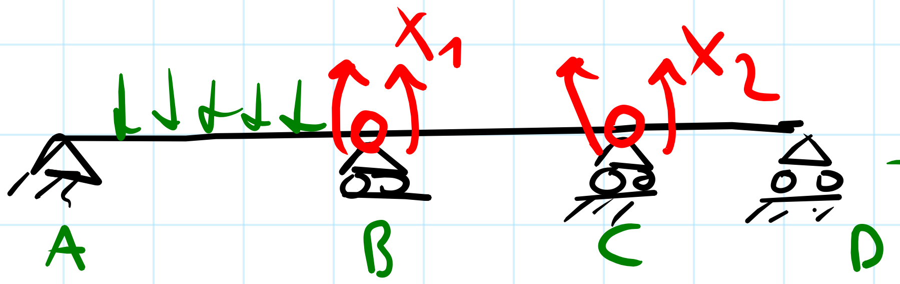
METHODS OF SOLUTION OF ELASTIC STRUCTURES (REVIEW) CSM 24/9/26

1°: METHOD OF FORCES (COMPATIBILITY ...)



REDUNDANT (HYPERSTATICS) → 2 TIMES

TO SOLVE (X_1, X_2 ARE THE UNKNOWNS!) $\Rightarrow$ COMPATIBILITY EQS: $\left.\begin{array}{l} \varphi_{BA} = \varphi_{BC} \\ \varphi_{CB} = \varphi_{CD} \end{array}\right\}$

LINEAR SYSTEM

LINEAR SYST. $\left[\ \underset{\sim}{C}\ \right]\left[\underline{X}\right] = \left[\underline{U}\right]$

FLEXIBILITY MATRIX

UNKOWNS

KNOWN TERMS (DISPLAC., ROTATION)

$$\underline{X} = \underset{\sim}{C}^{-1}\,\underline{U}$$

2°: DISPLACEMENT METHOD

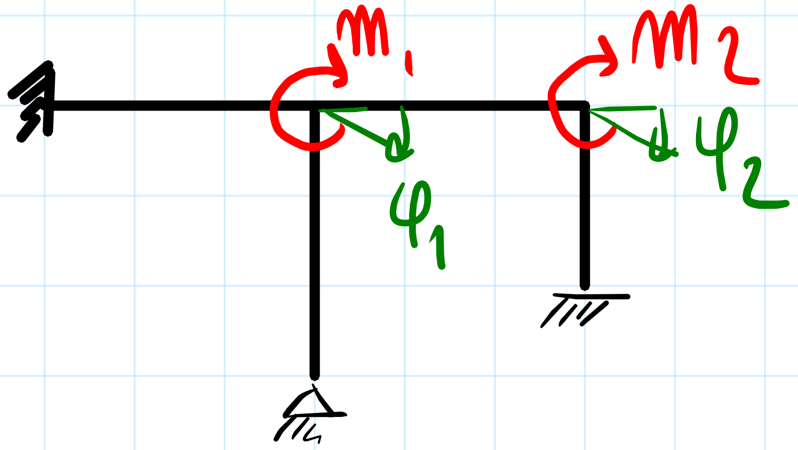
$$\left[\;\underset{\sim}{K}\;\right]\left[\underline{U}\right] = \left[\underline{F}\right]$$

SCHEMA II

NODAL FORCES (M_1, M_2)

STIFFNESS MATRIX

DISPLACEMENTS (UNKOWNS) (φ_1, φ_2)

