Lezione 5 - 18/9/2026

es.

$\mathbb{N} = \{0, 1, 2, \ldots\}$ numeri naturali

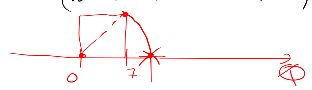
$\mathbb{Z} = \{\ldots -2, -1, 0, 1, 2, \ldots\}$ " interi

$\mathbb{Q} = \{$ numeri razionali $\frac{p}{q}$, $p \in \mathbb{Z}$, $q \in \mathbb{N} \setminus \{0\} \}$

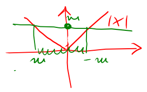
$\mathbb{R} = \{$ numeri reali $\}$

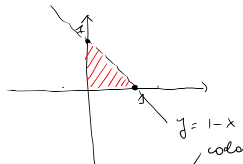
disegnare il grafico delle relazioni (su $\mathbb{Q}\times\mathbb{Q}$ o su $\mathbb{R}\times\mathbb{R}$)

$|x| + |y| \leq 1$



$|x| = \begin{cases} x & \text{se } x \geq 0 \\ -x & \text{se } x < 0 \end{cases}$

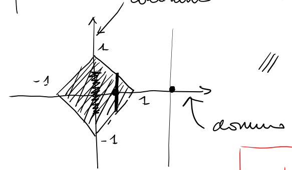




$x > 0$, $y > 0$

$x + y \leq 1$

$y \leq 1 - x$ $\quad f(x) = 1 - x$



/// = insieme $\{(x,y) : |x| + |y| \leq 1\}$

$|x + y| \leq 1$

$\Updownarrow$

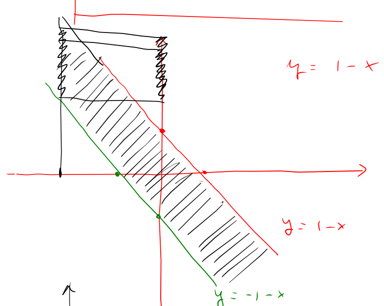
$-1 \leq x + y \leq 1$

$-1 - x \leq y \leq 1 - x$

$|a| \leq m$ (con $m \geq 0$)

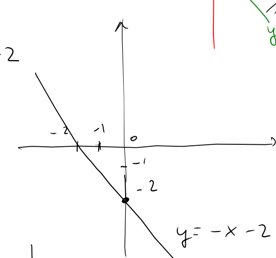
$\Updownarrow$

$-m \leq a \leq m$



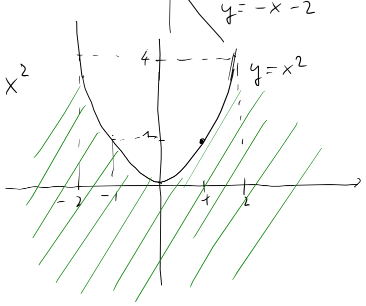
$x + y + 2 = 0$

$y = -x - 2$



funzione !

$y \leq x^2$



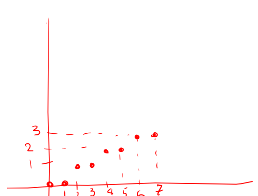
$\forall x \in \mathbb{R}$

$x^2 \geq 0$

$(x^2 - y)^2 \leq 0 \iff (x^2 - y)^2 = 0$

$\Updownarrow$

$x^2 - y = 0 \qquad x^2 = y$



$f(n) = \begin{cases} n/2 & \text{se } n \text{ è pari} \\ \frac{n-1}{2} & \text{se } n \text{ è dispari} \end{cases}$

sia $f: \mathbb{N} \to \mathbb{N}$ $\quad f(n) = 2n+2 = 2(n+1)$

$f(1) = 4$

$f(\{3,4,5\}) = \{6, 10, 12\}$

f^{-1} $\quad f^{\leftarrow}(\{3,4,5\}) = \{1\}$ $\qquad 2(n+1) = 4$

$n+1 = 2$

$n = 1$

5) $f(x) = 1+x$, $\quad g(x) = \dfrac{x^2}{1+x^2}$

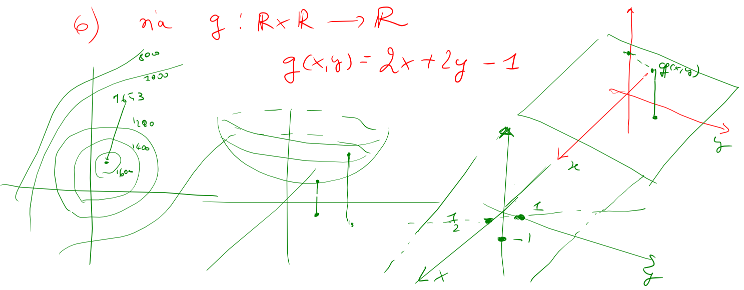
$f \circ g$ e $g \circ f$

$x \xrightarrow{g} \dfrac{x^2}{1+x^2} = y \xrightarrow{f} 1+y$ $\qquad f\circ g(x) = \dfrac{x^2}{1+x^2} + 1$

$x \xrightarrow{f} 1+x = y \longmapsto \dfrac{y^2}{1+y^2} = \dfrac{(1+x)^2}{1+(1+x)^2} = \dfrac{1+2x+x^2}{2+2x+x^2}$

