



UNIVERSITÀ
DEGLI STUDI
DI TRIESTE

Applied seismology

Instrumentation

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SISMOMETRIA – Cronologia storica

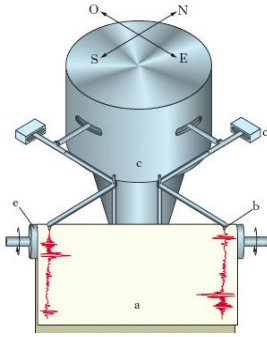
- 132- Sismoscopio di CHANG HENG
- 1703 – Sismoscopio di la Hante Feuille (bacinella con mercurio)
- 1747 – Sismoscopio di Nicola Cirillo (primo strumento su base meccanica)
- 1850 - Inizio ERA STRUMENTALE propriamente detta. Robert Mallet propone una rete di osservazione estesa a tutto il pianeta
- 1856 – Sismoscopio di Palmieri (“Sismografo elettromagnetico, in realtà una serie di sismometri assemblati assieme per la misura di direzione, intensità, durata dei terremoti; misura sia movimenti orizzontali che verticali).
- 1875 - Cecchi, primo sismografo propriamente detto. Due pendoli per le componenti orizzontali; una massa appesa a una molla a spirale per la verticale. Dotato di un dispositivo per il rilevamento di moti rotatori.
- 1889 – Potsdam I pendoli orizzontali di Pashwitz registrano il primo telesisma.
- 1892 – John Milne (Ewing, Gray, Omori) realizza il primo sismografo ad alta definizione.
- 1900-1910 – E. Wiechert: sismografo meccanico a carta affumicata: circa 20 ingrandimenti masse pendolari fino a 17.000 kg (Gottingen e Tacubaya. B.B. Galitzin: sismografo meccanico fotografico. Il sismografo Wiechert si basa su modelli costruiti da scienziati italiani: De Rossi, Agamennone, Vicentini, tutti meccanici con carta affumicata e ingrandimenti superiori a 150.
- 1935 - H. Benioff realizza il sismografo a “deformazione lineare” indipendente dal principio del pendolo.
- 1925 – Sismoscopio a torsione Wood-Anderson per la misura della magnitude.
- 1970-80- Sensori a banda larga di concezione moderna.

SISMOMETRIA – Cronologia storica

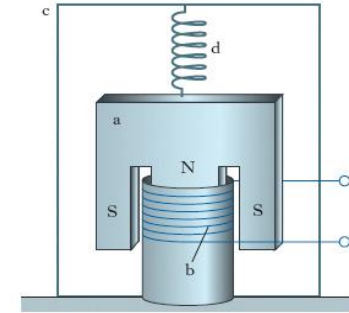
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TRASDUTTORI

MECCANICI



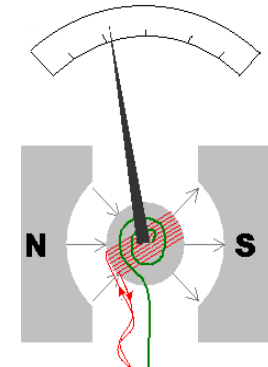
GEOFONI



ELETTROMAGNETICI

A GALVANOMETRO

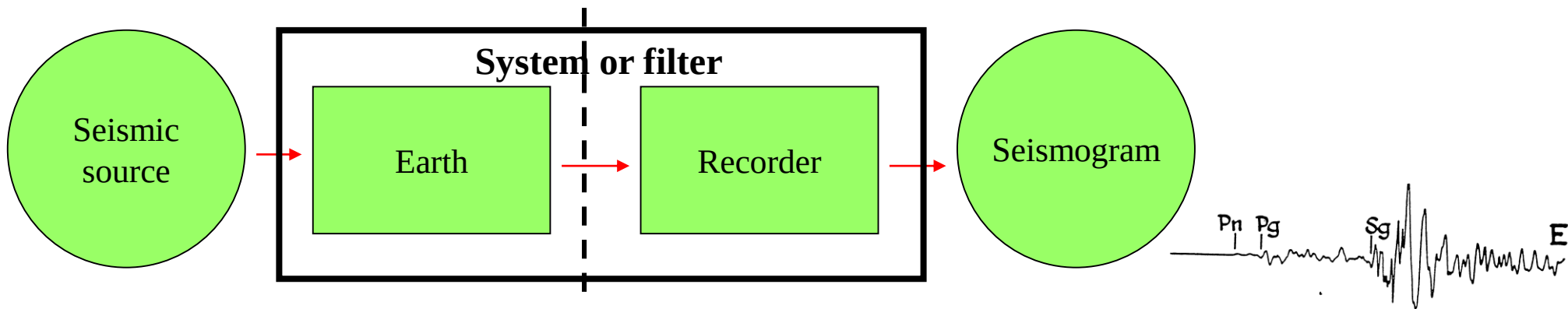
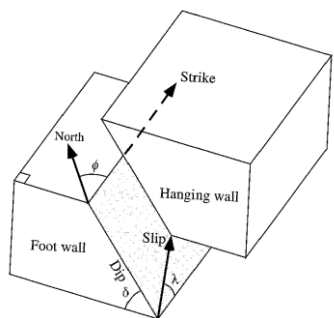
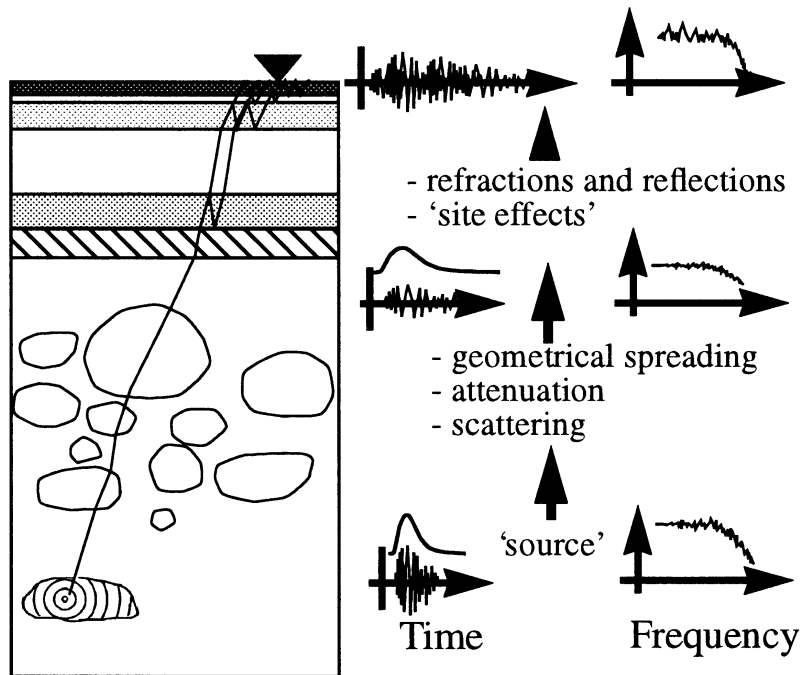
ELETTRONICI A BANDA LARGA



SISMOMETRO: uscita proporzionale allo spostamento

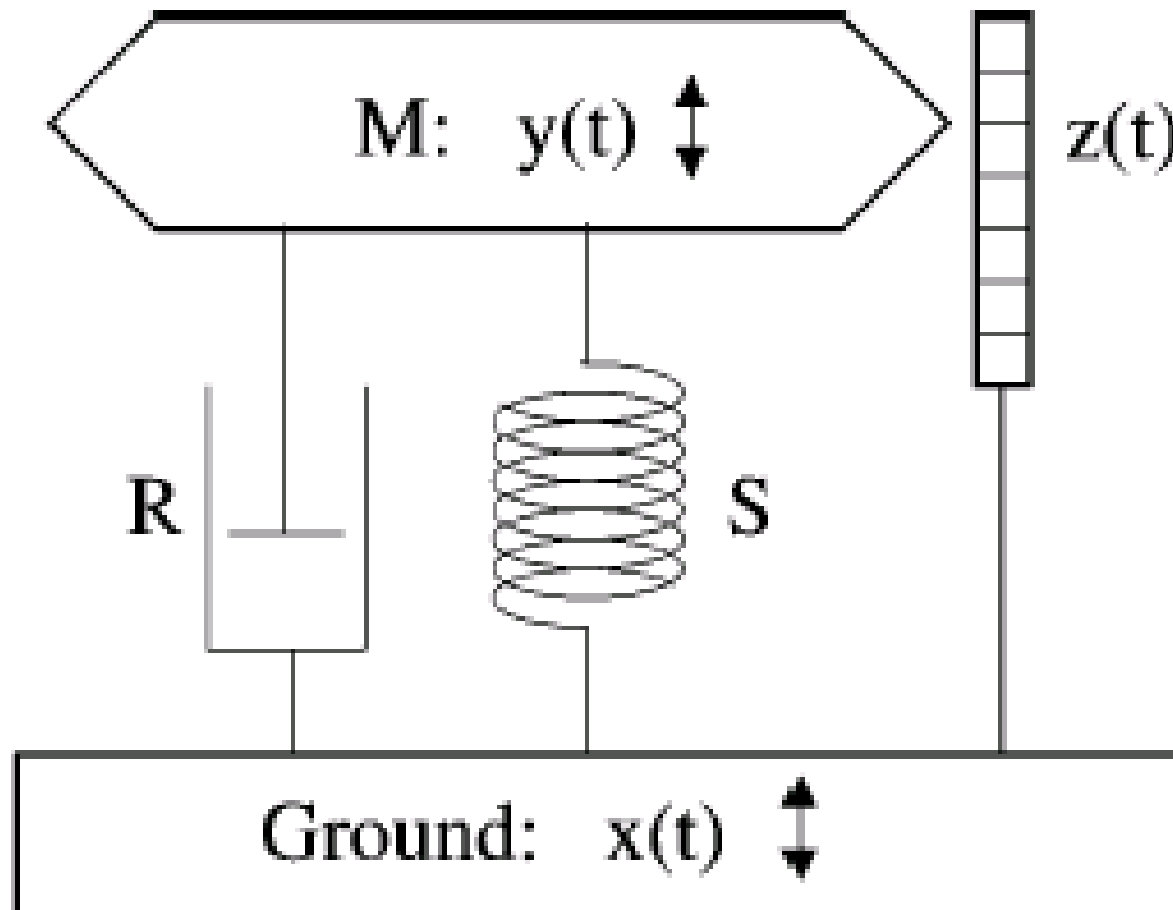
VELOCIMETRO: uscita proporzionale alle velocità

ACCELEROMETRO: uscita proporzionale all'accelerazione



MECHANICAL PENDULUM

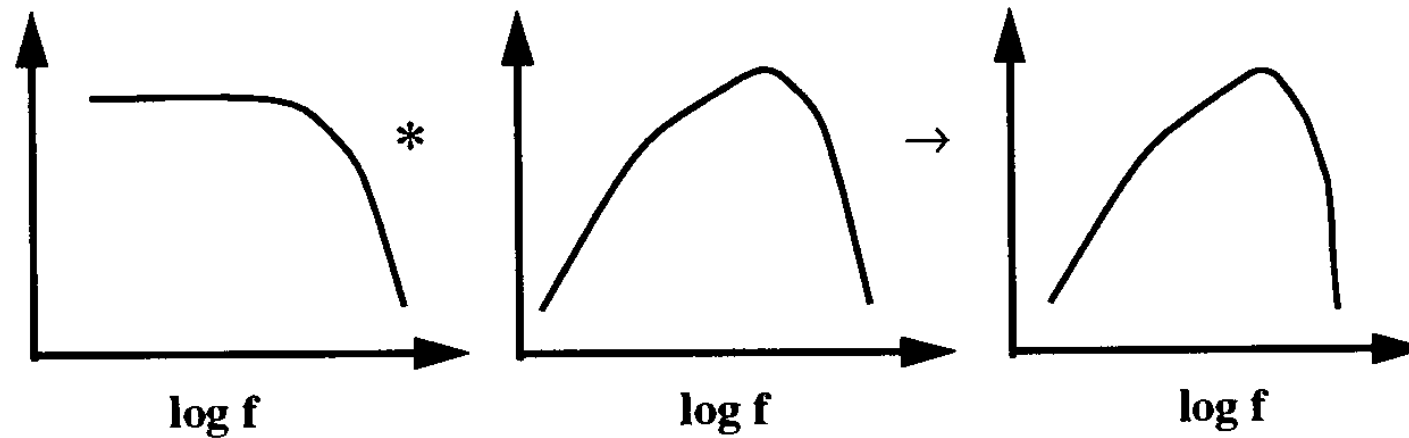
Damped harmonic oscillator



$$z(t) = y(t) - x(t)$$

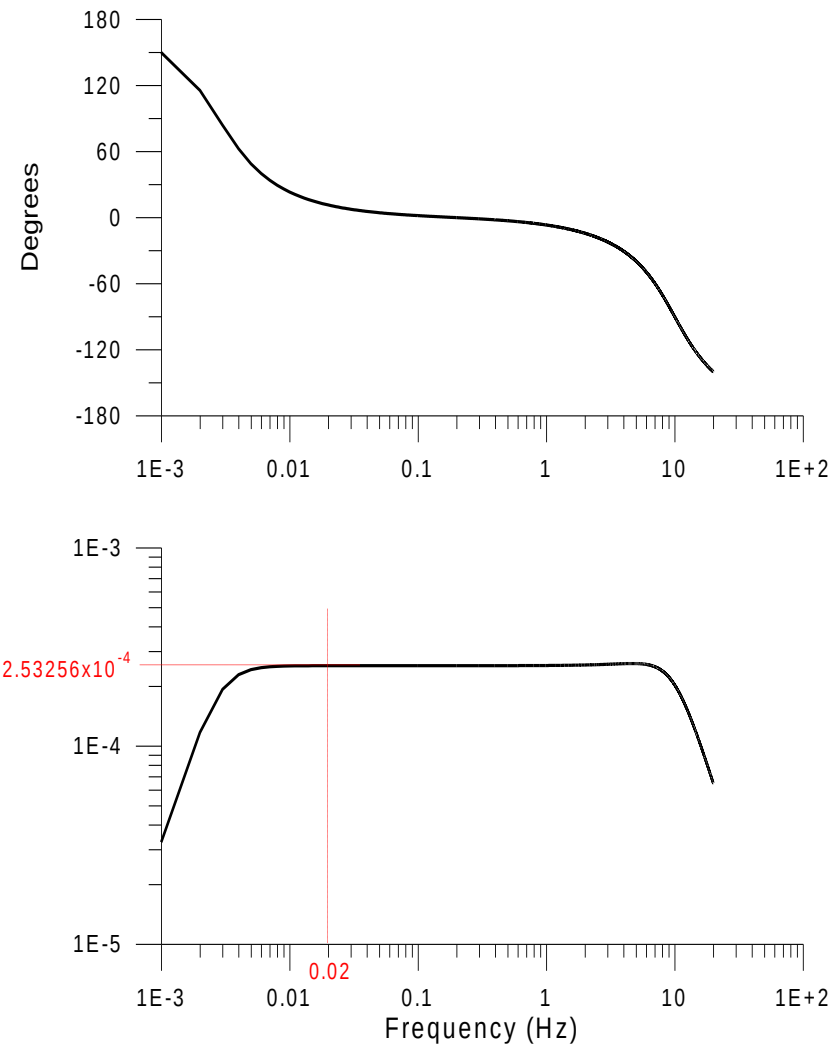
$$G(f) = H(f) \cdot F(f)$$

source spectrum * recording system → recorded spectrum

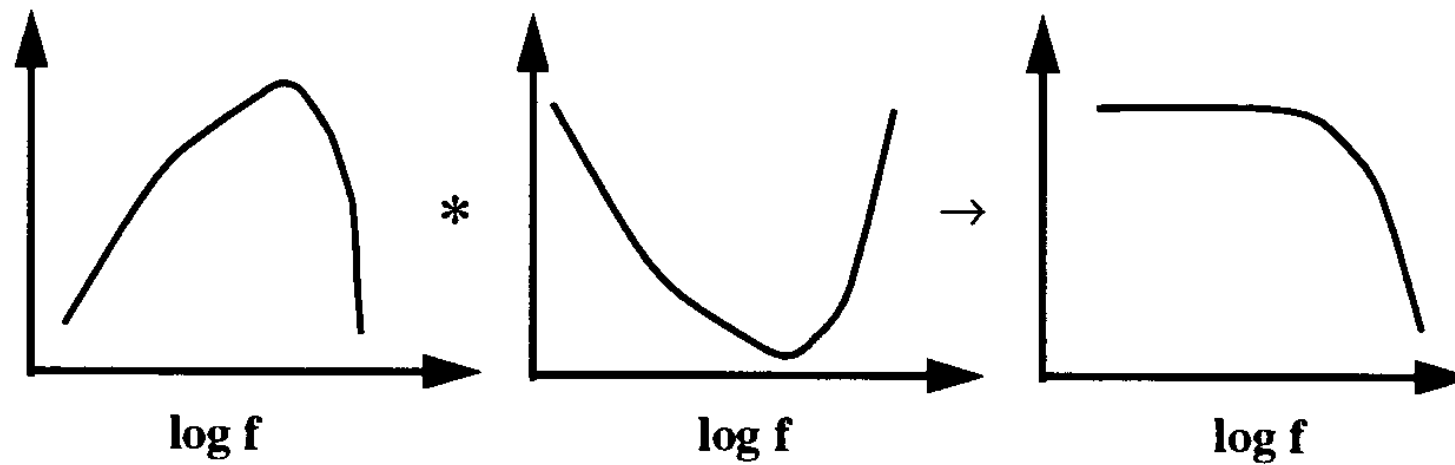


Recording the displacement spectrum of an idealized earthquake source

The most commonly used description of an instrumental response is the "amplitude response curve," which is the frequency-dependent amplification of ground motion. Mathematically, this is the modulus (absolute value) of the complex frequency response, usually called the amplitude response. It specifies the steady-state harmonic responsiveness (amplification, magnification, conversion factor) of the seismograph as a function of frequency. However, for the correct interpretation of seismograms, the phase response of the recording system must also be known. In principle it can be calculated from the amplitude response, but it is normally specified separately or derived together with the amplitude response from the mathematical description of the system from its complex transfer function or complex frequency response.



recorded spectrum * inverse filter → source spectrum



Restoration of source spectrum by inverse filter in the noise-free case

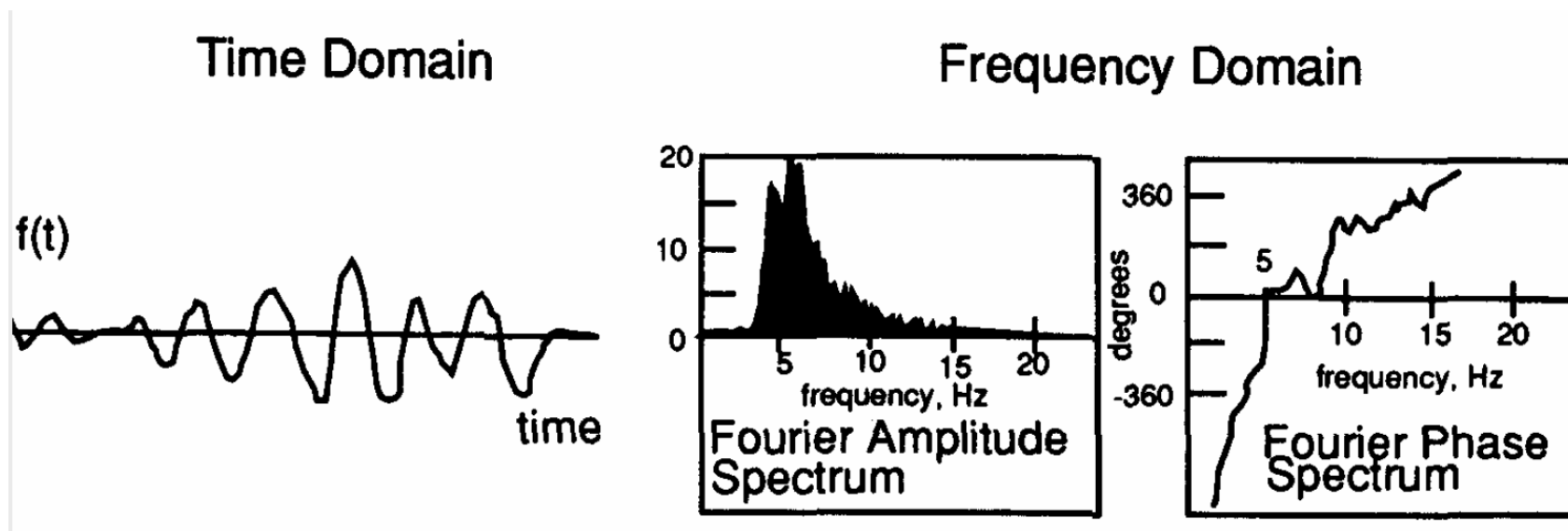
According to the Fourier theorem any arbitrary transient function $f(t)$ in the time domain can be represented by an equivalent function $F(\omega)$ in the frequency domain, that is, the Fourier transform of $f(t)$. The following relationships apply:

$$f(t) = (2\pi)^{-1} \int_{-\infty}^{\infty} F(\omega) \exp(i\omega t) d\omega$$

$$F(\omega) = \int_{-\infty}^{\infty} f(t) \exp(-i\omega t) dt = |F(\omega)| \exp(i\phi(\omega))$$

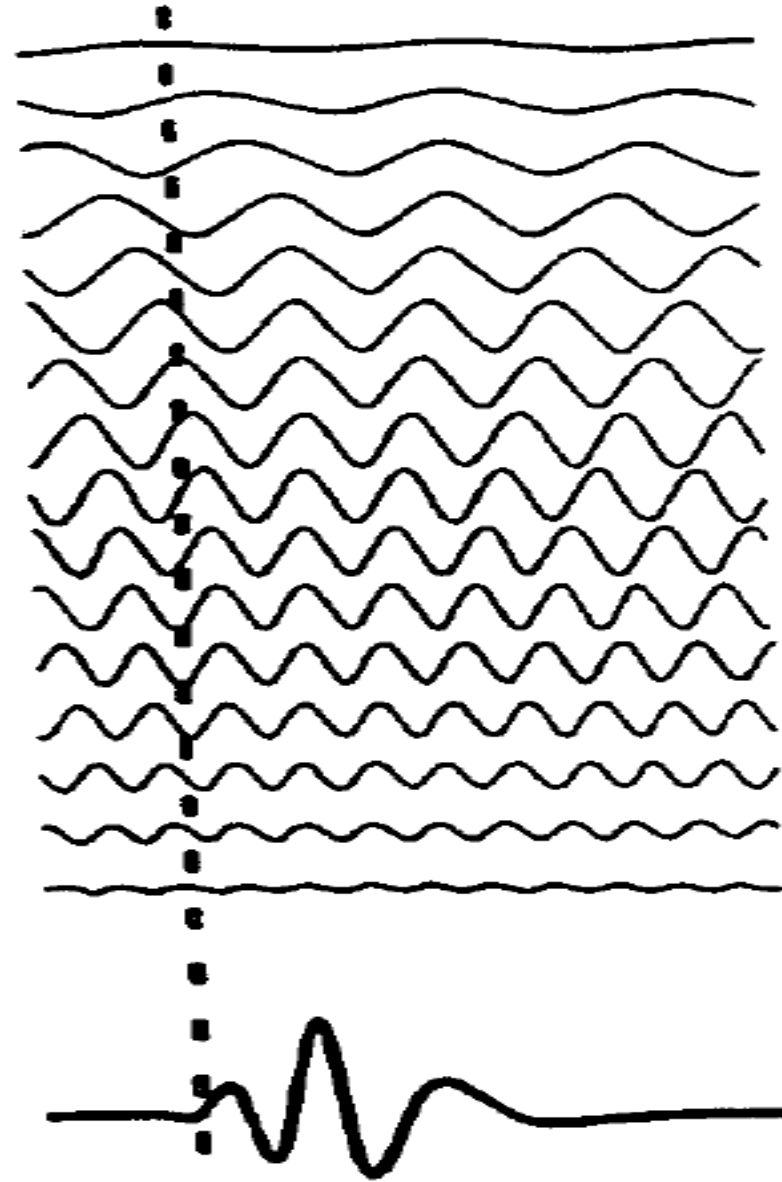
$|F(\omega)| = A(\omega)$ is the spectral density of the amplitude with unit m/Hz, $\omega = 2\pi f$ the angular frequency (with f - frequency with unit Hz) and $\phi(\omega)$ the phase spectrum with units deg, rad or $2\pi_{\text{rad}}$. The integral is equivalent to a sum. Thus, Fourier's theorem states that an arbitrary finite time series, even an impulsive one, can be expressed as a sum of monochromatic periodic functions, viz.