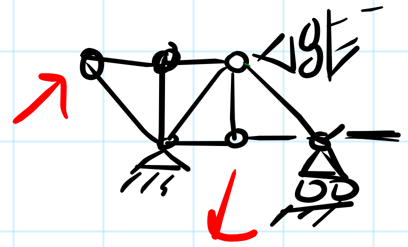


$$\underline{U} = \underset{\sim}{K}^{-1}\,\underline{F}$$

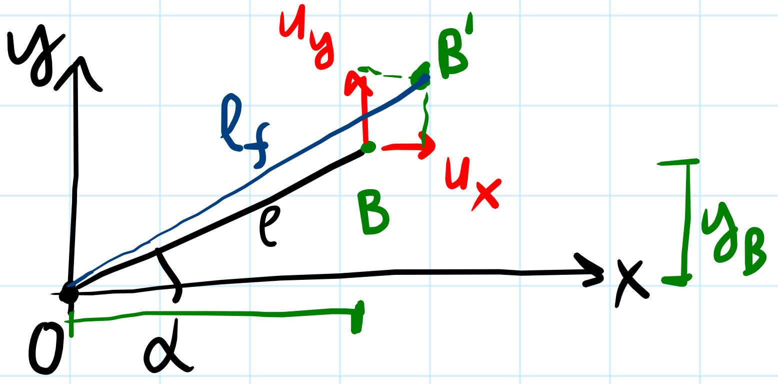
FEMs FORMULATED FROM STATIONARITY OF THE TOTAL POTENTIAL ENERGY LEAD TO A LINEAR SYSTEM THAT IS EXACTLY

$$\boxed{\underset{\sim}{K}\,\underline{U} = \underline{F}}$$

DISPLACEMENT METHOD FOR PLANE TRUSS STRUCTURES



KINEMATICS OF A BAR IN THE PLANE



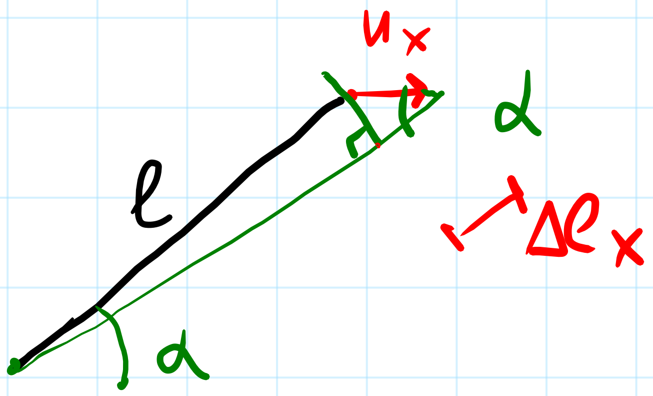
$$\ell = \sqrt{x_B^2+y_B^2} \; ; \; \ell_f = \sqrt{(x_B+u_x)^2+(y_B+u_y)^2} = \ell_f(u_x,u_y)$$

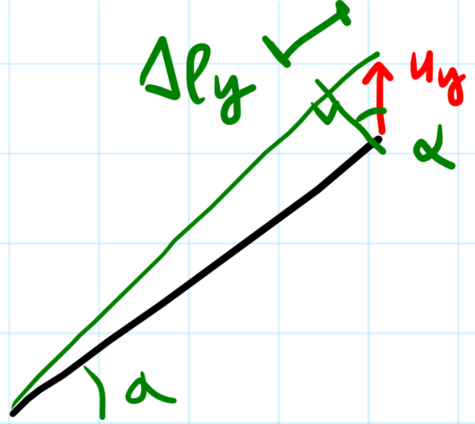
GOAL: LINEARIZE $\ell_f(u_x, u_y)$

$$\ell_f = \ell_f(0,0) + \frac{\partial \ell_f}{\partial u_x}\Big|_{(0,0)} u_x + \frac{\partial \ell_f}{\partial u_y}\Big|_{(0,0)} u_y + O\left(\sqrt{u_x^2+u_y^2}\right)$$

$$= \ell + \left[\frac{1}{\cancel{2}\sqrt{\cdots}}\,\cancel{2}(x_B+u_x)\right]_{(0,0)} u_x + \left[\frac{1}{\cancel{2}\sqrt{\;}}\,\cancel{2}(y_B+u_y)\right]_{(0,0)} u_y + \cdots$$

$$\ell_f \cong \ell + \underbrace{\frac{1}{\ell}\, x_B}_{\cos\alpha} u_x + \underbrace{\frac{1}{\ell}\, y_B}_{\sin\alpha} u_y \longrightarrow \ell_f = \ell + \overbrace{\underbrace{\cos\alpha\, u_x}_{\Delta\ell_x} + \underbrace{\sin\alpha\, u_y}_{\Delta\ell_y}}^{\text{CHANGE IN LENGTH}}$$





