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11 Investment analysis

Outline

Objectives

1 To explain the nature and significance of capital budgeting.
2 To describe and distinguish between different types of investment or capital expenditure.
3 To explain the process and principles of cash flow analysis.
4 To explain the different methods of evaluating investment projects.
5 To explain the concept and measurement of the cost of capital.
6 To explain the nature and significance of risk and uncertainty in investment appraisal.
7 To examine the measurement of risk.
8 To explain the different ways of incorporating risk into managerial decision-making in terms of investment analysis.
9 To explain the concept of the optimal capital budget and how it can be determined.

11.1 Introduction

11.1.1 The nature and significance of capital budgeting

So far in the analysis of the previous chapters we have concentrated largely on the aspects of managerial decision-making that relate to making the most efficient use of existing resources. It is true that some aspects of decision-making in the long run have been considered, for example determining the most appropriate scale for producing a given output (Chapter 6), and the decision to expand capacity in a duopolistic market (Chapter 9), but many factors were taken as given in these situations. This chapter examines these long-run decisions in more detail, and explains the various factors that need to be considered in determining whether to replace or expand a firm's resources. As has been the case throughout the book, it will normally be assumed that the firm's objective is to maximize shareholder wealth, but certain aspects of public sector decision-making will also be considered, and these will be examined in further detail in the final chapter.

First of all, what do we mean by **capital budgeting**? Textbooks on both economics and finance tend to use the terms capital budgeting and investment analysis interchangeably. They both refer to **capital expenditure** by the firm, as opposed to current expenditure. Capital expenditure is ***expenditure that is expected to generate cash flows or benefits lasting longer than one year***, whereas current expenditure yields benefits that accrue within a one-year time period. Capital budgeting and investment analysis refer to the process of planning and evaluating capital expenditures.

Why is capital budgeting important? Unlike many other management decisions, capital budgeting decisions involve some commitment by the firm over a period of years, and as seen in Chapter 9, the nature of such decisions is that they are difficult or costly to reverse. Bad decisions can therefore be very costly to the firm. If a firm overinvests, there are resulting financial losses due to low revenues relative to high depreciation charges, and therefore there is a poor return to shareholders' capital. However, if a firm underinvests, the firm is often left with obsolete equipment and low productivity, with the additional problem that it may not be able to satisfy demand in peak periods, thus losing customers to competitors. Both of these problems are examined in more detail in Case Studies 11.2 and 11.3.

11.1.2 Types of capital expenditure

There are a number of different reasons for a firm to invest, and these can be classified in different ways. In each case the considerations, depth of analysis, and level of decision-making are different. The following seven-category classification is useful:

a. *Replacement*. This is the simplest type of investment decision because it involves replacing existing equipment with identical goods. Some decisions are as basic as changing a light bulb, while others, like replacing a photocopier, involve rather more expenditure. These investments must be made if the firm is to continue to operate efficiently with its current products in its current markets. Often such investments do not require a detailed analysis, and do not involve top management.

b. *Expansion*. This refers to expansion involving existing products and markets, thus increasing the scale or capacity of the firm. This is normally in response to an increase in demand, or in anticipation of an increase in demand. Such investments usually involve considerable expense and more uncertainty relating to the future; therefore, a more detailed analysis is generally required, and a higher level of management involved.

c. *New technology*. This type of investment may also involve the replacement of existing equipment, but, in this case, with newer, more productive equipment. The spur to this may be either cost reduction or demand expansion. The latter is relevant if the use of the new technology is seen as being important in attracting new customers. The new technology may therefore be used to produce existing products more cheaply, or to produce new products that are superior in some aspect of quality. There is a wide variation within this category in terms of cost, and therefore in depth of analysis and level of management involvement. The decision by car manufacturers to develop electric cars is obviously at the top end of the cost scale.

d. *Diversification*. This again involves expansion, but into new products or markets. This can change the whole nature of the firm's business, and involve very long-term and large expenditures. In many cases, mergers and acquisitions are involved. Therefore, very thorough and detailed analysis is required, and such decisions generally involve top management.

e. *Research*. This type of investment is sometimes ignored, or included in other categories, but it does have certain distinct features that merit a separate category. The most important of these is that such investment gives the firm options in the future, in terms of possible further investment opportunities. This is best explained by means of an example. If a firm conducts market research into the development of a new product, such research involves certain costs, but unlike any of the previously mentioned categories of investment it is not directly associated with any revenues. Only if the research indicates a favourable consumer response will the firm undertake the further investment necessary to produce and market the new product.

f. *Legal requirement*. Governments often make and change laws relating to such issues as the environment and working conditions. Thus firms may have to change either processes of production or the nature of the products they are selling if they are to continue in business. For example, the introduction of the EU Working Time Directive regarding a maximum working week in the UK has led companies to invest in more equipment of various types, both in order to maintain output levels, and to monitor the working schedules of employees. Even changes in tax conditions can result in such decisions; the high tax on petrol in the UK, including diesel fuel, may lead some firms to invest in converting their vehicles to operating on natural gas.

g. *Ancillaries*. These refer to investment projects that are not directly related to the core activities of the firm. They may include car parks for employees, cafeteria facilities, sporting facilities and suchlike. In many cases there are no direct increases in revenues in terms of cash flow, but there are measurable benefits to the firm that have to be evaluated. In the absence of such benefits there would be no reason for a firm to invest in such facilities. This aspect is examined in some detail in Case Study 11.1.

11.1.3 A simple model of the capital budgeting process

There are a number of steps involved in the capital budgeting process, which parallel those that are used in valuing securities like stocks and bonds. For each potential investment project that is identified by management the following steps need to be taken:

1 The initial cost of the investment must be determined.
2 The expected cash flows from the investment must be estimated, including the value of the investment asset at the end of its expected life.
3 The riskiness of the investment must be assessed.
4 The appropriate cost of capital for discounting the cash flows must be determined.
5 Some criterion must be applied in order to evaluate whether the investment should be undertaken or not. This involves calculating the **net present value** (NPV) and/or **internal rate of return** (IRR) and making the appropriate comparisons.

In practice the last three steps are interdependent, as will be seen, but it is convenient to discuss them in the above order. This is, therefore, the subject matter for the next four sections. Subsequently, the issue of the optimal capital budget for the firm is discussed, before finishing with the usual problem-solving approach.

11.2 Cash flow analysis

This aspect is the most fundamental, and also the most difficult, of all the processes involved in capital budgeting. It relates to both of the first two steps mentioned above, determining the initial cost outlay of the investment project, and estimating the annual cash inflows and outflows associated with it once operation begins. Various departments within the firm are usually involved: the initial cost outlay is often estimated by engineering, design and product development managers; operating costs are estimated by accountants and production, personnel and purchasing managers; revenues are estimated by sales and marketing managers.

A large amount of uncertainty is inevitable in such estimation, even concerning initial cost outlay. Many large-scale projects, for example the Montreal Olympics in 1976 and the Channel Tunnel, have been notorious in coming in at around five times the initial budget estimate. Some projects have exceeded even this. The uncertainty and inaccuracy becomes even greater with estimates of future operational cash flows. This aspect is dealt with in the next section. At this stage we are concerned with the principles of identification and measurement of cash flows.

11.2.1 *Identification of cash flows*

There are two main points that need to be clarified here.

a. *Cash flows not accounting income and expenses.* The income and expenses that appear in accounting records of profit and loss do not necessarily correspond to cash flows. For example, sales on credit are recorded as an income, but do not result in a cash flow in the corresponding period. Similarly, capital costs are cash flows, but are not recorded as expenses; depreciation on the other hand is recorded as an expense, but is not a cash flow. This creates some complications in terms of measuring cash flows, since the amount of a firm's tax liability is based on profit, not cash flow, yet tax does represent a cash flow. This complication is discussed in the next subsection on measurement. It is vital that cash flows, not income and expenses, are used in order to make the correct investment decision; the reason for this will be seen more clearly in Section 11.5 when evaluation criteria are explained.

b. *Incremental flows not actual cash flows*. The correct cash flows to consider are the differences between the cash flows if the investment project is undertaken and the cash flows if the project is not undertaken:

$$CF_t = CF_t \text{ with project} - CF_t \text{ without project} \qquad (11.1)$$

Only in this way can the effect of the project on the firm be properly seen and the correct investment decision made. The principle will be seen more clearly in the example in the next subsection.

11.2.2 *Measurement of cash flows*

Again there are a number of factors that have to be taken into consideration here. One, taxes, has just been mentioned, and some of the others have been discussed in Chapter 6, in the context of the relevant costs for decision-making. These factors are best explained in terms of a practical example, so a solved problem is now presented for this purpose, and this is further developed in later sections.

SP11.1 Cash flow estimation

Maxsport produces nutritional supplements for athletes and sports participants. They have developed a new bottled soft drink called Slimfuel, which claims both to provide nutrition and energy and to act as an aid to losing bodyfat. The marketing department has estimated sales to be 30 million bottles a year at a price of £2 per bottle. Research and development costs have already amounted to £500,000. The new product can be produced from the existing plants, but new machinery is required costing £4 million in each of five plants in the year 2002. Production and sales would begin in 2003. Advertising and promotion costs in the first year are estimated at 30 per cent of sales revenues, going down to 20 per cent in later years, with the product having a life of four years. Variable production costs are estimated at 40 per cent of sales revenues, with fixed overhead costs being £5 million per year, excluding depreciation.

Estimate the cash flows from the operation in order to evaluate the investment project, stating any necessary assumptions.