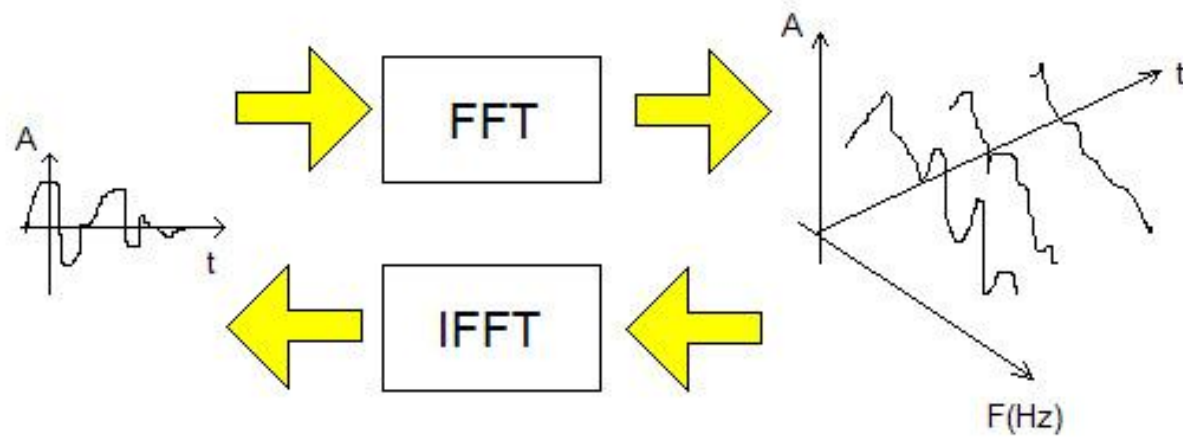
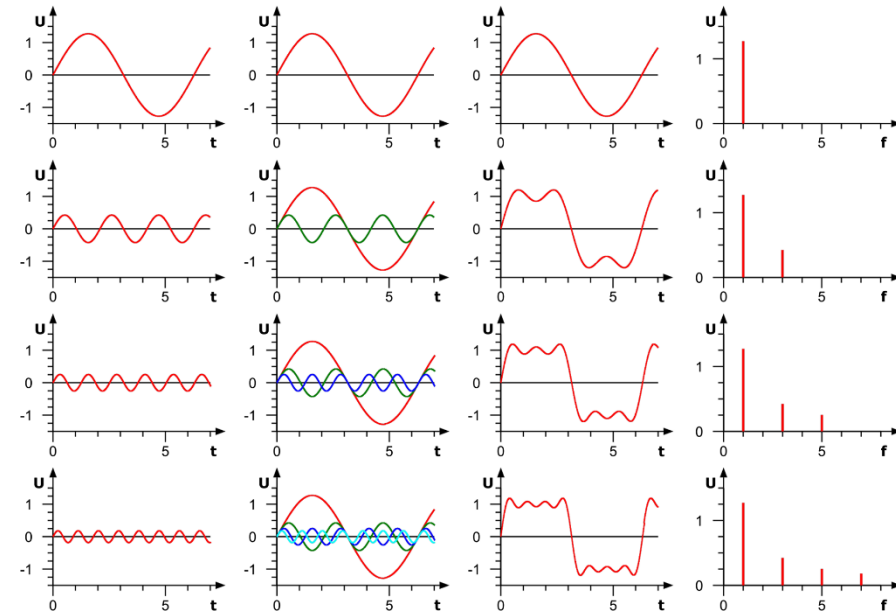
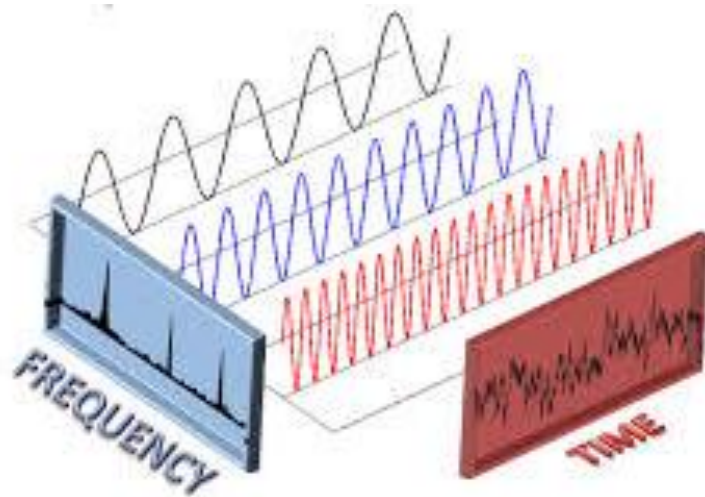
$$f(t) = 2\pi^{-1} \sum |F(\omega)| \exp(i[\omega t + \phi(\omega)]) \Delta\omega.$$

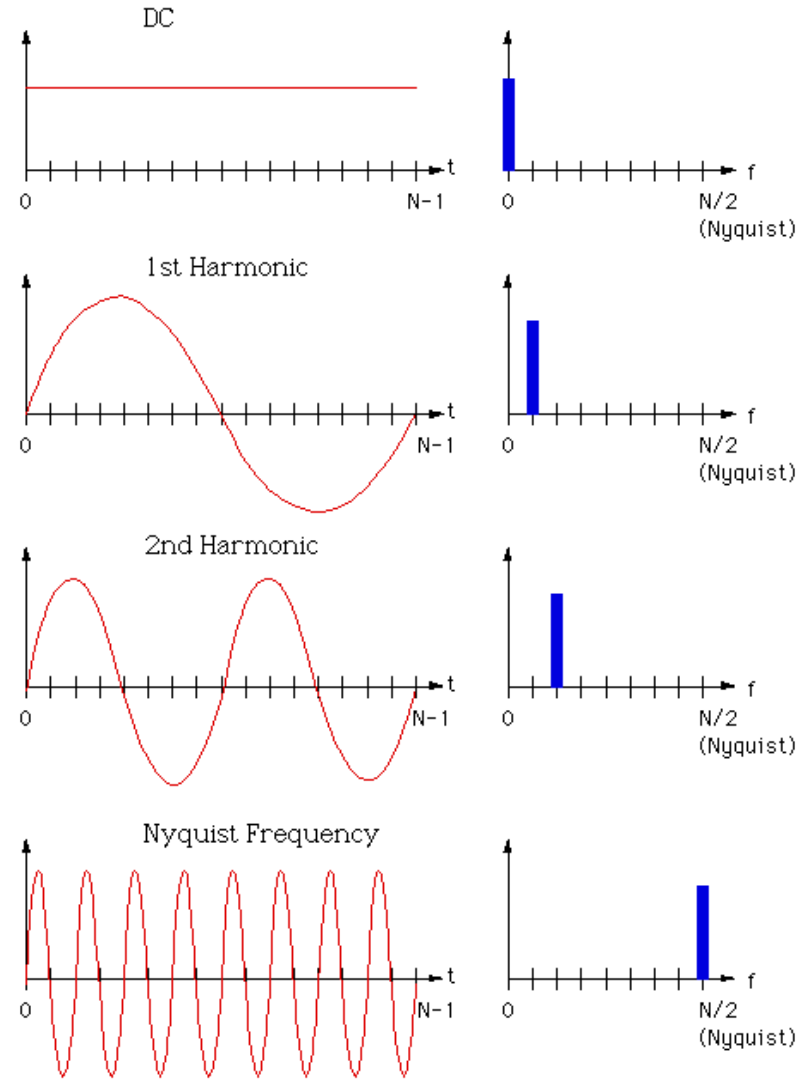


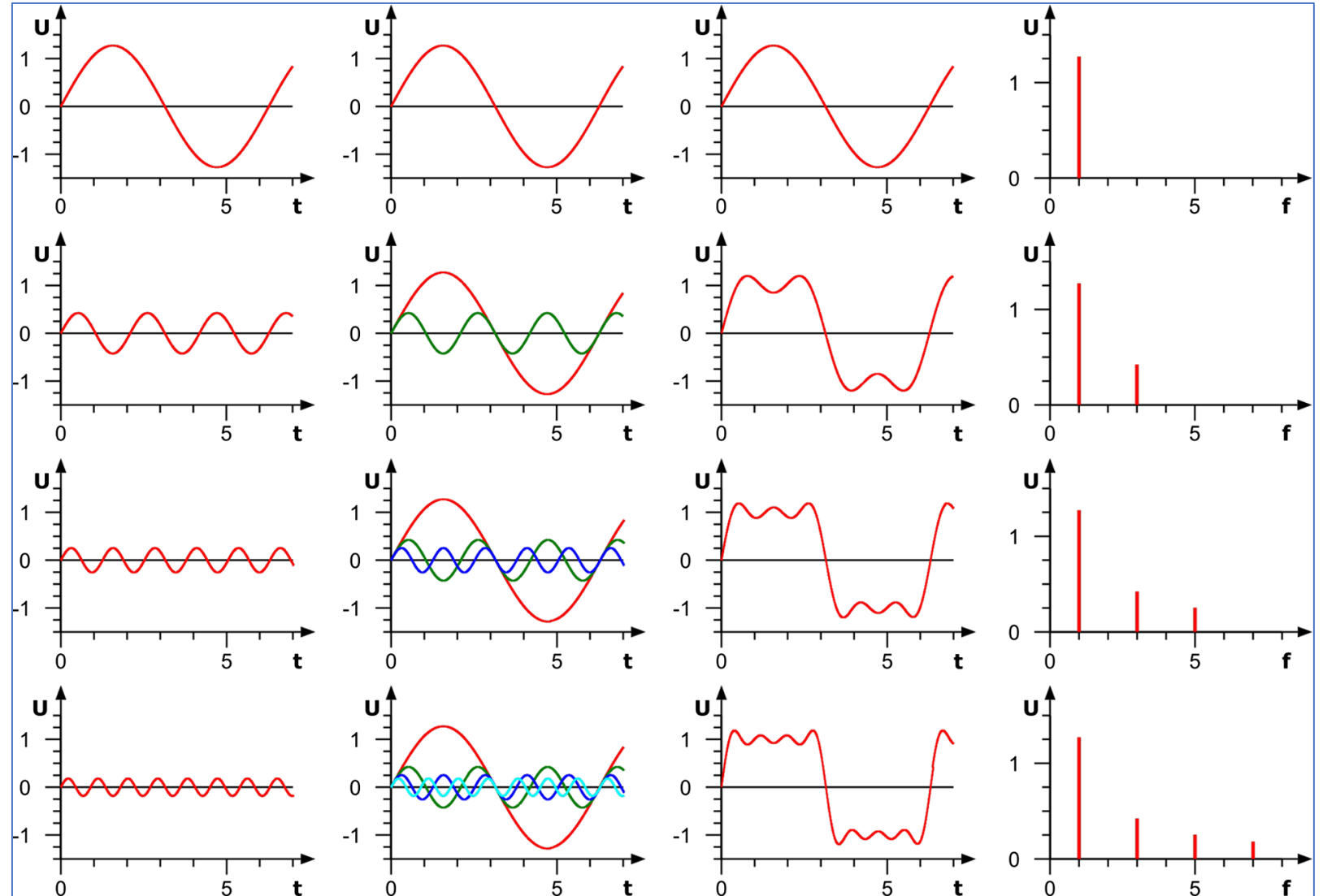
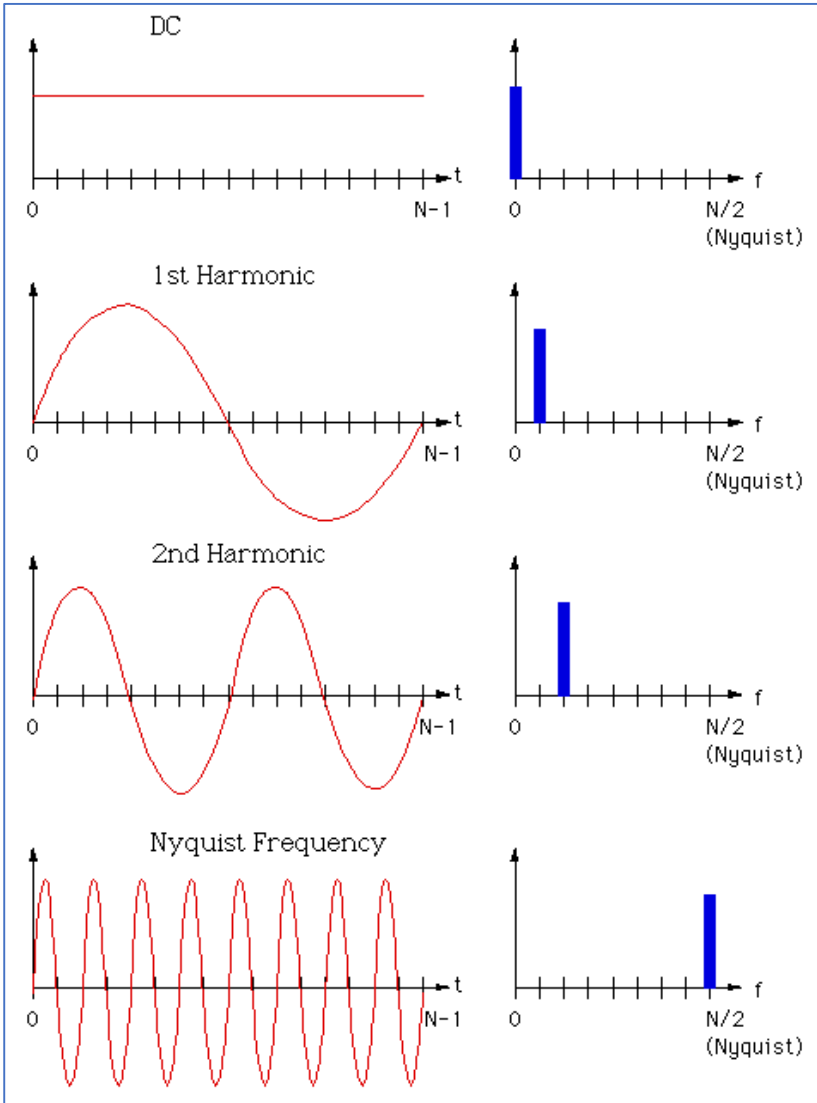
Un segnale registrato in funzione del tempo (a sinistra) può essere rappresentato in modo equivalente nel dominio delle frequenza dal suo spettro di Fourier. L'ampiezza (in centro) e lo spettro di fase (a destra) sono entrambi necessari per ottenere la serie temporale completa.

he transient $f(t)$ is formed infinite harmonic terms. The amplitudes of each harmonic term vary, being described by the amplitude spectrum. The shift of the the phase of each harmonic term is given by the phase spectrum.



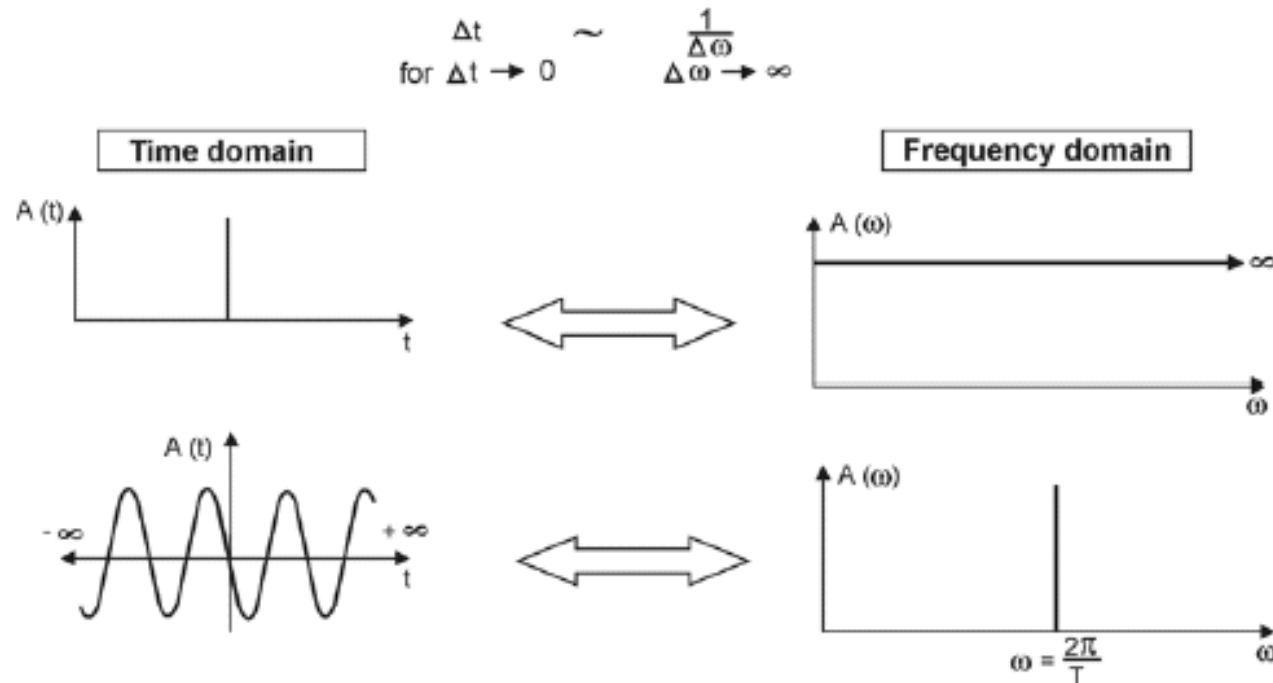


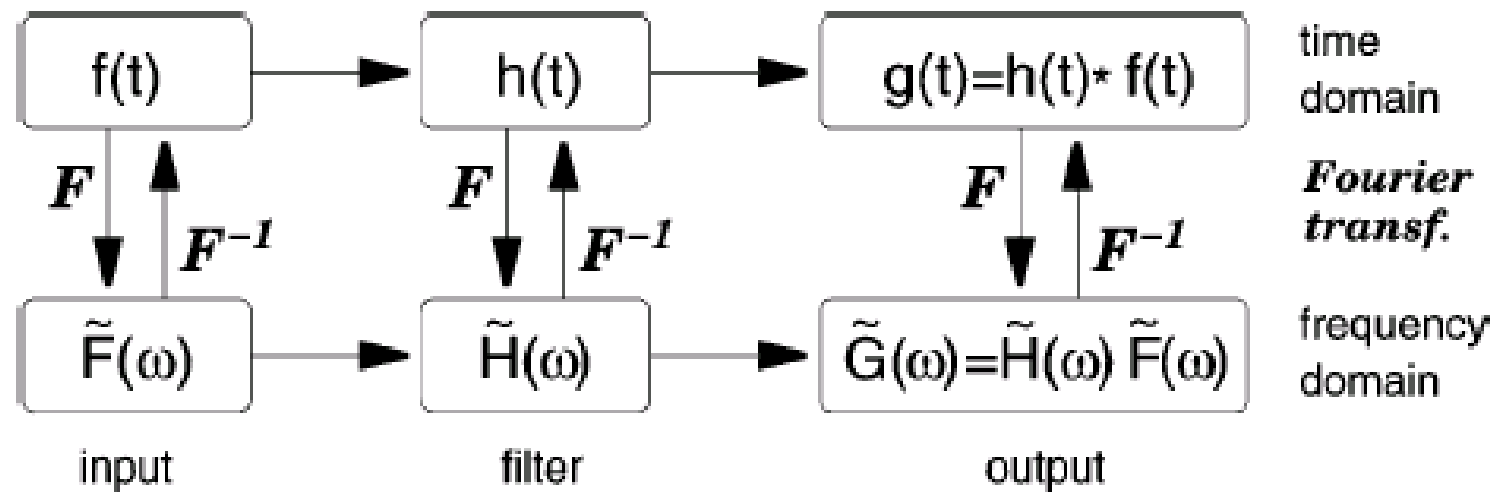




The impulsive response

The function $\delta(t)$ (Dirac delta) is an infinitely short, infinitely high positive function centered at time 0. Both its Laplace and Fourier transforms have the constant value 1. Its amplitude spectrum contains all frequencies.