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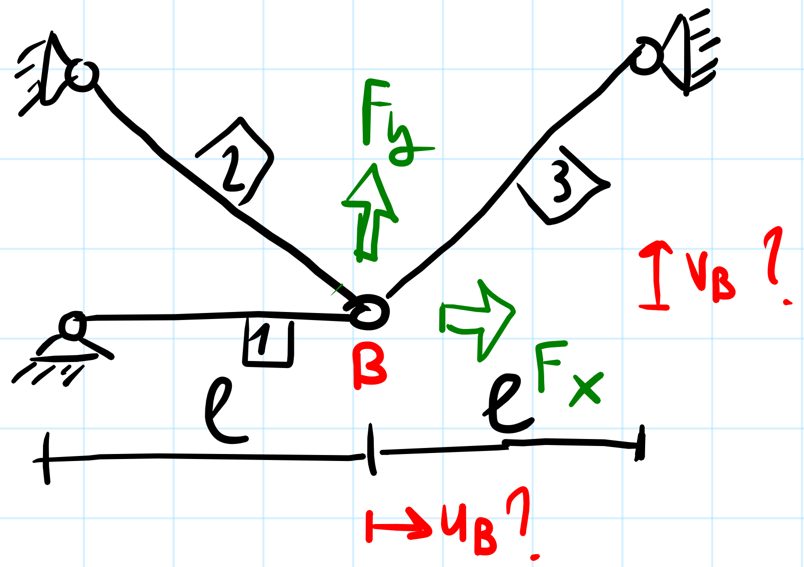
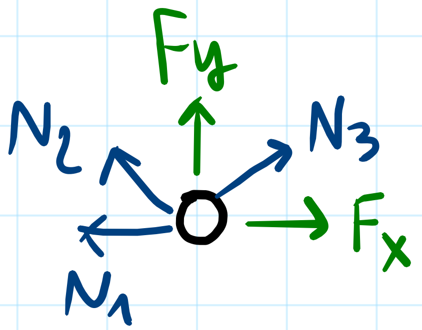


SOLVE VIA DISPL. METHOD ; OBTAIN u_B, v_B



$$\left.\begin{aligned} \xrightarrow{+} &: -N_1 - N_2 \frac{1}{\sqrt{2}} + N_3 \frac{1}{\sqrt{2}} + F_x = 0 \\ +\uparrow &: N_2 \frac{1}{\sqrt{2}} + N_3 \frac{1}{\sqrt{2}} + F_y = 0 \end{aligned}\right\}$$

2 DOFs : u_B, v_B

$\ell_1 = \ell \qquad A_1 = A$

$\ell_2 = \ell\sqrt{2} \qquad A_2 = \sqrt{2}A$

$\ell_3 = \ell\sqrt{2} \qquad A_3 = \sqrt{2}A$

E : CONST

USE CONSTITUTIVE EQS

$$N_1 = \frac{EA}{\ell} \Delta\ell_1$$

$$N_2 = \frac{E\sqrt{2}A}{\ell\sqrt{2}} \Delta\ell_2 = \frac{EA}{\ell} \Delta\ell_2$$

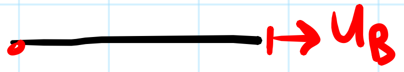
$$N_3 = \frac{E\cancel{\sqrt{2}}A}{\ell\cancel{\sqrt{2}}} \Delta\ell_3 = \frac{EA}{\ell} \Delta\ell_3$$

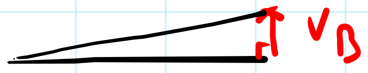
$$\Delta\ell_i = \Delta\ell_i(u_B, v_B) \quad \gtrless$$

$$K = \frac{EA}{\ell}$$

u_B v_B Δl_i

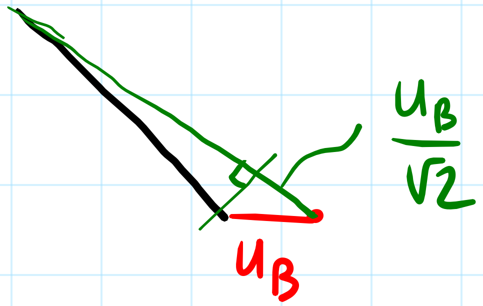
1)

