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Corso di Termofluidodinamica Computazionale

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Problem statement

A plan fin of uniform cross-section, as shown in figure 1, is made with a uniform, isotropic material with a thermal conductivity $k = 50$ [W/(m K)]; it has a thickness t and a length L and its width w is large in comparison with its thickness.
The fin is cooled only by convection, with a convective heat transfer coefficient $h = 200$ [W/(m^2 K)], and the surrounding fluid temperature is $T_\infty = 25$ [°C]. The base of the fin is maintained at a temperature $T_b = 200$ [°C], while the tip of the fin is assumed perfectly insulated.
The fin is also subject to internal heat generation, equal to $\dot{q}_g = 1 \times 10^7$ [W/m^3].

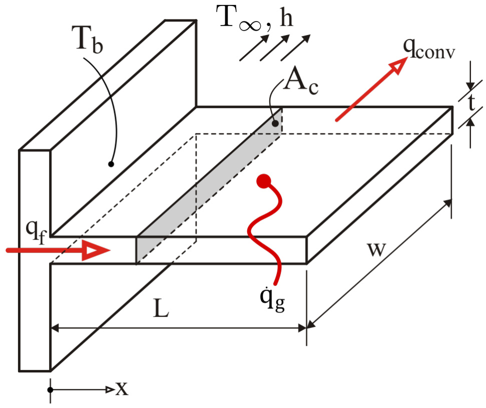

Figure 1: Straight fin with uniform cross-section and internal heat generation.

Assigned tasks

- Assuming a *1D* temperature distribution, i.e. $T \approx T(x)$, compute, with the Finite Volume method (FV), the heat flux from the fin q'_{num} [W/m], using a number N of FVs equal to $N = 10$, 20, 40, and 80.

  Consider the two following cases:

  1. $L = 20$ mm and $t = 1$ mm.
  2. $L = 20$ mm and $t = 5$ mm.

- Plot in a *log-log* graph the behavior of the error vs N, verifying its quadratic trend. The error is defined as the difference between the numerical value of the computed heat flux per unit width q'_{num} and its analytical solution q'_{teor}, reported in [2, 3] and given below.

- In addition, compare, for N = 20, the computed temperature profile $T_{num}(x)$ with the theoretical one, also given below.

- Heat flux at the fin base [W/m]:

$$q'_{teor} = 2\frac{(T_b - T_\infty)\, h\, y_0}{\sqrt{h\, y_0/k}} \left[\left(1 - \frac{\dot{q}_g\, y_0}{h\,(T_b - T_\infty)} \right) \tanh\left(\sqrt{\frac{h\, y_0}{k}}\, \frac{L}{y_0} \right) \right] \tag{1}$$

- Temperature profile $T(x)$ [°C]:

$$T_{teor}(x) = T_\infty + \frac{q_{teor}}{\sqrt{2\, h\, k\, t}} \left(\frac{\cosh\,(m\, x)}{\tanh\,(m\, L)} - \sinh\,(m\, x) \right) + \frac{\dot{q}_g\, t}{2\, h} \tag{2}$$

with

$$y_0 = t/2$$
$$m = \sqrt{\frac{2\, h}{k\, t}}$$

References

[1] F. P. Incropera, D. P. Dewitt, T. L. Bergman, A. S. Lavine, *Fundamentals of Heat and Mass Transfer*, 6th Ed., Wiley, (2007).

[2] A. D. Kraus, A. Aziz, J. Welty, EXTENDED SURFACE HEAT TRANSFER, J. Wiley & Sons, (2001).

[3] W.S. Minkler, W.T. Rouleau, The Effects of Internal Heat generation on Heat Transfer in Thin Fins, *Nuclear Science Eng.*, **7**, pp. 400-406, (1960).