Multi-criteria decision analysis methods

Economic evaluation of industrial projects

Introduction

- In economic methods, the characteristics of solutions are measured in terms of their outcome on a single economic criterion (e.g. the NPV).
- The decision-making process has three foundations:
  - a formal definition of candidate solutions
  - a formal definition of the characteristics or consequences of solutions
  - a model of the preference system.

- The candidate (feasible) solutions or the conditions of their acceptability (feasibility space) are given.
- Some characteristics are selected to evaluate the actions.
- The preferences of the decision maker are expressed through the economic value and the concept of opportunity cost.
- A common structure of single-criteria optimisation methods is as follows:
  - an objective function is defined and applied to actions g(*a*)
  - the set A to which the candidate actions ($a \in A$) may belong is defined
  - the model allows for a complete ranking of candidate actions.

However, consider the following problem

- A metalworking company selects a new welding station that should provide a cleaner process and good ergonomics.
- Key goals:
  - low energy consumption
  - low emissions
  - low process waste
  - improve the ergonomics of the workstation.

- The working group identified 6 evaluation criteria in line with the project goals: 5 quantitative and 1 qualitative:

Definition	Acronym	Unit	Notes
Energy Consumption	ENE	kWh/m	Electrical energy per unit weld length
Greenhouse Gas Emissions	GRG	g CO_2/s	CO_2 equivalent per welding time
Waste generation	WST	kg	Solid waste (kg) per kg of weld deposit
Capital cost	CAP	€	Purchase price of welding equipment
Defect rate	DFR	%	Ratio between total defective weld length and total weld length inspected, as a percentage
Ergonomics	ERG	?	Accessibility of controls; workstation adjustability

- If several solution are feasible, how can the company select one?

The concept of dominance

- Given
  - a set of alternatives $A = \{a_i, i = 1, \ldots, m\}$
  - a set of criteria $G = \{g_j, j = 1, \ldots, n\}$
- each alternative will have a *performance* with respect to the set G: $g_j(a_i),\ j = 1, \ldots, n$.

- Definition:
  a_h is *dominated* if there exists (in the set *A*) at least one other alternative a_i that performs better than a_h in at least one criterion and performs no worse than a_h in the remaining criteria.
- Formally:

  a_i *dominates* a_h if and only if

  $$g_j(a_i) \geq g_j(a_h) \text{ for } j = 1, \dots, n$$

  and

  $$g_j(a_i) > g_j(a_h) \text{ for at least one } j = 1, \dots, n$$

- These concepts can be applied when $g_j(a)$ is at least measured on an *ordinal scale*.

Non-compensatory methods

- Non-compensatory methods do not allow the positive performance of one criterion to compensate the negative performance of another.
- Each criterion is considered separately from the others.
- In *dominance analysis*, the alternatives are compared in pairs against each criterion.

- The identification of dominated alternatives *does not* require the decision maker to judge the importance of the different criteria.
- A rational decision maker will not choose a dominated alternative: therefore, they could be removed from the set of alternatives initially available.
- A dominance graph can be constructed in which the dominated alternatives are noted.

Example

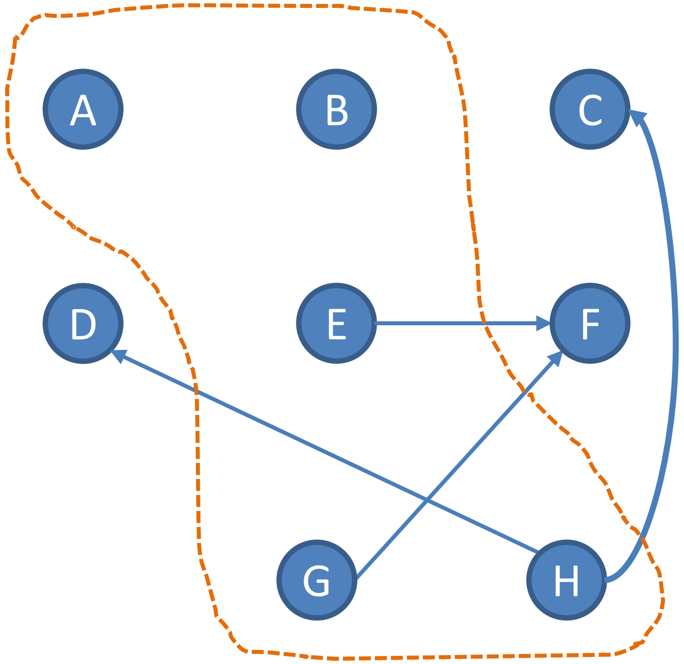
- A company wants to select suppliers of consumable materials.
- They are evaluated on three criteria (C, fixed annual cost of supply; T, average post-order delivery time; QS, quality of customer service):