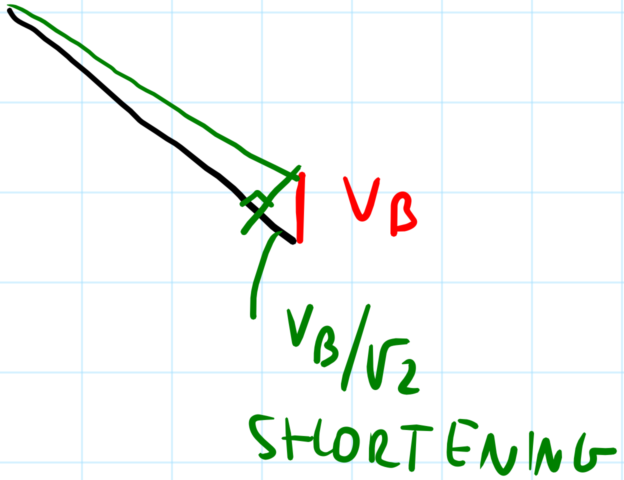




$$\Delta l_1 = +u_B + 0$$

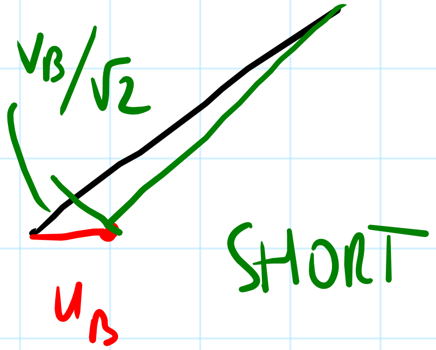
2)

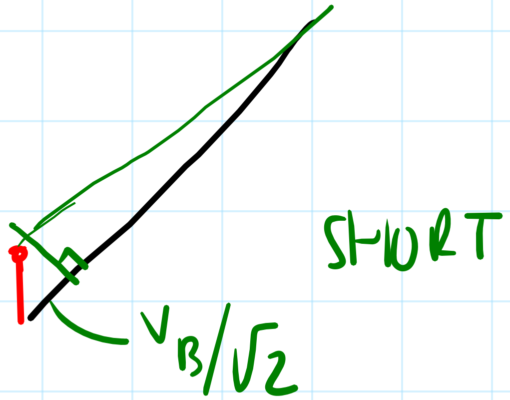




$$\Delta l_2 = +\frac{u_B}{\sqrt{2}} - \frac{v_B}{\sqrt{2}}$$

3)