Percorsi di elaborazione del segnale nei domini di tempo e frequenza. L'asterisco tra $f(t)$ e $g(t)$ indica una convoluzione.

$$\mathbf{G}(f) = \mathbf{H}(f) \cdot \mathbf{F}(f)$$

The impulsive response

The function $\delta(t)$ (Dirac delta) is an infinitely short, infinitely high positive function centered at time 0. Fourier transforms has the constant value 1. Its amplitude spectrum contains all frequencies.

$$\mathbf{G}(f) = \mathbf{H}(f) \cdot \delta(t)$$

$$\mathbf{G}(f) = \mathbf{H}(f)$$

Consequently, $\mathbf{H}(f)$ is the Fourier transform of the impulsive response $\mathbf{G}(f)$

The convolution theorem

Any signal can be understood as consisting of a sequence of pulses. This is evident in the case of sampled signals, but it can be generalized to continuous signals by representing the signal as a continuous sequence of Dirac pulses. We can construct the response of a linear system to an arbitrary input signal as a sum of appropriately delayed and scaled impulse responses.

This process is called convolution:

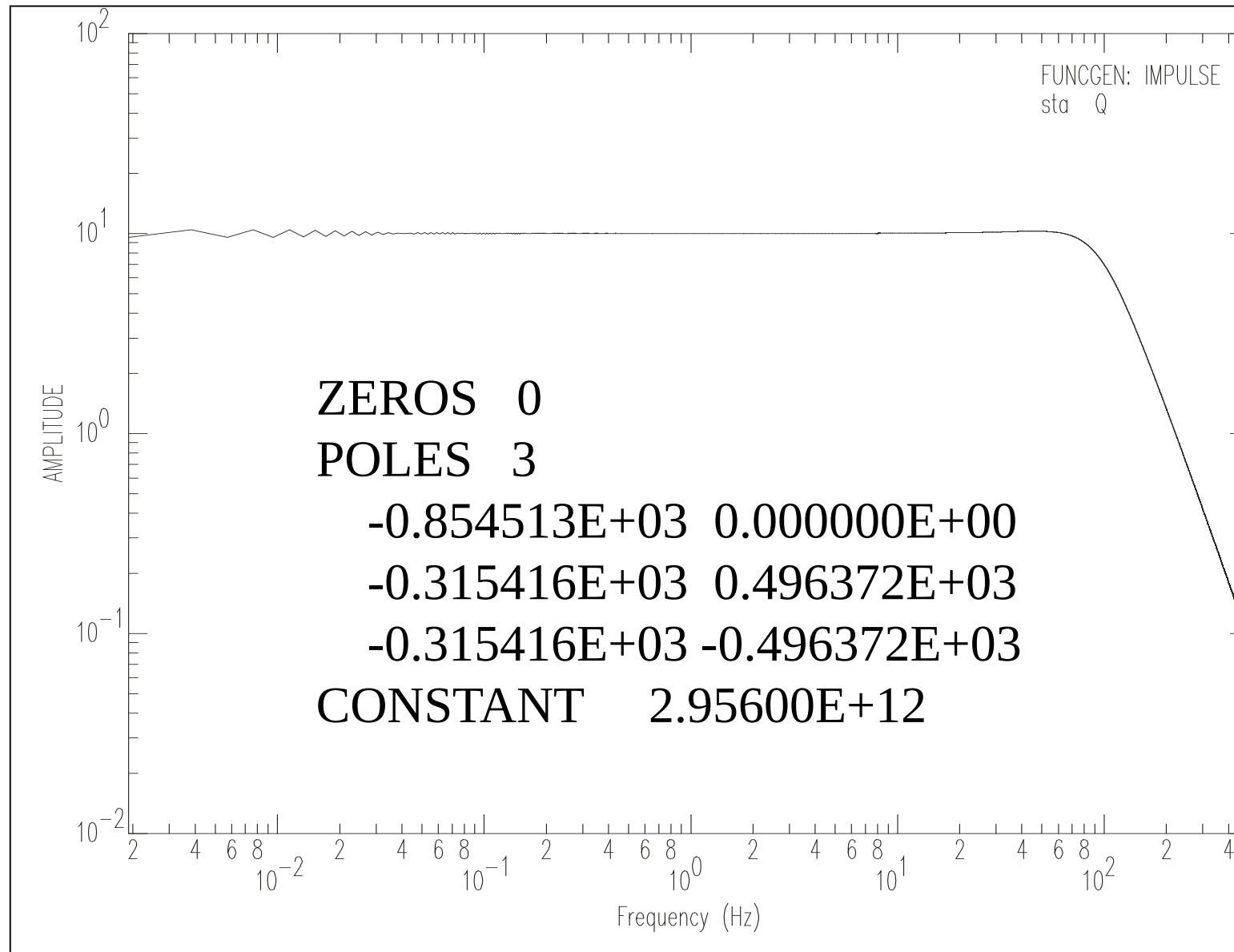
$$g(t) = \int_0^{\infty} h(t') f(t - t') dt' = \int_0^{\infty} h(t - t') f(t') dt'$$

The convolution theorem

$$f(t) = \delta(t),$$

$$g(t) = \int h(t') \delta(t - t') dt' = h(t)$$

$h(t)$ is the impulsive response to the system



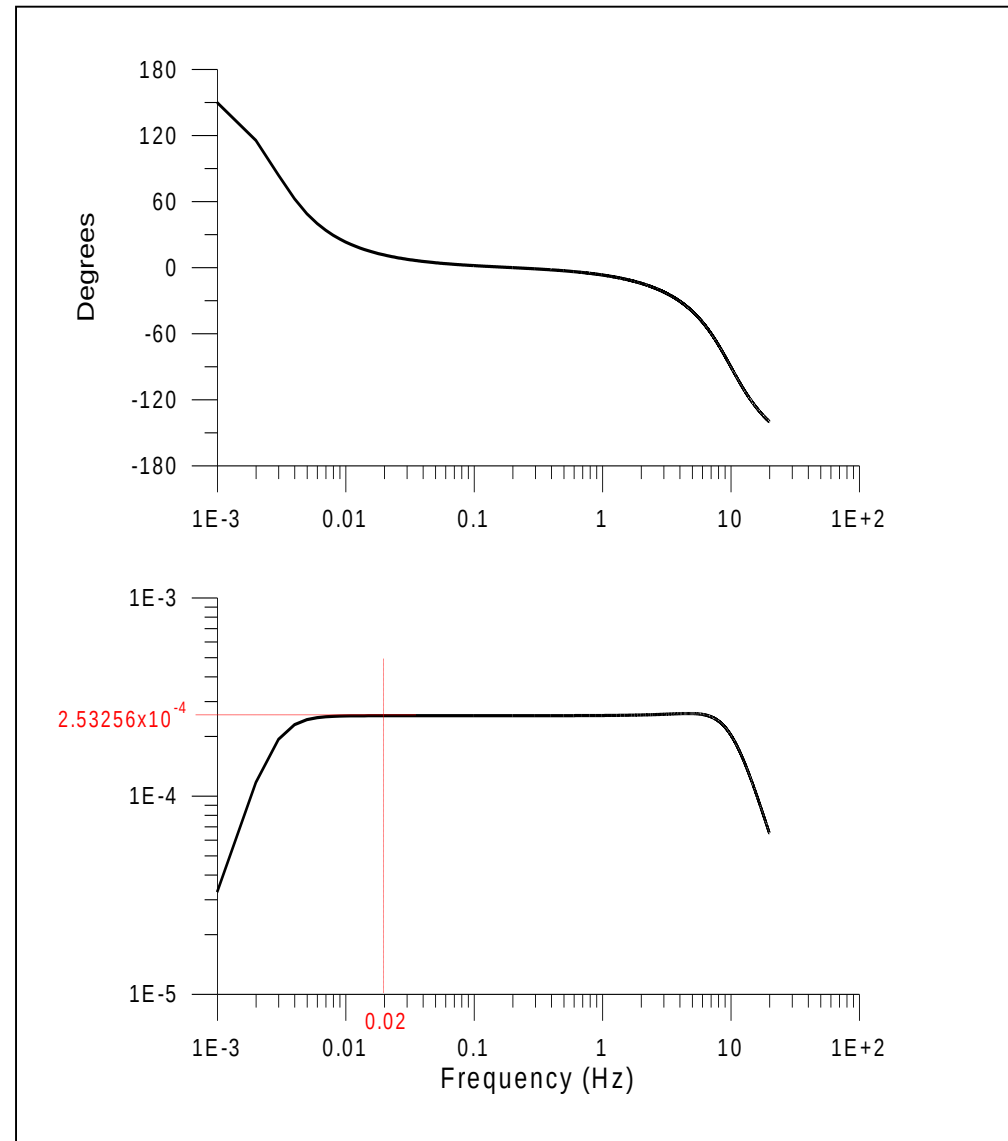
POLES: 4 (rad/sec)

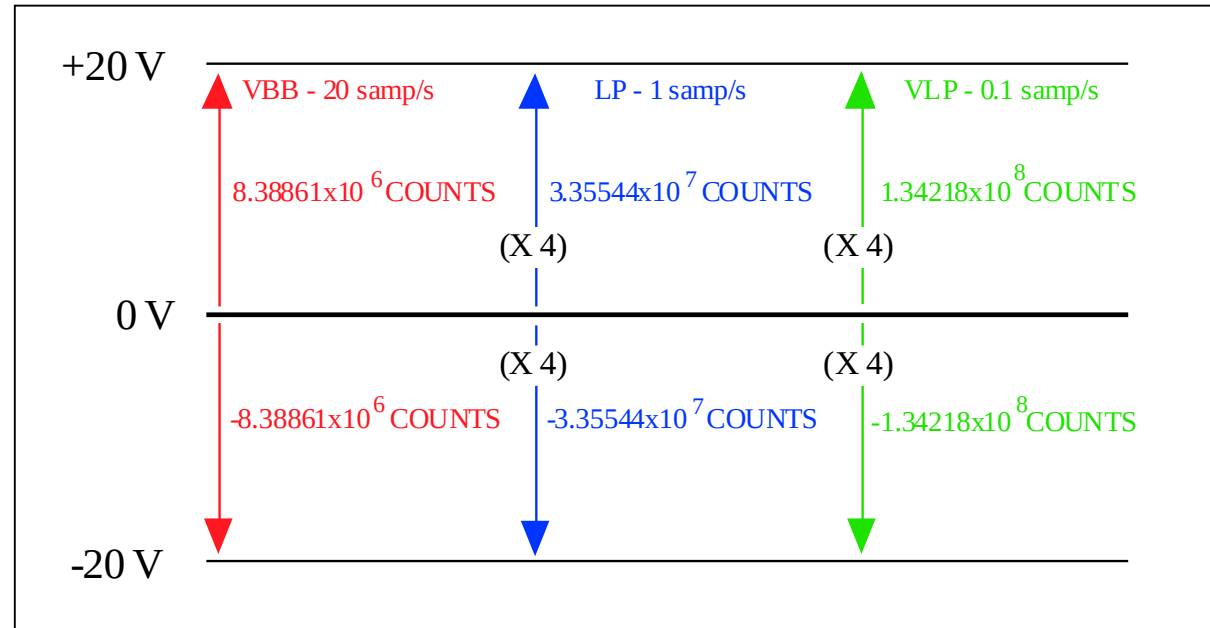
-0.123400d-01	0.123400d-01
-0.123400d-01	-0.123400d-01
-0.391800d+02	0.491200d+02
-0.391800d+02	-0.491200d+02

ZEROS: 2

0.000000d+00	0.000000d+00
0.000000d+00	0.000000d+00

NORMALIZATION FREQUENCY: 0.02 Hz
NORMALIZATION FACTOR: 3948.573





VBB digital data

- DIGITIZER: range 40 V P-P; 2²⁴ counts P-P; 2.384185x10⁻⁶ V/count

- VBB: 2.38419x10⁻⁶ V/counts

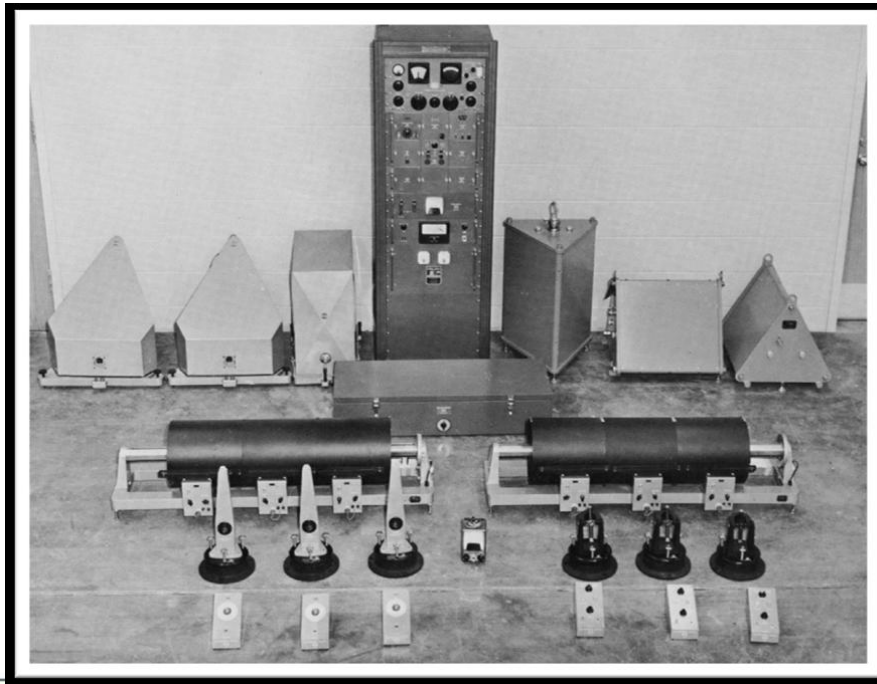
LP: 5.96047x10⁻⁷ V/counts

VBB: 1.49011x10⁻⁷ V/counts

$$\frac{\text{counts}}{\text{m/s}} = \frac{\frac{\text{V}}{\text{m/s}}}{\text{Coun}}$$

The long-period seismographs of the now obsolete WWSSN (Worldwide Standard Seismograph Network) consisted of a long-period electrodynamic seismometer with an own period of 15 seconds and a long-period mirror galvanometer with a free period of about 90 seconds.

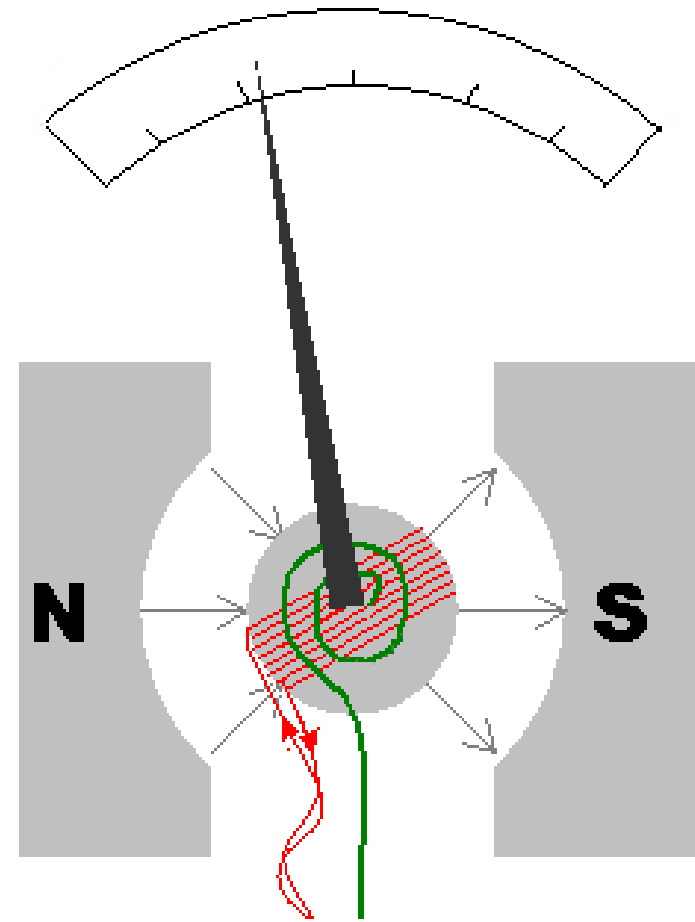
WWSSN seismograms were recorded on photographic paper rotating on a drum. We will now derive several equivalent forms of the transfer function for this system. In our example, the damping constants are chosen as 0.6 for the seismometer and 0.9 for the galvanometer.



Components of the worldwide Standardized Seismograph Network system. Long-period seismometers are shown in the upper left, galvanometers and recorders in the lower left. Short-period seismometers are shown at upper right, galvanometers and recorder at lower right. Electronic console contains timing system, controls and power supply

GALVANOMETER

The most common use of the galvanometer is as a measuring instrument or direct current detector. The device consists of a moving coil that can partially rotate within a magnetic field, integral with an indicating pointer superimposed on a graduated scale. A spring holds the coil in the zero position. When a current flows through the coils, the solenoid generates a magnetic field, which opposing the external field produces a force that rotates the coil and thus the indicator needle. The spring counteracts the rotation, with the result that the angle of deflection is proportional to the intensity of the current.



The transfer function of a WWSSN-LP seismograph.

The transfer function of an electromagnetic seismometer(input: displacement, output: voltage) is

$$H_s(s) = Es^3 / (s^2 + 2s\omega_s h_s + \omega_s^2)$$

$\omega_s = 2\pi / T_s$ angular cut-off frequency

h_s numerical damping

E constant of the electromagnetic transducer generator (200 Vsec/m)

The galvanometer is a second-order low-pass filter and has the transfer function:

$$H_g(s) = \gamma\omega_g^2 / (s^2 + 2s\omega_g h_g + \omega_g^2)$$

γ Galvanometer amplification (393.5 m/V) that gives the desired overall magnification

$$H_s(s) = Es^3 / (s^2 + 2s\omega_s h_s + \omega_s^2)$$

$$H_g(s) = \gamma\omega_g^2 / (s^2 + 2s\omega_g h_g + \omega_g^2)$$

The overall transfer function H_d of the seismograph is obtained in our simplified treatment as a product of the factors

$$H_d(s) = \frac{Cs^3}{(s^2 + 2s\omega_s h_s + \omega_s^2)(s^2 + 2s\omega_g h_g + \omega_g^2)}$$

$$C = E\gamma\omega_g^2 \quad 383.6/\text{sec}$$

$$2\omega_g h_g \quad 0.1257/\text{sec}$$

$$2\omega_s h_s \quad 0.5027/\text{sec}$$

$$\omega_g^2 \quad 0.00487/\text{sec}^2$$

$$\omega_s^2 \quad 0.1755/\text{sec}^2$$

$$H_d(s) = 383.6s^3 / (s^4 + 0.6283s^3 + 0.2435s^2 + 0.0245s + 0.000855)$$

Since both input and output are displacements, $|H_d(s)|$ is the frequency-dependent amplification of the seismograph. The gain C has physical dimensions sec^{-1} , therefore, $H_d(s)$ is dimensionless.

To find the amplification at the angular frequency ω :

$$M(\omega) = |H_d(j\omega)|:$$

$$M(\omega) = \frac{C\omega^3}{\sqrt{(\omega_s^2 - \omega^2)^2 + 4\omega^2\omega_s^2h_s^2} \sqrt{(\omega_g^2 - \omega^2)^2 + 4\omega^2\omega_g^2h_g^2}}$$

$$H_d(s) = \frac{Cs^3}{(s^2 + 2s\omega_s h_s + \omega_s^2)(s^2 + 2s\omega_g h_g + \omega_g^2)}$$

A triple zero exists for $C=0$

Each factor $s^2 + 2s\omega_0 h + \omega_0^2$ in the denominator has zeros

$$s_0 = \omega_0(-h \pm j\sqrt{1-h^2}) \quad \text{for } h < 1$$

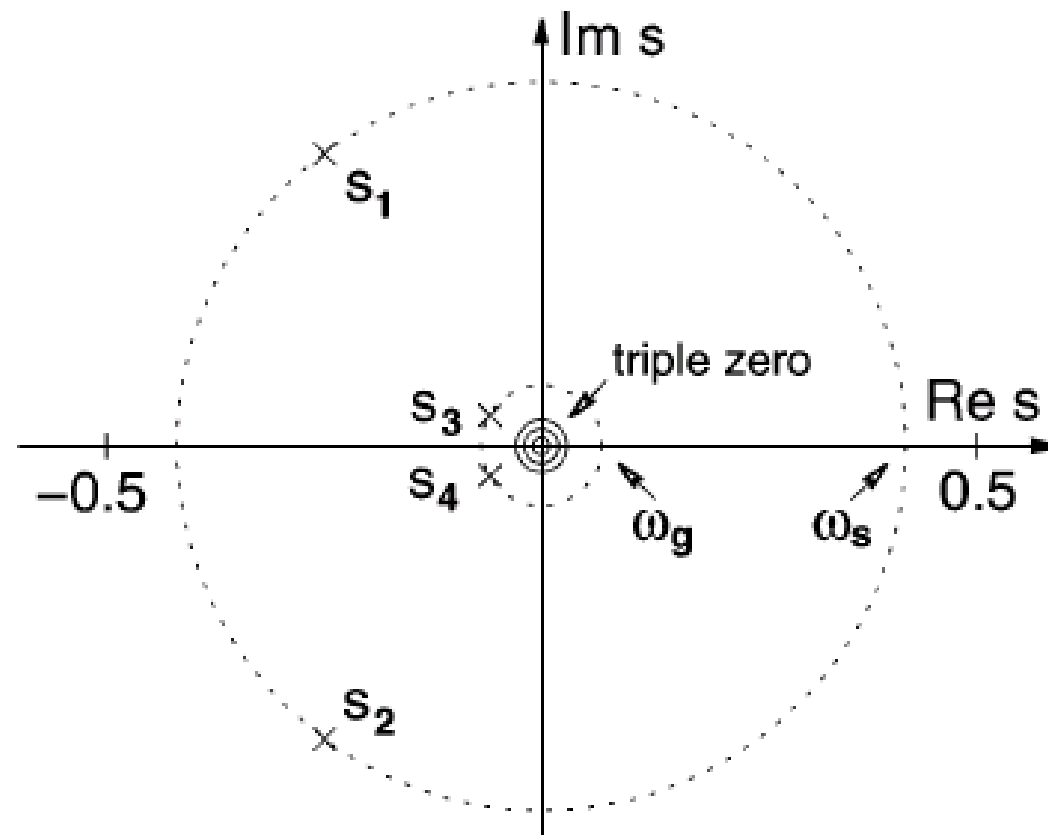
$$s_0 = \omega_0(-h \pm \sqrt{h^2 - 1}) \quad \text{for } h \geq 1$$

$$s_1 = \omega_s(-h_s + j\sqrt{1-h_s^2}) = -0.2513 + 0.3351j \quad [\text{sec}^{-1}]$$

$$s_2 = \omega_s(-h_s - j\sqrt{1-h_s^2}) = -0.2513 - 0.3351j \quad [\text{sec}^{-1}]$$

$$s_3 = \omega_g(-h_g + j\sqrt{1-h_g^2}) = -0.0628 + 0.0304j \quad [\text{sec}^{-1}]$$

$$s_4 = \omega_g(-h_g - j\sqrt{1-h_g^2}) = -0.0628 - 0.0304j \quad [\text{sec}^{-1}]$$