Solution

We can now consider the relevant factors in estimating the cash flows.

a. *Timing*. The timing of cash flows is important because of the **time value of money**. This concept is explained in more detail in section 11.5, but at this point it is sufficient to appeal to intuition that to receive £100 today has more value than receiving £100 in one year's time, which in turn has more value than receiving £100 in two years' time. Strictly speaking, cash flows should be analysed on a daily basis, but in practice some simplification is in order; in evaluating projects most firms assume that cash flows occur on a yearly basis, usually at the end of each year, or in some cases quarterly or monthly. The present example is typical in the sense that there is a considerable outlay at the start of the project, in 2002. Cash inflows begin in 2003 and continue until 2006.

b. *Sunk costs*. As already explained in Chapter 6, sunk costs are not incremental costs and therefore should not be included in the analysis. In this case the R & D costs of £500,000 have no bearing on the decision of whether to undertake the project or not, and should not be included as a cash flow.

c. *Opportunity costs*. These were also considered in Chapter 6, and were seen as being relevant to the decision-making process. Thus in the above situation the firm has spare capacity if it is capable of producing the new product with the same plant. This spare capacity may have other uses that could earn a profit for the firm; if this is the case then any net cash flows forgone by the decision to invest in the Slimfuel project can be regarded as opportunity costs and should be deducted from the cash flows directly generated by the project. We will assume for simplicity that there is no alternative use of the spare capacity, but we will need to return to this point in section 11.5, in the discussion regarding the evaluation of mutually exclusive and independent projects.

d. *Externalities*. This refers to any ***effects that the project may have on other operations of the firm***. For example, the production of Slimfuel may boost the sales of other products that are perceived as complementary, or it may detract from sales of existing products that are perceived as substitutes. Maxsport may be currently producing a similar product, Trimfuel, and net cash inflows from this product may be reduced by £2.5 million for the first two years of the project (not allowing for inflation).

e. *Net working capital*. It is often the case that investment projects require an increase in inventories, and sometimes in accounts receivable or debtors. Firms therefore have to consider not only the initial cost outlay in terms of fixed assets, but also any increase in current assets associated with the project. Maxsport may have to have inventories on hand of 10 per cent of the estimated cost of sales at the beginning of 2003. Therefore the initial cost outlay in 2002 will be:

C_0 = (£4 million × 5) + (10% × 40% × £60 million)

$\boldsymbol{C_0}$ **= £22.4 million**

This is assuming that the cash outflows associated with the inventory are related only to production costs, with no overheads, and that inventory levels are still at the 10% level at the end of the first year of operation.

f. *Taxes*. As mentioned under the identification of cash flows, the existence of taxes creates a complication because they are based on profit after allowing for depreciation. Since this measure of profit is not a cash flow, while taxes are a cash flow, the cash flows from a project have to be measured as follows:

$$CF_t = (R_t - C_t - D_t)(1 - T) + D_t \tag{11.2}$$

where CF_t represents incremental cash flows in a given time period, R_t represents incremental revenues, C_t represents incremental operating costs, D_t represents incremental depreciation, and T represents the firm's marginal tax rate. Thus in expression (11.2) the term $(R_t - C_t - D_t)$ represents profit before tax and the term $(R_t - C_t - D_t)(1 - T)$ represents profit after tax. Since depreciation does not represent a cash outflow, it then has to be added back to profit after tax in order to estimate the incremental cash flow. We can now apply this procedure to the first year of operation, 2003.

Year 1 (2003)

R_1 = (£2 × 30 million) − £2.5 million = £57.5 million
C_1 = 40% × £60 million + 30% × £60 million + £5 million = £47 million
D_1 = £20 million × 25% = £5 million (assuming a straight-line method of depreciation with no salvage value)
Profit before tax = £5.5 million
Profit after tax = £3.3 million (assuming a marginal tax rate of 40%)
CF_1 = £3.3 million + £5 million
$\boldsymbol{CF_1}$ **= £8.3 million**

The cash flows in the later years of operation are estimated after the discussion regarding adjustment for inflation.

g. *Inflation* Most countries experience inflation, meaning a continuing increase in the price level, to some degree. There are certain exceptions, Japan being the most notable in recent times, but even in cases of deflation or disinflation it is necessary to make allowances for changing prices in order to make correct capital budgeting decisions. As will be seen in section

11.4, the cost of capital is normally calculated on a market-determined basis, meaning allowing for inflation. Since we shall also see, in section 11.5, that cash flows are often discounted by this cost of capital in order to evaluate the investment project, it is also necessary to adjust the estimated cash flows to allow for inflation.[1] In reality this can be quite complicated, since not all cash flows are affected in the same way. For example, wage costs may increase more than material costs, and final prices may increase by a still different rate. Depreciation is normally not affected at all. We shall assume in SP11.1 that variable costs, overheads and prices all increase by 3 per cent per year. Therefore in the second and third years of operation the incremental cash flows are estimated as follows:

Year 2 (2004)

R_2 = £2.06 × 30 million) – £ 2.575 million = £59.225 million
C_2 = 40% × £61.8 million + 20% × £61.8 million + £5.15 million = £42.23 million
D_2 = £20 million × 25% = £5 million (assuming a straight-line method of depreciation with no salvage value)
Profit before tax = £11.995 million
Profit after tax = £7.197 million (assuming a marginal tax rate of 40%)
CF_2 = £7.197 million + £5 million
CF_2 = £12.197 million

Year 3 (2005)

R_3 = (£ 2.12 × 30 million) = £ 63.6 million
C_3 = 40% × £63.6 million + 20% × £63.6 million + £5.3045 million = £43.4645 million
D_3 = £20 million × 25% = £5 million (assuming a straight-line method of depreciation with no salvage value)
Profit before tax = £15.1355 million
Profit after tax = £9.0813 million (assuming a marginal tax rate of 40%)
CF_3 = £9.0813 million + £5 million
CF_3 = £14.0813 million

In year 4 of operation it is only necessary to produce 90 per cent of total sales because of starting inventories of 10 per cent of sales. Thus we have:

Year 4 (2006)

R_4 = (£2.18 × 30 million) = £65.4 million
C_4 = 40% × 90% × £65.4 million + 20% × £65.4 million + £5.4636 million = £42.0876 million
D_4 = £20 million × 25% = £5 million (assuming a straight-line method of depreciation with no salvage value)
Profit before tax = £18.3124 million
Profit after tax = £10.9874 million (assuming a marginal tax rate of 40%)
CF_4 = £10.9874 million + £5 million
CF_4 = £15.9874 million

Now that all the incremental cash flows have been estimated, the next stage of the capital budgeting process can be performed. Before this is examined, it is useful to consider a case study involving a situation where the nature of the benefits and cash flows is somewhat different.

Case study 11.1: Investing in a corporate fitness programme

Procal Co. is considering establishing a corporate fitness programme for its employees. The firm currently employs 500 workers, mainly managerial and administrative, in a number of offices in one local area. The type of programme being considered involves subsidizing employees by paying 50 per cent of any membership fees to a specific fitness centre. This subsidy represents the cost of operating the programme, while the main benefits expected are in terms of increased productivity, reduced sickness and absenteeism, and reduced staff turnover costs. The average salary paid to employees is £50, 000 per year, and employees work a forty-hour week for fifty weeks in the year. The firm has researched the extent of these costs and benefits and discovered the following information:

1. 10 per cent of employees can be expected to participate in the programme.
2. The membership fees are £400 per individual on a group scheme.
3. Workers who do not participate in any fitness programme suffer a drop in productivity of 50 per cent in their last two hours of work each day.
4. The normal sickness/absenteeism rate of eight days lost per year is reduced by 50 per cent for those workers on a fitness programme.
5. Staff turnover should be reduced from 20 per cent a year to 10 per cent.
6. Each new employee involves a total of twelve hours of hiring time.
7. Each new employee takes five days to train, and training is carried out in teams of five new employees at a time.
8. Each new employee has a productivity that is 25 per cent lower than average for their first six weeks at work.

Questions

1. Estimate the costs of operating the programme described above.
2. Estimate the benefits in terms of increased productivity.
3. Estimate the benefits from reduced sickness and absenteeism.
4. Estimate the benefits from reduced staff turnover.
5. What conclusion can you come to regarding the operation of the programme?

11.3 Risk analysis

In all the analysis so far it has been assumed that the cash flows are known with certainty. This is clearly an oversimplification; the existence of risk and uncertainty in the decision-making process was initially discussed in the context of the theory of the firm in Chapter 2, but we now need to discuss its implications in terms of investment analysis. The starting point of this discussion is an explanation of the nature of risk in the capital budgeting situation.

11.3.1 Nature of risk in capital budgeting

Previously we have discussed risk and uncertainty largely as if they related to the same situation, but it was mentioned in Chapter 2 that there was a technical difference between them. We can now consider these different types of scenario in more detail, and stress that it is important at this stage to differentiate between them.[2]

1 **Risk** refers to a decision-making situation where there are ***different possible outcomes and the probabilities of these outcomes can be measured in some way***.
2 **Uncertainty** refers to a decision-making situation where there are ***different possible outcomes and the probabilities of these outcomes cannot be meaningfully measured, sometimes because all possible outcomes cannot be foreseen or specified***.

As we shall see, different decision-making techniques have to be applied in each case.

It is also necessary to distinguish between different concepts of risk in terms of how they apply to the decision-making situation. There are three types of risk that relate to investment projects:[3] stand-alone risk, within-firm (or corporate) risk, and market risk.

a. *Stand-alone risk*. This examines the *risk of a project in isolation*. It is not usually important in itself, but rather as it affects within-firm and market risk. However, in the presence of agency problems, managerial decisions may be influenced by stand-alone risk; it may affect the position of individual managers, even though it does not necessarily affect the position of shareholders. Stand-alone risk is therefore the starting point for the consideration of risk in a broader context. The measurement and application of this aspect of risk is discussed in subsections 11.3.2 and 11.3.3.

b. *Within-firm risk*. This considers the *risk of a project in the context of a firm's portfolio of investment projects*. Thus the impact of the project on the variability of the firm's total cash flows is examined. It is possible that a project with high stand-alone risk may not have much effect on within-firm risk, or indeed may actually reduce the firm's within-firm risk if the project's cash flows are negatively correlated with the other cash flows of the firm. This issue will be discussed in more detail later.

c. *Market Risk*. This considers *a project's risk from the viewpoint of the shareholders of the firm, assuming that they have diversified shareholding portfolios*. It is sometimes referred to as *systematic risk*, as it relates to factors that affect the market as a whole. This is the most relevant concept of risk when considering the effect of a project on a firm's share price. Again it is possible that a project with high stand-alone risk may not represent high market risk to shareholders.

11.3.2 Measurement of risk

It was stated above that the concept of risk involves the measurement of probability. It is assumed that students already have an acquaintance with this topic, but it is worthwhile reviewing it here. Essentially, there are three approaches to measuring probability.