$$\Delta l_3 = -\frac{u_B}{\sqrt{2}} - \frac{v_B}{\sqrt{2}}$$

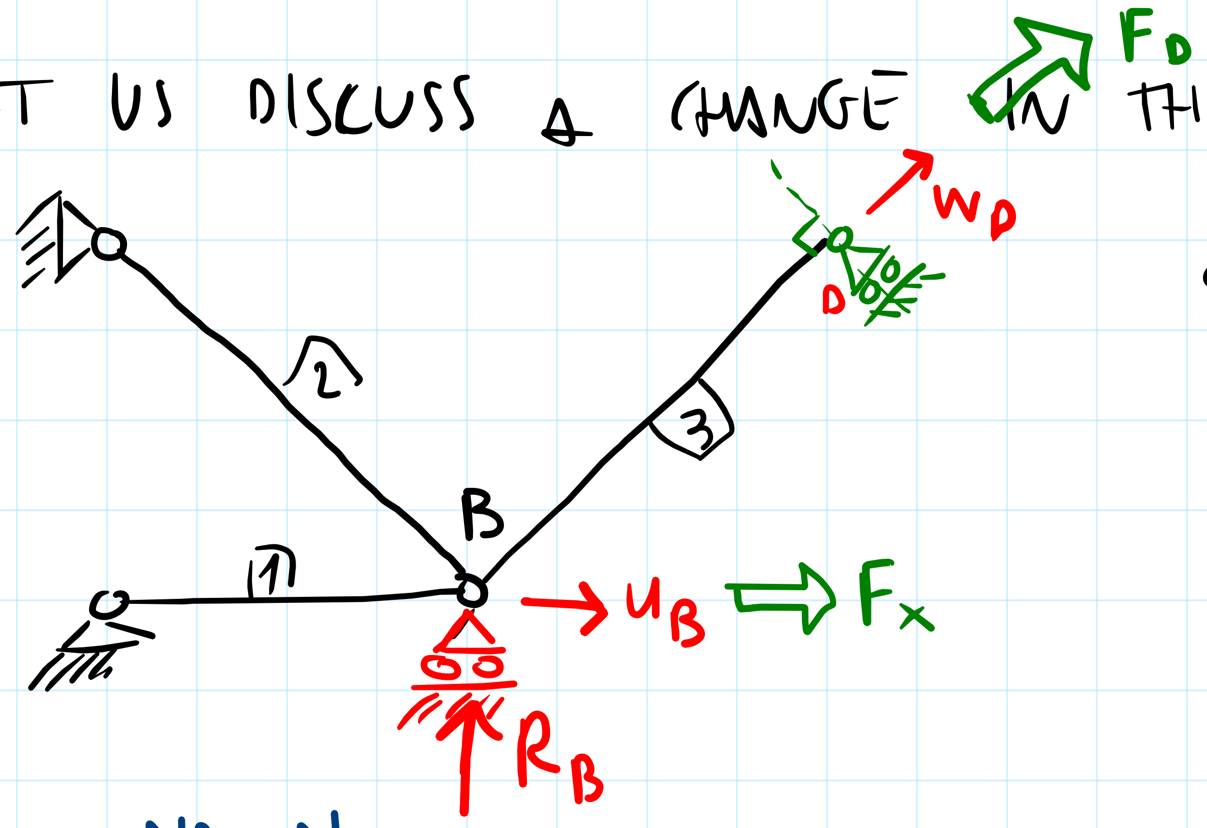
EQ. EQS :

$$\left.\begin{aligned} &-Ku_B-\frac{1}{\sqrt{2}}K\frac{1}{\sqrt{2}}(u_B-v_B)+\frac{1}{\sqrt{2}}K\frac{1}{\sqrt{2}}(-u_B-v_B)+F_x=0\\ &+\frac{1}{\sqrt{2}}K\frac{1}{\sqrt{2}}(u_B-v_B)+\frac{1}{\sqrt{2}}K\frac{1}{\sqrt{2}}(-u_B-v_B)+F_y=0 \end{aligned}\right\}\quad \begin{matrix}u_B\\ v_B\end{matrix}=f(K,F_x,F_y)$$

$$-\underbrace{\begin{bmatrix}2K & 0\\ 0 & K\end{bmatrix}}_{\underset{\sim}{K}\ 2\times 2}\underbrace{\begin{bmatrix}u_B\\ v_B\end{bmatrix}}_{\underline{u}}+\underbrace{\begin{bmatrix}F_x\\ F_y\end{bmatrix}}_{\underline{F}}=\begin{bmatrix}0\\ 0\end{bmatrix}\Longrightarrow\quad \underset{\sim}{K}\,\underline{u}=\underline{F}\ ;\quad \underline{u}=\underset{\sim}{K}^{-1}\underline{F}$$

$\underset{\sim}{K}$: SYMMETRIC, POSITIVE DEFINITE $\Big]$ $\underset{\sim}{K}^{-1}$ ALWAYS EXIST

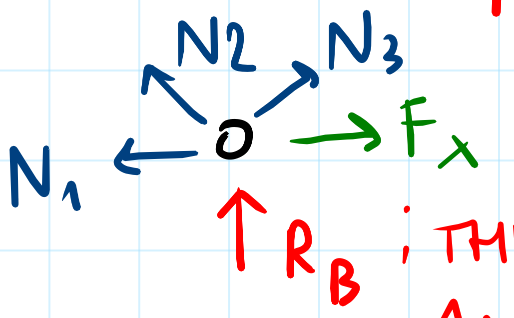
LET US DISCUSS A CHANGE IN THE PREVIOUS EXERCISE



