Net Present Value (NPV)

Example

- What sum invested today for 3 years at a rate of 10% will yield a sum of 100,000€ at the end?

$$PV = \frac{100,000}{(1+0.1)^3} \cong 75,130\,€$$

- If the agent can invest less than 75,130€, getting 100k€ after three years, the investment project is worth undertaking.
- If the investment needed is 75,130€ the investor would be indifferent to undertaking the project or the opportunity that yields a return of 10%.

- *Net Present Value* (NPV) is a measure of the *value added to* the company by the investment made.
- This concept is expressed through the *present worth* or *present value* of an investment project.
- It is not enough that the *algebraic* sum of income and expenditure is greater than zero, but the period in which they occur must be taken into account.
- The NPV represents the incremental wealth generated by the project, expressed *as if it were immediately available*.

- If r is equal to the *cost of capital* for the investor, an NPV>0 indicates that the cash flows are able to
  - remunerate capital at the rate r
  - repay the investment in full
  - generate additional resources that can be used for other purposes.
- Thus, a positive NPV is a measure of the value generated in excess of the *minimum* amount required by the investor and expressed through the discount rate.

- NPV can be calculated in different ways:

$$NPV = \sum_{t=0}^{n} \frac{R_t}{(1+r)^t} - \sum_{t=0}^{n} \frac{C_t}{(1+r)^t} - \sum_{t=0}^{n} \frac{I_t}{(1+r)^t} =$$

$$= \sum_{t=0}^{n} \frac{(R_t - C_t)}{(1+r)^t} - \sum_{t=0}^{n} \frac{I_t}{(1+r)^t} =$$

$$= \sum_{t=0}^{n} \frac{F_t}{(1+r)^t} - \sum_{t=0}^{n} \frac{I_t}{(1+r)^t}$$

(this while keeping the investments explicit).

- The NPV will be greater than 0 when the present value of cash inflows is greater than the present value of cash outflows.
- It is worth remarking that cost savings resulting from operating an asset are avoided cash outflows, so they are indeed positive cash flows.
- The condition for the acceptability of an investment, according to the NPV criterion, is that NPV > 0.
- If NPV = 0 there is indifference about the decision.
- If NPV < 0, the investment is not attractive.

Example [STEG]

- A food processing plant employs in the sterilization process a steam generator which produces 4000 kg/h of steam at medium pressure.
- The process operates 2400 hours per year.
- The old generator could be replaced by a new one with feedwater economizer, improved isolation and flow control.

Economic data		
Cost natural gas	0.56	€/Sm³
Electricity cost kWh	0.25	€/kWh
Useful life	8	years
Discount rate	8%	

Example [STEG]

Old generator data		
Efficiency at medium load	0.84	
Electricity cost per ton of steam	3€/t	
Water treatment cost per ton of steam	3€/t	
Maintenance cost	8000€/yr	

New generator data		
Efficiency at medium load	0.97	
Electricity cost per ton of steam	2€/t	
Water treatment cost per ton of steam	3€/t	
Maintenance cost	6250€/yr	

- Improved performance of the new generator:
  - the equipment and monitoring system ensure lower electricity consumption
  - the control system allows for a better maintenance programme.
- The investment (turn-key) cost of the new system is 250,000€, including design, installation and accessories.

Example [STEG]

	Old_genrt	New_genrt	
Gas consumption (Sm^3/h)	255.00	225.00	
Gas consumption (Sm^3/yr)	612,000.00	540,000.00	
Cash flow			
	Old_genrt	**New_genrt**	**Savings**
Cost_nat_gas (€/yr)	342,720.00	302,400.00	40,320.00
Cost_electr (€/yr)	28,800.00	19,200.00	9,600.00
Cost_maint (€/yr)	8,000.00	6,250.00	1,750.00
			51,670.00
Investment cost (€)		250,000.00	

- The economic assessment is used to evaluate if the investment required for the acquisition of the new generator is economically beneficial.
- This value is expressed through annual cost savings (51,670€), which is a constant inflow ($\bar{F}$).