1. *Theoretical*. These probabilities are sometimes referred to as *ex-ante* probabilities, because they *can be estimated from a purely theoretical point of view, with no need for observation*. Such probabilities can therefore be calculated before any experiments or trials are conducted. Tossing a coin or throwing a die are classic examples. The probability of success, for example getting a head or a six, is given by the following expression:

$$P(\text{success}) = \frac{\text{total number of favourable outcomes}}{\text{total number of possible outcomes}} \tag{11.3}$$

It is assumed here that the coin or die is unbiased, that is all possible outcomes are equally probable. Unfortunately, such situations rarely arise in business management, unless we are considering the management of gambling casinos.

2. *Empirical*. These are sometimes referred to as *ex-post* probabilities, because they *can only be estimated from historical experience*. This is something that actuaries and insurance companies do; by amassing large amounts of data relating to car accidents for example, it is possible to estimate the probability of someone having an accident in any given year. These probabilities can then be revised according to age group, location of residence, occupation, type of car and so on. The probabilities are still calculated according to expression (11.3), but the outcomes can only be determined from empirical observation. It should be noted that the term 'favourable' does not imply any state of desirability, it merely refers to the fulfilment of a specified condition. In the example just quoted possible outcomes refer to the total number of motorists, while 'favourable' outcomes refer to the number of motorists having accidents, paradoxical though that may seem.

3. *Subjective*. In practice, managers often have to resort to estimating probabilities subjectively, for the simple reason that they are *dealing with circumstances that have never occurred exactly before*. They usually have some background of relevant past experience to help them make such estimates, but they cannot rely on the purely objective empirical approach. It is important to realize in later analysis in this chapter that the probabilities discussed are therefore somewhat imprecise because of the subjectivity involved.

Now that the measurement of probability has been discussed we can move on to the measurement of risk, and in particular the risk involved in investment situations.

a. Stand-alone risk

The measurement of risk can first be considered from the point of view of an individual project. There are various sources of risk and uncertainty in this context:

1 The initial capital cost of the project; in practice this may be spread over several years, increasing uncertainty.

2 The demand for the output from the project.
3 The ongoing operational costs of the project.
4 The cost of capital.

These sources can be illustrated by considering the situation in SP11.1.Often the most important variable where there is variability in terms of outcomes is the demand for the output, as shown by projected sales figure of 30 million bottles per year, and also by the projected price. As we have seen in the chapter on demand estimation, such forecasts are often associated with a considerable margin of error. This sales figure can really be regarded as an **expected value** (EV). Since it is assumed that students have a familiarity with this concept and with the topic of probability in general, only a brief review is given here. The definition of an expected value is ***the sum of the products of the values of the different outcomes multiplied by their respective probabilities***:

$$\boldsymbol{EV = \sum p_i X_i} \tag{11.4}$$

Let us assume that there are considered to be three possible sales values, 20 million, 30 million and 40 million, and that the probabilities of each outcome are estimated (subjectively in this case) to be 0.25, 0.5 and 0.25 respectively. Therefore the expected value of sales is given by:

$$\begin{aligned} EV &= (0.25 \times 20\text{m}) + (0.5 \times 30\text{m}) + (0.25 \times 40\text{m}) \\ &= 5\text{m} + 15\text{m} + 10\text{m} \\ &= 30\text{m} \end{aligned}$$

This is a simplified case since it is assumed that the probability distribution of outcomes is discrete. A more realistic scenario is when the distribution is continuous, with a theoretically limitless number of possible outcomes. However, the expected value concept is still applicable to such a distribution, and this situation is represented in Figure 11.1.

The distribution in Figure 11.1 is assumed to be symmetrical but this need not be the case. Once the distribution of outcomes is estimated, not only can the expected value of the distribution be calculated, as above, but also measures of its variability. The standard deviation is the most common measure used here, and the higher the standard deviation of sales the greater the risk of the project in stand-alone terms. The general formula for calculating the standard deviation is given by:

$$\sigma_x = \sqrt{\left(\sum p_i X_i^2\right)} \tag{11.5}$$

In the above example the standard deviation is given by:

$$\sigma_x = \sqrt{[(0.25 \times 10^2) + (0.5 \times 0^2) + (0.25 \times 10^2)]} = 7.071 \text{ million}$$

So far we have concentrated on the uncertainty related to demand. In some projects, especially those where major capital expenditure is involved, there may be much uncertainty regarding this initial cost. In projects like the

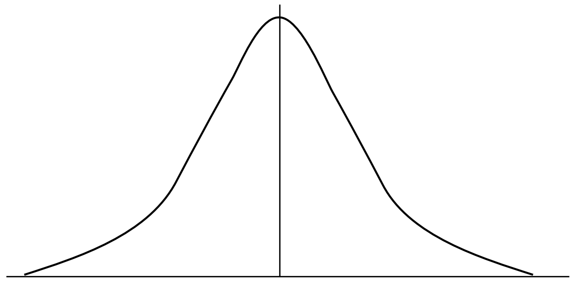

Figure 11.1. Continuous distribution of sales outcomes.

Channel Tunnel it has not been unknown for the eventual cost to be as much as five times the original estimate.

b. Within-firm risk

When a project is considered in the context of corporate risk it is important to consider the correlation between the project's cash flows and those of the firm as a whole. In practice this is often done subjectively: if the project is in the same line of business as the firm's other projects, then there will be high positive correlation and high stand-alone risk will also involve high corporate risk. On the other hand, if the project is in a different line of business then the correlation may be low and the firm's corporate risk may not be much affected. It is even possible, as mentioned earlier, that if the project is in a business area whose prospects are opposite to those of the firm's main line of business, that correlation may be negative and high stand-alone risk may actually reduce corporate risk. This situation is rare however.

c. Market risk

The relationship between stand-alone risk and market risk now needs to be discussed. Market risk is the most relevant type of risk as far as shareholders are concerned. It is also possible to measure this type of risk using an objective, though not necessarily accurate, method. This involves using one of the most important models in financial analysis, the **capital asset pricing model** (CAPM). The CAPM ***describes the risk–return relationship for securities, assuming that these securities are held in well-diversified portfolios***. There are a number of other assumptions involved in the model, but for the sake of simplicity these will be largely ignored in this text. Essentially the model shows that the higher the risk to the investor the higher the return required to compensate for that risk. Some government securities (depending on the government) are regarded as risk-free, and pay the risk-free rate k_{RF}. This then represents the minimum rate of return on investors' funds, and rates of return on other investments are correspondingly higher according to the amount of risk associated with holding that firm's securities. The general relationship is shown by the **security market line** (SML), which is depicted in Figure 11.2.

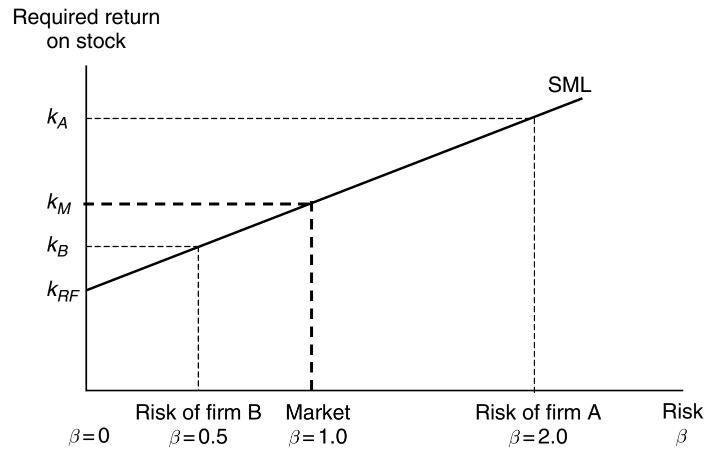

Figure 11.2. The security market line (SML).

Empirically the SML appears to be approximately linear. The problem with which we are now faced is: how can the risk of a security be objectively measured? This involves the concept of a **beta coefficient**. As seen in Chapter 4, a beta coefficient refers to the slope of a regression line. In the current financial context involving the SML, the beta coefficient represents ***the slope of the regression line between the returns on an individual security and the returns on the market as a whole***. This line is called the **characteristic line**. An example is given in Figure 11.3, for a firm with a beta coefficient of 2.0. In this case, observations are taken over a five-year period. In year 1 the return on the stock was about 12%, while the average return on the market was about 4%. In year 2, on the other hand, the stock gave a negative return of 4%, and the market also gave a negative return of 4%. From this illustration it can be seen that the greater the variability, or volatility, of the security the steeper the characteristic line and the greater the beta coefficient. The value of beta thus measures the relative volatility of the security compared with an average stock; a security with a beta of 1 has the same volatility as the market as a whole, securities with a beta more than 1, as in Figure 11.3, are more volatile than the market as a whole, while securities with a beta less than 1 are less volatile than the market as a whole. More specifically, a security with a beta coefficient of 2.0 has generally twice the volatility of the average stock; if the market returns rise by 1%, then such a security should find its return rising by 2%. Likewise, if market returns fall by 1%, the return on the security should fall by 2%.

The concept of the beta coefficient can now be applied to the CAPM. It can be seen from Figure 11.2 that the return on the market as a whole is given by k_M. The equation of the SML can also be seen. The intercept is given by the risk-free rate, k_{RF}, and the slope is given by $(k_M - k_{RF})$, by comparing the return with no risk with the return on the market. Thus the equation of the SML is:

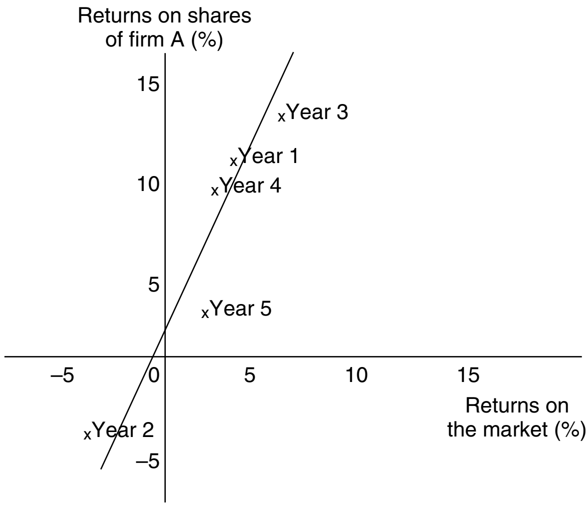

Figure 11.3. Calculation of beta coefficients.

$$k_i = k_{RF} + (k_M - k_{RF})\beta i \tag{11.6}$$

where k_i represents the rate of return on any individual security. We shall see that the CAPM model is also useful in considering the cost of capital in the next section. Finally, in Section 11.5, the aspects involving the application of the measurement of risk to decision-making will be examined.

