6. Systems of inequalities

A system of inequalities in one variable is a set consisting of two or more inequalities in the same variable.

Solving a system of inequalities means determining, if they exist, those values that verify all the inequalities simultaneously.

- We solve each inequality independently;
- We intersect the sets of solutions.

Example

We consider the following system of inequalities

$$\begin{cases} x^2 - 7x + 12 \geq 0 \\ x^2 - 11x + 24 \geq 0 \,. \end{cases}$$

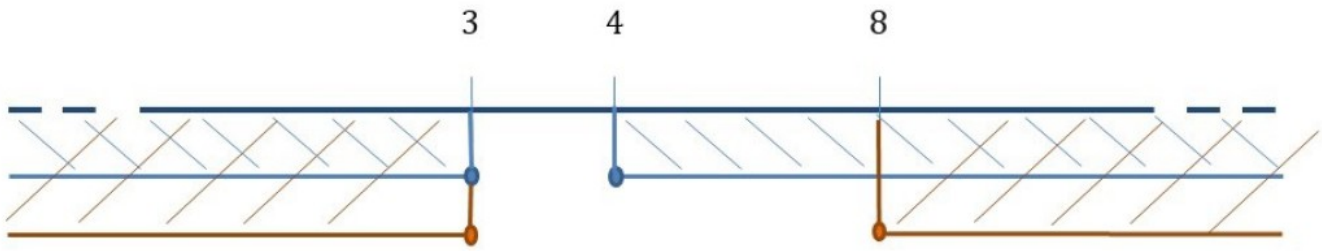
First, we solve $x^2 - 7x + 12 \geq 0$ for which $\Delta = 49 - 48 = 1$. Then,

$$x_1 = 3 \quad \text{and} \quad x_2 = 4 \,.$$

Second, we solve $x^2 - 11x + 24 \geq 0$ for which $\Delta = 121 - 96 = 25$.
Then,

$$x_1 = 3 \quad \text{and} \quad x_2 = 8\,.$$

Taking the intersection of the sets of solutions, we have



The system is solved by $x \in]-\infty, 3] \cup [8, +\infty[$.

Example

We consider the following system of inequalities

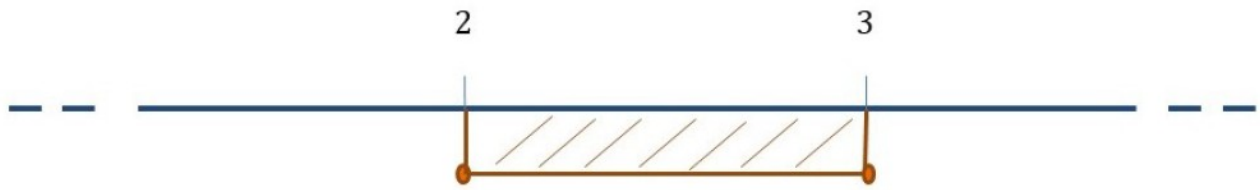
$$\begin{cases} x^2 - 2x - 15 \leq 0 \\ x^2 - 5x + 6 \leq 0\,. \end{cases}$$

First, we solve $x^2 - 2x - 15 \leq 0$ for which $\Delta = 4 + 60 = 64$. Then,

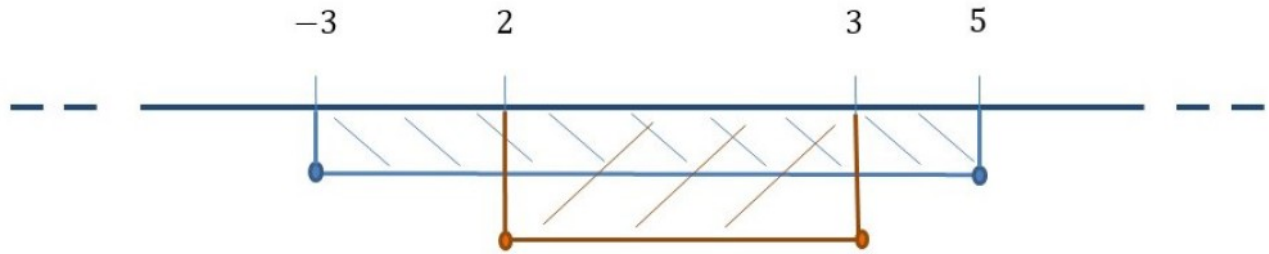
$$x_1 = -3 \quad \text{and} \quad x_2 = 5\,.$$

Second, we solve $x^2 - 5x + 6 \leq 0$ for which $\Delta = 25 - 24 = 1$. Then,

$$x_1 = 2 \quad \text{and} \quad x_2 = 3\,.$$



Taking the intersection of the sets of solutions, we have



The system is solved by $x \in [2,3]$.