	C (k€)	T (days)	QS (A,B,O*)
A	65	3	A
B	58	4	A
C	85	3	G
D	80	4	E
E	65	5	G
F	72	5	G
G	70	4	G
H	80	3	E

(*A: acceptable; G: good; E: excellent)

- The following dominance graph is obtained:

	C (k€)	T (days)	QS (A,B,O*)
A	65	3	A
B	58	4	A
C	85	3	G
D	80	4	E
E	65	5	G
F	72	5	G
G	70	4	G
H	80	3	E



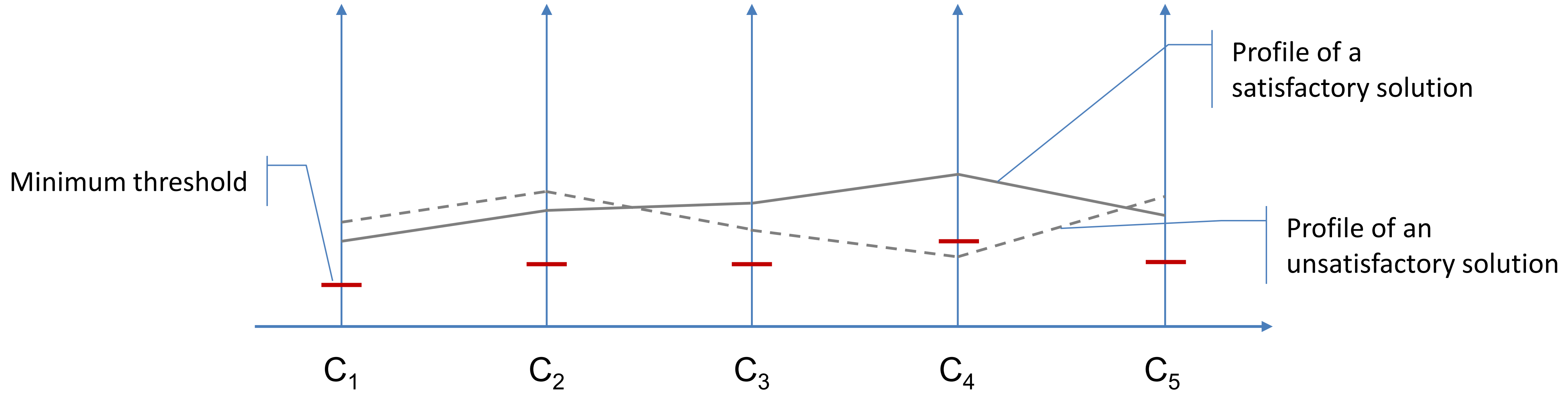
- Alternatives C, D, F can be removed from the evaluation.
- The set {A,B,E,G,H} consists of non-dominated alternatives (ND-A).

- The image of the set of ND-A in the space of consequences is called *efficient* or *Pareto frontier*.
- If a new alternative were added, the ND-A set could change.
- However, a dominated alternative cannot become non-dominated after the addition of a new alternative.

- Some non-compensatory methods are used to identify a set of solutions or candidates that have a satisfactory performance in *each* criterion.
- In *the conjunctive method, threshold values* (minimum or maximum) are set for each criterion (let J be the set of all attributes considered).
- A solution will be considered satisfactory if its performance
  - exceeds the minimum value for *all* attributes with a min threshold
  - does not exceed the maximum value for *all* attributes with a max threshold

- If all are minimum value thresholds, given the generic criterion j∈J (x_j^0), a solution a_i will be satisfactory if