11.4 Cost of capital

The cost of capital is an important concept for the firm, not just for evaluating investment projects but also for maximizing shareholder value in general. It was seen in Chapter 3 that a firm should discount its expected cash flows by its cost of capital in order to compute the value of the firm. We now need to consider this concept of the cost of capital in more detail.

11.4.1 Nature and components

There are essentially two ways of considering the cost of capital. From the point of view of the firm, representing the demand side, the cost of capital is what the firm has to pay for its sources of funds. These funds, which are liabilities on the balance sheet, are then used to finance new investments, which represent assets on the balance sheet. From a supply point of view, the cost of capital represents the return that investors, who provide the firm with funds, require in order to lend the firm money or buy its shares.

Strictly speaking the capital involved represents all the firm's liabilities, including short-term debt and other aspects of working capital. In practice, however, the main sources of funds that are relevant for most firms when considering investment projects are long-term debt (mainly bonds) and common equity.

11.4.2 Cost of debt

It is helpful, as usual, to make some simplifying assumptions in order to calculate this cost. We shall assume that only one form of debt is used, twenty-year bonds, that the interest rate on these bonds is fixed rather than floating, and that the payment schedule for this debt is known in advance of the issue. Most new bonds are sold at par value, meaning face value, and therefore the coupon interest rate is set at the rate of return required by investors. If we take a normal bond with a par value of £1,000 and a coupon rate of 8 per cent, the cost of debt capital can be obtained using a variation of the present-value formula in Chapter 2:

$$V_0 = \sum \frac{I_t}{(1+k_d)^t} + \frac{P}{(1+k_d)^n} \tag{11.7}$$

where V_0 is the current market value of the bond, P is the par value, k_d is the cost of debt, and I_t represents the annual interest payment in period t (the formula has to be slightly modified if interest payments are semi-annual). In the above example we obtain the following:

$$1{,}000 = \frac{80}{(1+k_d)} + \frac{80}{(1+k_d)^2} + \frac{80}{(1+k_d)^3} + \cdots + \frac{1{,}000}{(1+k_d)^{20}}$$

The value of k_d cannot be solved directly from this equation, and can only be estimated iteratively, but it can be shown on a calculator programmed to perform this kind of calculation that the cost of debt is 8 per cent, the same as the coupon rate of interest.

Two further complications can now be introduced. The most fundamental one concerns tax. The cost of debt shown above is a pre-tax cost; however, interest payments are deductible from the firm's taxable income. Therefore the firm's after-tax cost of debt is given by multiplying the pre-tax cost by 1 minus the firm's marginal tax rate:

$$k_a = k_d(1-t) \tag{11.8}$$

Assuming as before a marginal tax rate of 40 per cent:

$$k_a = 8(1-0.4) = 4.8 \text{ per cent}$$

The final complication that should be mentioned concerns flotation costs. The issuing institution, normally an investment bank, charges a fee for its services to the firm. If this is 1 per cent of the issue, this cost needs to be deducted from the proceeds of the sale of the bonds in order to calculate the cost of debt. In this case the firm would only receive £990 for each bond sold, so the value of V_0 in (11.7) would now be 990.

11.4.3 Cost of equity

In a similar way to the cost of debt the cost of equity represents the equilibrium or minimum rate of return required by the firm's common shareholders. These funds can be obtained in two ways: internally, from retained earnings, and externally, from issuing new stock. These two sources are now discussed.

a. Internal sources

The cost of retained earnings represents an opportunity cost to investors. These earnings could be paid out in the form of dividends to investors, who could then reinvest the funds in other shares, bonds or property. Unless the retained earnings can earn at least the same return within the firm as they can outside the firm, assuming the same degree of risk, it is not profitable to use them as a source of funds.

There is no simple way to calculate this internal cost; there are several alternative approaches, which are not mutually exclusive. Two main approaches will be discussed here: the **capital asset pricing model** (CAPM), and the **dividend valuation model** (DVM).

1. The capital asset pricing model (CAPM). The general nature of this model was discussed in the previous section. It was seen that the variability, or volatility, in returns to an individual security can be divided into two components: that part which is related to the corporate risk of the firm, sometimes called **unsystematic risk**, and that part which affects the market as a whole, the **systematic risk**. Rational investors will diversify their portfolios so as to eliminate unsystematic risk; such risk carries no benefit in terms of additional return, since investors can obtain the same returns through holding a diversified portfolio of securities with similar corporate risk, but with reduced market risk.

The cost of equity, in terms of retained earnings, can now be estimated using the equation of the SML in (11.6). Thus, assuming $k_M = 10$ per cent , $k_{RF} = 6$ per cent and a beta coefficient of 2.0, the cost of equity would be:

$$k_e = 6 + (10 - 6)2 = 14 \text{ per cent}$$

While the CAPM may appear to be an objective and precise method for estimating the cost of capital it is subject to a number of drawbacks, which arise from the nature of the assumptions underlying the model. One of the most important of these is the use of beta coefficients based on historical data. In practice this is the only objective method for estimating such coefficients, but conceptually the cost of capital should be based on a model involving expected beta coefficients for the future. It is beyond the scope of this text to examine these assumptions and drawbacks in more detail, but they are discussed in most texts on financial management.

2. The dividend valuation model (DVM). This model is also known as the DCF model since it involves the now familiar present-value formula from Chapter 2, and is similar to expression (11.7). The value of a shareholder's wealth is the sum of expected future returns, discounted by their required rate of return. These returns come in the form of dividends and an increase in the market value of the firm's shares. Thus the present value of the share is given by:

$$V_0 = \sum \frac{D_t}{(1+k_e)^t} + \frac{V_n}{(1+k_e)^n} \tag{11.9}$$

where D_t is the dividend paid by the firm in period t. Since the future value of the share, V_n, is in turn determined by the sum of expected future dividends, equation (11.9) can be rewritten as the sum to infinity of all expected future dividends:

$$V_0 = \frac{\sum D_t}{(1+k_e)^n} \tag{11.10}$$

We now make the assumption that the dividends of the firm grow at a constant rate of g per year. Thus the value of the shares is given by:

$$V_0 = \frac{D_1}{(1+k_e)} + \frac{D_1(1+g)}{(1+k_e)^2} + \frac{D_1(1+g)^2}{(1+k_e)^3} + \cdots \tag{11.11}$$

where D_1 is the dividend that is expected to be paid in the following period. This is a geometric series which can be summed to infinity as long as the terms become smaller, in other words as long as $g < k$. The sum is given by:

$$V_0 = \frac{\frac{D_1}{(1+k_e)}}{1 - \frac{(1+g)}{(1+k_e)}} = \frac{D_1}{k_e - g} \tag{11.12}$$

This equation can be rearranged to solve for the cost of equity as follows:

$$k_e = \frac{D_1}{V_0} + g \tag{11.13}$$

For example, if a firm has a current share price of £20, the dividend next year is expected to be £1.20 and dividends have been growing on average at 4 per cent per year, then the cost of equity is given by:

$$k_e = \frac{1.20}{20} + 0.04 = 0.10, \text{ or 10 per cent}$$

Just as the CAPM has its problems, so does the DVM also have drawbacks. Its main failing is essentially the same as with the CAPM: it looks backwards rather than forwards. While historical data can be used to estimate the average growth rate of dividends over the last ten or twenty years say, such information is not a reliable indicator of future dividend growth rates. Furthermore, current share prices can be highly volatile, and many firms do not pay dividends at all if management believes that the funds can be more profitably reinvested in the firm than returned to shareholders. It can therefore readily be seen that many fast-growing high-tech firms would on this basis have very uncertain estimates of their cost of capital.

b. External sources

It is more expensive for a firm to raise equity externally rather than internally for two reasons:

1 There are flotation costs, as discussed with the cost of debt.
2 New shares have to be sold at a price lower than the current market price, in order to attract buyers. The reason for this is that the current price normally represents an equilibrium between existing demand and supply. A new issue involves an increase in supply, thus reducing the equilibrium price.

The result of these factors is that equation (11.13) has to be modified in order to provide an estimate of the cost of external equity as follows:

$$k_{ne} = \frac{D_1}{V^*} + g \tag{11.14}$$

where k_{ne} represents the cost of new equity, and V^* is the net proceeds to the firm from the new issue (per share) after deducting flotation costs.

11.4.4 *Weighted average cost of capital*

Now that the two main components of the cost of capital have been examined, the overall cost of capital to the firm can be estimated. Since firms generally rely on both debt and equity to finance new projects, some kind of average cost is involved. However, the financial managers need to consider two factors in calculating this cost:

1 Historical costs of capital are not relevant; it is the marginal cost of capital, meaning the cost of raising new capital, which should be used. As already seen, this involves more uncertainty.
2 The relative proportions of debt and equity to be raised need to be estimated; since again this may not be known with certainty beforehand, it is common practice for managers to use the proportions that have been determined in the firm's long-term capital structure. These proportions need to be estimated in order to provide weights for the costs involved.

Once these two issues have been addressed the firm can estimate its weighted average cost of capital (WACC) as follows:

$$k = \frac{D}{D+E} \times k_a + \frac{E}{D+E} \times k_e \tag{11.15}$$

where D and E refer to the amounts of new debt and equity involved.

For example, if a firm estimates its costs of debt and equity to be 4.8 per cent and 10 per cent, and that 30 per cent of its new capital will be from long-term debt, its WACC will be:

$$k = 0.3(4.8) + 0.7(10) = 8.44 \text{ per cent}$$

Now that the methods for estimating the cost of capital have been examined, we can consider how the cost of capital is relevant in the capital budgeting process.

