ES 2)

$P: \quad x = R\cos\varphi \qquad \dot{x} = -R\dot{\varphi}\,\mathrm{sen}\,\varphi$

$\quad\;\; y = R\,\mathrm{sen}\,\varphi \qquad \dot{y} = R\dot{\varphi}\cos\varphi$

$\quad\;\; z = a\cos 2\varphi \qquad \dot{z} = -2a\dot{\varphi}\,\mathrm{sen}\,2\varphi$

$Q: \quad x = R\cos\theta$

$\quad\;\; y = R\,\mathrm{sen}\,\theta$

$\quad\;\; z = 0$

$$a = \begin{pmatrix} m(R^2 + 4a^2\,\mathrm{sen}^2 2\varphi) & 0 \\ 0 & MR^2 \end{pmatrix}$$

1) $T = \frac{m}{2}\dot{\varphi}^2(R^2 + 4a^2\,\mathrm{sen}^2 2\varphi) + \frac{M}{2}\dot{\theta}^2R^2$

$V = mga\cos 2\varphi + \frac{k}{2}\Big[R^2(\cos\varphi - \cos\theta)^2 + R^2(\mathrm{sen}\,\varphi - \mathrm{sen}\,\theta)^2$

$\qquad + a^2\cos^2 2\varphi\Big]$

$= mga\cos 2\varphi + \frac{ka^2}{2}\cos^2 2\varphi + kR^2(1 - \cos(\varphi - \theta))$

3) $k = 0 \rightarrow \theta$ ciclica $\rightarrow p_\theta = \frac{\partial L}{\partial\dot{\theta}} = MR^2\dot{\theta}$ cost. del moto

$a = 0 \rightarrow$ shift sym. $\varphi \mapsto \varphi + \alpha$
$\qquad\qquad\qquad\qquad\; \theta \mapsto \theta + \alpha$

cost. del moto di Nöther: $1\cdot p_\varphi + 1\cdot p_\theta = m(R^2 + 4a^2\,\mathrm{sen}^2 2\varphi)\dot{\varphi}$
$\qquad + MR^2\dot{\theta}$

4) $V = mga\cos 2\varphi + \frac{Ka^2}{2}\cos^2 2\varphi - KR^2\cos(\varphi-\theta)$

$\partial_\varphi V = -2mga\,\text{sen}\,2\varphi \underbrace{-2Ka^2\cos 2\varphi\,\text{sen}\,2\varphi}_{-Ka^2\text{sen}\,4\varphi} + KR^2\,\text{sen}(\varphi-\theta)$

$\partial_\theta V = KR^2\,\text{sen}(\theta-\varphi)$

$\partial V = 0 \rightarrow \text{sen}(\theta-\varphi) = 0 \qquad \theta = \varphi,\ \varphi+\pi$

$\text{sen}\,2\varphi\left(\frac{mg}{Ka} + \cos 2\varphi\right)$

- $\varphi = 0, \frac{\pi}{2}, \pi, \frac{3}{2}\pi$
- $\varphi = \pm\varphi^*$ con $\cos 2\varphi^* = -\frac{mg}{Ka}$ se $\frac{mg}{Ka} \leq 1.$

$(\varphi,\theta) = (0,0)\ (\frac{\pi}{2},\frac{\pi}{2})\ (\pi,\pi)\ (\frac{3}{2}\pi,\frac{3}{2}\pi)\ (\varphi^*,\varphi^*)\ (-\varphi^*,-\varphi^*)$

$(0,\pi)\ (\frac{\pi}{2},\frac{3}{2}\pi)\ (\pi,0)\ (\frac{3}{2}\pi,\frac{\pi}{2})\ (\varphi^*,\varphi^*+\pi)\ (-\varphi^*,\pi-\varphi^*)$

$$\partial^2 V = \begin{pmatrix} \overbrace{-4mga\cos 2\varphi - 4Ka^2\cos 4\varphi}^{\beta} + KR^2\cos(\varphi-\theta) & -KR^2\cos(\varphi-\theta) \\ -KR^2\cos(\varphi-\theta) & KR^2\cos(\varphi-\theta) \end{pmatrix}$$