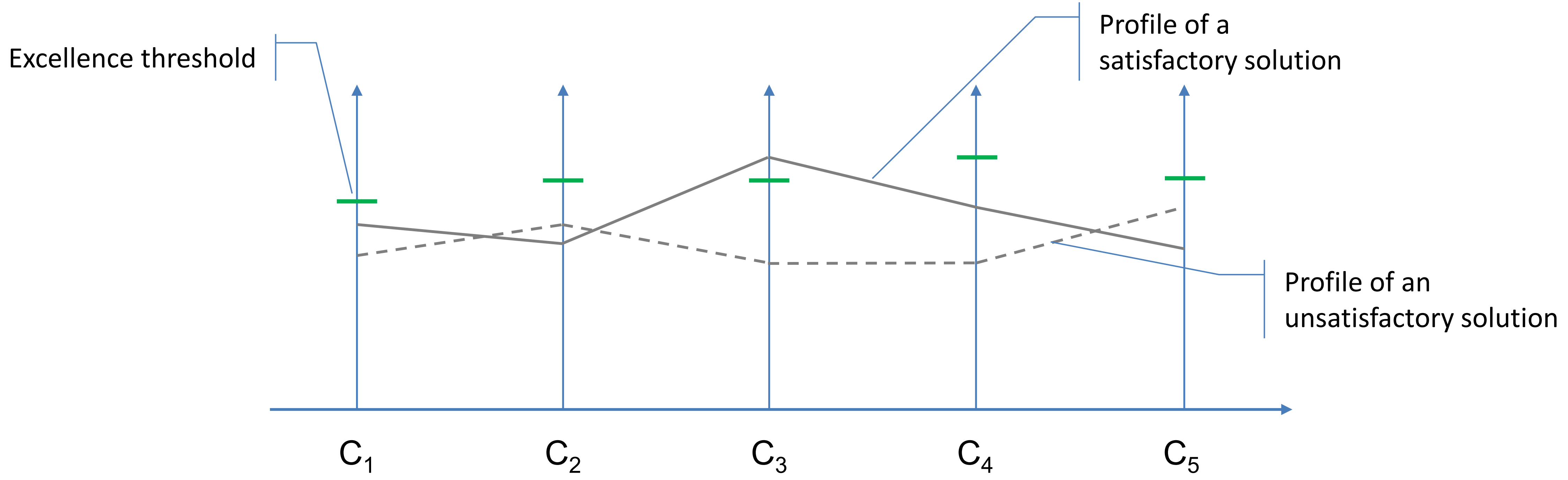
$$x_{ij} \geq x_j^0 \quad \forall j, j \in J$$



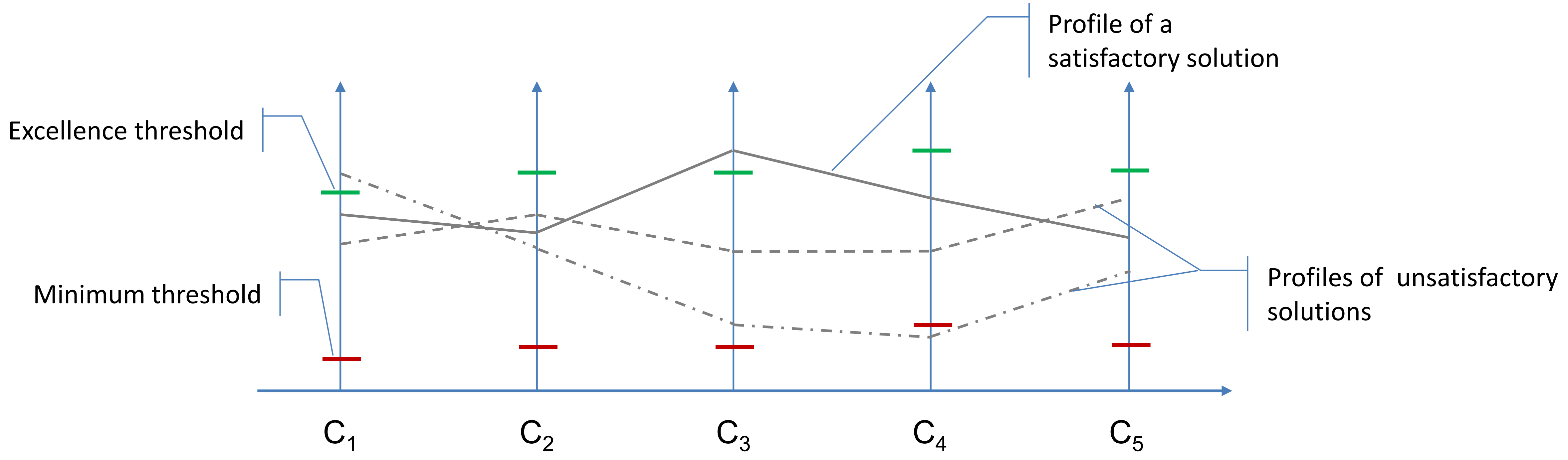
- The *disjunctive method* considers the possibility that a candidate may perform particularly well *in at least one criterion* (excellence threshold).
- This approach emphasises the excellence that a candidate or solution may demonstrate even in just one criterion.
- A solution will be considered valuable if its performance
  – is below the minimum value in *at least one* criterion with min threshold
  – exceeds the maximum value in *at least one* criterion with max threshold

- If excellence values are upper thresholds and given the threshold value of the generic criterion j∈J (x_j^S), a solution a_i will be satisfactory if

$$\exists j, j \in J : x_{ij} \geq x_j^S$$



- It is possible to use both methods together to define more precisely the criteria that candidates must meet in order to be considered suitable.