11.5 Evaluation criteria

Ultimately, managers must decide whether to invest in new projects or not. Once the preliminary stages of estimating the cash flows, assessing the relevant risks and estimating the cost of capital have been performed, some criterion or decision rule must be applied in making the investment decision. There are two main criteria that can be used here, **net present value** and **internal rate of return**, although firms sometimes also use other criteria, usually on a supplementary basis. These criteria are now discussed, and further consideration is given to risk and uncertainty, in terms of how these affect investment decisions.

11.5.1 Net present value

The concept of net present value (NPV) again takes us back to Chapter 2. In that context it was applied to the valuation of shareholder wealth; expected future profits were discounted and summed in order to find the value of the firm, as shown in equation (2.1). The same concept can be applied to an individual investment project, in this case the net present value of the project being the sum of discounted net cash flows (DNCF), as follows:

$$NPV = \sum \frac{NCF_t}{(1+k)^t} \qquad (11.16)$$

The cost of capital is used to discount the cash flows. It should now be clear that any project that has a positive NPV will automatically increase the value of the firm and therefore should be undertaken. Likewise, any project that has a negative NPV will decrease the value of the firm and should not be undertaken. However, this simple rule only applies to **independent** projects, and we now need to distinguish between two main categories of project:

1 *Independent*. These are projects where *the operation of one project has no bearing on whether the other project(s) should be carried out*.
2 *Mutually exclusive*. These are projects where *the operation of one project automatically eliminates the need for the other one(s)*. This situation occurs when there are alternative ways of achieving the same objective, this issue being discussed in more detail in subsection 11.5.5. The rule in this case is that if two or more projects have a positive NPV, managers should select the project with the highest NPV.

We can now develop the example given earlier in section 11.2, involving Maxsport. The cost of capital is assumed to be 8.44 per cent, as estimated in the previous section. Table 11.1 shows the estimated net cash flows and the

Table 11.1. NPV calculations

Year	NCF (£m)	DNCF (£m) ($k = 8.44\%$)
2002	(22.4)	(22.4)
2003	8.3	7.654
2004	12.197	10.372
2005	14.0813	11.043
2006	15.9874	11.562
Total		18.231

expected discounted net cash flows from the investment project, with the sum of the latter giving the NPV of £18.231 million. The conclusion is that the project should be accepted if it is independent, since it is expected to increase shareholder wealth by £18.231 million. It should only be rejected if it is mutually exclusive with another project that has a higher NPV.

11.5.2 *Internal rate of return*

The concept of the internal rate of return (IRR) on an investment project corresponds to the concept of **yield to maturity** (YTM) for investors buying securities. The **IRR** is defined as ***the discount rate that equates the present value of project's expected net cash flows to zero***. In mathematical terms it is the interest rate, i, that satisfies the following equation:

$$\sum \frac{NCF_t}{(1+i)^t} = 0 \qquad (11.17)$$

Thus the IRR calculation essentially makes use of the same equation (11.16) as the NPV calculation, but instead of taking the value of k as given (the cost of capital) and calculating the value of the NPV, it takes the value of NPV as given (zero) and calculates the discount rate.

The criterion for acceptance in this case is that any project that has an IRR greater than the cost of capital should be accepted, since this will generate a surplus that will increase shareholder wealth. Likewise, any project that has an IRR less than the cost of capital will reduce shareholder wealth. This criterion for independent projects must be slightly modified for mutually exclusive projects; in this situation the project with the highest IRR should be accepted, assuming this IRR is greater than the cost of capital.

The solution of the equation to calculate the IRR is, however, more difficult than finding the NPV. In the case of Maxsport we obtain the following equation:

$$-22.4 + \frac{8.3}{(1+i)} + \frac{12.197}{(1+i)^2} + \frac{14.0813}{(1+i)^3} + \frac{15.9874}{(1+i)^4} = 0 \qquad (11.18)$$

This kind of equation is best solved using a financial calculator or computer. When this is done the solution obtained is that $i = 0.3744$, or **37.44** per cent.

11.5.3 Comparison of net present value and internal rate of return

At this stage certain questions may well be asked regarding the two criteria described above:

1 Do both criteria result in the same investment decision in all cases?
2 If not, which criterion is better, or are they both equally valid?

The answer to the first question is that for independent projects the two criteria will always yield the same result. Any project with a positive NPV will automatically have an IRR greater than the cost of capital, and any project with a negative NPV will automatically have an IRR less than the cost of capital. Complications arise, however, with mutually exclusive projects. It is possible for a conflict to arise between the two approaches if the two projects are of different size, meaning that their initial cost outlays are different, or if the timings of the cash flows are different, with one project getting high early returns while another project gets high returns later on. In these situations a project with a higher IRR than another project will not necessarily have a higher NPV if the cost of capital is much less than the IRR. This situation arises because of different assumptions made by the two approaches regarding the opportunity cost of reinvestment of cash inflows. These assumptions are summarized below:

1 The NPV approach assumes that inflows can be reinvested at the cost of capital.
2 The IRR approach assumes that inflows can be reinvested at the same rate as the IRR.

These assumptions are inherent in the mathematical calculations for each measure.

This leads us to the second question. It can be seen that the opportunity cost for reinvestment is in fact the cost of capital, assuming that this remains the same for the future, meaning that any future projects can be financed at this same rate. For example, if the cost of capital is 8 per cent this is the opportunity cost for reinvestment purposes, even if projects arise in the future with IRRs of 20 per cent; these future projects can still be financed at a cost of 8 per cent.

Our conclusion therefore is that *the NPV criterion is superior to the IRR criterion*, and that if a conflict arises between the two approaches for mutually exclusive projects, the NPV approach should be used. Having said this, it should also be stated that managers often prefer the IRR approach, since it indicates profitability in percentage terms rather than in money terms, and this is often a more meaningful indicator when comparing different projects.

11.5.4 Other criteria

The two approaches discussed above are by far the most common in sophisticated capital budgeting analysis. However, there are other approaches that are sometimes used by managers, varying from very simple methods to quite complex ones. Four of these are now described briefly.

a. *Payback method*. This is by far the simplest criterion. It simply calculates *the length of the period it takes for cash inflows to exceed cash outflows*, and compares this with some basic yardstick, for example four years. If the **payback period** is shorter than the yardstick the project is accepted; if it is longer than the yardstick the project is rejected. In the case of Maxsport the payback period is a little over two years, so the project would be accepted if the yardstick were four years. There are a number of obvious drawbacks with this approach: it fails to take into account the time value of money by discounting, it fails to consider cash flows after the payback period, and the selection of the yardstick is entirely arbitrary. However, because of its simplicity, it is still popular with managers, at least as a supplementary guide to decision-making.

b. *Discounted payback*. This is essentially similar to the ordinary payback method, the only difference being that the cash flows are discounted at the cost of capital. However, the approach still suffers from the other problems mentioned above.

c. *Profitability index (PI)*. This is sometimes referred to as the benefit–cost ratio. It is calculated as follows:

$$PI = \frac{\text{present value of benefits}}{\text{present value of costs}} \tag{11.19}$$

where benefits refer to cash inflows and costs refer to cash outflows. The criterion for acceptance is that the PI should be greater than one, meaning that the present value of the benefits exceeds the present value of the costs. In the case of Maxsport, the PI $= 40.631/22.4 = 1.814$. This project would therefore be accepted, if it were an independent project. For mutually exclusive projects the one with the highest PI would be accepted.

With independent projects the PI approach will always yield the same result as the NPV and IRR methods. For mutually exclusive projects, conflict is again possible when comparing projects of different sizes. A large project may have a higher NPV than a smaller project, but a lower PI. Again the NPV method should take precedence in these cases.

d. *Modified internal rate of return (MIRR)*. This approach is designed to eliminate the problem discussed earlier with the IRR, that it assumes cash inflows can be reinvested at the same rate as the IRR. It is also more complex than the methods discussed so far. The MIRR is the interest rate that equates the present value of the cash outflows with the present value of the **terminal value** of the cash inflows. The terminal value is ***the future value of the cash inflows at the end of the project, assuming that the inflows are reinvested at the cost of capital***. Thus the terminal value of the cash inflows for Maxsport is given by:

$$8.3(1 + .0844)^3 + 12.197(1 + .0844)^2 + 14.0813(1 + .0844) + 15.9874 = 56.184$$

We then have to solve the equation:

$$22.4 = \frac{56.184}{(1+i)^4}$$

This gives: $(1+i)^4 = 2.5082, i = 0.2585,$ or **25.85 per cent**.

This measure of the MIRR is superior to the ordinary IRR as an indicator of a project's real rate of return, but it can still give results which conflict with those using the NPV criterion when comparing mutually exclusive projects of different sizes. Once again, only the NPV approach should be used in these circumstances, as it is the only measure that gives a direct indicator as to how the value of the firm is affected by the investment project.

11.5.5 Decision-making under risk

In the analysis in this section so far we have ignored the existence of risk. The measurement of risk was discussed in section 11.3, but we have not yet examined how measures of risk can be incorporated into the decision-making process. Four techniques will be examined in this context: sensitivity analysis, scenario analysis, decision tree analysis, and simulation.

a. Sensitivity analysis

In considering the nature and measurement of risk it was seen that a number of variables that determine a project's profitability are not known with certainty, but have some variability which might be expressed in terms of a probability distribution with a measurable standard deviation. Sensitivity analysis examines the responsiveness of a project's NPV and IRR to a given change in a particular input variable. We might want to know the sensitivity in response to a 10 per cent fall in sales below the expected level, or to a 20 per cent increase in operating costs. Sometimes these aspects of responsiveness are shown graphically, where the whole relationship between the input variable and the NPV is shown. Projects may have much greater sensitivity to changes in some input variables than to changes in others; for example, a project may be much more sensitive to changes in sales volume than to changes in the cost of capital. In general, projects showing greater sensitivity demonstrate more risk.

b. Scenario analysis

This is really a development of sensitivity analysis. The development is that the amount of the likely variation in a variable is specified, often in terms of its probability distribution, as well as the effect of this variation. Thus worst- and best-case scenarios are often depicted; for example, a worst-case scenario for sales volume might be 20 million units, with a probability of 25 per cent, and a best-case scenario might be 40 million units, again with a probability of 25 per cent. The most likely outcome of 30 million units may have a probability of 50 per cent. The same scenarios and their probabilities can be estimated for other input variables. Resulting worst-case and best-case NPVs can then be calculated, taking into account all the worst-case input variables and all the best-case input variables. The expected NPV of the project can then be calculated, along with its standard deviation.