To reconstruct $H_d(s)$ from its poles and zeros we write:

$$H_d(s) = \frac{Cs^3}{(s - s_1)(s - s_2)(s - s_3)(s - s_4)}.$$

$$H_d(s) = \frac{Cs^3}{(s^2 - s(s_1 + s_2) + s_1s_2)(s^2 - s(s_3 + s_4) + s_3s_4)}.$$

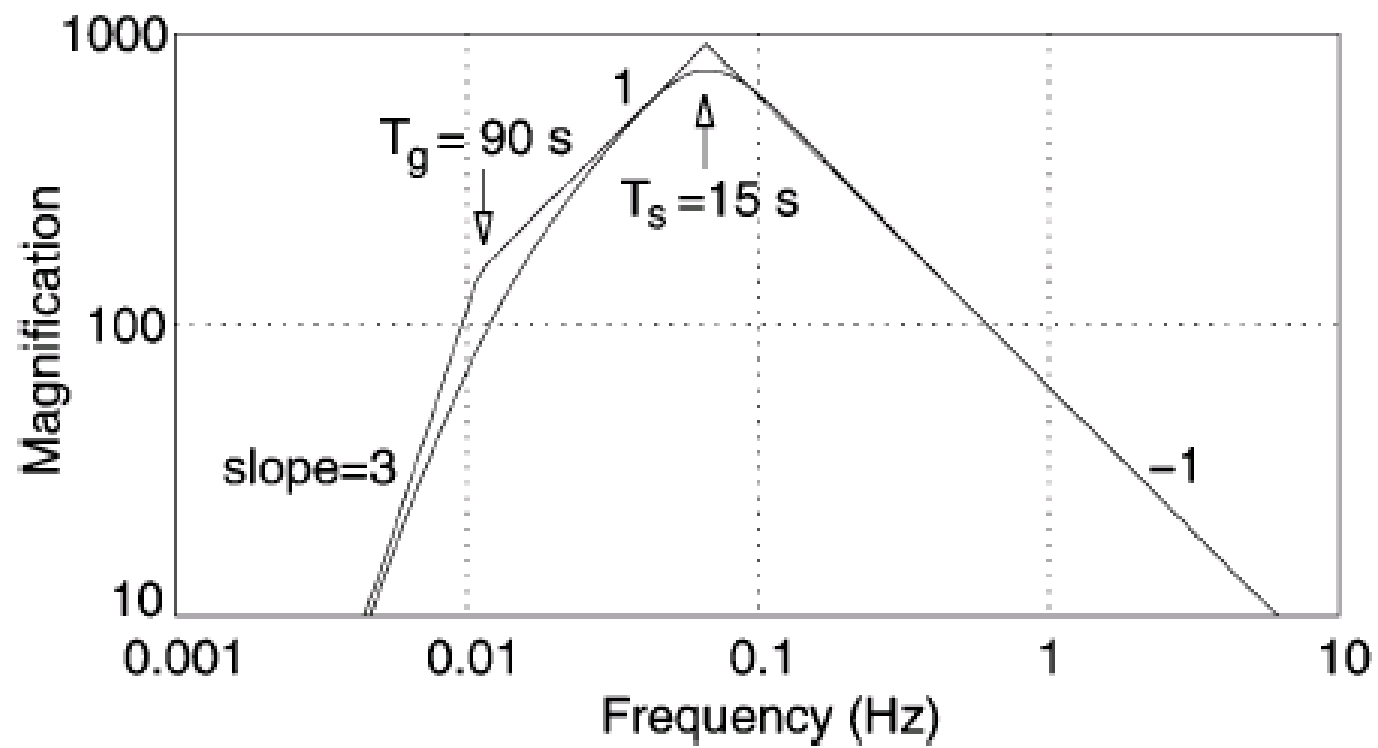
In this way we have all the real factors as:

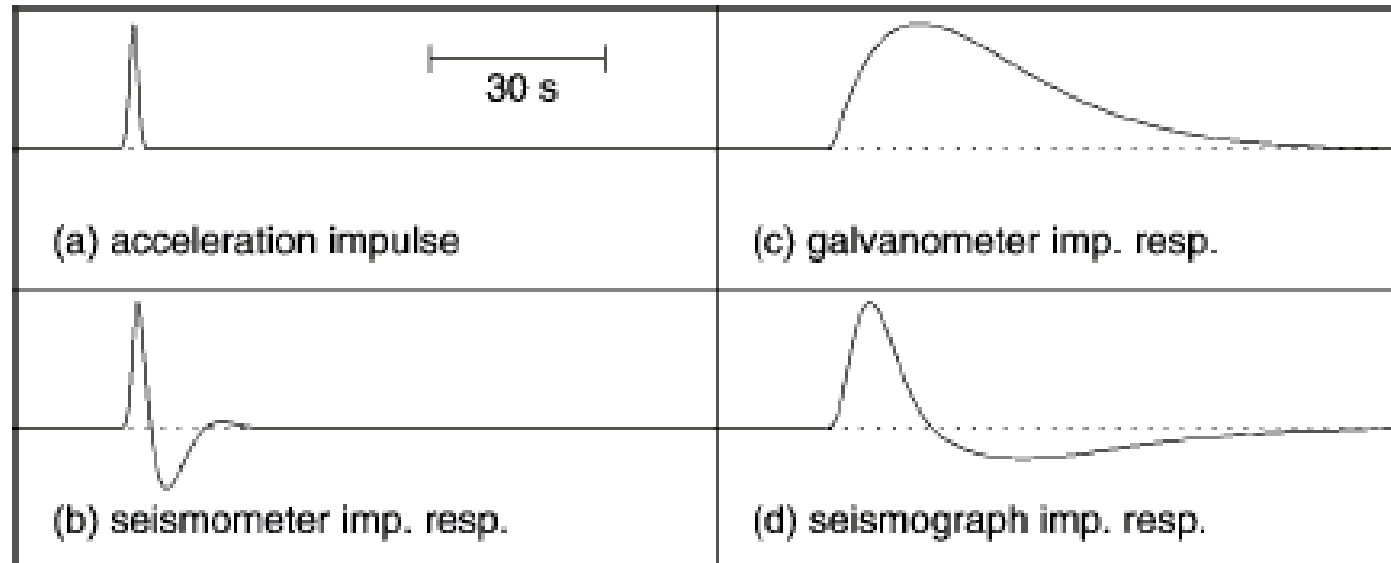
$$s_2 = s_1^* \qquad s_4 = s_3^*.$$

$$H_d(s) = \frac{Cs^3}{(s^2 - s(s_1 + s_2) + s_1s_2)(s^2 - s(s_3 + s_4) + s_3s_4)}.$$

$$H_d(s) = \frac{Cs^3}{(s^2 + 2s\omega_s h_s + \omega_s^2)(s^2 + 2s\omega_g h_g + \omega_g^2)}$$

$$H_d(s) = \frac{383.6s^3}{(s^2 + 0.5027s + 0.1755)(s^2 + 0.1257s + 0.00487)}$$





If $P(s)$ it is a polynomial of s and $P(\alpha) = 0$ then $s = \alpha$ is called a root, or a zero, of the polynomial

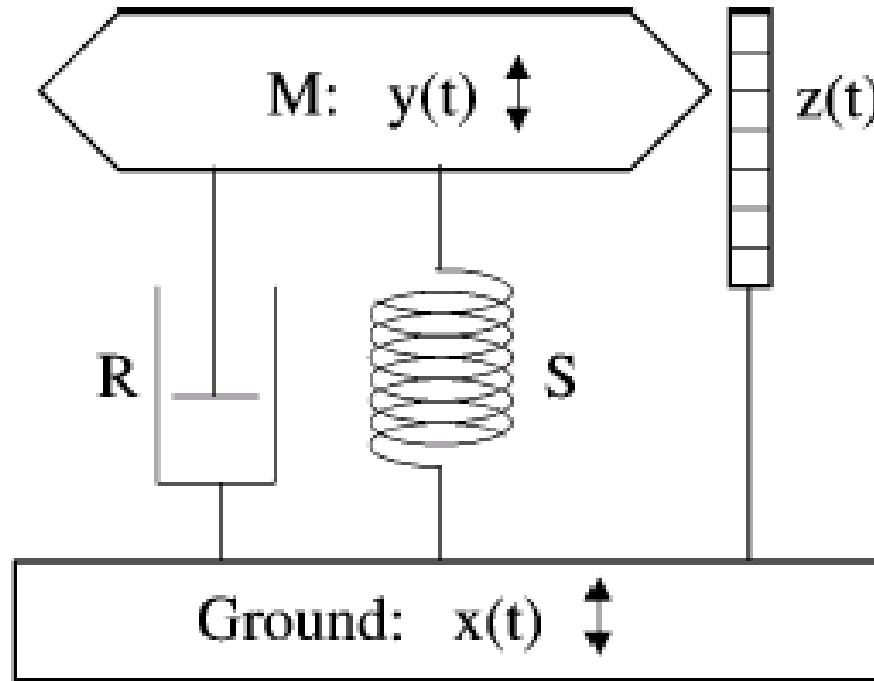
A polynomial of degree n has n complex zeros and can be factorized as:

$$P(s) = p \cdot \prod (s - s_i) .$$

The zeros of a polynomial, with the p -factor, completely determine the polynomial. Since our transfer functions $H(s)$ are the ratio of two polynomials, they can be specified by their zeros (the numerator zeros $G(s)$), their poles (the denominator zeros $F(s)$) and a gain factor (or equivalently the total gain at a given frequency). The whole system, provided it remains within its linear operating range and does not produce noise, can, therefore, be described by a small number of discrete parameters.

$$H(s) = \frac{c_2 s^2 + c_1 s + c_0}{d_2 s^2 + d_1 s + d_0}$$

$$H(s) = A \frac{\prod_{n=1}^N (s - z_n)}{\prod_{m=1}^M (s - p_m)}$$



$$z(t) = y(t) - x(t)$$

$x(t)$ = ground motion

$y(t)$ = absolute motion of mass

$z(t)$ = relative ground motion of the mass

$f(t)$ = external force acting on the mass

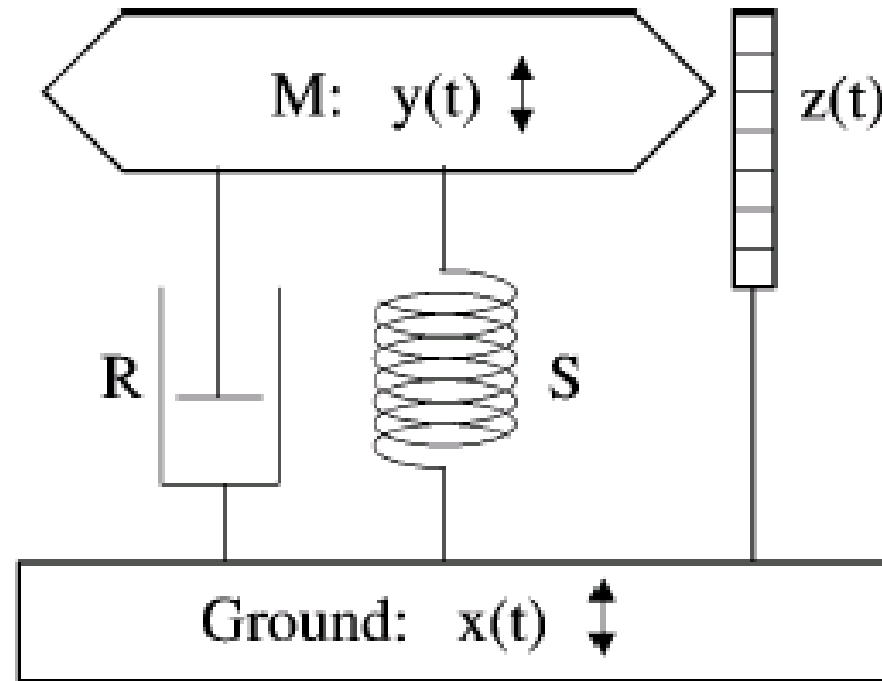
$$M \ddot{y}(t) = f(t) - S z(t) - R \dot{z}(t) .$$

M = mass (kg)

S = spring constant (Newtons per meter)

R = damping constant (Newtons per meter per second)

From each external force $f(t)$ acting on the mass and the forces transmitted by the spring and damper will correspond an acceleration $\ddot{y}(t)$



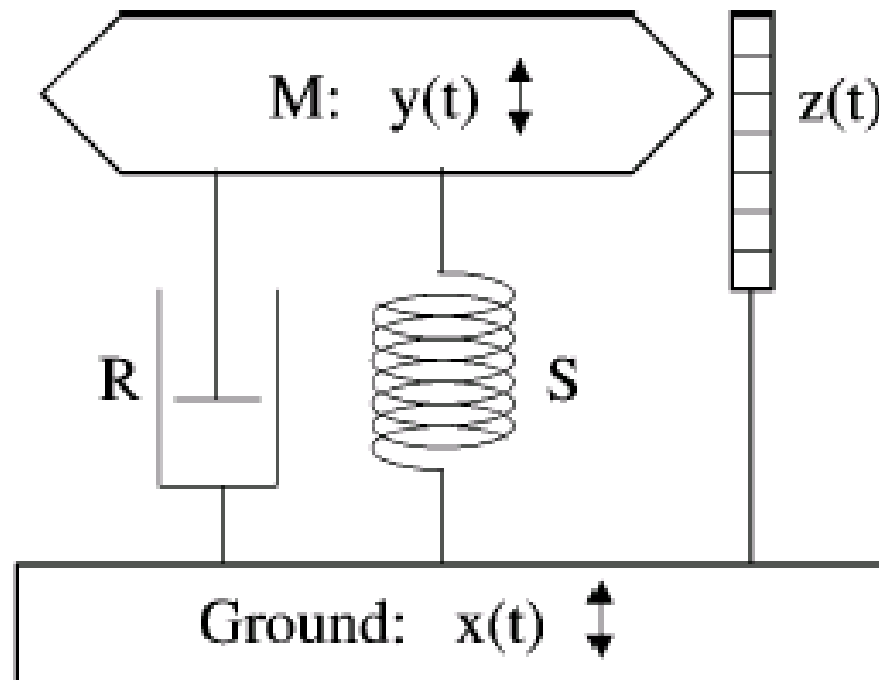
$$z(t) = y(t) - x(t)$$

$$M \ddot{y}(t) = f(t) - S z(t) - R \dot{z}(t).$$

Since we are interested in the relationship between $z(t)$ e $x(t)$:

$$M \ddot{z}(t) + R \dot{z}(t) + S z(t) = f(t) - M \ddot{x}(t).$$

A ground acceleration $\ddot{x}(t)$ corresponds to an acceleration $f(t) = -M \ddot{x}(t)$ applied to the mass in the absence of ground acceleration

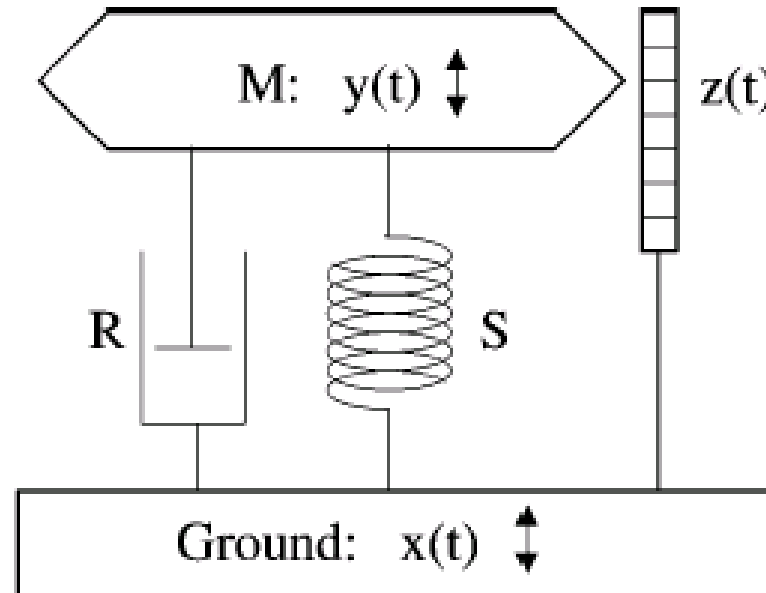


$$z(t) = y(t) - x(t)$$

$$M \ddot{z}(t) + R \dot{z}(t) + S z(t) = f(t) - M \ddot{x}(t)$$

Using the Fourier transform assuming harmonic motion

$$x(t) = \tilde{X} e^{j\omega t} / 2\pi \text{ ed a harmonic force } f(t) = \tilde{F} e^{j\omega t} / 2\pi,$$



$$z(t) = y(t) - x(t)$$

$$M \ddot{z}(t) + R \dot{z}(t) + S z(t) = f(t) - M \ddot{x}(t)$$

$$(-\omega^2 M + j\omega R + S)\tilde{Z} = \tilde{F} + \omega^2 M \tilde{X}$$

$$\tilde{Z} = (\tilde{F} / M + \omega^2 \tilde{X}) / (-\omega^2 + j\omega R / M + S / M).$$

$$\tilde{Z} = (\tilde{F} / M + \omega^2 \tilde{X}) / (-\omega^2 + j\omega R / M + S / M).$$

Frequenza di taglio

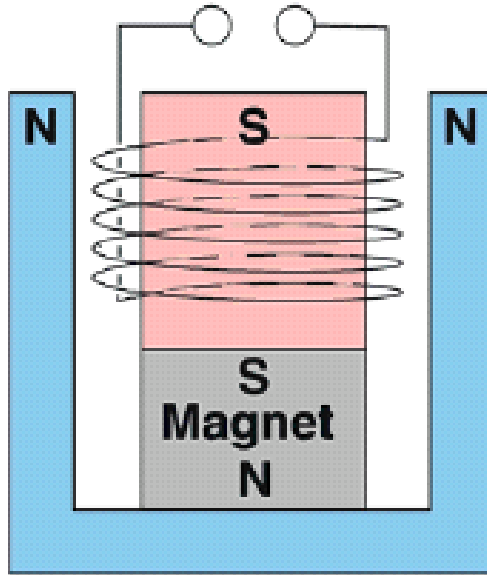
$$f_0 = \omega_0 / 2\pi \quad \omega_0 = \sqrt{S / M}.$$

smorzamento numerico

$$h = R / (2\omega_0 M)$$

$$\tilde{Z} = (\tilde{F} / M + \omega^2 \tilde{X}) / (-\omega^2 + 2j\omega\omega_0 h + \omega_0^2)$$

Alla frequenza angolare ω_0 il moto del terreno viene amplificato di un fattore $\omega_0 M / R$ ed uno sfasamento di $\tilde{X}$



To convert the motion of the mass into an electrical signal, the mechanical pendulum is equipped with an electromagnetic velocity transducer.

$$\tilde{Z} = (\tilde{F} / M + \omega^2 \tilde{X}) / (-\omega^2 + 2j\omega\omega_0 h + \omega_0^2)$$

o convert the movement of the mass into an electrical signal, the mechanical pendulum in the simplest case is equipped with an electromagnetic velocity transducer, and we denote by U its output voltage. Thus we have an electromagnetic seismometer, also called a geophone, designed for seismic exploration. When the reactivity of the transducer is E (volts per meter per second; $\tilde{U} = -Ej\omega\tilde{Z}$) we get

$$\tilde{U} = -j\omega E(\tilde{F} / M + \omega^2 \tilde{X}) / (-\omega^2 + 2j\omega\omega_0 h + \omega_0^2)$$

In the case of the absence of external forces

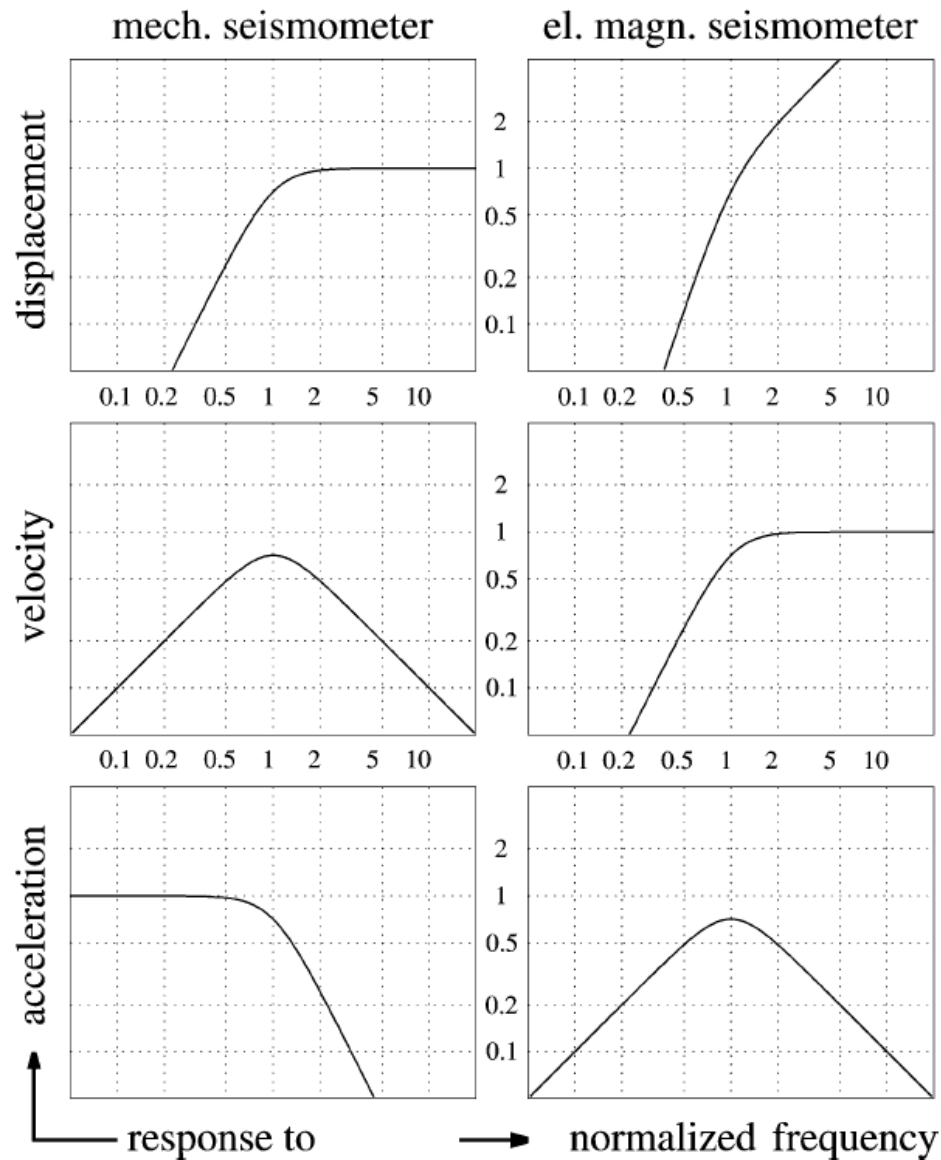
$$f(t) = 0, \tilde{F} = 0$$

We obtain the frequency-dependent complex response function

$$\tilde{H}_d(\omega) := \tilde{U} / \tilde{X} = -j\omega^3 E / (-\omega^2 + 2j\omega\omega_0 h + \omega_0^2)$$