Example [STEG]

$$PB = \frac{I_0}{\overline{F}} = \frac{250000}{51670} \simeq 4.8\,(\text{yr})$$

- As $\overline{F}$ is an annuity over n=8 years, we can obtain the corresponding present value:

$$PV_{\overline{F},n} = \overline{F}\,\frac{(1+r)^n - 1}{r\cdot(1+r)^n} = 51670\,\frac{(1+0.08)^8 - 1}{0.08\times(1+0.08)^8} \simeq 296{,}928.83\,(€)$$

Example [STEG]

- We can then calculate the PI and the NPV for the project:

$$PI = \frac{PV_{\bar{F},n}}{I_0} = \frac{296928.83}{250000} \simeq 1.19$$

$$NPV = PV_{\bar{F},n} - I_0 = 296928.83 - 250000 \simeq 46,928.83 \text{ (€)}$$

- Using Excel’s function “Goal seek”, we can obtain IRR ≅ 12.8%

- The NPV value depends on
  - the cash flow profile
  - the value of the discount rate.
- At equal r, investments with high positive cash flows in the first years have an advantage.
- Low r values (2-6%) favour projects with medium- to long-term benefits.
- High r values (>10%) favour projects with short payback periods.

- Consider the value of 1€ generated n years in the future; for different values of r we have:

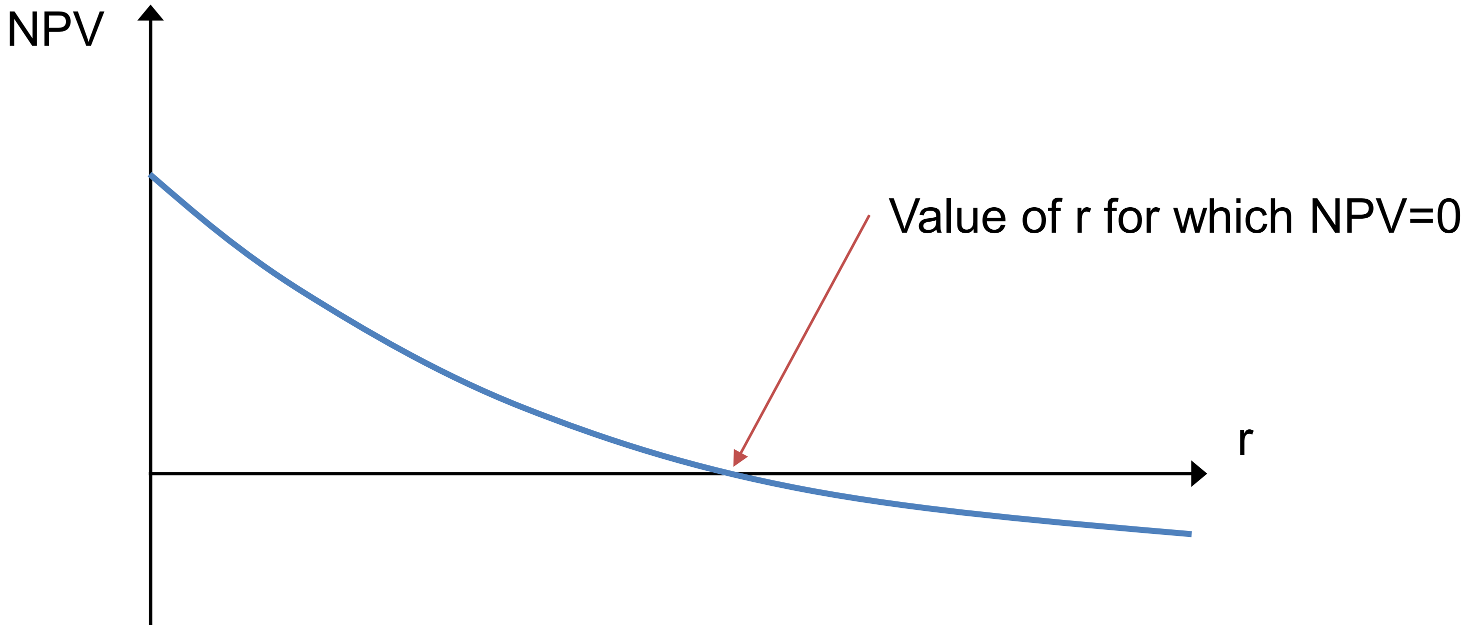
	5 years	10 years	20 years	50 years	100 years
1%	0.95	0.91	0.82	0.61	0.37
5%	0.78	0.61	0.38	0.088	0.0076
10%	0.62	0.038	0.15	0.0085	0.000072
20%	0.40	0.16	0.026	0.00011	0.00000001

