1. Logic notions

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All mathematical “arguments” consist of “statements” or, equivalently, “propositions”.

These are declarative sentences that can be either true or false and not both.

This is the so called *Principle of the Excluded Middle* or *Tertium non datur*.

Any proposition must be formulated in a non-ambiguous way: we must be able to decide whether it is true or false.

Definition

A *proposition* is a statement which may be *true* (T) or *false* (F).

Definition (Union)

The *union* $p \vee q$ ("*p or q*") of two propositions p, q is the proposition which is true if at least one of the propositions p, q is true, and false otherwise.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Definition (Intersection)

The *intersection* $p \wedge q$ ("*p and q*") of two propositions p, q is the proposition which is true if both the propositions p, q are true, and false otherwise.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Definition (Implication)

Given two propositions p, q, the *implication* $p \Rightarrow q$ ("*p implies q*", or else "*if p then q*") is the proposition which is false if p is true and q is false, and true otherwise.

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

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Definition (Bi-implication)

Given two propositions p and q, the *bi-implication* $p \Leftrightarrow q$ ("*p is equivalent to q*", or else "*p if and only if q*") is the proposition which is true if p, q have the same logic value, and false otherwise.

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Example

We will see now the technique of proving an implication by using a "*reduction to the absurd*".

Such a terminology was often used by Aristotle (384-322 b.C.) procedure, also called "*by contraposition procedure*":

$$(p \Rightarrow q) \Leftrightarrow (\neg q \Rightarrow \neg p).$$

p	q	$\neg p$	$\neg q$	$p \Rightarrow q$	$\neg q \Rightarrow \neg p$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

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