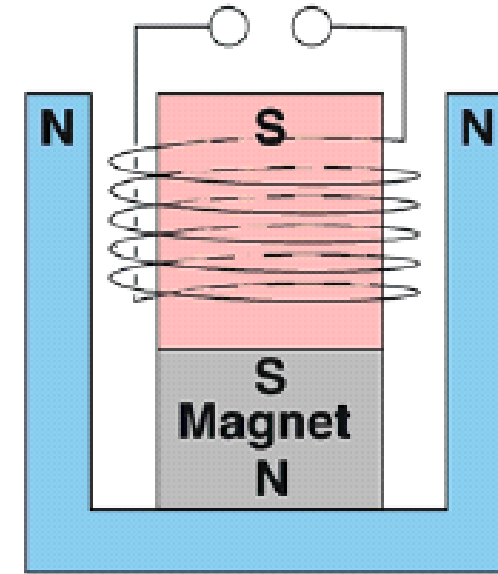
$$\tilde{H}_v(\omega) := \tilde{U} / (j\omega\tilde{X}) = -\omega^2 E / (-\omega^2 + 2j\omega\omega_0 h + \omega_0^2)$$

$$\tilde{H}_a(\omega) := \tilde{U} / (-\omega^2 \tilde{X}) = j\omega E / (-\omega^2 + 2j\omega\omega_0 h + \omega_0^2)$$



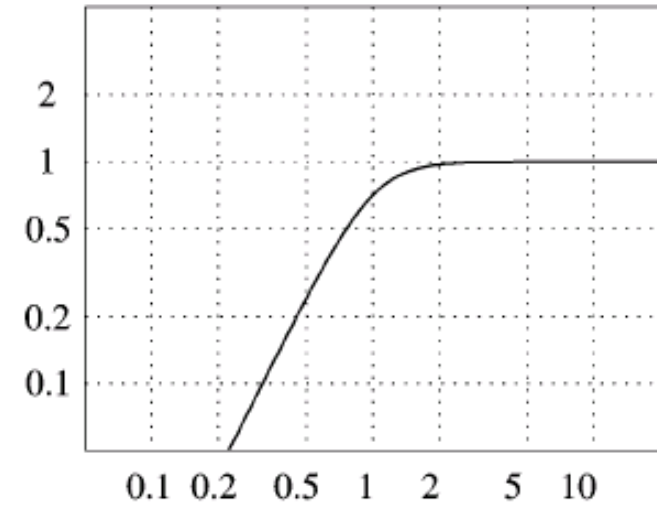
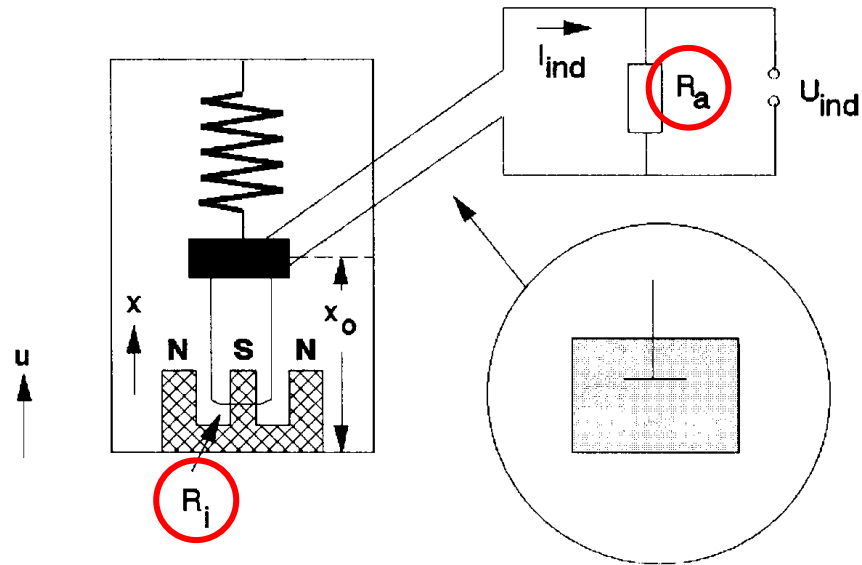
Response curves of a mechanical seismometer (pendulum and spring, left) and electrodynamic seismometer (geophone, right) with respect to different types of input signals (displacement, velocity and acceleration, respectively). The normalized frequency is the frequency signal divided by the self-frequency (corner frequency) of the seismometer.

The simplest transducer for both detecting motion and exerting forces is an Electromagnetic (electrodynamic) device in which a coil moves in the field of a permanent magnet, as in a loudspeaker. Movement induces a voltage in the coil; a current flowing through the coil produces a force. It follows from conservation of energy that the reactivity of the coil-magnet system as a force transducer, in Newtons per Ampere, and its reactivity as a velocity transducer, in Volts per meter per second, are identical. In fact, the units are the same (remember that $1\text{Nm} = 1\text{Joule} = 1\text{VAs}$). When the velocity transducer is loaded with a resistor, thus allowing a current to flow, it generates a force, opposed to motion. This effect is used to dampen the mechanical free oscillation of the passive seismic sensor (geophones and electromagnetic seismometers).



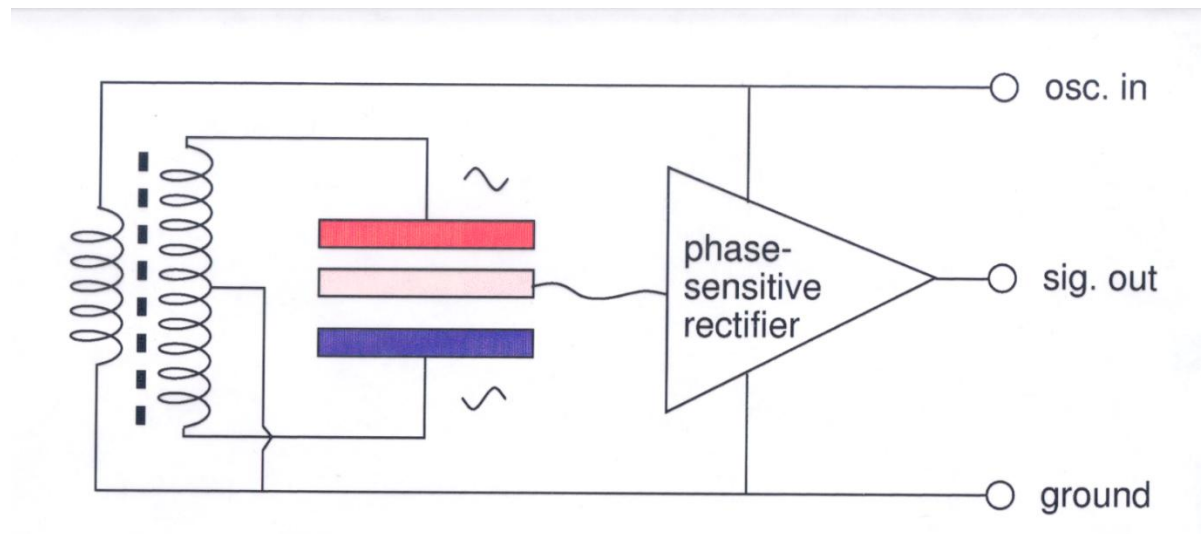
In this case the damping is given in small part by the mechanical damping and mainly by the electromagnetic transducer:

Where $h_{el} = E^2 / 2M\omega_0 R_d$ is the total damping (the sum of the coil resistance and the external resistance R_d)

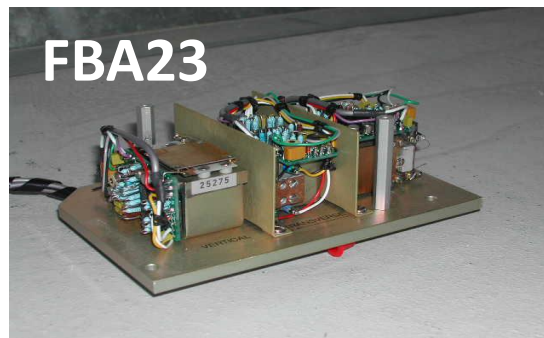
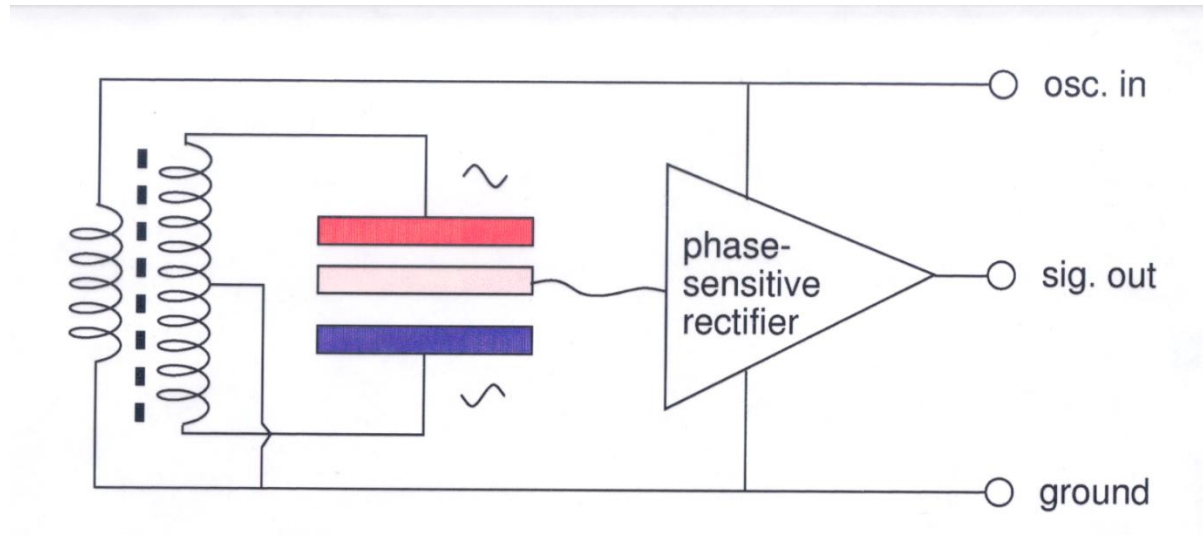


The total damping is chosen equal to $1/\sqrt{2}$ so as to obtain a flat response.

At very low frequencies, the output signal of electromagnetic transducers becomes too small to be useful for seismic detection. Therefore, active electronic transducers are used in which a carrier signal, generally in the audio frequency range, is modulated by the motion of the seismic mass. The basic modulating device is an inductive or capacitive half-bridge. Inductive half-bridges are separated by a moving magnetic core. Capacitive half-bridges are made as three-plate capacitors in which the center plate or outer plates move with the seismic mass. Their sensitivity is limited by the ratio of the electrical noise of the demodulator to the electric field strength.

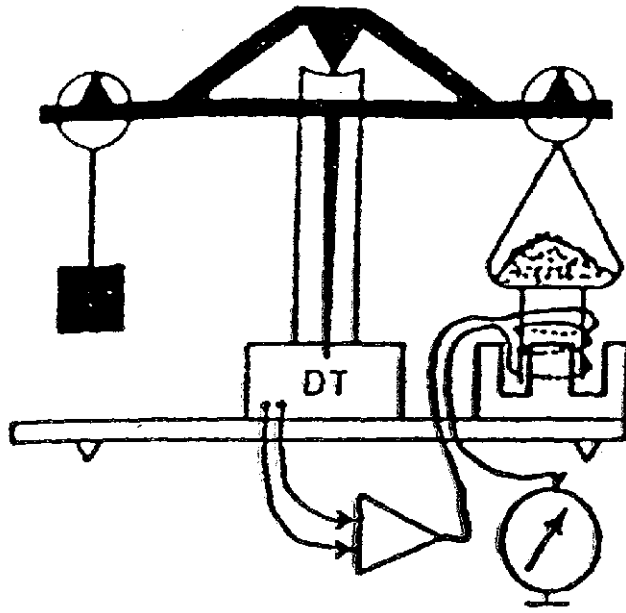


Capacitive displacement *transducer* (Blumlein bridge).

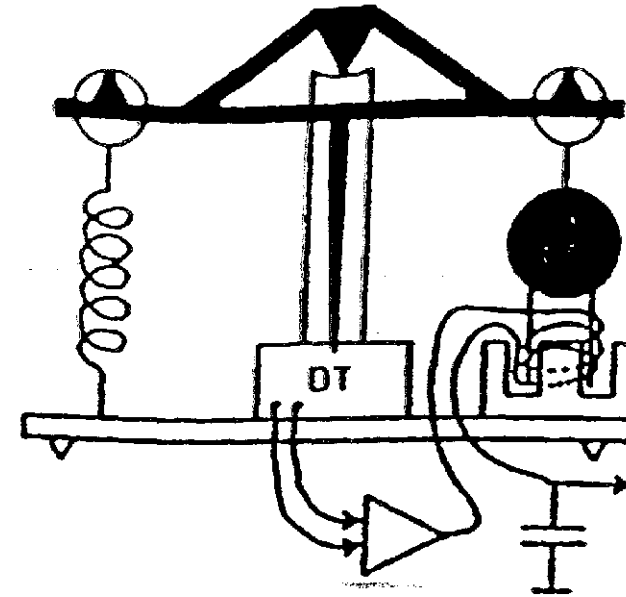


"Force balance" seismometers and accelerometers

The inertial force is compensated (or balanced) with an electrically generated force so that the mass moves as little as possible. A small amount of movement is still necessary otherwise the inertial force cannot be observed.



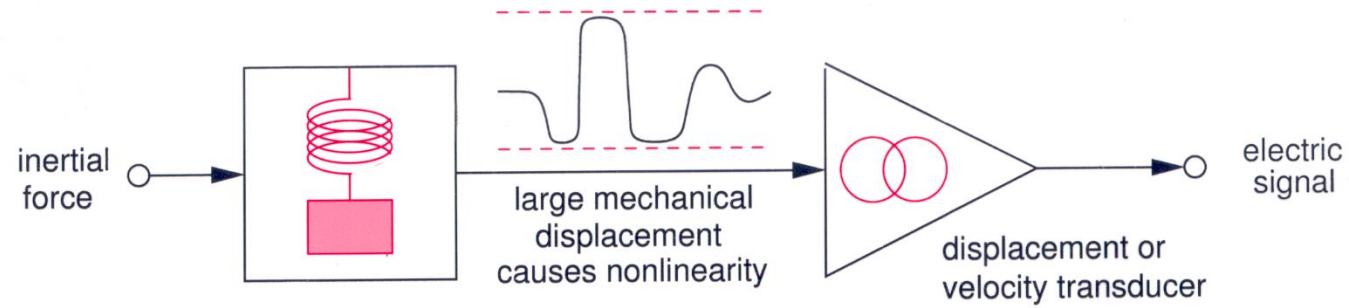
Scales with automatic
weight replacement



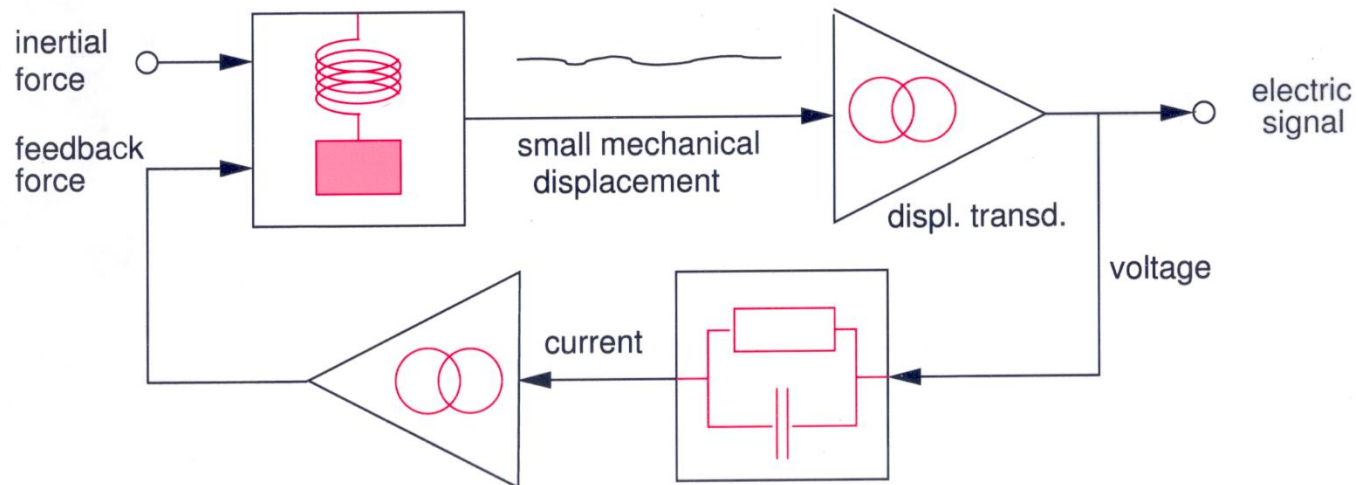
Feedback
seismometer

Figure 1.10 Automatic weight replacement and feedback seismometer

Seismometer without feedback

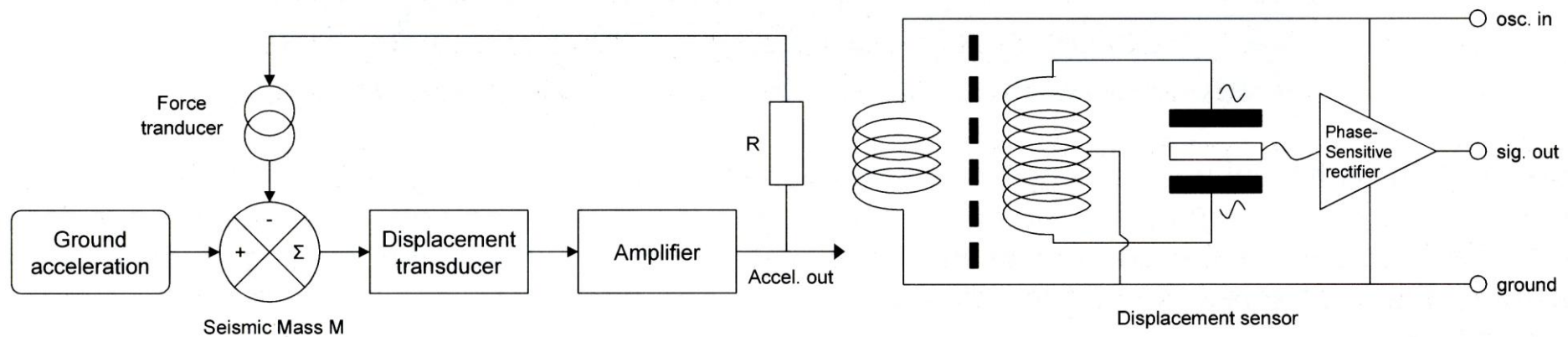
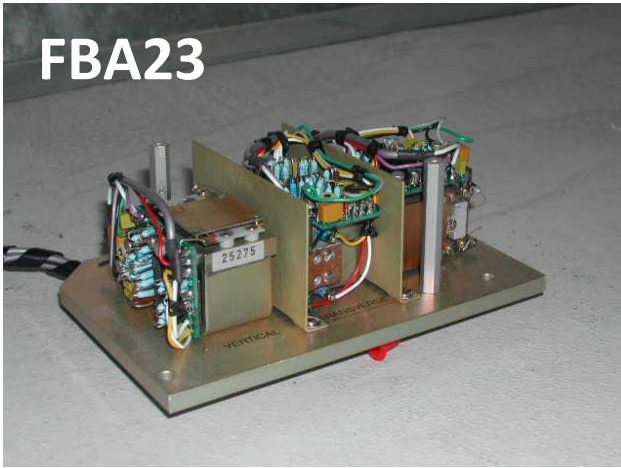


Force-Balance Seismometer

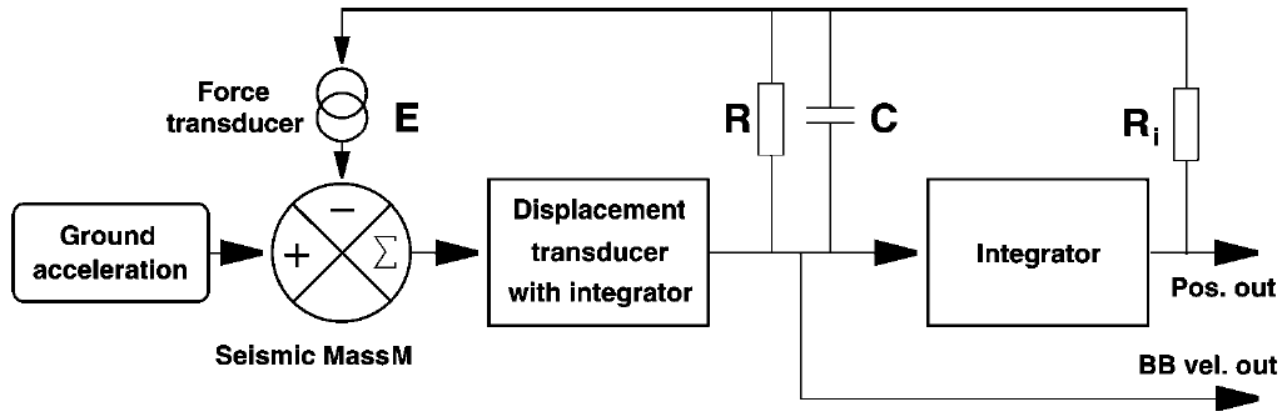


ideally, the mechanical sensor "doesn't know" how large the seismic signal is

FBA23



The motion of the mass is controlled by the sum of 2 forces: the inertial force due to ground acceleration and the negative counter-reaction force.