	Project A			Project B		
Year	Ft k€	Ft (4%) k€	Ft (10%) k€	Ft k€	Ft (4%) k€	Ft (10%) k€
0	-2500	-2500	-2500	-2500	-2500	-2500
1	300	288	273	500	481	455
2	400	370	331	600	555	496
3	400	356	301	900	800	676
4	600	513	410	600	513	410
5	700	575	435	400	329	248
6	600	474	339	300	237	169
7	500	380	257	200	152	103
8	300	219	140	200	146	93
9	200	141	85	100	70	42
10	100	68	39	100	68	39
NPV		**884**	**107**		**850**	**231**

PB of A = 6 years; at 4% $NPV_A > NPV_B$

PB of B = 4 years; at rate 10% $NPV_B > NPV_A$

- More generally, for the same cash flow, the NPV decreases as r increases.
- In the case of profiles where the flows change from negative to positive only once, there is just one value of r where NPV=0:



Remark: PI and NPV

- We have seen that when comparing several alternatives, the one with the highest PI should be chosen (as long as PI > 1).
- However, it is possible that PI and NPV give conflicting results.
- Recall that, in the calculation of PI, in the denominator all investment outlays, at any time, for the completion, renewal, commissioning or overhaul of the project, are included.
- Cash flows from operations and management are not included.

Project	I (k€)	ΣPVA (k€)	PI	NPV (k€)
A	5,000	7,582	1.52	2,582
B	20,000	24,072	1.20	4,072

- In the example above, both projects are profitable, but the two indices give conflicting results.
- Note that:
  - PI gives a relative measure of the profitability (or efficiency) of a project
  - NPV expresses the economic advantages (or disadvantages) in absolute terms.

Project	I (k€)	ΣPVA (k€)	PI	NPV (k€)
A	5,000	7,582	1.52	2,582
B	20,000	24,072	1.20	4,072

- We can use PI to value if B's higher investment (compared to A) produces incremental benefits:

$$PI = \frac{24072 - 7582}{20000 - 5000} = \frac{16490}{15000} = 1.10$$

- PI > 1 indicates that the "additional" investment produces positive economic effects for each additional € invested.

Equivalent annual worth

- The *equivalent annuity* (EA) is the uniform monetary amount that occurs at the end of each period t=1,...,n and is *equivalent* to the profile of cash flows of the project.
- It can be associated with the total of cost and revenue flows (*equivalent annual worth*) or with the cost flows alone (*equivalent annual cost*).

$$EA(r) = NPV(r)\frac{r(1+r)^n}{(1+r)^n - 1}$$

- With constant annuities A and an outlay at t=0 equal to I_0:

$$EA = I_0 \frac{r(1+r)^n}{(1+r)^n - 1} + A$$

- If cash flows are variable, the corresponding PV can be obtained and then EA can be determined.

Example [SLOG]

- SLOG company wants to evaluate two different automated storage systems with the same level of service (k€):

Project A	0	1	2	3	4	5	6	7	8
Initial outlay	- 200								
Ordinary maintain.		- 15	- 15	- 15	- 15	- 15	- 15	- 15	- 15
Refurbishment									
Salvage value									30
Project B	0	1	2	3	4	5	6	7	8
Initial outlay	- 160								
Ordinary maintain.		- 10.5	- 10.5	- 10.5	- 10.5	- 10.5	- 10.5	- 10.5	- 10.5
Refurbishment					- 60				
Salvage value									10

Example [SLOG]

