Simple additive weighting (SAW) method

- The weighted sum is an additive model in the form:

$$V(a) = \sum_{j=1}^{n} w_j \cdot g_j(a)$$

where $g_j(a)$ is the performance of *a* in criterion j.

- A *synthesis value* V(*a*) (*overall score*) is assigned to alternative *a* from the performance value assigned to it in each criterion j.

- This is acceptable if the criteria can be considered mutually independent.
- If the performance of a_i with respect to g_j is a numerical value x_{ij} then:

$$V(a_i) = \sum_{j=1}^{n} w_j \cdot x_{ij}$$

- In cases where the x_{ij} are “objective” measures of performance, the expression thus seems to separate the analytical phase (which produces the x_{ij}) from the evaluative phase (where the w_j come into play).

Typical industrial problems in which the method is applied.*

- Location of production facilities
- Selection of technological solutions for plants or processes
- Selection of production or service processes
- Selection of materials or components for production
- Selection of energy management solutions
- Evaluation of green manufacturing projects
- Evaluation and selection of suppliers of materials or services

(* In practical cases, other types of multi-criteria methods are also used).

Example [COAT]

- A furniture manufacturer must replace the current coating line for flat components with a newer automated technology.
- The new line will use powder coatings because they are essentially VOC-free and solvent free.
- Five lines satisfy all the company's requirements and will be evaluated against six criteria [to be maximised or minimised]:
    1. Transfer efficiency (percent of sprayed powder that adheres) [max]
    2. Productivity [max]
    3. Maximum coating thickness [max]
    4. Capital cost (turnkey cost) [min]
    5. Maintenance (ordinary) and downtime (minutes per 8h shift) [min]
    6. Flexibility [max]

Example [COAT]

Description	Acronym	Units	Notes
Transfer efficiency	TRF	%	
Productivity	PRD	m^2/h	Factors: cure time, conveyor speed, gun layout
Maximum coating thickness	THK	mm	Max thickness without issues such as orange peel, poor adhesion, and uneven curing.
Capital cost	CAP	k€	
Maintenance and downtime (minutes per 8h shift)	MNT	min	Operations including filter replacement intervals, oven maintenance schedule, and ease of cleaning
Flexibility	FLX	score (1-10)	The ease of switching colours or products without long (>1h) changeovers.

Example [COAT]

- The performance table (or matrix) of the five alternatives is:

	TRF	PRD	THK	CAP	MNT	FLX
	(%)	(m^2/h)	(mm)	(k€)	(min)	(-)
A1	95	600	0.23	160	10	8
A2	98	550	0.23	180	15	8
B	92	800	0.21	120	30	6
C1	96	700	0.22	140	20	7
C2	94	800	0.24	180	20	9

Normalisation

- A first step is to convert the performance table into a normalised one with column values between 0 and 1.
- This is always required if the rating scales are different (or have different orders of magnitude).
- We use the following notation:
  - x_{ij} the value of $g_j(a_i)$
  - x_{ij}' the normalised value
  - x_j^{min} the minimum value among x_{ij} , for each g_j
  - x_j^{max} the maximum value among x_{ij} , for each g_j

Linear max-min

- For the benefit criteria (the DM wants the maximum):

$$x_{ij}' = \frac{x_{ij} - x_j^{\min}}{x_j^{\max} - x_j^{\min}}$$

- For the cost criteria (the DM wants the minimum):

$$x_{ij}' = \frac{x_j^{\max} - x_{ij}}{x_j^{\max} - x_j^{\min}}$$

- (Note that in the latter relationship x_j^{max} represents the maximum “cost” among the alternatives).

- In the example [COAT] (TRF criterion), we have:

$$x_{ij}' = \frac{x_{ij} - 92}{98 - 92}$$

- For the CAP criterion, we have:

$$x_{ij}' = \frac{180 - x_{ij}}{180 - 120}$$

- Linear max-min normalisation (Weitendorf, 1976) respects the characteristics of non-absolute scales.
- They do not have an absolute origin but only a conventional one.
- In such scales, differences between values have meaning but not ratios between values (which require an absolute zero).
- The ratios between differences in values (prior and after normalisation) are preserved.
- This normalisation can be problematic:
  - when the value 0 has no concrete meaning
  - when in a criterion the alternatives have just either the maximum or the minimum values.

Linear max

- Linear max normalisation (Van Delft and Nijkamp, 1977) uses the equations:
  - for benefit criteria

$$x_{ij}' = \frac{x_{ij}}{x_j^{\max}}$$

  - for cost criteria

$$x_{ij}' = 1 - \frac{x_{ij}}{x_j^{\max}}$$

- For benefit criteria, min values ≠ 0 are transformed into normalised values ≠ 0. Cost criteria have normalised min values < 1.