Circuito di retroazione di un sismometro VBB (velocità-banda larga). La massa sismica è la somma della forza inerziale e della forza di controreazione negativa.

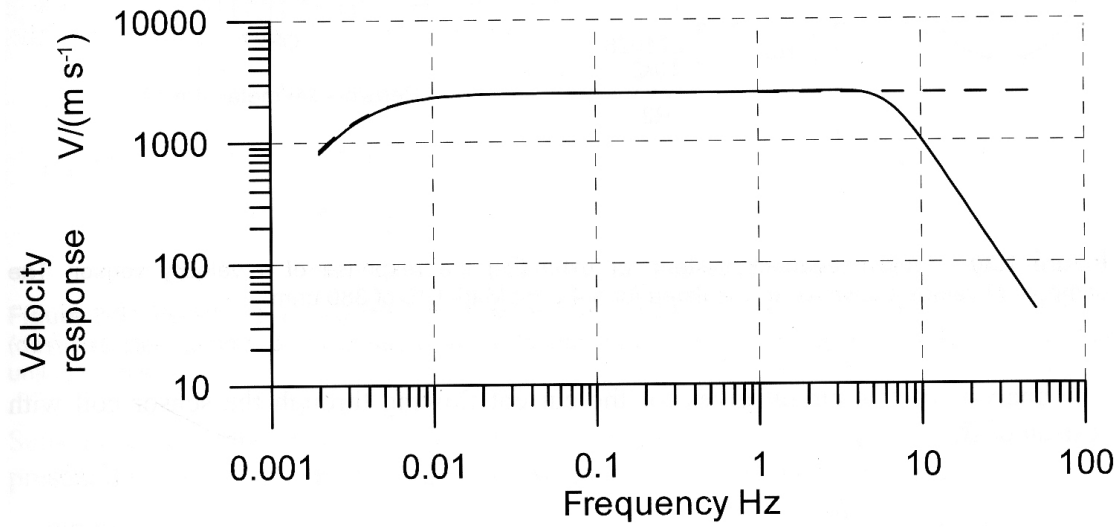
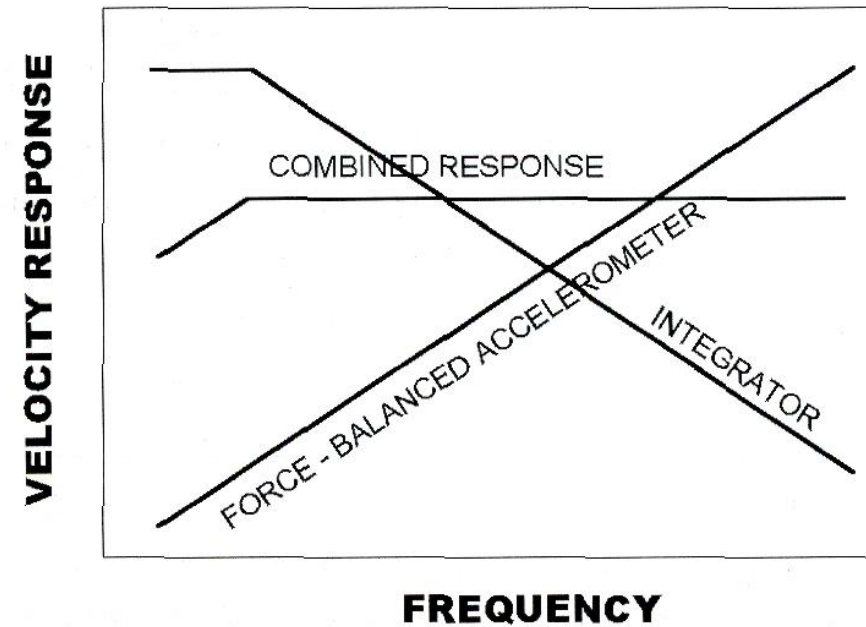
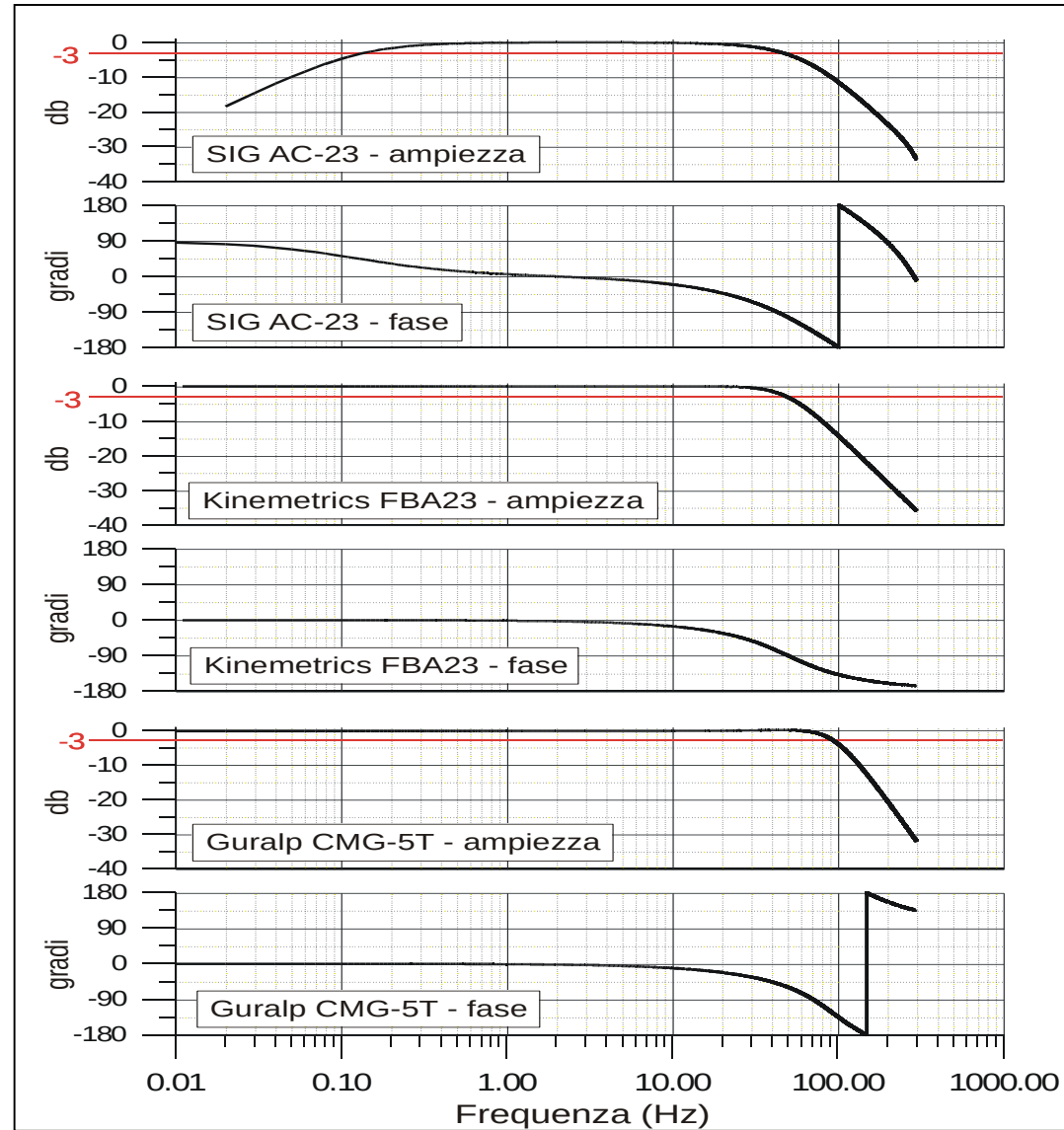
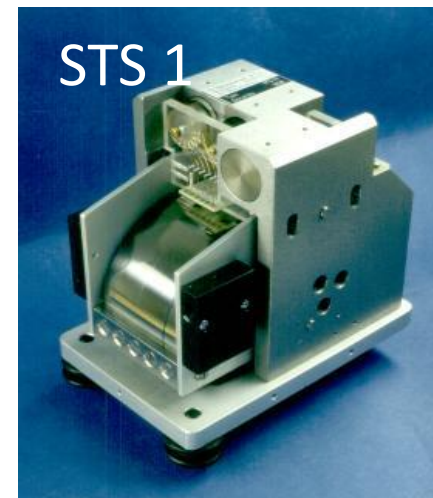
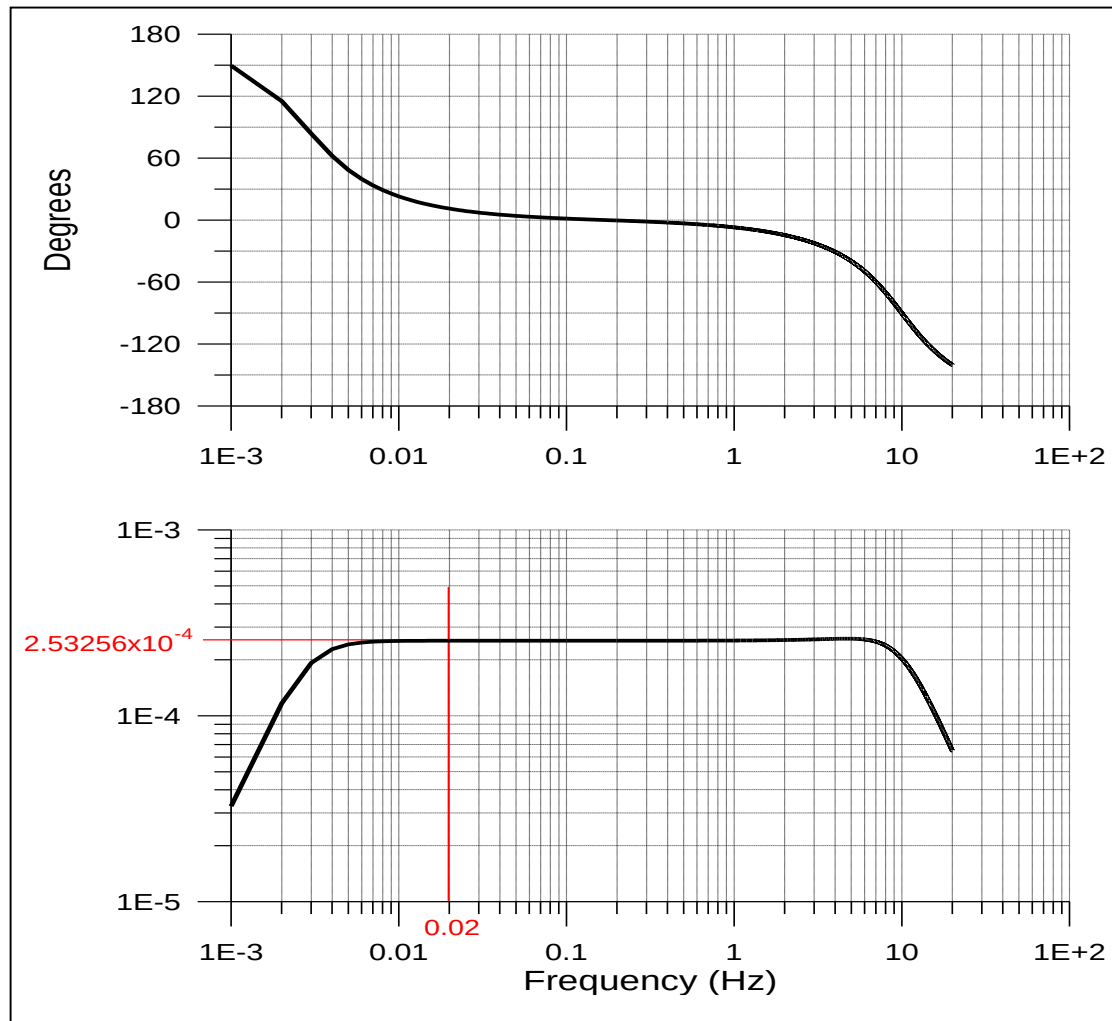


Figure 2.29 Frequency response of a BB system







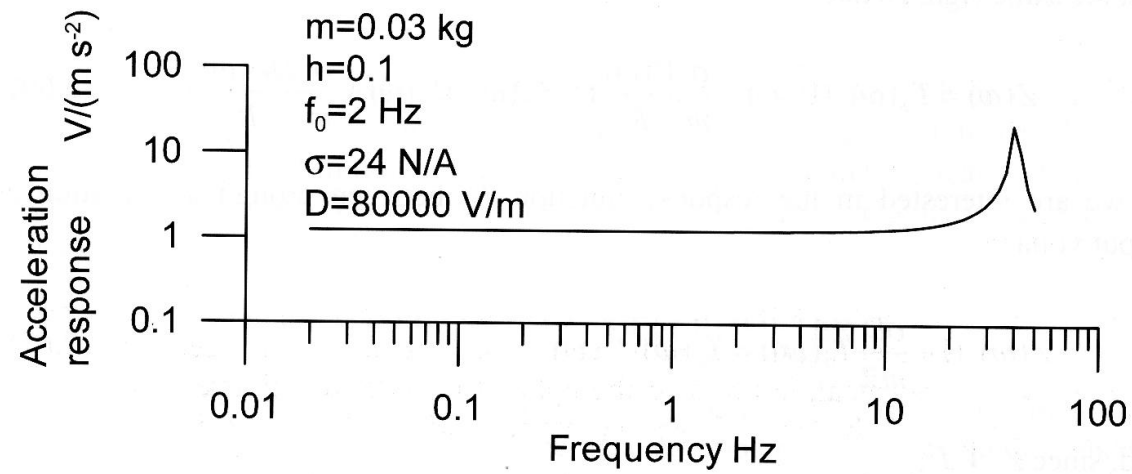


Figure 2.26 Example of the effect of force feedback on the sensor response.

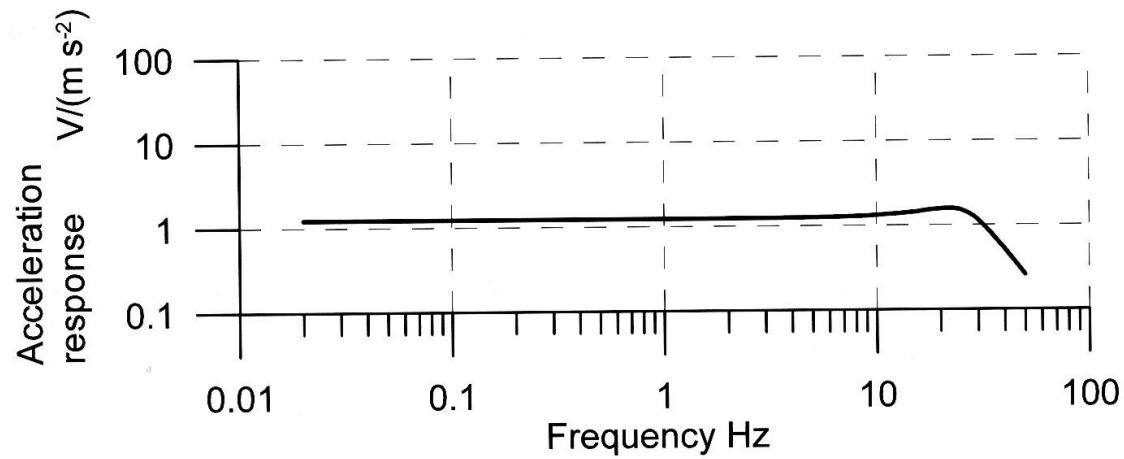


Figure 2.27 Frequency response of filtered FBA.

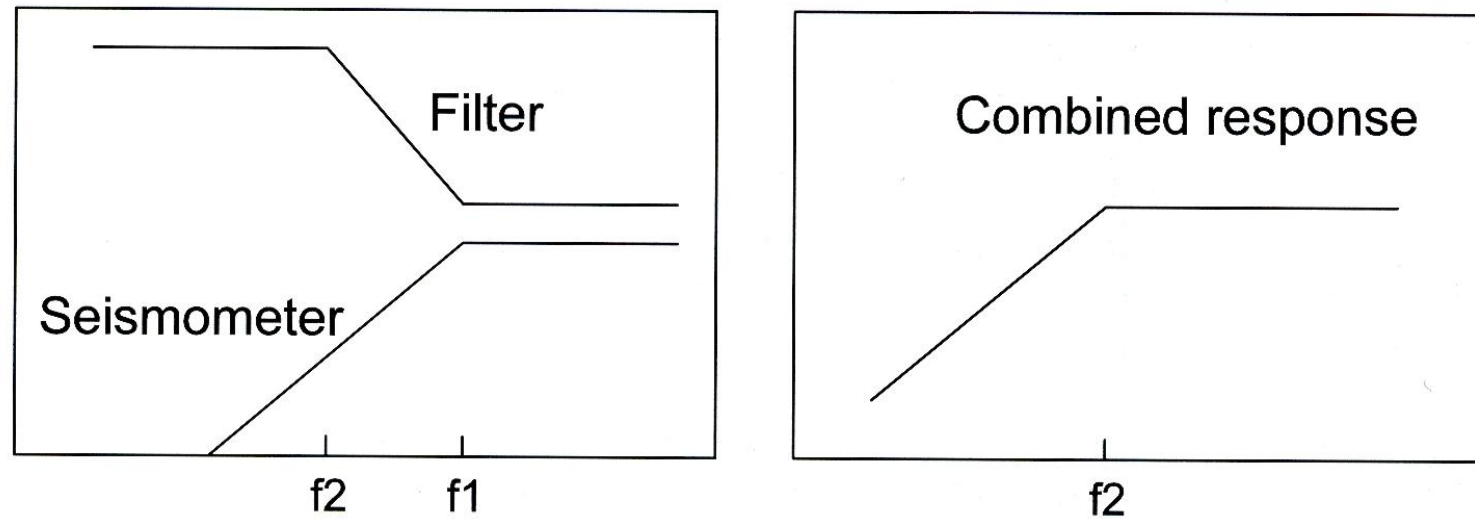
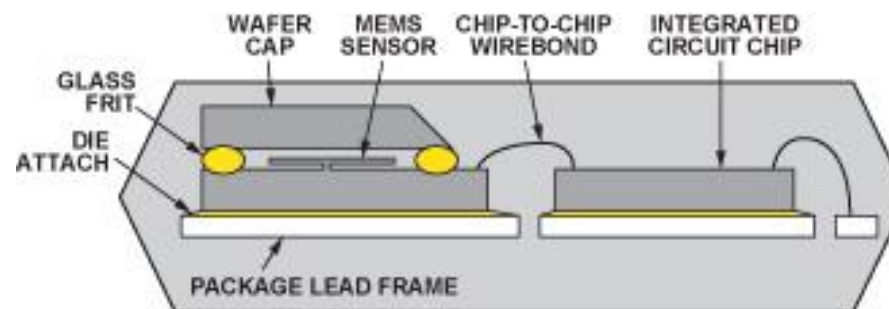
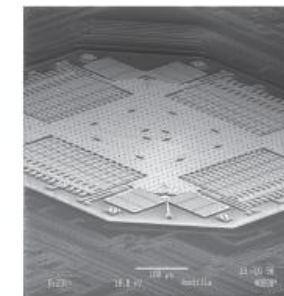
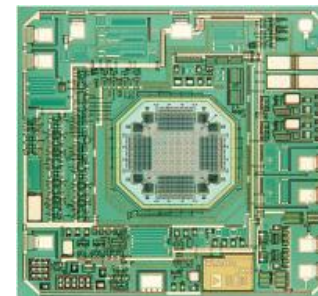
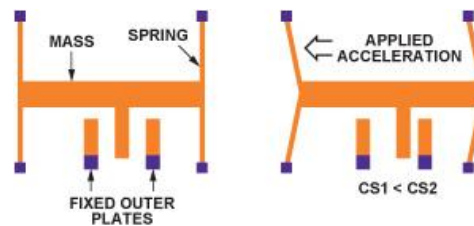
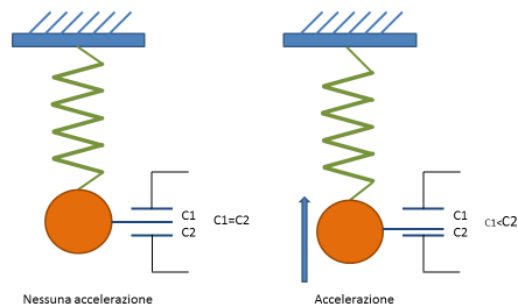


Figure 2.22 Enhancing frequency response by inverse filtering. The response functions are shown as log gain versus log frequency. The seismometer has corner frequency f_1 and the special filter corner frequencies f_1 and f_2 (left). To the right is seen the combined response.