2 DOFs ; $u_B , w_D \Rightarrow$ EQUIL EQS

$$\sum F^B_{(u_B)} = 0 \quad \rightarrow +$$

$$\sum F^D_{(w_D)} = 0 \quad \nearrow +$$

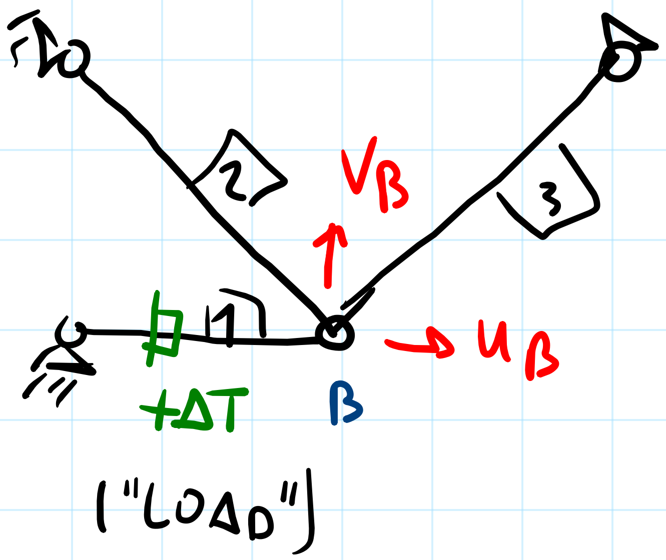


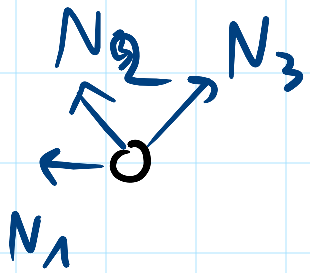
; THIS IS UNKNOWN AND CAN BE SOLVED ONLY AFTER HAVING OBTAINED u_B, w_D, N_1, N_2, N_3

THROUGH $\sum F^B = 0 \uparrow +$ (R_B IS THE ONLY UNKNOWN)

$$R_B + \frac{N_2}{\sqrt{2}} + \frac{N_3}{\sqrt{2}} = 0$$

HOW TO DEAL WITH A THERMAL VARIATION





$\xrightarrow{+}$: $-N_1 - \frac{N_2}{\sqrt{2}} + \frac{N_3}{\sqrt{2}} = 0$

$+\uparrow$: $\frac{N_2}{\sqrt{2}} + \frac{N_3}{\sqrt{2}} = 0$

NO APPLIED FORCES

CONST. EQS: $N_2 = \frac{E\sqrt{2}A}{\sqrt{2}\,\ell}\Delta\ell_2$; $N_3 = \frac{E\sqrt{2}A}{\sqrt{2}\,\ell}\Delta\ell_3$

$$N_1 = \frac{EA}{\ell}\Delta\ell_1^{elast} = \frac{EA}{\ell}\left(\Delta\ell_1 - \alpha\Delta T\,\ell\right)$$

SEE PREVIOUS EXERCISE (u_B, v_B)

$$\begin{bmatrix} 2K & 0 \\ 0 & K \end{bmatrix}\begin{bmatrix} u_B \\ v_B \end{bmatrix} = \begin{bmatrix} K\alpha\Delta T\ell \\ 0 \end{bmatrix}$$

$$\underset{\sim}{K} \qquad \underline{u} \qquad \underline{F}$$

$\underline{u}$ IS A FUNCTION OF THE "THERMAL LOAD"

$$\underset{\sim}{K}\,\underline{u} = \underline{F}^{el} + \underline{F}^{INELASTIC} + \underline{F}^{DISP*}$$

* $\underline{F}^{DISP}$: DISPLACEMENT IMPOSED TO CONSTRAINED