Reading

- **Robert Gibbons, *A Primer in Game Theory*, Harvester Wheatsheaf 1992.**
- Additional readings could be assigned from time to time. They are an integral part of the class and you are expected to read them.

Structure of the course

Normal Form Games and Nash Equilibrium, Economic applications

Dynamic games of complete information, either with perfect of imperfect information. Repeated games. Economic applications.

Static and dynamic games of incomplete information. Economic applications.

Basic Theory, Normal Form Games and Iterated dominance

Game Theory - Motivation

- Game Theory is a tool to understand situations in which two or more decision-makers interact.
- Outcomes of (economic) decisions frequently depend on others' actions
  - effect of price policy depends on competitors
  - outcome of wage negotiations depends on choices of both sides
  - outcome of elections depends on others' votes

- Decision makers should thus take expectations of others’ decisions into account
- Such situations are plausibly modeled as a “*game”*, a model of interactions where the outcome depends on others’ as well as one’s own actions
- this definition and the scope of game theory is much broader than the everyday definition of a game
  - e.g., game theory is not only concerned with “winning” a competitive game

Useful Definitions

A fact F is *common knowledge* if:

Each player know F

Each player knows that the others know F

Each player knows that the every other players knows that each player know F

And so on

A player is *rational* if he select the action that leads to the outcome he most prefers

(or he is maximizing his expected utility)

Beliefs: player's assessment about the behaviour of the others in the game

Classification of Games

- According to the timing
  - Static games (or simultaneous move game):
    - Actions are taken "simultaneously" (i.e. without information about others' moves)
    - Normal form representation
  - Dynamic games (or sequential games):
    - Actions are taken in according a "sequence"
    - extensive form representation

- Information, complete versus incomplete:
  - Complete information: each player’s payoff function is common knowledge
  - Incomplete information: some player is uncertain about another player’s payoff function.
- Information, perfect versus imperfect
  - Perfect information: when players have to move, they know the full history of play of the game
  - Imperfect information: some player has to move without knowing the full history of the play of the game

Normal form representation

A normal form specifies:

1. the agents in the game,
2. for each agent a set of available actions (or strategies): $\boldsymbol{S_i}$ denotes the set of strategies available to player i and $\boldsymbol{s_i}$ is an element of $\boldsymbol{S_i}$
3. The payoff received by each player for each combination of strategies.
   - u_i $(s_1$ … s_i … $s_n)$ denotes the payoff function of player i where $(s_1$ … s_i … $s_n)$ are the actions chosen by the players
     - $G = \{S_1, \ldots.. S_n; u_1 \ldots u_n\}$

Example 1: the Prisoner's Dilemma

- Static games sometime can be represented using tables

		Player 2	
		Not confess	Confess
Player 1	Not confess	-1, -1	-9, 0
	Confess	0, -9	-6, -6

- This game captures many situations where players prefer to defect, but prefer both to cooperate over both to defect

Example 2: the "Battle of the Sexes"

		Player 2	
		Ball	Theatre
Player 1	Ball	2,1	0,0
	Theatre	0,0	1,2

- This game captures many situations where players agree that they want to **coordinate** but disagree about the action to coordinate on.

Example 3: Matching Pennies

		Player 2	
		Head	Tail
Player 1	Head	1,-1	-1,1
	Tail	-1,1	1,-1

- Player 1 wants both to choose the same action, player 2 to choose different actions.
- This is an example of a **strictly competitive game,**

Example 4: Stag-Hunt

		Player 2	
		Stag	Hare
Player 1	Stag	2,2	0,1
	Hare	1,0	1,1

- Stag-Hunt models situation where players have a common interest to cooperate, but may want to play a safe strategy if they are not sure whether the other will cooperate

Iterated elimination of strictly dominated strategies

Example: Prisoner Dilemma

		Player 2	
		Not confess	Confess
Player 1	Not confess	-1, -1	-9, 0
	Confess	0, -9	-6, -6

- If player 2 is going to play "*Not confess*", then player 1 prefer "*Confess*"
- If player 2 is going to play "*Confess*", then player 1 prefer "*Confess*"
- Player 1 prefers "*Confess*" in both cases
- We say that for player 1 playing "*Not confess*" is dominated by playing "*Confess*"

Definition of ***dominated strategy***

s_i' is **strictly dominated** by s_i if:

$$u_i(s_i, s_{-i}) > u_i(s_i', s_{-i})$$

for each feasible combination of other players' strategies (s_{-i})

Note:

1) rational players do not play strictly dominated strategies (because a strictly dominated strategy is not optimal for all possible beliefs)

2) s_i' is **dominated** by s_i if:

$$u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i})$$

Example 1

		Player 2	
		L	R
Player 1	T	2,3	5,0
	M	3,2	1,1
	~~B~~	~~1,0~~	~~4,1~~

		Player 2	
		L	R
Player 1	T	2,3	5,0
	M	3,2	1,1

		Player 2	
		L	~~R~~
Player 1	T	2,3	~~5,0~~
	M	3,2	~~1,1~~

		Player 2
		L
Player 1	~~T~~	~~2,3~~
	M	3,2

		Player 2
		L
Player 1	M	3,2

- This process is called "*iterated elimination of strictly dominated strategies*"

  *(*or *iterated dominance)*

The strategies that survive iterated dominance is called **rationalizable strategies**

- It is based on the idea that rational players do not play dominated strategies
- Two problems:
  - This process requires that it is common knowledge that player are rational
  - This process often produces no accurate predictions (see the following example)

Example 2

		Player 2			
		L	C	~~R~~	
Player 1	~~T~~	~~2,3~~	~~2,2~~	~~5,0~~	3
	Y	3,2	5,3	~~3,1~~	
	Z	4,3	1,1	~~2,2~~	
	~~B~~	~~1,2~~	~~0,1~~	~~4,4~~	1
				2	

Definition of *"Best Response"*

The ***Best Response*** of a player is his preferred action given the strategies played by the other players.

- Consider the n-player normal form game

$$G = \{S_1, \ldots\ldots S_n;\ u_1 \ldots u_n\}$$

- The best response of player *i* to the strategies s_{-i} = $(s_1, \ldots, s_{i-1}, s_{i+1}, s_n)$ solves the following problem:

$$\max_{s_i \in S_i} u(s_1, \ldots, s_i, \ldots, s_n)$$

Nash Equilibrium

- It is a prediction about the strategy each player will choose
- This prediction is correct if each player's predicted strategy is a best response to the predicted strategies of the other players.
- Such prediction is *strategically stable* or *self enforcing*: no player wants to change his/her predicted strategy
- We call such a prediction a *Nash Equilibrium*.

Definition of Nash Equilibrium

Consider the n-player normal form game

$$G = \{S_1, \ldots.. S_n; u_1 \ldots u_n\}$$

The strategy profile $(s_1^*, \ldots, s_n^*)$ is a **Nash equilibrium** if:

for every player i and every action $s_i \in S_i$:

$$u_i(s_1^*, \ldots, s_i^*, \ldots, s_n^*) \geq u_i(s_1^*, \ldots, s_i, \ldots, s_n^*)$$

The strategy profile $(s_1^*, \ldots, s_n^*)$ is a **Strict Nash equilibrium** if:

for every player i and every action $s_i \in S_i$:

$$u_i(s_1^*, \ldots, s_i^*, \ldots, s_n^*) > u_i(s_1^*, \ldots, s_i, \ldots, s_n^*)$$

Note

1. in a Nash equilibrium there are no players that want to deviate

2. s_i^* solves the following problem:

$$\max_{s_i \in S_i} u_i(s_1^*, \dots, s_i, \dots, s_n^*)$$

3. in a Nash equilibrium, each player strategy is a best response to the other players' strategies

Example 1: the Prisoner's Dilemma

		Player 2	
		C(ooperate)	D(efect)
Player 1	C(ooperate)	2,2	0,$\underline{3}$
	D(efect)	$\underline{3}$,0	$\underline{1}$,$\underline{1}$

- The unique Nash equilibrium is (D,D)
- For every other profile, at least one player wants to deviate

Example 2: the “Battle of the Sexes”

		Player 2	
		Ball	Theatre
Player 1	Ball	2,1	0,0
	Theatre	0,0	1,2

- There are two Nash equilibria: (Ball, Ball) and (Theatre, Theatre)

Example 3: Matching Pennies

		Player 2	
		Head	Tail
Player 1	Head	<u>1</u>,-1	-1,<u>1</u>
	Tail	-1,<u>1</u>	<u>1</u>,-1

- There is no Nash equilibrium (of the game with ordinal preferences)

Example 4: “Stag-Hunt”

		Player 2	
		Stag	Hare
Player 1	Stag	<u>2</u>,<u>2</u>	0,1
	Hare	1,0	<u>1</u>,<u>1</u>

- There are two equilibria:
- (Stag, Stag) and (Hare, Hare)