Scenario analysis is a widely used technique among managers, but suffers from two main shortcomings:

1 It assumes discrete probability distributions for the input variables, whereas continuous distributions are more realistic. The approach, therefore, usually considers only a small number of possible outcomes; furthermore, it is usually unlikely that all the worst outcomes for the different variables will occur simultaneously, and the same applies to the best outcomes.
2 The probabilities of the different scenarios are usually estimated subjectively and are therefore prone to considerable error.

c. Decision tree analysis

This approach shares a number of characteristics with scenario analysis. Different **states-of-nature** are described, such as high sales or low sales of a product, with associated probabilities. Expected monetary values (EMV) are then calculated, which correspond to expected NPVs in multiperiod situations, and decisions are made based on maximizing EMV.

The main use of decision tree analysis is in situations where sequential decision points are involved, often over many periods. For example, a firm like Maxsport may face an initial decision regarding whether to conduct a market research survey or not. Depending on the results of such a survey they may choose to test-market the product, launch the product nationally, or drop the project. If they test-market the product, they may then face a choice regarding scale of operation, and so on. The probabilities of different states-of-nature may be conditional on previous events. Thus the probability of high sales may depend on the results of the market survey or of the test-marketing process. The objective of the analysis is to calculate the EMVs at each state-of-nature node, and thus determine the optimal decision-making path. A simple example of the use of decision tree analysis follows.

SP11.2 Decision tree analysis

Maxsport is now considering whether to test-market its new product, Slimfuel. The results of the test-marketing can then be used to decide whether to launch the product nationally or drop it. Alternatively, the firm can skip the test-marketing stage, which costs £3 million, and go straight to national launch. Maxsport estimates that the probability of good test-marketing results is 0.6 and the probability of bad results is 0.4. If the results are good, management estimates that the probability of high sales is 0.8 and the probability of low sales is 0.2. If the results are bad, management estimates that the probability of high sales is 0.3 and the probability of low sales is 0.7. High sales in the situation where test-marketing is conducted represents an estimated NPV of £20 million and

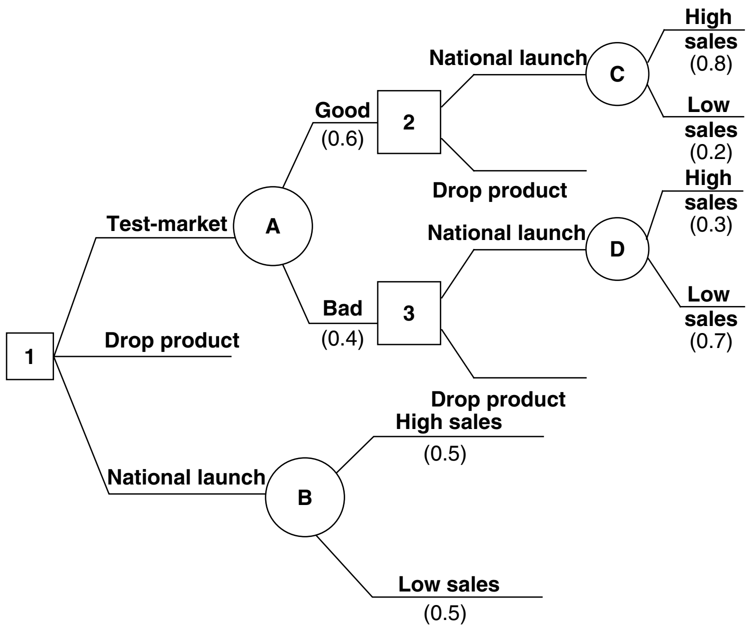

Figure 11.4. Decision tree for Maxsport.

low sales means an estimated NPV of −£10 million. These NPVs do not take into account the cost of test-marketing. If no test-marketing is conducted, there is reckoned to be a fifty-fifty chance of high or low sales, with the NPV of high sales being £23 million and the NPV of low sales being −£7 million. These values are higher than if test-marketing is performed because of the greater advantage gained over competitors. If the product is dropped there is zero NPV from that stage.

Draw a decision tree representing the situation, and determine the optimal decision path.

Solution

There are two types of node in the decision tree (Figure 11.4):

1 *Decision nodes*. These are shown by squares, and are numbered.
2 *State-of-nature nodes*. These are shown by circles, and are lettered; the following states-of-nature have their probabilities shown in brackets.

It is important to realize that ***the decision tree is drawn from left to right, but the analysis of it is performed backwards, working from right to left***, meaning from the end of the tree to the beginning. The first stage in the analysis is to calculate the expected NPVs at each state-of-nature node. These NPVs are calculated from the point of view of the start of the project, meaning from decision node 1.

$$\text{At C: NPV} = 0.8(20) + 0.2(-10) - 3 = 11 \text{ million}$$
$$\text{At D: NPV} = 0.3(20) + 0.7(-10) - 3 = -4 \text{ million}$$

We can now start to determine the decision path, bearing in mind that dropping the product after test-marketing results in the NPV of –£3 million.

At 2: if test-marketing results are good, go for national launch.
At 3: if test-marketing results are bad, it is better to drop the product because the negative NPV of £3 million is preferable to the negative NPV of £4 million if the product is launched.

The expected NPVs at A and B can now be calculated.

$$\text{At A: NPV} = 0.6(11) + 0.4(-3) = 5.4 \text{ million}$$
$$\text{At B: NPV} = 0.5(23) + 0.5(-7) = 8 \text{ million}$$

Therefore, at decision node 1 the firm should go straight for a national launch and skip the test-marketing process. This means that the decisions at nodes 2 and 3 will not arise.

d. Simulation

One common feature regarding sensitivity analysis, scenario analysis and decision tree analysis is that they all simplify the decision-making situation by restricting the key decision and state-of-nature variables to certain discrete values. In SP11.2, for example, only a national launch and test-marketing were considered, whereas in practice various scales of investment might be possible, on a multiperiod basis. Likewise, only the states-of-nature of high sales and low sales were considered. A more realistic situation is where such variables can assume any value according to some continuous probability distribution. This situation often makes the mathematical aspects of analysis intractable, but it is still amenable to analysis by computer simulation.

Simulation approaches, sometimes referred to as **Monte Carlo methods** because of their original application to casino gambling, have become widely used in various business situations in recent years, as software packages have become more powerful and prolific. In terms of capital budgeting, the following stages are involved:

1 Specify probability distributions for each variable in the analysis, such as sales volume, price and unit costs; this involves specifying the means and standard deviations of the distributions, and also their shapes.
2 Select random values for each variable, according to their probability distributions. This is performed by the software package.
3 Calculate the resulting net cash flows and NPV for this set of values.

4 Repeat the previous two steps a large number of times, usually 1,000 or so, thus building up a probability distribution for the NPV.
5 The expected value and the standard deviation of the NPV can then be calculated.

Simulation techniques are not without their problems. First of all, it can be difficult to perform the first stage; the characteristics of the relevant probability distributions often have to be estimated subjectively. Second, the distributions may not be independent of each other because some of the variables may be correlated; these correlations can be specified as inputs into the software package in selecting random values for the variables, but it is difficult to estimate the values that should be specified.

To conclude this subsection, we should mention another problem that is common to all the techniques discussed above. Although they show the effect of risk on the NPV, in terms of giving a standard deviation or similar measure, they do not in themselves provide any definitive decision rule. For one thing they only provide a measure of stand-alone risk, and, as we saw in section 11.3, it is the market risk that is the primary concern of well-diversified shareholders. It has also been seen that high stand-alone risk does not necessarily lead to high market risk; the relationship depends on the correlation between the project's returns and the returns on other assets owned by the firm's shareholders. Therefore, in order to fully incorporate risk into the decision-making process, a **risk-adjusted cost of capital** (RACC) should be applied. The RACC ***estimates this correlation so that the effect on market risk can in turn be estimated. This can then be reflected in the cost of capital that is used to discount the cash flows and calculate the NPV***. In this whole procedure it is obvious that there are many possible sources of inaccuracy. In practice, firms often use a general rule, such as: for projects with high stand-alone risk add 2 per cent to the cost of capital, while for projects with low stand-alone risk subtract 2 per cent. It is therefore a bold financial manager who can estimate a major investment project's NPV with a high degree of confidence.

11.5.6 Decision-making under uncertainty

Sometimes managers are reluctant to give even subjective estimates of the probabilities of various events or states-of-nature. This tends to be the case when there is very little information to go on regarding the success or failure of a project, because the characteristics of the situation are entirely new and cannot be easily compared with previous projects. Table 11.2 illustrates this situation, where payoffs can be estimated but not their associated probabilities.There are a number of decision rules that can be used in this type of situation, but there is no single best criterion that is widely used. The two main rules are now discussed.

a. *Maximin criterion*. This criterion concentrates entirely on the worst possible outcome, meaning the minimum payoff, from each possible decision, and

Table 11.2. Payoff matrix under uncertainty

		States of nature	
		Success	Failure
Alternative decisions	Invest	80	−60
	Do not invest	0	0

Table 11.3. Regret matrix under uncertainty

		States of nature	
		Success	Failure
Alternative decisions	Invest	0	60
	Do not invest	80	0

selects the decision that maximizes this minimum payoff. Given the situation in Table 11.2, the minimum payoff from investing is −60 and the minimum payoff from not investing is 0. Therefore the maximin criterion would dictate the decision not to invest. It can be seen that this is a very conservative decision rule, and many people would find it inappropriate in most cases. For example, taking an everyday situation, this decision rule would mean that we would never cross a road; taking the decision whether to cross a road or not to cross, crossing would always involve a lower possible payoff (death) than not crossing. Note, however, that this is really an inappropriate situation for using such a criterion; although people do not actually consciously estimate probabilities of success or failure in crossing a road, it is quite possible to do so on the basis of historical experience. This issue is discussed further in the next chapter, in connection with government policy.

b. *Minimax regret criterion*. Regret in this context refers to opportunity cost. The opportunity cost of each decision and each state-of-nature is calculated, and this can be shown in a regret or opportunity cost matrix. The regret matrix corresponding to Table 11.2 is shown in Table 11.3. The decision rule in this case is to select the decision that minimizes the maximum regret or opportunity cost; since the maximum regret from investing is 60 and the maximum regret from not investing is 80, the decision in this case would be to invest. As with the previous criterion, a number of objections can be made to its use.[4]

11.6 The optimal capital budget

We have assumed so far in this chapter that each investment project can be evaluated separately, according to the firm's cost of capital. However, the total capital budget and the cost of capital should be calculated simultaneously, in the same sort of way that price and quantity are determined simultaneously by demand and supply. The resulting cost of capital can then be applied to each individual project. The demand for capital is shown by a firm's **investment opportunity schedule**, and the supply of capital is given by the **marginal cost of capital schedule**. These are now discussed in turn.