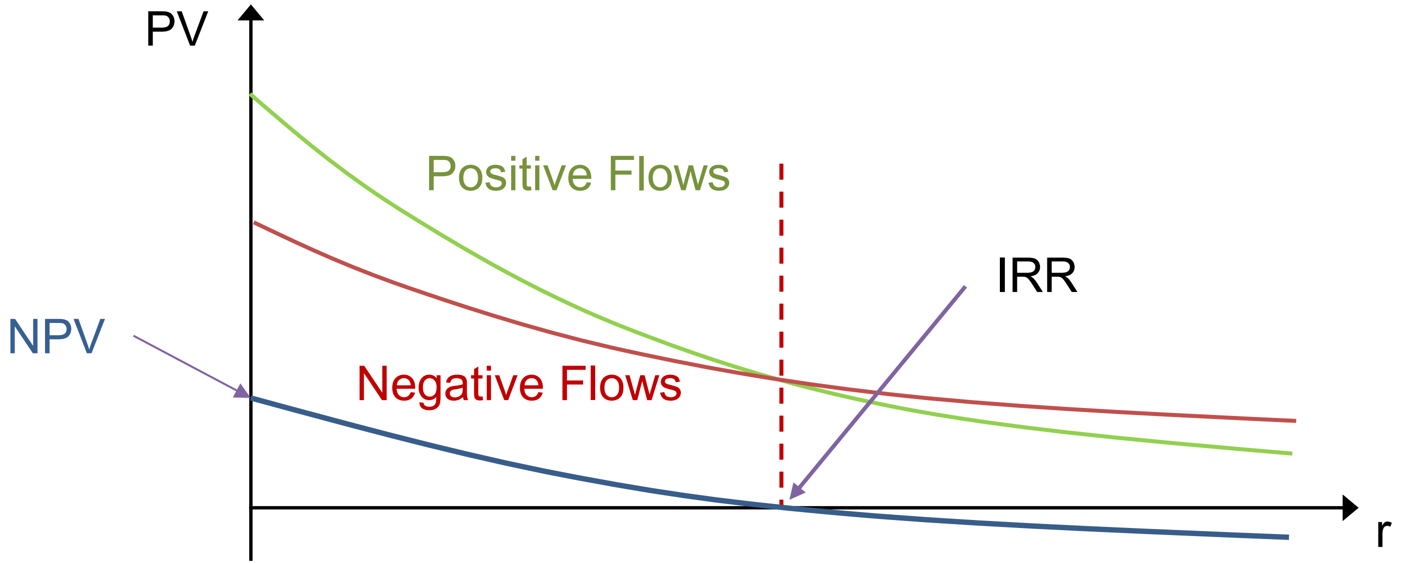
- The following annuity values are obtained (k€, r=8%):

	Investment	A	Refurbish.	Salvage	EA
Prog. A	- 34.8	- 15	-	2.8	- 46.9
Prog. B	- 27.8	- 10	- 7.7	0.9	- 44.6

- Project B has lower *equivalent annual cost* than Project A.

Internal Rate of Return (IRR)

- We have seen that NPV decreases as r increases.
- Indeed, the investment can be assessed by determining the value of r at which the NPV becomes zero.
- Such financial indicator is called *internal rate of return*.
- Consider an investment in which from 0 to t’ net cash flows are negative and from t’+1 net cash flows are positive.
- IRR is the rate at which the present value of positive flows (PV+) equals that of negative flows (PV-):
  - for r < IRR, PV+ > PV-
  - for r > IRR, PV+ < PV-



- In the case of investments in the first years and then positive F_t flows, we have:

$$\sum_{t=0}^{n} \frac{F_t}{(1+IRR)^t} - \sum_{t=0}^{n} \frac{I_t}{(1+IRR)^t} = 0$$

$$\sum_{t=0}^{n} \frac{F_t}{(1+IRR)^t} = \sum_{t=0}^{n} \frac{I_t}{(1+IRR)^t}$$

- A positive IRR alone does not guarantee the profitability of a project.
- The value of the IRR must be compared with a “threshold” value (*hurdle rate* – HR), set by the investor.
- The *opportunity cost of* capital, i.e. the rate of return given by the most promising investment that is foregone, is often adopted as HR.
- If IRR > HR the investment project is worthwhile
- If IRR < HR the investment is not sufficiently profitable.

Existence and uniqueness of the IRR

- One of the biggest problems in the use of IRR is related to its existence and uniqueness.
- The determination of IRR is done through the solution of the equation NPV(r)=0.
- If the useful life of the investment is n years, this is an equation of degree n in the unknown x=1/(1+r):

$$F_0 \cdot \frac{1}{(1+r)^0} + F_1 \cdot \frac{1}{(1+r)^1} + \ldots + F_n \cdot \frac{1}{(1+r)^n} = 0$$

$$F_0 \cdot x^0 + F_1 \cdot x + \ldots F_n \cdot x^n = 0$$

- The solution of the equation can consist of:
  - no real solution
  - only one real solution
  - several real solutions (acceptable, from a financial point of view, only if they are positive).
- The existence and number of real solutions depend on:
  - the degree n (if n is odd there is at least one real solution)
  - the number of sign changes of the addends.

Nostrom's Theorem

- Let S(t) denote the cumulative sum of the undiscounted cash flows at time t of the project.
- If S(0)<0 and if S(t) only changes sign *once*, then there is only one r*>0 for which NPV(r*)=0 (*sufficient condition*).

