_{C'} v_{C'}] \in L(B,C) \cap L(B',C')$$

$$[\underbrace{-\alpha_A v_A + \alpha_B v_B}_{v_3}] = [\alpha_{A'} v_{A'} - \alpha_{B'} v_{B'}] \in L(A',B') \cap L(A,B)$$

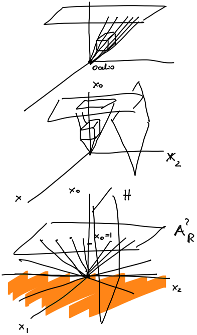
Infine $v_1 + v_2 + v_3 = 0$

$\Rightarrow$ sono LIN DIP $\Rightarrow$ $[v_1], [v_2], [v_3] \in \mathbb{P}^3_{\mathbb{R}}$ sono 3 punti allineati

Beato angelico, annunciazione di cortona, 1434-36, da s. domenico, predella 06 presentazione al tempio



La prospettiva, ovvero rappr. ad esempio il pavimento del quadro, equivale a fare una opportuna proiettività del pavimento reale a quello della tela

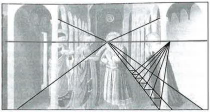
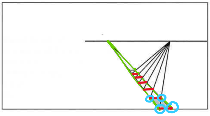


se H ha eq. $ax_1+bx_2+cx_0=d$

è un piano affine la cui retta all'infinito

è $ax_1+bx_2+cx_0=0$

APPLIC. del Teo di Struttura per proiettività

$$\mathbb{P}^2_{\mathbb{R}} \to \mathbb{P}^2_{\mathbb{R}}$$