11.6.1 The investment opportunity (IO) schedule

This shows the relationship between the internal rates of return on different potential projects and the amount of new capital required. The concept, and the steps involved in deriving it, is best explained by an example. We shall derive an IO schedule for Maxsport, assuming for the sake of simplicity that the different projects considered all involve the same degree of risk. The following steps have to be performed:

1 Identify all the various possible capital projects that the firm can feasibly undertake, specifying which are independent and which are mutually exclusive.
2 Estimate the initial cash outlays, net cash flows and IRRs for each of these potential projects. This is shown in Table 11.4. Note that we do not calculate NPVs at this stage (or MIRRs), since these require a knowledge of the cost of capital, which is what we are trying to estimate.
3 These IRRs are then plotted in descending order against cumulative initial outlay. This is shown for Maxsport in Figure 11.5. Note that there are two IO schedules, since projects B and C are mutually exclusive, and these overlap in places.

11.6.2 The marginal cost of capital (MCC) schedule

The concept of the weighted average cost of capital (WACC) has already been discussed in section 11.4, and it was also stated at that point that it was the cost of new capital, not the historical cost of capital, that was important. However, this cost is not constant; as the firm tries to raise more and more capital it will find that this cost of new capital will rise. There are two main reasons for this:

1 The cost of equity will rise as the firm is forced into issuing new equity rather than relying on retained earnings. As has already been seen, the cost of new equity is greater because of flotation costs.
2 The cost of debt may rise, as higher interest rates are required to attract additional investors to supply funds to the firm.

It is now necessary to estimate the following three measures:

1 The current WACC without issuing new capital.
2 The **retained earnings breakpoint** where it becomes necessary to raise new capital.
3 The WACC of issuing new capital.

Table 11.4. Capital budgeting information for IO Schedule

	Potential investment projects					
	A	B*	C*	D	E	F
Initial outlay (£m)	20	10	15	5	30	10
IRR (%)	15	18	12	10	8	13

*Note: Projects B and C are mutually exclusive.

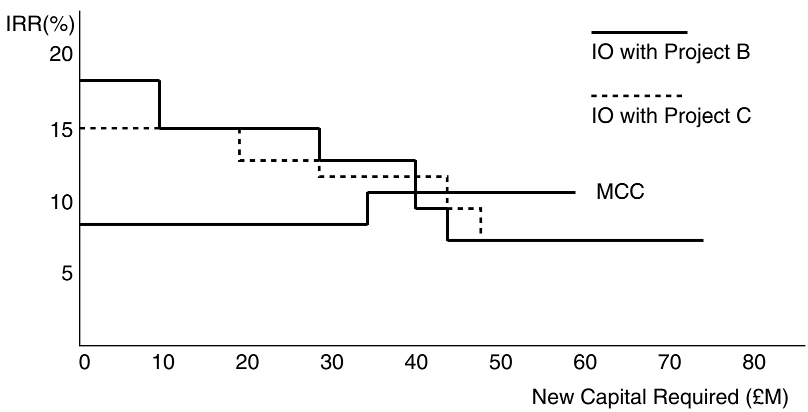

Figure 11.5. IO and MCC schedules for Maxsport.

Let us now assume that the cost of debt is constant at 8%, that 40% of the firm's capital is debt and 60% equity, that the tax rate is 40% and the cost of retained earnings is 11.8%. The current cost of capital is thus:

$$k_a = 0.4(8\%)(0.6) + 0.6(11.8\%) = 9\%$$

In order to estimate the breakpoint we need to estimate the amount of retained earnings that the firm will have, plus any other cash flows, for example from depreciation. Let us assume that Maxsport has estimated retained earnings of £18 million and £5 million in depreciation cash flow during the planning period. We know that the £18 million must be 60% of the total capital raised, with the other 40% being debt; therefore the breakpoint is given by:

$$B = 18\text{m}/0.6 + 5\text{m} = 35 \text{ million}$$

In order to estimate the WACC of issuing new capital we need to estimate the cost of issuing new equity, taking into account flotation costs. Assuming that this cost is 15%, the new WACC is given by:

$$k_a = 0.4(8\%)(0.6) + 0.6(15\%) = 10.92\%$$

The resulting MCC schedule is shown in Figure 11.5.

11.6.3 Equilibrium of IO and MCC

The optimal capital budget and the cost of capital are given by the intersection of the IO and MCC curves. There is an added complication in this case because there are two intersection points, caused by having two different IO curves. In order to obtain the optimal capital budget we can see first of all that the equilibrium marginal cost of capital will be 10.92 per cent. We then have to estimate the total NPV of undertaking projects A, B and F and compare this with the total NPV of projects A, C and F; both strategies involve projects A and F, so we have to compare the NPVs of projects B and C at the cost of capital of 10.92 per cent. The strategy involving the higher NPV is then selected, thus determining the optimal capital budget. If project B has the higher NPV the optimal capital budget is £40 million, while if project C has the higher NPV the optimal capital budget is £45 million.

If the different projects involve different degrees of risk, either from each other or from the firm's existing assets, this complicates the analysis since the MCC has to be adjusted accordingly; this may in turn affect the decision whether to invest in a particular project or not.

11.7 A problem-solving approach

All problems related to the capital budgeting process essentially involve one or more of the five stages described in the first section of this chapter, and examined at length in the remaining sections. The two solved problems, SP11.1 and SP11.2, illustrate particularly important and problematical aspects: cash flow analysis and decision tree analysis. In addition, the three case studies cover similar aspects, concentrating on the following areas:

1 Cash flow analysis when benefits are intangible.
2 Appropriate use of evaluation criteria.
3 Application of risk analysis to high-tech firms.

Case study 11.2: Under-investment in transportation infrastructure

Grinding to a halt[5]

As well as the familiar signs greeting London's Monday morning commuters, such as 'station closed' and 'train cancelled', motorists were confronted by some more unusual sights. Parts of the A40, the main road into London from the west, were under water, as were bits of the M25, London's orbital motorway. Many commuter rail services into London simply stopped, and mainline railway stations emptied. After just one particularly bad night's weather, the transport system of one of the world's biggest and richest cities seemed close to collapse.

Of course, it did not help that Britain's train system was already reeling from the speed limits and track replacements put into place after a deadly rail crash at Hatfield, the previous week. But a consensus is emerging about what is wrong with Britain's transport system. The system is old, and not enough has been spent to keep it up to date.

Population pressure alone is not a sufficient explanation for the travails of the transport system. Take London – 1.1m people travel into Greater London every day, 270,000 to the City of London alone; 394,000 travel in on the Underground. These are similar to the commuting figures for Paris and New York. As Tony Travers of the London School of Economics points out, rather than being uniquely crowded, the London region is very similar in size and population density to New York or the central area of the Netherlands between The Hague and Amsterdam. But London's transport suffers from its age, and from the persistent under-investment in its infrastructure. In this respect, Mr Travers argues that a better comparison with London would be Moscow.

It is probably no coincidence that the most notoriously inefficient of London's underground lines, the Northern, is also the oldest deep-level line in the world. It was first opened in 1890, and has proved hard to modernise. Whereas the number of passengers that the Underground carries has increased in line with the economic boom in the South-East since the mid-1980s, little new track has been laid. In 1982, at the bottom of a recession, there were only 498m passenger journeys a year. The latest figure, for 1998, is 832m passenger journeys. But in the past 30 years, only one new line, the Jubilee, has been built for the Underground.

If history plays its part, so does under-investment. On both the railways and the roads, the disruption that bad weather causes is often the direct result of cutting costs. Take the famous and much derided excuse of the 'wrong kind of snow', trotted out to explain the failure of rail services to run. This was because snow was getting into electric train motors. The filters that could have prevented the fine snow from getting into the motors were deemed to be too expensive and were not used. Chris Nash, a professor of transport economics at the University of Leeds, also cites the example of the electrification of the east coast line. The cost of the overhead electrical equipment was 'cut to the bone', so the system is not as robust in high winds as it should be.

Since the Beeching Report of 1963, which recommended massive cuts in the network, British railways have been reduced to what has been described as a 'lean system'. Felix Schmid, of the department of mechanical engineering at the University of Sheffield, argues that the system has been 'reduced to the absolute minimum for operating'. This means that it is near full capacity most of the time. In normal times this is efficient. But even quite small disruptions can have serious knock-on effects and the system just 'collapses in a crisis'.

Questions

1 What are the features of a 'lean system'?
2 In terms of the steps described in this chapter, how did the under-investment in transportation infrastructure come about?
3 In what ways is a transportation infrastructure different from other types of investment? How might such differences affect the investment decision?

Case 11.3: Over-investment in fibre optics

Drowning in glass[6]

Can you have too much of a good thing? The history of technology says not, but that was before the fibre-optic bubble.

Dreamy it may seem, but 'build it and they will come' is one of the most fundamental and lasting laws of technology. Each year the labs of Silicon Valley find ways to increase the capacity of everything, from processors to storage space, seemingly beyond all sense and reasonable demand. Yet somehow ways are always found to use it all. In technology, capacity drives demand, rather than the other way round.

The same has been true for communications capacity, which has been growing quickest of all, thanks to fibre optics. But here, the recent stockmarket bubble changed the picture. Investors threw tens of billions of dollars at new telecoms companies that were laying fibre networks in competition with the incumbents. The pace of new fibre laying, already fast, became frenetic: sales growth at leading fibre makers such as Corning hit 50% last year, nearly three times the previous rate. The race to lay new fibre reached such extremes that one company, 360networks, rose to fame not for its network technology but because it invented a railway cable-laying machine that could rise up to let trains pass underneath, saving it from having to waste valuable time scooting off to a siding.