Comments

- The alternatives can have *conflicting* performances in the criteria.
- A DM could prefer the alternative that has the best “overall” performance.
- One way to obtain an overall (or “global”) performance is to *aggregate* the scores in the criteria.
- The simplest way, if all criteria have the same unit of measurement, is to add up the scores, which implies giving equal importance to them.
- For example:

	g_1	g_2	
Alternative A	8	4	12
Alternative B	5	4	9
Alternative C	3	8	11

Example

- The result is that now one of the alternatives appears less satisfactory than the others and there is a preferable one.
- However, if the decision-maker weighs the two criteria differently, different results can be obtained.
- For example:

	g_1	g_2	
	0,4	0,6	
Alternative A	3,2	2,4	5,6
Alternative B	2	2,4	4,4
Alternative C	1,2	4,8	6

Comments

- Note that aggregation in the form of a sum makes it possible to *trade off* the performance of the criteria:

  Alternative C with a low score in criterion 1 compensates for its *overall performance* by means of a higher score in criterion 2.
- Criteria weights determine the impact of the criteria on overall performance.
- Thus, alternative C, which has 5 points less than A in criterion 1 ($5 \times 0.4=2$), recovers in total performance due to the 4 points more in criterion 2 ($4 \times 0.6=2.4$).

	g_1	g_2
	0.4	0.6
Alternative A	8	4
Alternative B	5	4
Alternative C	3	8

- In other words, when criteria contribute *equally* to total performance (same scale and weights), one can "give up" a unit of one criterion to obtain a unit of another without losing in overall performance.
- If the scales are different or the weights are different, the value of a unit of the criterion that is decreasing must be determined by expressing it in the units of the one that is increasing (*trade-off*).
- This "conversion factor" is usually called *substitution* (or *trade-off*) *rate*.
- The substitution rate between two criteria c_j and c_k at the point

$$\mathbf{c}^0 = (c_1^{\,0}\,,\ \mathrm{c}_2^{\,0}\,,\ ...,\ c_{\mathrm{n}}^{\,0})$$

- is the variation $r_{\mathrm{j,k}}$ in the criterion c_j whereby the alternative characterised by $\mathbf{c}^0$ is *indifferent* (not worse not better) to the alternative characterised by the vector

$$(c_1^{\,0}\,,\ c_2^{\,0}\,,\ ...,\ c_j^{\,0}+r_{\mathrm{j,k}}\,,\ ...,\ \mathrm{c}_{\mathrm{k}}^{\,0}\ -1,\ ...,\ c_{\mathrm{n}}^{\,0})$$

- In the case of the simple weighted sum, we have
  $r_{j,k} = w_k / w_j$ where w_j are the weights of the criteria.
- In fact, the overall performance of an alternative 0
  $[\ldots, w_j \cdot c_j, \ldots, w_k \cdot c_k, \ldots]$
  is equivalent to the overall performance of an alternative 1
  $[\ldots, w_j \cdot (c_j + r_{j,k}), \ldots, w_k \cdot (c_k - 1), \ldots]$:

$$w_j \cdot c_j + w_k \cdot c_k = w_j \cdot (c_j + r_{j,k}) + w_k \cdot (c_k - 1)$$

$$0 = w_j \cdot r_{j,k} - w_k$$

$$r_{j,k} = \frac{w_k}{w_j}$$

Notation used

- A: $(a_1 , a_2 , ..., a_i , ..., a_h , ..., a_m)$
  set of admissible alternatives
- I: $(1, 2, ..., j, ..., n)$
  set of indices of the n criteria $g_j(.)$
- W: $(w_1 , w_2 , ..., w_j , ..., w_n)$
  set of weights assigned to the criteria
- $g_j(a_i)$ evaluation of a_i with respect to g_j
- if $g_j(a_i)$ is a numerical value, we will denote it by x_{ij}

	g_1	…	g_j	…	g_n
a_i	$g_1(a_i)$	…	$g_j(a_i)$	…	$g_n(a_i)$

- Typically, weights are normalised: they have values between 0 and 1 and respect the following relationship

$$\sum_{j=1}^{n} w_j = 1$$