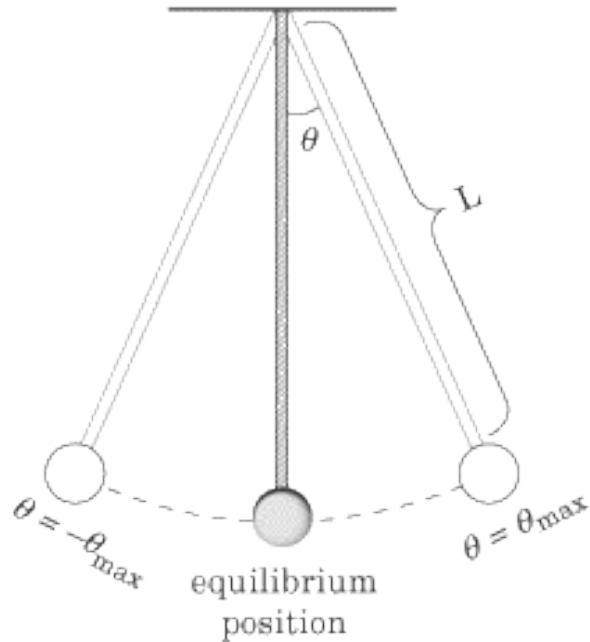
LE-3D/5s	
Power Supply	10...16 V DC, unstabilized
Power Consumption	10 mA at 12 V DC
Output Voltage	400 V/(m/s) precisely adjusted on all components
Full scale output voltage	± 5 V (differential output)
Damping	0.707 critical (internal damping; independent of datalogger input resistance)
Dimensions	195 mm diameter 165 mm height
Weight	6.5 kg
Temperature Range	-15...+60 °C
Housing	Painted aluminium, splash proof, with level adjustment feet and water bubble level control
Eigenfrequency	0.2 Hz
Upper Corner Frequency	> 40 Hz
RMS Noise at 1 Hz	< 1 nm/s
Dynamic Range (typical)	140 dB
Poles	3 poles: -0.885/+0.887j -0.855/-0.887j -0.427/0.000j
Suggested waiting time after connecting power (does not include time required to reach ambient temperature)	30 seconds



The differential equation is that of harmonic motion with period:

$$T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{dz}{dg}}$$

Si vede che la sensibilità è direttamente proporzionale al quadrato del periodo.
Costruttivamente è difficile ottenere $T > 1s$.



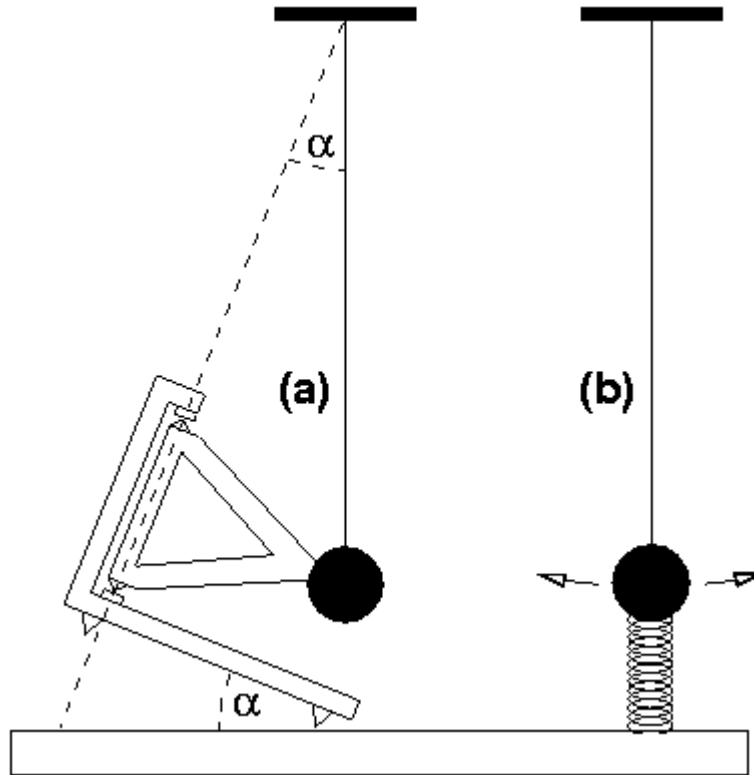
$$L = 1 \text{ m}$$

$$T = 2 \text{ s}$$

$$L = 100 \text{ m}$$

$$T = 20 \text{ s}$$

Astatization Suspension garden-gate



Equivalence between an inclined "garden gate" pendulum and a rope pendulum. For a free period of 20 sec, the rope pendulum must be 100 m long. The angle of inclination α of a garden-gate the pendulum with the same free period and a length of 30 cm is about 0.2° . The longer the period considered, the less stable it will be under the influence of small changes in inclination. (b) Lengthening of the period with an auxiliary compressed spring.

$$T_0 = 2\pi \sqrt{\ell / g \sin \alpha}$$

$$T_0 = 20$$

$$\ell = 30 \text{ cm}$$

$$\alpha = 0.2^\circ$$

The principle is that of the dynamometer. Consider a mass m suspended from a spring with constant k . Under the equilibrium conditions we will have:

$$mg = kz_0$$

If gravity varies, it will be:

$$mdg = kdz$$

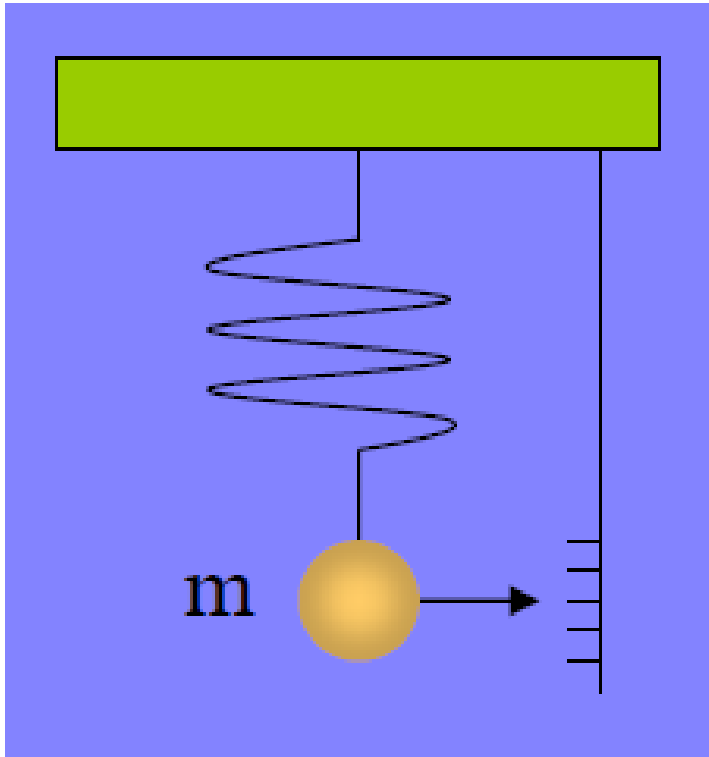
da cui:

$$\frac{dz}{dg} = \frac{m}{k}$$

The ratio is the sensitivity of the instrument. Moving the mass from equilibrium conditions and applying Newton's law gives:

$$mg - kz = m \frac{d^2(z - z_0)}{dt^2}$$

$$\frac{d^2(z - z_0)}{dt^2} + \frac{k}{m}(z - z_0) = 0$$



ASTATIZATION

The mass is supported by a rigid arm of length b rotating about an axis O . We will therefore have equilibrium of moments. At the equilateral position, the moment of the applied forces $M(\varphi, g(\varphi))$ will be zero.

$$M(\varphi_0) = 0$$

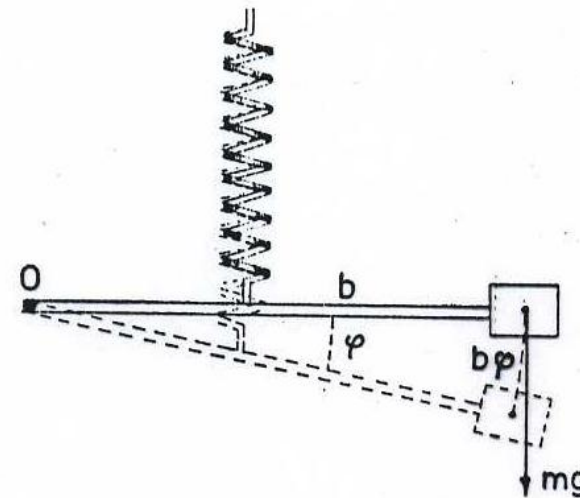
Deriving:

$$\frac{\delta M}{\delta \varphi} d\varphi + \frac{\delta M}{\delta g} dg = 0$$

It results in:

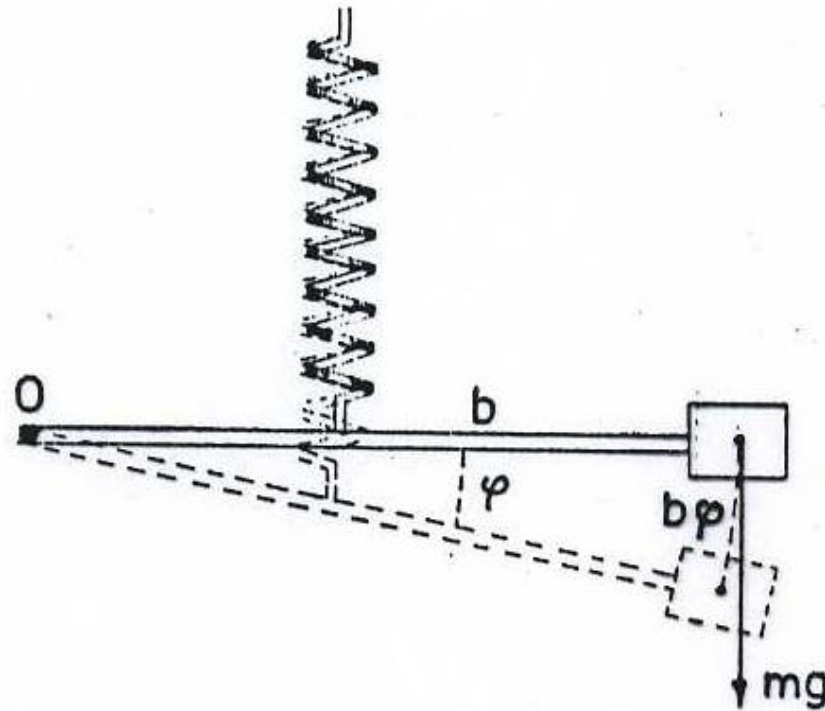
$$\frac{d\varphi}{dg} = - \frac{\delta M / \delta g}{\delta M / \delta \varphi}$$

Sensibility



The total moment is given by the stabilizing component (elastic) and the destabilizing component (gravitational)

$$M = M_e - M_g$$



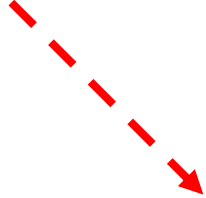
For small values of $Mg = mgb \cos\varphi \approx mgb$ and we will have $\delta Mg/(\delta g) = mb$ while $\delta Me/(\delta g) = 0$ because Me does not depend explicitly on g . So it will be:

$$d\varphi = - \frac{-mb}{\delta M / \delta \varphi} dg$$

For an infinitesimal displacement of mass resulting from a rotation $d\varphi$ there will be:

$$ds = bd\varphi = \frac{mb^2}{\delta M / \delta \varphi} dg$$

Moving the mass from the equilibrium point φ_0 will cause it to oscillate around the point φ_0 . By d'Alembert's principle, the equation of motion will be obtained by equating the moments of the applied forces to those of the forces of inertia. The moment of the forces applied around the equilibrium point (φ_0 small and g constant) can be written:



$$M(\varphi) = M(\varphi_0) + \left(\frac{\delta M}{\delta \varphi} \right)_0 \varphi + \dots$$

0

While the moment of inertia forces, tangential acceleration being $b\ddot{\varphi}$, will be given by:

$$M_i = b \cdot m a_t = m b^2 \ddot{\varphi}$$

From which it follows that the equation of motion will be

$$\frac{d^2 \varphi}{dt^2} + \frac{\left(\frac{\partial M}{\partial \varphi}\right)_0}{m b^2} \varphi = 0$$

Which still gives a harmonic motion of period

$$T = 2\pi \sqrt{\frac{m b^2}{\left(\partial M / \partial \varphi\right)_0}}$$

Therefore also in this case the sensitivity is proportional to the square of the period. To obtain large periods and high sensitivity it will suffice to realize:

$$\left(\frac{\delta M}{\delta \varphi}\right)_0 \cong 0$$

Il procedimento di rendere $\left(\frac{\delta M}{\delta \varphi}\right)_0 \cong 0$ viene detto astatizzazione e può essere realizzato in vari modi

Per avere:

$$\frac{\delta M}{\delta \varphi} = \frac{\delta M_e}{\delta \varphi} = \frac{\delta M_g}{\delta \varphi} \approx 0$$

Si può procedere in due modi.

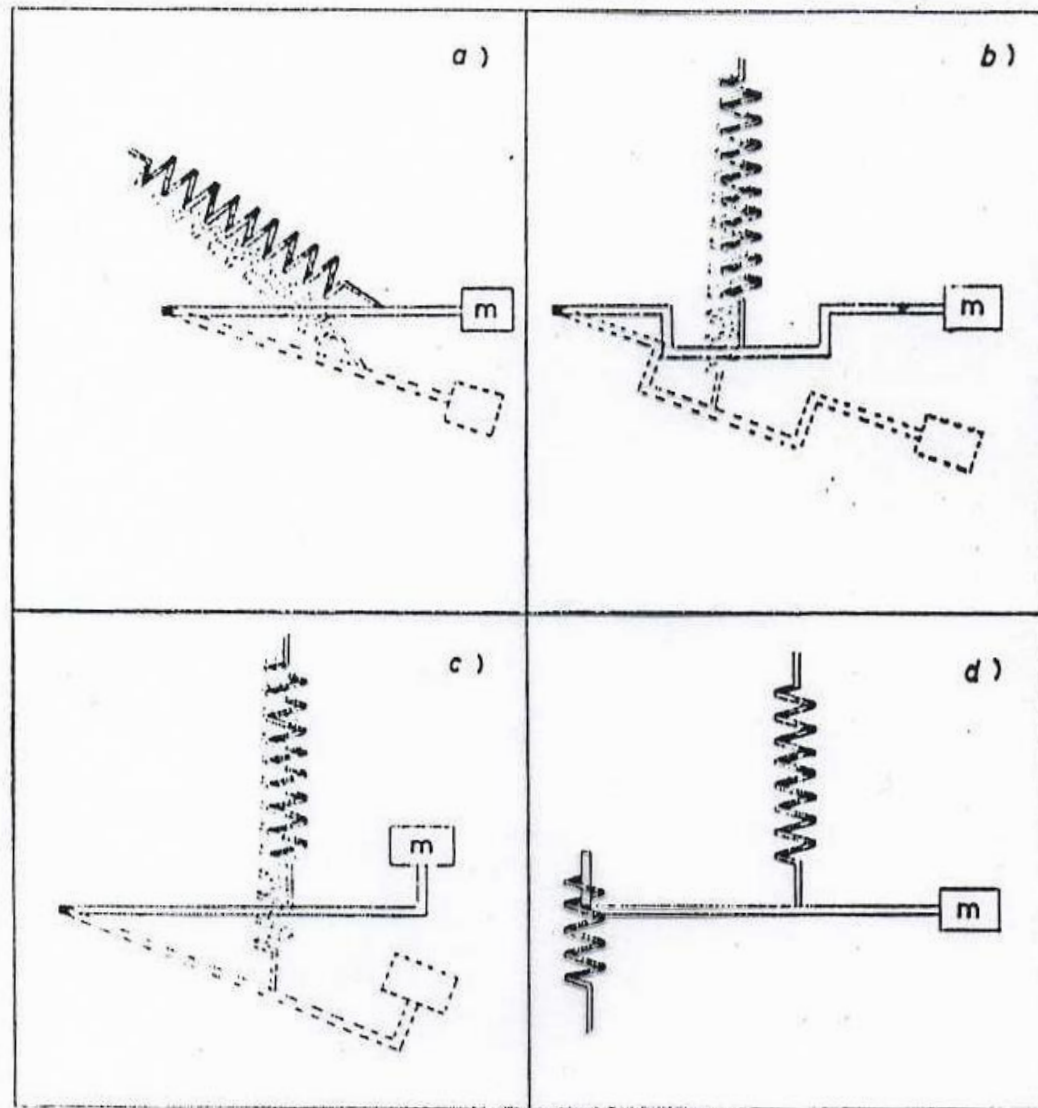
1
Make M_g independent of φ (for which $\frac{\delta M_g}{\delta M_\varphi} \approx 0$) And realize the geometric conditions of the system so that it is $\frac{\delta M_e}{\delta M_\varphi} \approx 0$

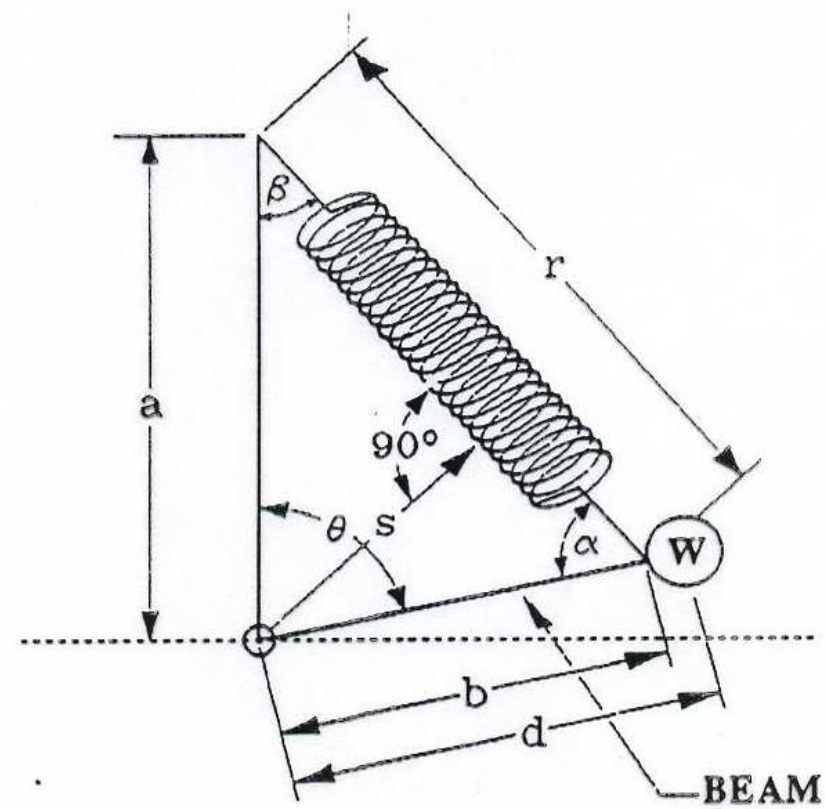
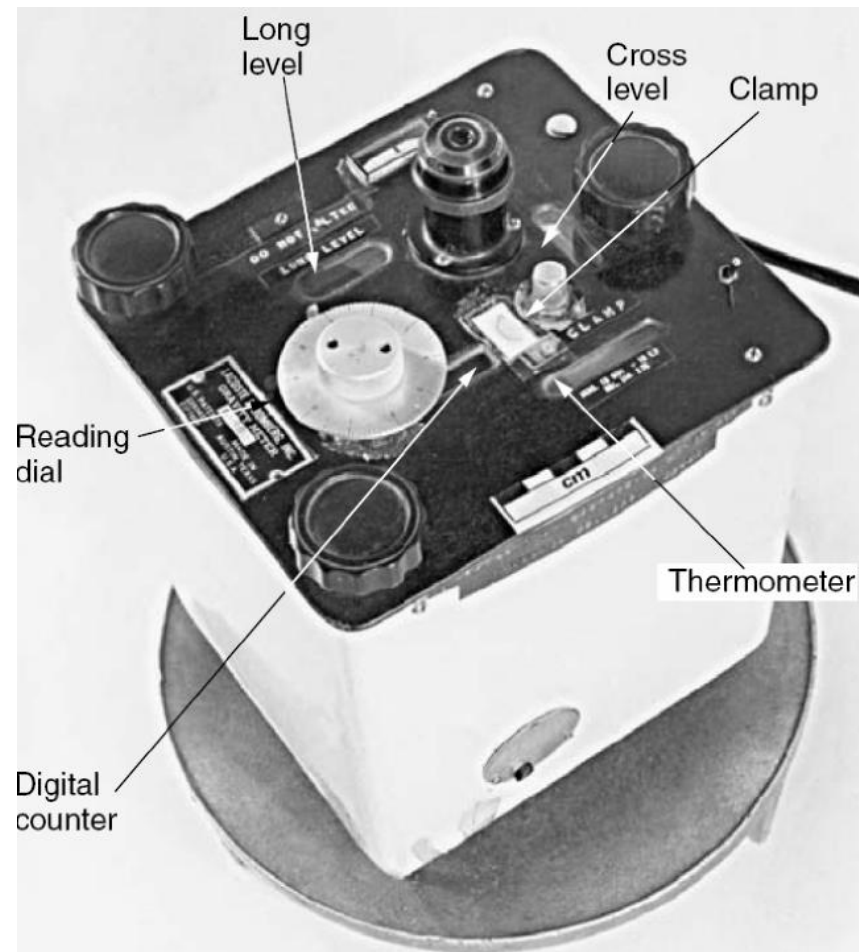
- a) Spring length varies very little
- b) Increases tension, but decreases arm

2

Make $\frac{\delta M_e}{\delta \varphi} \approx \frac{\delta M_g}{\delta \varphi}$

- a) Increase M_e , but increase M_g as well
- b) An increase in M_e is opposed by an opposite moment due to another spring