Linear

- Linear normalisation (Stopp, 1975) preserves the same distribution of values before and after normalisation.
- For benefit criteria

$$x_{ij}' = \frac{x_{ij}}{x_j^{\max}}$$

- For cost criteria

$$x_{ij}' = \frac{x_j^{\min}}{x_{ij}}$$

- Best values are equal to 1 after normalisation, and normalised worst values (if the originals are ≠ 0) are ≠ 0.

- The linear normalisations discussed, apart from linear max-min, produce normalised values that may be distorted in the case of cost criteria.
- Some authors have proposed a variant of linear normalisation that overcomes this potential problem.
- For benefit criteria

$$x_{ij}' = \frac{x_{ij}}{x_j^{\max}}$$

- For cost criteria

$$x_{ij}' = 1 - \frac{x_{ij} - x_j^{\min}}{x_{ij}^{\max}}$$

Linear sum

- The *linear sum* (Voogd, 1983) normalises the value of performance of an alternative in terms of its contribution to the total value, sum of values in j given by all the alternatives:

$$g_j^*(a_i) = \frac{g_j(a_i)}{\sum_{h=1}^{m} g_j(a_h)}$$

- Here, it is first necessary to convert the values of cost criteria ($x_{ij}^*=1/x_{ij}$).
- With this method normalised values cannot be 1 or 0.

Weights

- The weights (W_j) are also normalised (w_j) by means of the linear sum:

$$w_j = \frac{W_j}{\sum_{k=1}^{n} W_k}$$

- In [COAT] the weights were determined by *direct rating*: a score between 1 and 10 was assigned to each criterion, according to its importance.
- Then they were normalised using the linear sum

	TRF	PRD	THK	CAP	MNT	FLX
scores	10	10	8	9	6	7
normalised	0.20	0.20	0.16	0.18	0.12	0.14

- With linear max-min normalisation, the following values are obtained:

	TRF	PRD	THK	CAP	MNT	FLX	
	0.20	0.20	0.16	0.18	0.12	0.14	
A1	0.500	0.200	0.667	0.333	1.000	0.667	0.520
A2	1.000	0.000	0.667	0.000	0.750	0.667	0.490
B	0.000	1.000	0.000	1.000	0.000	0.000	0.380
C1	0.667	0.600	0.333	0.667	0.500	0.333	0.533
C2	0.333	1.000	1.000	0.000	0.500	1.000	0.627

- The weighted sum in the form: $V(a_i) = \sum_{j=1}^{6} w_j \cdot x_{ij}'$ produces the ranking

$$C2 \succ C1 \succ A1 \succ A2 \succ B$$

- When linear sum normalisation is used, the matrix of relative values for the example [COAT] is:

	TRF	PRD	THK	CAP	MNT	FLX	
	0.20	0.20	0.16	0.18	0.12	0.14	
A1	0.200	0.174	0.204	0.190	0.333	0.211	0.211
A2	0.206	0.159	0.204	0.169	0.222	0.211	0.192
B	0.194	0.232	0.186	0.254	0.111	0.158	0.196
C1	0.202	0.203	0.195	0.218	0.167	0.184	0.197
C2	0.198	0.232	0.212	0.169	0.167	0.237	0.204

- The weighted sum in the form: $V(a_i) = \sum_{j=1}^{6} w_j \cdot g_j^{*}(a_i)$
  produces the ranking:

$$A1 \succ C2 \succ C1 \succ B \succ A2$$

- As the example shows, it is possible to obtain different rankings using different normalisation methods.
- In [COAT] we have:

	lin. max-min	lin. max	linear	lin. sum
A1	3	2	1	1
A2	4	5	5	5
B	5	4	3	4
C1	2	3	3	3
C2	1	1	2	2

- This can happen when the performances of the alternatives have similar values, particularly in the most important criteria.
- In [COAT] it can be suggested to further analyse alternatives A1 and C2.

Weight estimation

- Criteria weights generally express the *importance of* one criterion in relation to the others.
- However, their meaning is broader and *depends on* the method used.
- For compensatory methods, the weights indicate:
  - the marginal contribution of a criterion in V(a)
  - the trade-off between criteria
  - a scaling factor with respect to attribute measurements.

Some practical methods to estimate weights

- We have seen the *direct rating* method, in which the DM assigns each criterion a value on a discrete scale (e.g. 1-10).
- In the *rank sum* method, the n criteria are ordered by importance and the first is assigned the value n, the second (n-1) and so on.

- In the *distributive* method, the DM has a total number of points (e.g. 100) at their disposal which they distribute among the criteria: the more points are allocated to a g_j the fewer points remain for the others (trade-off).
- In the *ratio* method, the DM orders the criteria by importance. The least important criterion is assigned a weight of 10; the others are assigned multiples of 10 according to their importance.

- These methods require the normalisation of weight values.