Program of the course

DIFFERENTIAL GEOMETRY I

Academic Year 2025-2026 – Fabio Vlacci

Preliminaries Surfaces in $\mathbb{R}^3$: compatibility equations.
Local version of Gauss–Bonnet Theorem.
Triangulation of surfaces and global version of Gauss–Bonnet Theorem with applications.

Introduction to differentiable manifolds as generalizations of surfaces: differentiable structures, local coordinates, charts, atlases, orientability and partition of unity.
Differentiable mappings between differentiable manifolds.
Immersions and embeddings.
Vector fields and Tangent bundle. Lie brackets.
Jacobi identity. Lie groups and Lie algebras.
Riemannian metrics on a differentiable manifold: definition and existence.
Length of a curve on a differentiable manifold. Existence and examples of Riemannian metrics.
Isometries.

Intrinsic derivativation on a differentiable manifold

Linear connections. Parallel transport and covariant derivative.
Torsion and symmetric connections. Levi–Civita connection associated with a Riemannian manifold.
Geodesics and the corresponding differentiable equations.
Gauss Lemma and the esponential map. Local caracterization of geodesics as curves minimizing the distance between two points.

Tensors on manifolds

Introduction to the notion of curvature of a differentiable manifold: definition, main properties and Bianchi identity.
Introduction to Jacobi fields: definition, main properties and geometric interpretation as rate of spreding of geodesics.
Conjugate points of Jacobi fields and their main properties.
Ricci and sectional curvatures: definition, main properties and

Schur Lemma (only statement).
(Covariant) tensors on a differentiable manifold.
Isometric immersions. Fundamental equation: Gauss, Ricci and Codazzi equations.
Complete manifolds. Hopf–Rinow Theorem.
Covering spaces. Hadamard Theorem and applications.
Short overview on Bonnet Myers Theorem, on Synge-Weinstein Theorem, on Cartan Theorem, on Comparison Theorems (Rauch Theorem), on Space Forms (Hopf-Killing Theorem), on Riemann Surfaces and on Minimal Surfaces

Main references

Beside the material on the Moodle page
`https://moodle2.units.it/course/view.php?id=16632`
Riemmanian Geometry, M. P. do Carmo, Birkhäuser, (1992)
Riemmanian Geometry, S. Gallot – D. Hulin – J. Lafontaine, Springer–Verlag, (1987)
Notes on Differential Geometry, N.J. Hicks, Van Nostrand, (1965)
Calculus in Manifolds, M. Spivak, Benjamin, (1965)
Foundations of Differential Geometry, S. Kobayashi – K. Nomizu, Wiley–Interscience, (1969)
Differential Geometry, Lie Groups and Symmetric Spaces, S. Helgason, Academic Press, (1978)