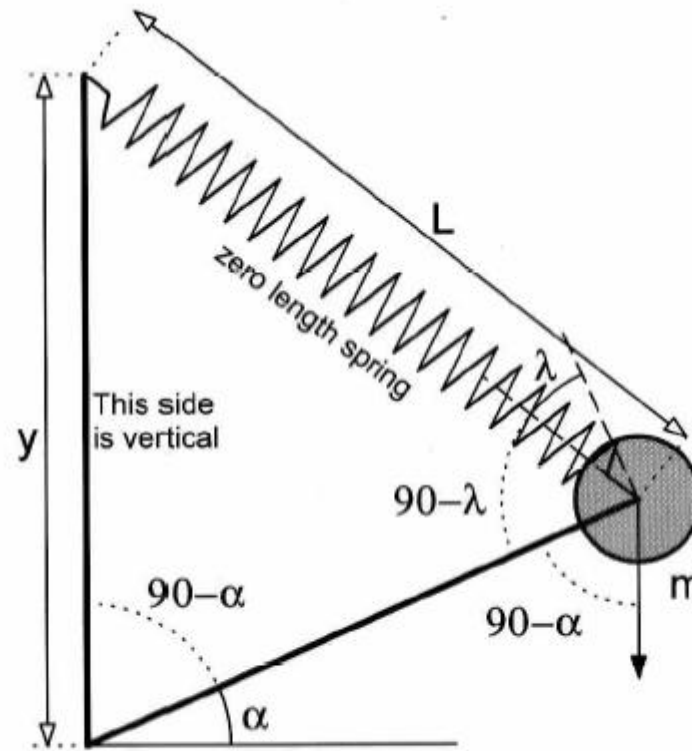
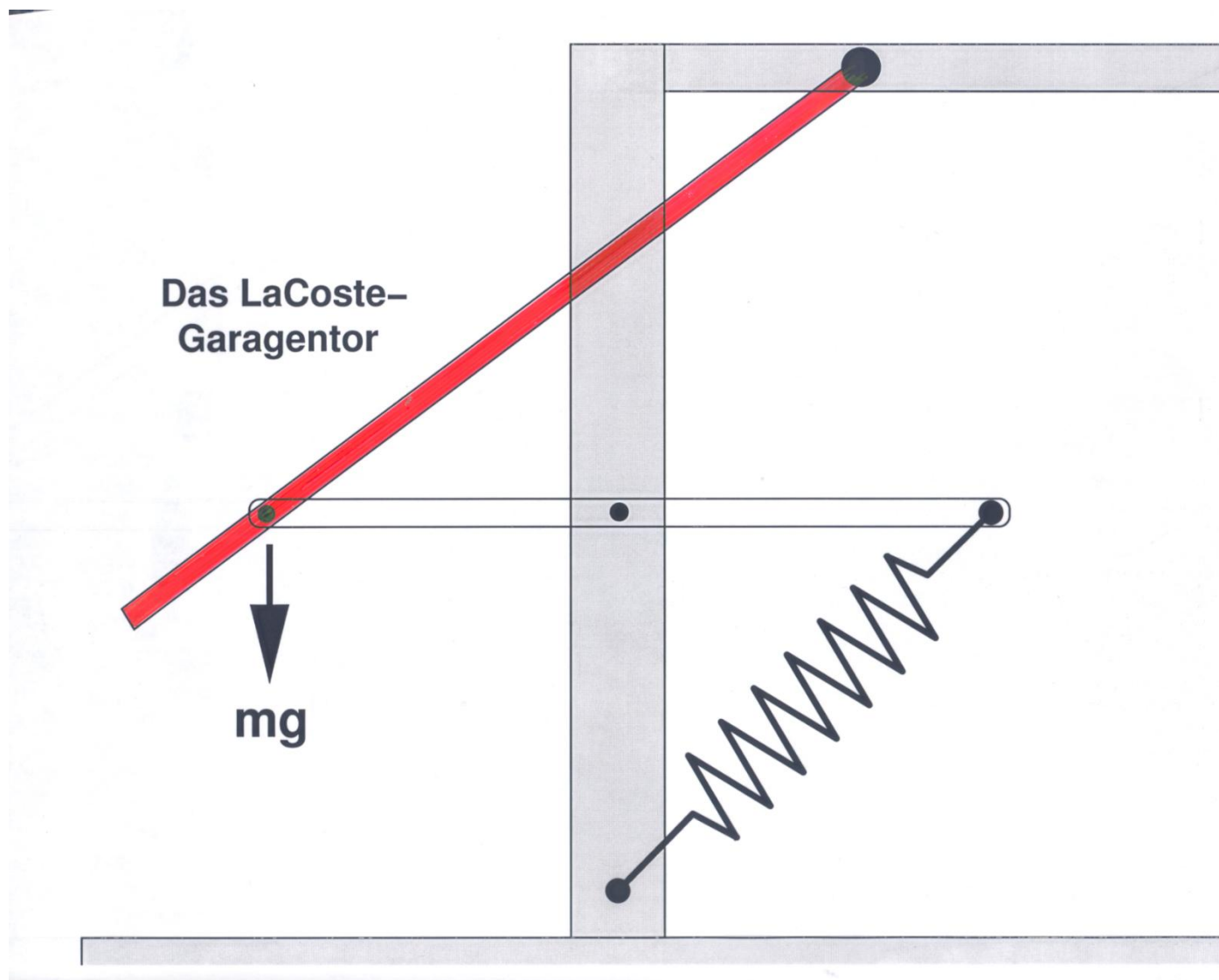
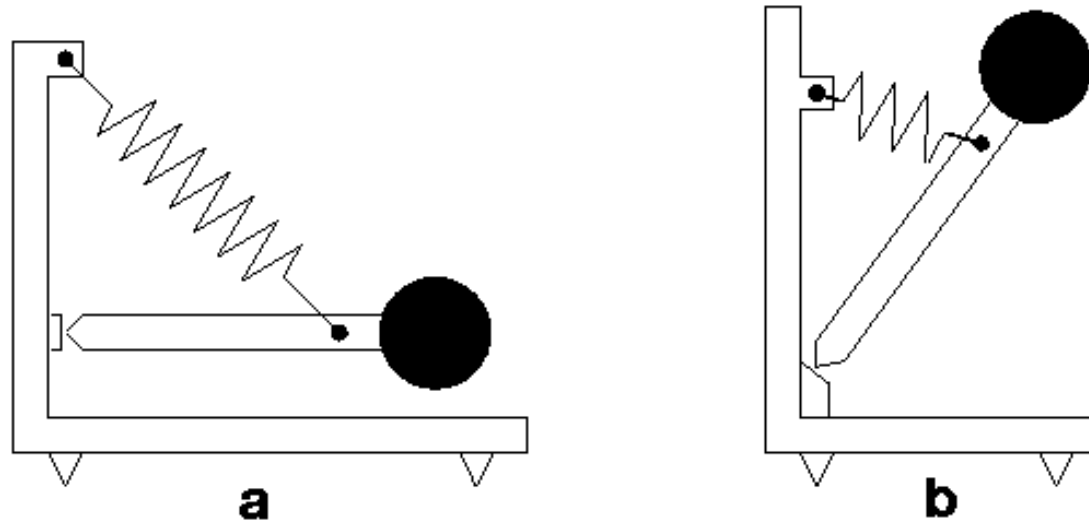


Figure 2.16 The principle behind the LaCoste suspension. The mass m is sitting on a hinge, which has an angle α with the horizontal and suspended by a spring of length L .

$$F = k \cdot L,$$

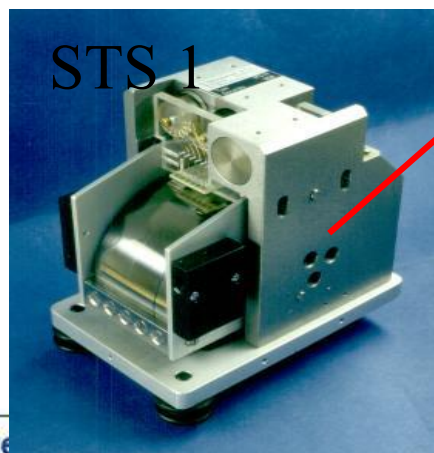
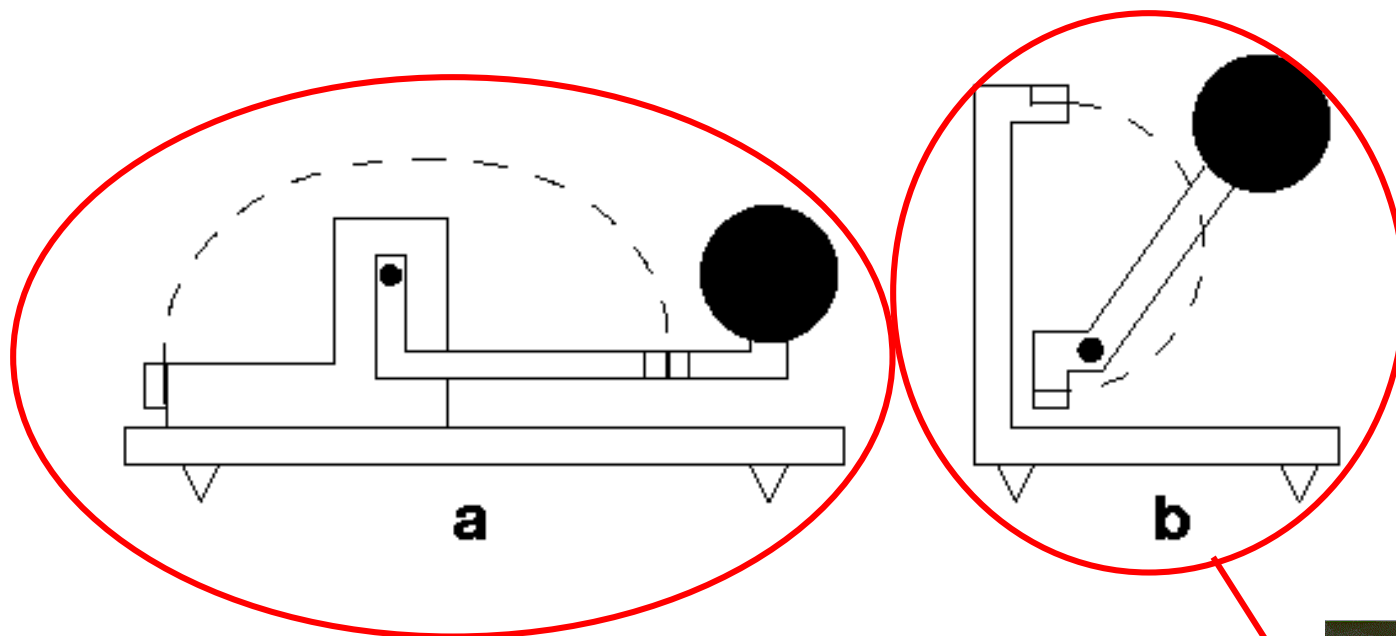
$$F = k \cdot \Delta L,$$



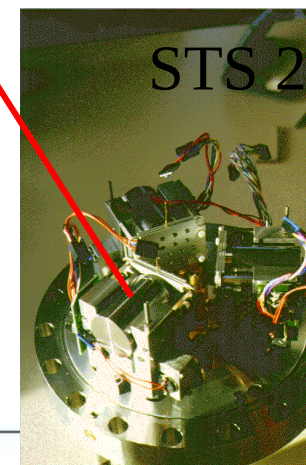


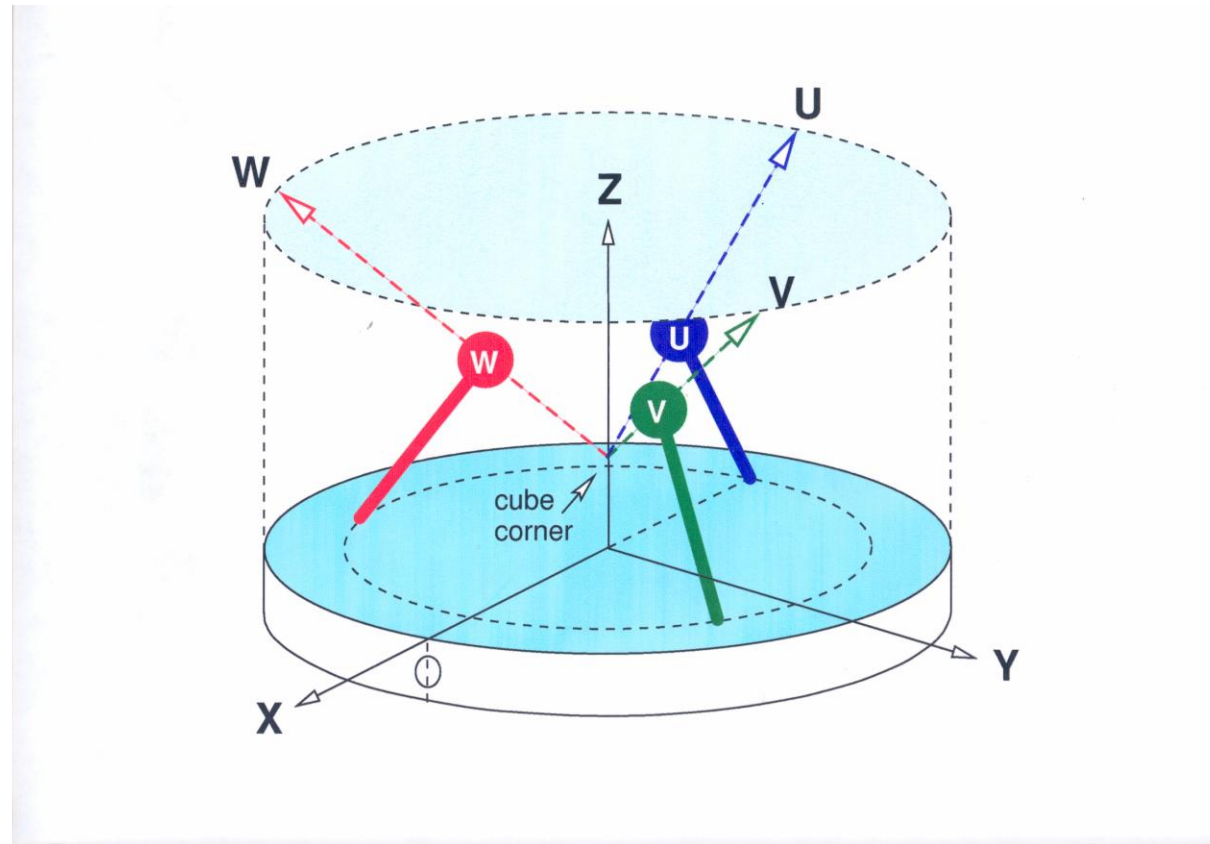
Sospensione La Coste

- verticale
- orizzontale



- a) Wielandt
- b) Streckeisen





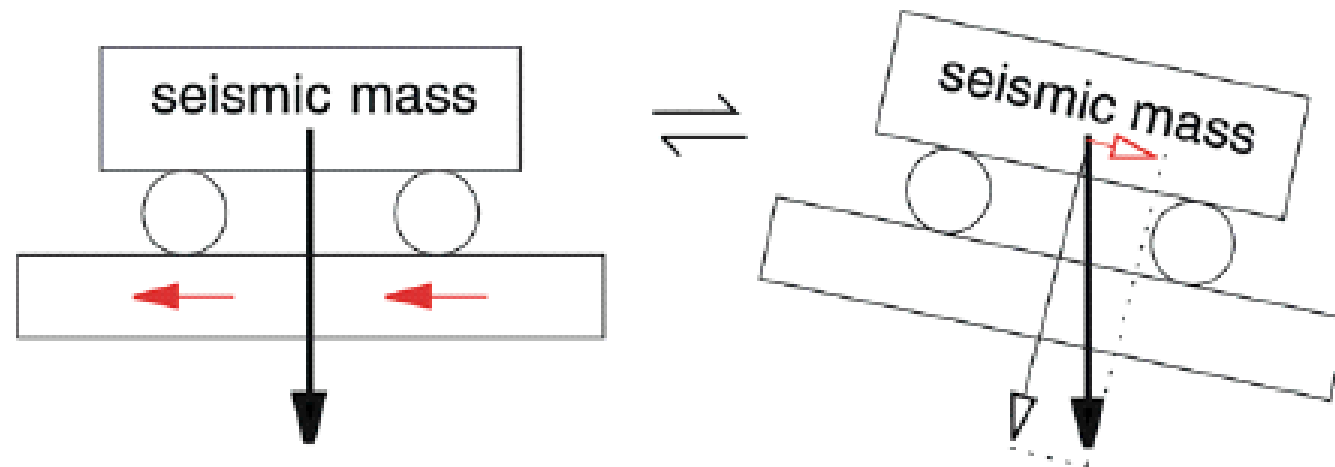
$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \frac{1}{\sqrt{6}} \begin{pmatrix} -2 & 1 & 1 \\ 0 & \sqrt{3} & -\sqrt{3} \\ \sqrt{2} & \sqrt{2} & \sqrt{2} \end{pmatrix} \begin{pmatrix} U \\ V \\ W \end{pmatrix}$$

STS 1

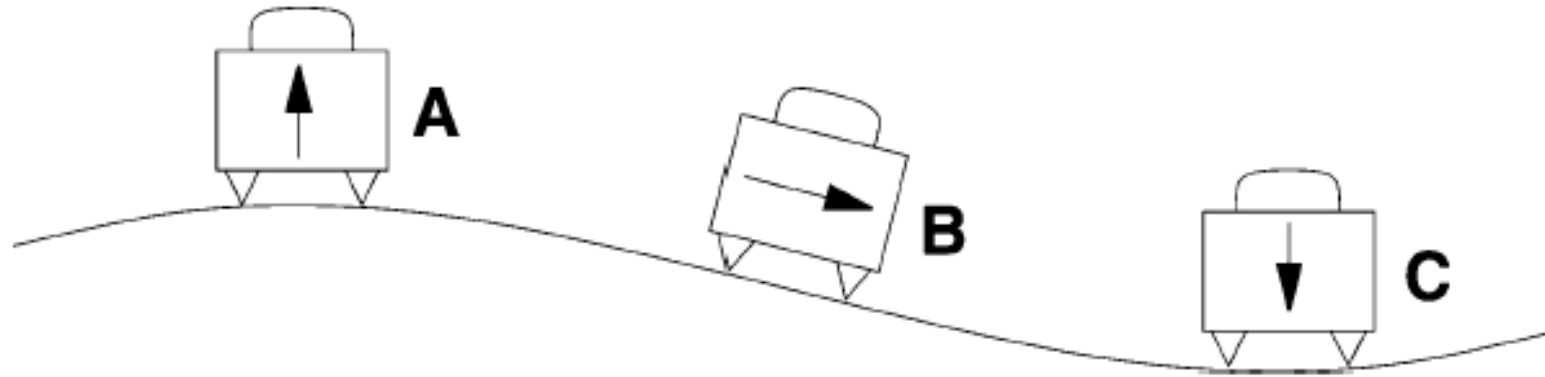
TRI



A seismic acceleration of the ground has the same effect on the seismic mass as an external force. The largest such force is gravity. It is normally nullified by suspension, but when the seismometer is tilted, the projection of the gravity vector on the sensitivity axis changes, producing a force that in most cases is indistinguishable from a seismic signal. The undesirable tilt at seismic frequencies can be caused by varying surface loads such as cars, people or atmospheric pressure. The resulting disturbances are a second-order effect in well-installed vertical seismometers but otherwise a first-order effect. This explains why long-period horizontal seismic traces are always noisier than vertical ones. A short, impulsive tilt is equivalent to a ground velocity step and will therefore cause a long-period transient in a broadband horizontal seismometer. For periodic signals, the apparent horizontal displacement associated with a given tilt increases with the square of the period



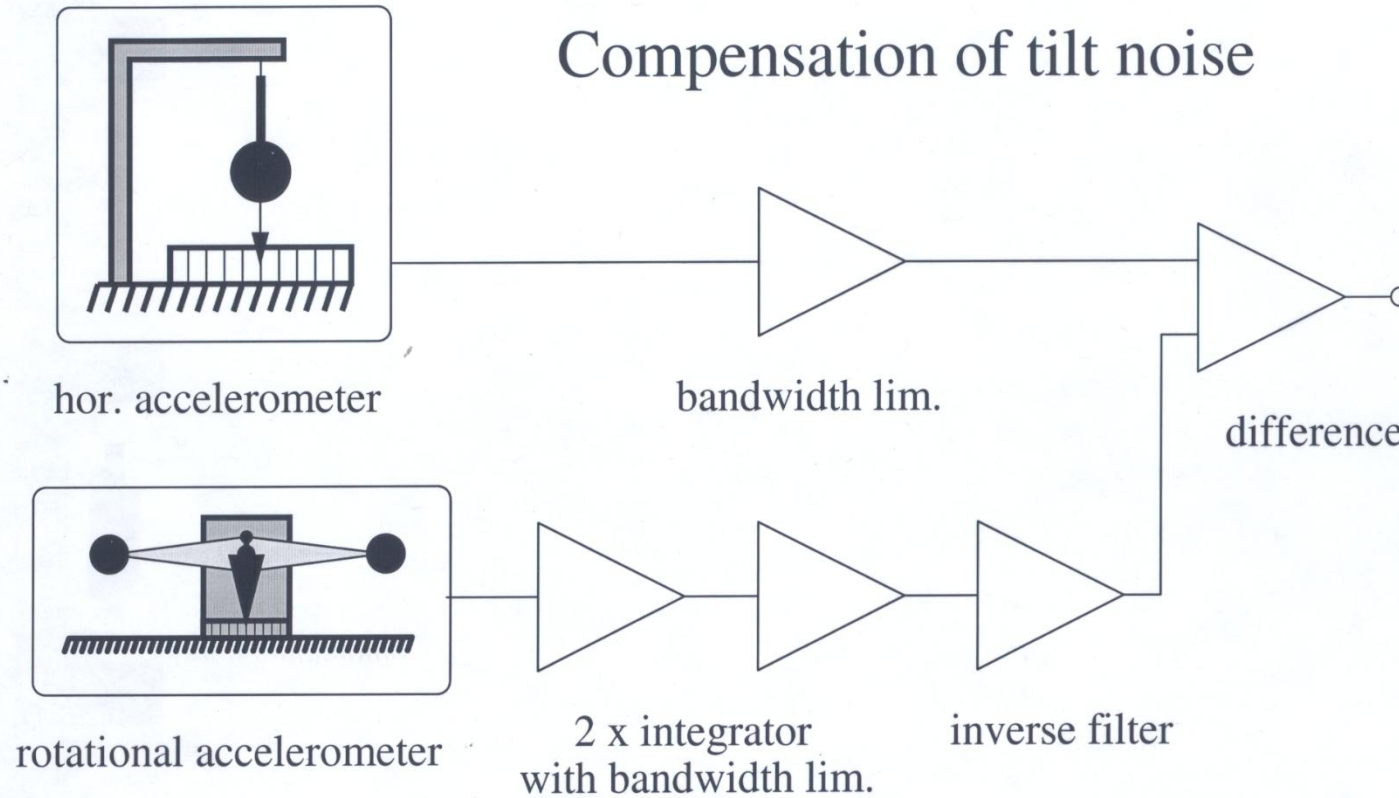
$\pm 1\mu m$ $3km$ $T = 10min$



A e C $\pm 10^{-10} m/s^2$ B $\pm 10^{-8} m/s^2$

The tilt of the ground caused by atmospheric pressure is the main source of a very long period on horizontal seismographs.

Compensation of tilt noise



Another integration is required for a VBB output!

