CAPITOLO 4

MOTI IN 2D → velocità ed accelerazione non giacciono necessariamente sulla stessa retta

→ moto avviene nel piano xy

→ estensione a 3D abbastanza automatica

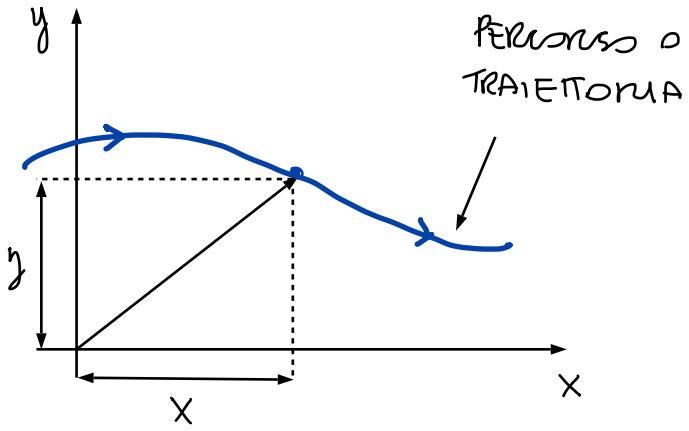
VELOCITÀ ED ACCELERAZIONE IN 2D

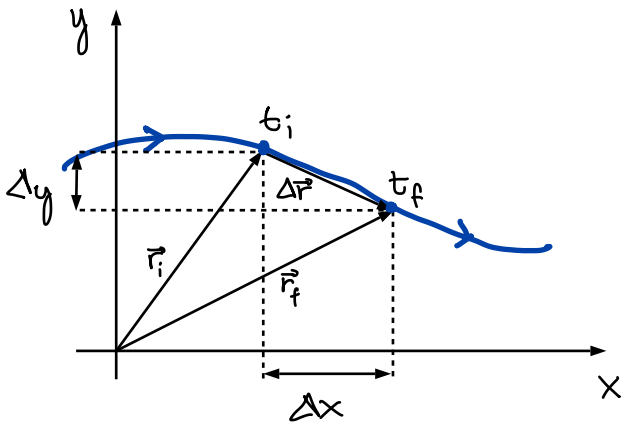
POSIZIONE → $\vec{r} = x\hat{i} + y\hat{j}$

SPOSTAMENTO → $\Delta\vec{r} = (x_f\hat{i} + y_f\hat{j}) - (x_i\hat{i} + x_f\hat{j})$

$= (x_f - x_i)\hat{i} + (y_f - y_i)\hat{j}$

$= \Delta x\hat{i} + \Delta y\hat{j}$



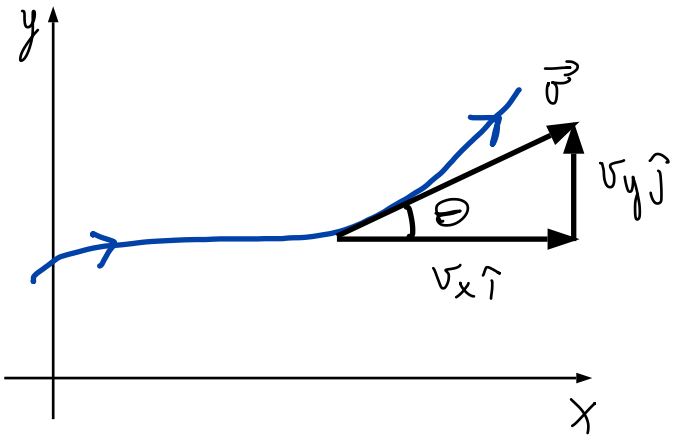


VELOCITÀ → $\langle\vec{v}\rangle = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} = \langle v_x\rangle\hat{i} + \langle v_y\rangle\hat{j}$ VELOCITÀ MEDIA

$$\vec{v} = \lim_{\Delta t\to 0}\langle\vec{v}\rangle = \lim_{\Delta t\to 0}\frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$ VELOCITÀ ISTANTANEA