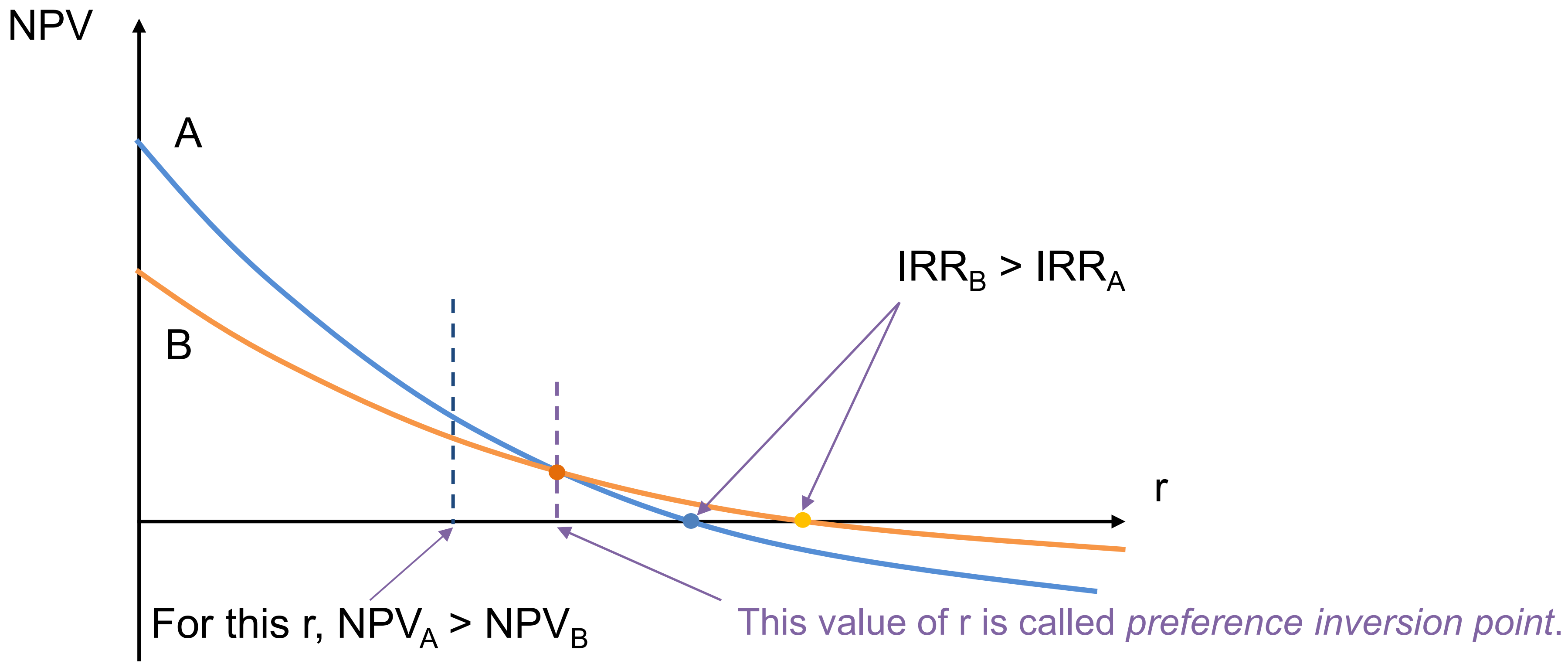
- *Example:*

	0	1	2	3	4	5
Cash flows	-4000	-1000	3000	4000	-2000	4000
Cumulative sum	-4000	-5000	-2000	2000	0	4000