6) sia $g: \mathbb{R}\times\mathbb{R} \to \mathbb{R}$

$g(x,y) = 2x+2y-1$

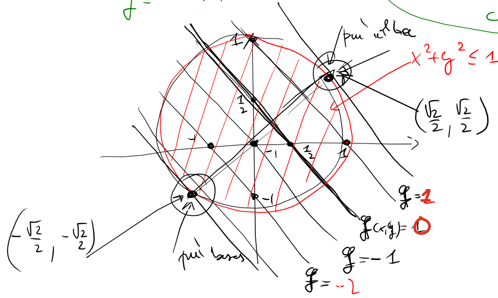


curve di livello quindi $2x+2y-1 = \text{costante}$

$2y = 1 + \text{cost} - 2x$

$y = \underbrace{\dfrac{1+\text{cost}}{2}}_{c'} - x$

le curve di livello son $y = -x + \text{cost.}$



$f(x,y) = 2x+2y-1$

$y = -x + c$

$g(\{x^2+y^2 \le 1\})$

$\|$

$x^2+y^2 = 1$

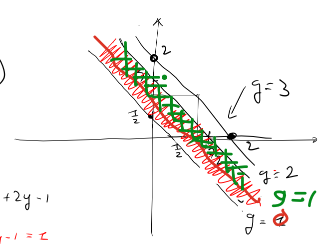
$x = y < 0$

$f\left(-\frac{\sqrt2}{2}, -\frac{\sqrt2}{2}\right) = 2\left(-\frac{\sqrt2}{2}\right) + 2\left(-\frac{\sqrt2}{2}\right) - 1 = -2\sqrt2 - 1$

$f\left(\frac{\sqrt2}{2}, \frac{\sqrt2}{2}\right) = 2\frac{\sqrt2}{2} + 2\frac{\sqrt2}{2} - 1 = 2\sqrt2 + 1$

$f(\{x^2+y^2 \le 1\}) = [-2\sqrt2 - 1, 2\sqrt2 - 1]$

trovare $g^{\leftarrow}(\{1 < x < 2\})$



$y = \frac{1}{2} - x$

$y = \frac{3}{2} - x$

$t \to (x(t), y(t))$

$g(x,y) = 2x+2y-1$

$g(x,y) = 2x+2y-1 = 1$

$\Updownarrow$

$2x+2y = 2$ $\qquad y = 1-x$

$g(x,y) = 2$

$2x+2y-1 = 2$

$2x+2y = 3$

$y = \frac{3}{2} - x$

perché sia suriettiva

$$\begin{cases} ax+by=z_1 \\ cx+dy=z_2 \end{cases}$$

z_1, z_2 interi devo trovare x,y interi

$$f: \mathbb{Z}\times\mathbb{Z} \longrightarrow \mathbb{Z}\times\mathbb{Z}$$
$$(x,y) \longmapsto (ax+by,\ cx+dy)$$

in quali casi è iniettiva?

$$f(x_1,y_1) = f(x_2,y_2) \Rightarrow (x_1,y_1)=(x_2,y_2) \Leftrightarrow \begin{cases} x_1=x_2 \\ y_1=y_2 \end{cases}$$

$$f(x_1,y_1) = (ax_1+by_1,\ cx_1+dy_1)$$
$$f(x_2,y_2) = (ax_2+by_2,\ cx_2+dy_2)$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\begin{cases} ax_1+by_1 = ax_2+by_2 \\ cx_1+dy_1 = cx_2+dy_2 \end{cases}$$

idea: cerco una condizione per cui alla fine si abbia $x_1=x_2 \wedge y_1=y_2$

$$\Updownarrow$$

$$\begin{cases} a(x_1-x_2)+b(y_1-y_2)=0 \\ c(x_1-x_2)+d(y_1-y_2)=0 \end{cases}$$

$\Updownarrow$ ← moltiplico per c la prima e per a la seconda

$$\begin{cases} ac(x_1-x_2)+bc(y_1-y_2)=0 \\ ac(x_1-x_2)+ad(y_1-y_2)=0 \end{cases}$$

sottraggo la ~~prima~~ seconda dalla ~~seconda~~ prima

$$bc(y_1-y_2) - ad(y_1-y_2) = 0$$

$(bc-ad)(y_1-y_2)=0$ se $\boxed{bc-ad\neq 0}$ allora $y_1=y_2$

$$\begin{cases} a(x_1-x_2)+b(y_1-y_2)=0 \\ c(x_1-x_2)+d(y_1-y_2)=0 \end{cases}$$

molt la 1ª per d e la seconda per b

$$\begin{cases} ad(x_1-x_2)+bd(y_1-y_2)=0 \\ cb(x_1-x_2)+db(y_1-y_2)=0 \end{cases}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

sottraggo

$(ad-bc)(x_1-x_2)=0$ se $\boxed{ad-bc\neq 0}$ $x_1=x_2$

Condizione se $ad-bc\neq 0$ allora f è iniettiva!

(oss. si può vedere che se $ad-bc=0$ allora non è iniettiva)

supponiamo $ad-bc=0$

$f(x,y)=(ax+by,\ cx+dy)$

abbiamo $f(0,0)=0$

$$f(-b,a) = (-ab+ba,\ -cb+da) \overset{\shortparallel}{=} (0,0)$$

$$f(0,0) = f(-b,a)$$

f suriettiva $\Longleftrightarrow$ $ad-bc = 1$ o -1