$\det = (\beta + KR^2\cos)KR^2\cos - (KR^2\cos)^2 =$
$= \beta KR^2\cos$

$\cos(\varphi-\theta) = \pm 1$. Se -1 INSTAB perché $\partial^2 V_{22} < 0$

prendiamo $\cos(\varphi-\theta) = +1 \quad (\theta = \varphi)$ $\qquad \cos 2x = 2\cos^2 x - 1$

$$\det \partial^2 V = \beta K R^2 \qquad \mathrm{tr} = \beta + 2KR^2$$

$$\frac{\beta}{4a} = -mg\cos 2\varphi - Ka\cos 4\varphi = -mg\cos 2\varphi + Ka - 2Ka\cos^2 2\varphi$$

$$\frac{\beta}{4a}\Big|_{\varphi=0} = -mg - Ka < 0 \rightarrow \text{INSTAB.}$$

$$\Big|_{\varphi=\frac{\pi}{2}} = mg - Ka \quad \begin{cases} \rightarrow \text{STAB se } \frac{mg}{Ka} > 1 \\ \rightarrow \text{INSTAB se } \frac{mg}{Ka} < 1 \end{cases}$$

$$\Big|_{\varphi=\pi} = -mg - Ka < 0 \rightarrow \text{INSTAB}$$

$$\Big|_{\varphi=\frac{3}{2}\pi} = mg - Ka \quad \begin{cases} \rightarrow \text{STAB se } \frac{mg}{Ka} > 1 \\ \rightarrow \text{INSTAB se } \frac{mg}{Ka} < 1 \end{cases}$$

$$\Big|_{\varphi=\pm\varphi^*} = Ka - mg\cos 2\varphi_* - 2Ka\cos^2 2\varphi_* =$$

$$= Ka - mg\left(-\frac{mg}{Ka}\right) - 2Ka\frac{(mg)^2}{(Ka)^2} =$$

$$= Ka - \frac{(mg)^2}{Ka} = \frac{1}{Ka}(Ka - mg)(Ka + mg)$$

$$\rightarrow \text{STAB. se } \frac{mg}{Ka} < 1$$

5) $mg = \frac{11}{8} kR$

$\theta = \varphi = \frac{\pi}{2}$

$$\rightarrow \beta = 4a(mg - ka) = 4R(mg - kR) = \frac{12}{8} kR^2$$

$$a = \begin{pmatrix} m(R^2 + 4a^2 \sin^2(2\varphi)) & 0 \\ 0 & MR^2 \end{pmatrix}$$

$$A = mR^2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$B = \left.\begin{pmatrix} \beta + kR^2 \cos(\theta - \varphi) & -kR^2 \cos(\theta - \varphi) \\ -kR^2 \cos(\theta - \varphi) & kR^2 \cos(\theta - \varphi) \end{pmatrix}\right|_{(\varphi, \theta) = (\frac{\pi}{2}, \frac{\pi}{2})}$$

$$= \begin{pmatrix} \frac{5}{2}\cancel{\frac{20}{8}} kR^2 & -kR^2 \\ -kR^2 & kR^2 \end{pmatrix}$$

$$\det(B - \lambda A) = (mR^2)^2 \det \begin{pmatrix} \frac{5}{2}\frac{k}{m} - \lambda & -\frac{k}{m} \\ -\frac{k}{m} & \frac{k}{m} - \lambda \end{pmatrix} =$$

$$= (mR^2)^2 \quad \frac{3}{2}\left(\frac{k}{m}\right)^2 + \lambda^2 - \frac{7}{2}\left(\frac{k}{m}\right)\lambda$$

$$\lambda_{1/2} = \frac{7}{4}\frac{k}{m} \pm \frac{1}{2}\sqrt{\left(\frac{49}{4} - 6\right)\left(\frac{k}{m}\right)^2} =$$

$$= \frac{k}{m}\left(\frac{7}{4} \pm \frac{5}{4}\right) \begin{cases} \frac{3k}{m} \\ \frac{k}{2m} \end{cases}$$