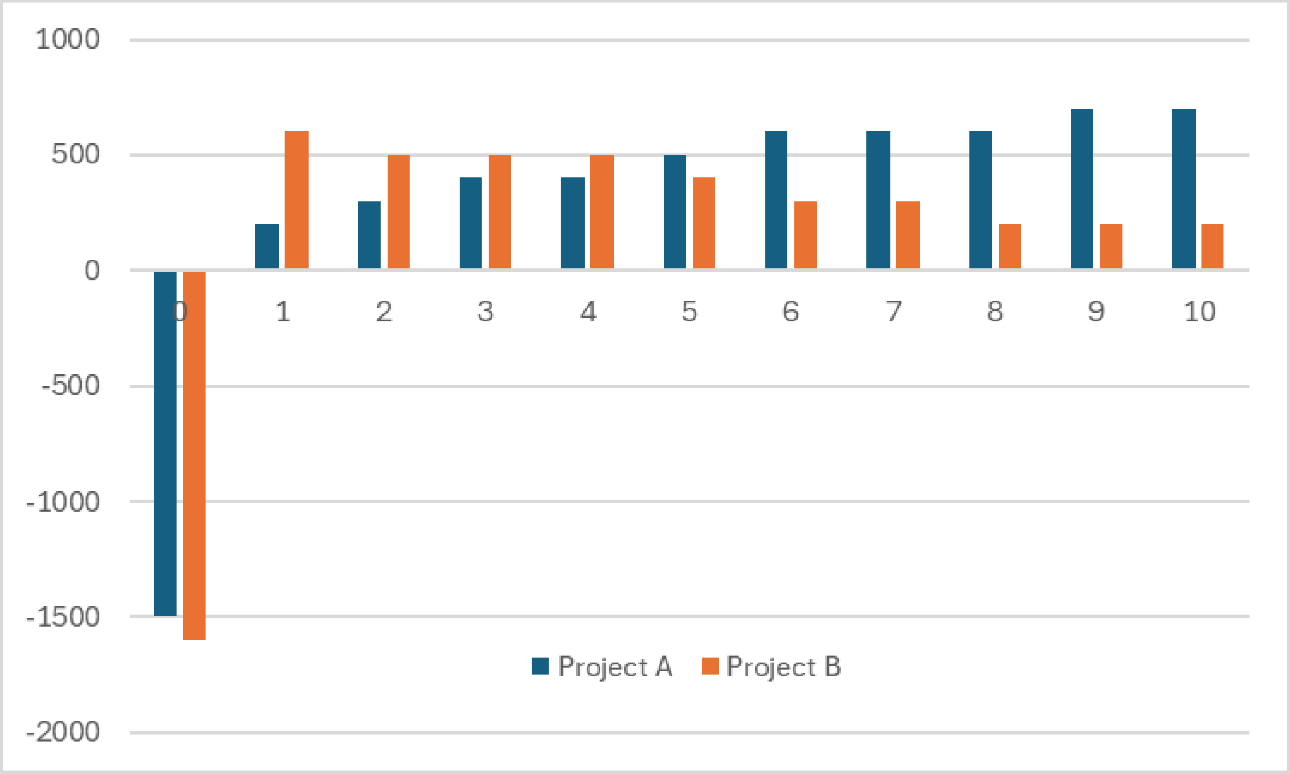
- In this case, IRR=21.8%.

- NPV and IRR can give conflicting indications.
- That is, it is possible that, given two investments A and B and a discount rate r, it results:

$$NPV_A > NPV_B \text{ and } IRR_A < IRR_B$$



- Typical situations where this may happen:
  - alternatives with initial investment outlays of very different sizes
  - alternatives with similar initial outlays, but with operating cash flows with opposite timing (see figure)



Example 1

- Consider the two alternatives with the following profiles:

	0	1	2	3	4	5
Project A	- 10,000	3,000	3,000	3,000	3,000	3,000
Project B	- 23,500	6,800	6,800	6,800	6,800	6,800

- If r=10%:

	NPV	IRR
Project A	1,372	15.2%
Project B	2,277	13.7%

- If HR=10%, both A and B are profitable.
- IRR does not consider the *size* of the initial investment and flows.

Example 1

- One way to deal with this problem is to perform an *incremental analysis*.
- Project B requires a larger investment but generates larger cash flows.
- We can calculate the IRR for the differential (incremental) cash flow:

	0	1	2	3	4	5
∆BA	- 13,500	3,800	3,800	3,800	3,800	3,800

- This gives an IRR of 12.6%.

Example 1

- If HR=10% the incremental investment is desirable, then Project B should be chosen (in agreement with the indication given by the NPV).
- In practice, agreeing to undertake the incremental project basically means realising A and ΔBA.
- This is practically done by implementing B.

Example 2

- Consider the two projects with the following profiles:

	0	1	2	3
Project A	- 2,000	1,500	1,000	500
Project B	- 2,000	500	1,000	2,000

- This results in (r=10%):

	NPV	IRR
Project A	566	28.9%
Project B	784	26.7%

Example 2

- Since the initial investment has the same value, the PI can be used:

	NPV	IR
Project A	566	1.28
Project B	784	1.39

- Project B offers a higher return per € invested (for the same initial investment).
- The indication provided by PI is consistent with that provided by the NPV.

Example 2

- Applying the IRR to the incremental investment ∆BA also yields a concordant result:

	NPV	IRR
∆BA	218	22.5%

- The incremental investment gives an IRR higher than 10%, so B is economically more beneficial than A.