When the stockmarket tumbled, the industry realised that it was looking at an unprecedented overhang of raw fibre. As expensive as it is to lay fibre, it is far more expensive to 'light' it with lasers, amplifiers and other optical equipment, and thus turn potential capacity into usable bandwidth. To light the new fibre that American carriers have already announced they are adding to their networks would cost more than $500 billion over the next three years, more than ten times current spending rates, according to Level 3 Communications, a carrier. Needless to say, that sort of money is no longer available.

Telecoms carriers tend to lay fibre speculatively, but only light it when they have an actual buyer. Now, with the stockmarket in a spin, they do not have as many of those as they were counting on. On March 19th, Corning warned that the growth of its fibre sales this year would be less than half last year's level – and even that will be propped up by a huge backlog of orders from last year, which it will now be able to fill. Over the past six months, concern that the white-hot optics industry was going to slow dramatically has savaged the share prices of its leaders, leaving stars such as JDS Uniphase more than 80% off their peaks.

There is plenty of evidence to support the fear of a fibre glut. Technologies that were expected to consume huge amounts of capacity have been slow to arrive. Fast mobile-data networks using so-called 3G technologies will be delayed for years, a victim of disappointment with the present technologies and a drying-up of the capital markets. Gigabit Ethernet, which allows companies to connect their office networks at blazing speeds, has been held back by slowing corporate technology investment. And Napster, which accounted for an estimated 4% of total Internet traffic at its peak (and much of the demand for home DSL and cable modem connections), now risks being shut down.

Many of the companies that were expected to be the main consumers of new fibre have also been hit by the market downturn. So-called competitive local-exchange carriers, such as ICG, which build fibre networks in cities to compete with big incumbents, are sagging under heavy debt loads; ICG itself is under bankruptcy protection. Most of the upstart firms that planned to offer high-speed DSL connections to homes and small businesses, such as Covad, are also now on the ropes. All carriers have been hurt by the over-investment of the past few years, which brought more competitors to the market than demand could bear.

One consequence of all this is a gap between the main supply of potential bandwidth capacity (the long-haul networks between cities) and the main sources of new demand (small businesses and homes). From now on, there will be fewer companies connecting these consumers to networks than before, and at slower rates. This "last mile" bottleneck keeps millions of homes and businesses using dial-up modems, consuming trickles of bandwidth when they might want floods, and leaves much of the fibre in long-haul networks unused.

But there is a big difference between a temporary mismatch in supply and demand and a rejection of the 'build it and they will come' rule of technology consumption. The industry clearly overshot in the heady days when money was easy and growth was everything. Yet hardly anybody doubts that almost all the fibre in the ground today will be used eventually. The question is whether the companies that made the investment will be able to stay in business long enough to see the day.

Even in the current slump, Internet and other data traffic continues to more than double each year. Sadly, fibre investments in recent years implied a belief in even higher growth than that. Along with the growth in fibre itself, the optical-equipment industry was developing new gear that could send many more wavelengths down each fibre strand, multiplying the capacity of even existing cables a hundredfold or more. All told, carriers in the United States planned to increase their capacity almost seventyfold over the next three years, according to Level 3. At current rates of growth, demand would have only risen about fourfold over the same period.

But here, price elasticity may help the industry's plight. One of the good things about the fibre glut is that the price of unused fibre, which had remained relatively stable (since it reflects the cost of construction workers more than technology), is now falling quickly. As more companies get in trouble and are forced to dump capacity, the price will fall even faster. The result may be that once the shakeout is over, the survivors will be able to offer unprecedented amounts of bandwidth for unheard-of prices. Companies such as Narad Networks are developing technology that will allow them to offer homes up to 100 megabits of raw bandwidth at less than $100 a month.

With that kind of capacity, applications such as video-on-demand suddenly become economically attractive. If people start watching TV over the Internet, the fibre now in the ground may no longer be enough. And so the cycle will start again, just as it does in Intel's chips and Seagate's hard drives. The only difference is that billions of dollars of investment will have been burned up waiting for that day. Fibre is not so different from other technologies, except for the cost of getting it wrong.

Questions

1 Explain what is meant by the 'last mile' bottleneck; what is its cause, and what effects does it have?
2 In terms of the steps described in this chapter, what has been the main cause of the over-investment in fibre-optic technology?
3 Explain the relevance of price elasticity in the industry's current situation.
4 Explain the nature of the cycle described in the last paragraph. What are the causes of this cycle? Does it happen in all industries?

Summary

1 Capital budgeting is important because it involves long-term decisions and commitment by firms; mistakes can be very costly.
2 There are various types of capital expenditure, whose importance varies considerably in extent; therefore decision-making processes also vary from one type to another.
3 For major decisions there are several steps involved: estimating the initial cost, estimating future cash flows, estimating the degree of risk, estimating the cost of capital, and applying some evaluation criterion.
4 In the identification and measurement of cash flows it is the incremental cash flows that are relevant.
5 There are three types of risk: stand-alone risk, within-firm risk and market risk. The last is the most important in determining the effect of a project on a firm's share price. High stand-alone risk does not necessarily imply high within-firm risk or high market risk.
6 The cost of capital can be considered from both the demand and supply points of view: it represents the cost of raising funds for investment as far as the firm is concerned, and it also represents the rate of return required by the providers of these funds.
7 The two main components of long-term funds for investment are debt and equity; each has a different cost. A firm therefore needs to estimate the weighted average of these costs of capital.
8 There are two main evaluation criteria that are applied to capital budgeting situations: NPV and IRR. The former is the better measure, since any project with a positive NPV is expected to increase shareholder value.
9 However, managers still often prefer the IRR measure since it is expressed as an interest rate and can make for easier comparisons between projects.

10 It is important for managers to estimate the optimal capital budget for the firm. This indicates the total amount that should be spent on capital projects, and can only be estimated simultaneously with the cost of capital when the IO (demand) and MCC (supply) schedules are combined.

Review questions

1 Define and explain the following terms:

a. Beta coefficient
b. Stand-alone risk; within-firm risk; market risk
c. *Ex-ante* and *ex-post* measures
d. Decision tree analysis
e. IRR
f. WACC
g. SML
h. IO schedule.

2 Explain the role of simulation in capital budgeting.
3 Explain why the NPV criterion is preferable to the IRR criterion.
4 Explain why the cost of capital can only be accurately estimated when the IO schedule is known.
5 Explain why the stand-alone risk of a project is not of primary concern to shareholders.

Problems

11.1 Blatt Packing Co. is examining the investment in a new air-conditioning system in its factory. The initial cost is £100,000, and it is expected to sell the system for scrap after five years at a salvage value of £20,000. The equipment will be depreciated on a straight-line basis for tax purposes. The tax rate is 40 per cent, with no tax payable on the salvage value. The investment requires an increase in net working capital of £5,000 at the outset. There is no increase in revenues expected, but there is expected to be a saving of £40,000 per year in before-tax operating costs.

a. Estimate the cash flows involved in the project.
b. If the firm's cost of capital is 8 per cent, should the firm invest in the system?
c. How would the decision above be affected if the firm's bond rating was reduced and its cost of capital changed to 10 per cent?

11.2 Moon Systems is considering investing in a new computer system. This has a net cost of £450,000, and is expected to increase pre-tax operating profit (allowing for depreciation) by £300,000 each year. Depreciation has been calculated on a straight-line basis, over three years, with no residual value. Taxes are at 40 per cent. The marginal cost of capital for the firm is 12 per cent. As financial director you are uncertain how long the economic life of the system is likely to be; however, recent indications are that the life may be only two years, or possibly even as little as eighteen months. Estimate the effect of the uncertainty regarding the economic life on the NPV and IRR of the investment project.

11.3 Wilson Products has analysed its investment opportunities for the future as follows:

Project	Cost (£, 000)	IRR (%)
A	100,000	15
B	60,000	12
C	80,000	11
D	50,000	10

The firm expects to achieve retained earnings of £96,000, plus £80,000 in cash flows resulting from depreciation. Its target capital structure involves 25 per cent debt and 75 per cent equity. It can borrow at a rate of 9 per cent. The firm's tax rate is 40 per cent and the current market price of its shares is £40. The last dividend was £2.26 per share, and the firm's expected constant growth rate is 6 per cent. New equity can be sold with a flotation cost of 10 per cent.

a. Calculate the WACC using retained earnings and the WACC issuing new stock.
b. Draw a graph of the IO and MCC schedules.
c. Determine which projects the firm should accept.

11.4 Safetilok is considering producing a new anti-theft device for cars. The initial stage would involve an investment of £20,000 to design the product and apply for approval from the insurance industry. Management believes that there is 75 per cent chance that the design will prove successful and approval will be given. If the product is rejected at this stage the project will be abandoned, with a salvage value of £5,000 after a year. The next step after approval is to produce some prototypes for testing. This would cost £300,000 in one year's time. If the tests are successful the product will go into full production; if not, the prototypes will be sold at scrap value for £50,000 after two years. Management believes that there is an 80 per cent chance of this stage proving successful. If the product goes into production this will cost £2 million after two years. If the market is favourable the net revenues minus operating costs are estimated to be £4 million, occurring after three years. If the market is unfavourable the net revenues minus operating costs are estimated at £1.5 million. Management estimates that there is a fifty-fifty chance of the market being favourable. The firm's marginal cost of capital is 10 per cent.

a. Draw a decision tree for the project.
b. Calculate the expected NPV for the project; should the firm undertake it?

Notes

1 J. C. Van Horne, 'A note on biases in capital budgeting introduced by inflation', *Journal of Financial and Quantitative Analysis*, **6** (January 1971): 653–658.
2 R. D. Luce and H. Raiffa, *Games and Decisions*, New York: Wiley, 1957, p. 13.
3 E. F. Brigham and L. C. Gapenski, *Financial Management: Theory and Practice,* 6th edn, Orlando: Dryden Press, 1991, p. 390.
4 Luce and Raiffa, *Games and Decisions*, p. 281.
5 'Grinding to a halt', *The Economist,* 2 November 2000.
6 'Drowning in glass', *The Economist,* 22 March 2001.