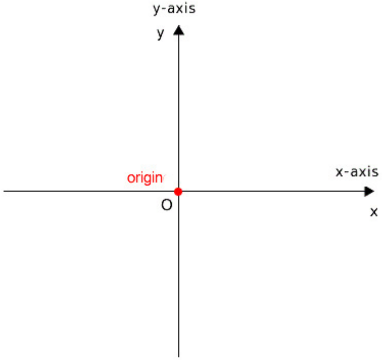
4. Geometrical notions in a plane

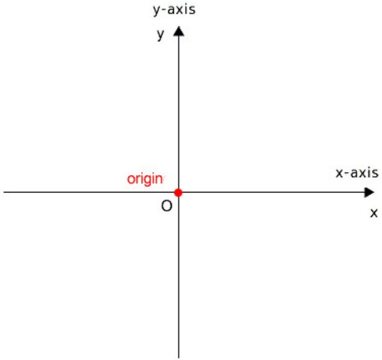
Cartesian plane

In the plane $\mathbb{R}^2$, we introduce a system of Cartesian axes that define the so-called *Cartesian plane*.

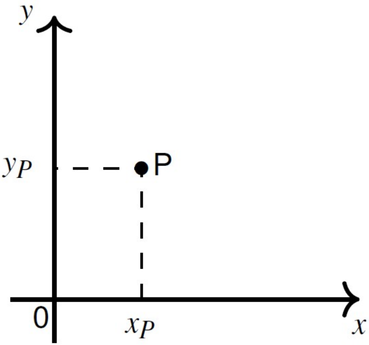
The Cartesian axes are two lines perpendicular to each other: the horizontal x-axis, and the vertical y-axis.



The point where the Cartesian axes intersect is called the *origin* $O(0, 0)$ of the Cartesian plane.



A point P in the plane is uniquely determined by a couple of real numbers (x_P, y_P), called the *coordinates* of point P, where x_P represents the x-coordinate and y_P represents the y-coordinate of P.



Given two points $A(x_1, y_1)$ and $B(x_2, y_2)$, these uniquely identify a segment.

- The *length* (i.e., the distance between A and B) is given by
$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}\,;$$
- The *midpoint* is
$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

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Straight lines

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be two distinct points, these uniquely determine a straight line.

- if $x_1 \neq x_2$ and $y_1 \neq y_2$, then we have an oblique line

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}\,;$$

- if $x_1 \neq x_2$ and $y_1 = y_2 = y_0$, then we have an horizontal line

$$y = y_0\,;$$

- if $x_1 = x_2 = x_0$ and $y_1 \neq y_2$, then we have a vertical line

$$x = x_0\,.$$

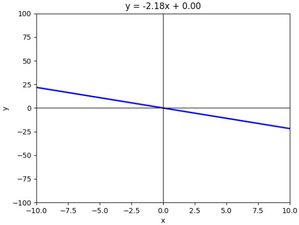
We can write every straight line in the implicit form

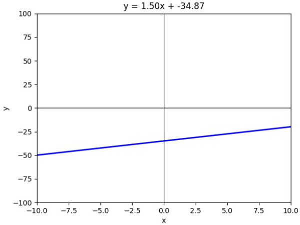
$$ax + by + c = 0$$

or in the explicit form

$$y = mx + q\,.$$

If we look at $y = mx + q$





- m is called the *slope* and depends on the angle that the line makes with the x-axis;
- q is called the *intercept* and it is the y-coordinate of the point where the line intersects the y-axis.

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Example

Given the points $A(x_1, y_1) = (2, 3)$ and $B(x_2, y_2) = (5, 8)$, we have the straight line given by

$$\frac{y-3}{8-3} = \frac{x-2}{5-2}$$

$$\Rightarrow \frac{y-3}{5} = \frac{x-2}{3} \Rightarrow 3(y-3) = 5(x-2) \Rightarrow 3y - 9 = 5x - 10.$$

Therefore,

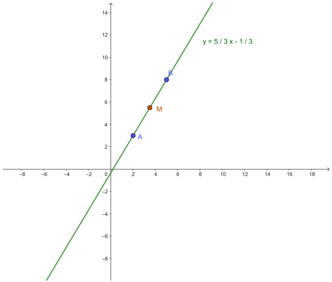
$$\underbrace{5x - 3y - 1 = 0}_{\text{implicit form}} \Rightarrow \underbrace{y = \frac{5}{3}x - \frac{1}{3}}_{\text{explicit form}}.$$

- The distance between A and B is given by

$$\overline{AB} = \sqrt{(2-5)^2 + (3-8)^2} = \sqrt{9+25} = \sqrt{34};$$

- The midpoint is

$$M\left(\frac{2+5}{2}, \frac{3+8}{2}\right) = \left(\frac{7}{2}, \frac{11}{2}\right).$$

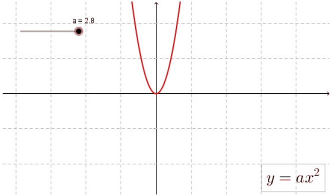


Parabola

Now, we consider the *quadratic function*

$$y = ax^2 + bx + c$$

which represents a parabola in the $\mathbb{R}^2$ plane.



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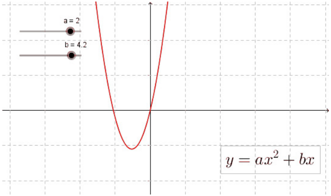

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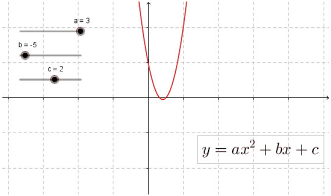
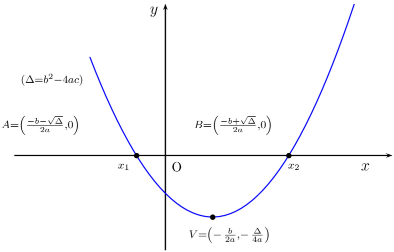


In particular, we can have the following case ($a > 0$, $\Delta > 0$)



where

$$\Delta = b^2 - 4ac\,.$$

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