Che possiamo scrivere anche analiticamente

$$\vec{v} = \lim_{\Delta t\to 0}\left(\frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j}\right) = \hat{i}\left(\lim_{\Delta t\to 0}\frac{\Delta x}{\Delta t}\right) + \hat{j}\left(\lim_{\Delta t\to 0}\frac{\Delta y}{\Delta t}\right) = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$



$$|\vec{v}| = \sqrt{v_x^2 + v_y^2}$$

$$\tan\theta = \frac{v_y}{v_x} \rightarrow \theta = \tan^{-1}\frac{v_y}{v_x}$$

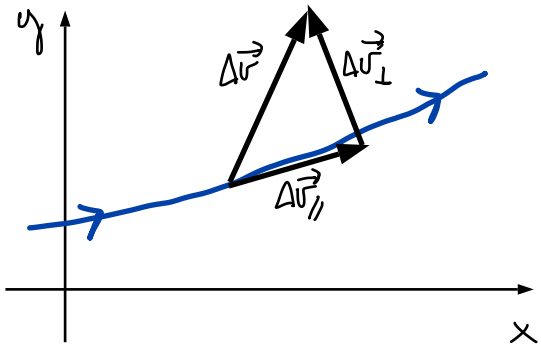
$$v_x = |\vec{v}|\cos\theta, \quad v_y = |\vec{v}|\sin\theta$$

ACCELERAZIONE $\longrightarrow \langle\vec{a}\rangle = \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j} = \langle a_x\rangle\hat{i} + \langle a_y\rangle\hat{j}$ ACCELERAZIONE MEDIA

$$\vec{a} = \lim_{\Delta t\to 0}\langle\vec{a}\rangle = \lim_{\Delta t\to 0}\frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$ ACCELERAZIONE ISTANTANEA

Di nuovo, possiamo esplicitarla analiticamente

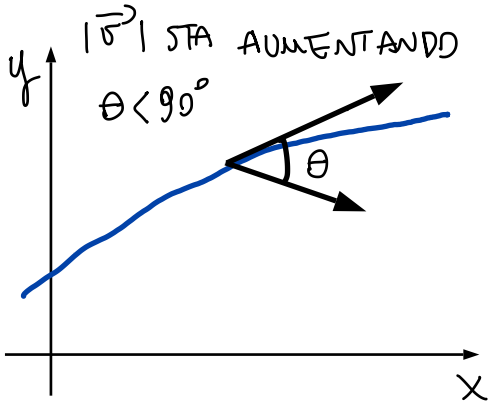
$$\vec{a} = \lim_{\Delta t\to 0}\left(\frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j}\right) = \hat{i}\left(\lim_{\Delta t\to 0}\frac{\Delta v_x}{\Delta t}\right) + \hat{j}\left(\lim_{\Delta t\to 0}\frac{v_y}{\Delta t}\right) = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j}$$

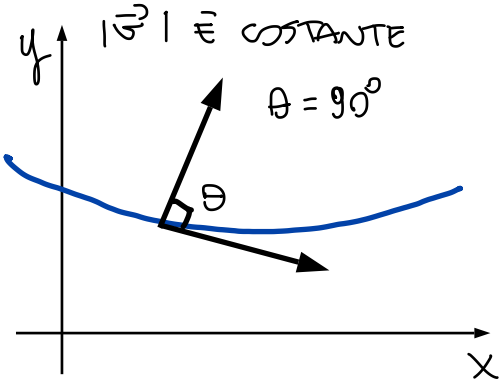


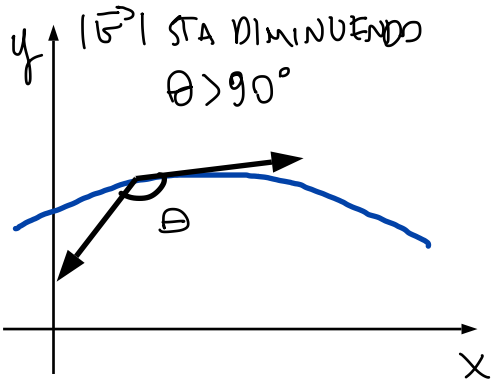
Per rimanere a comprendere meglio $\vec{a}$, possiamo scomporre $\Delta\vec{v}$ in componenti $\Delta\vec{v}_\perp$ e $\Delta\vec{v}_{//}$ alla traiettoria

Lo posso scomporre $\vec{a}$ in $\vec{a}_\perp$ e $\vec{a}_{//}$

$\Delta\vec{v}_\perp = \vec{a}_\perp = \vec{0}$ TRAIETTORIA RETTILINEA







ACCELERAZIONE COSTANTE = MOTO DEI PROIETTILI

$$\vec{a} = \langle \vec{a} \rangle = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v} - \vec{v}_0}{t - t_0} \longrightarrow \vec{v} = \vec{v}_0 + \vec{a}\, t$$

$$= (v_{0x}\,\hat{\imath} + v_{0y}\,\hat{\jmath}) + (a_x\hat{\imath} + a_y\hat{\jmath})\,t$$

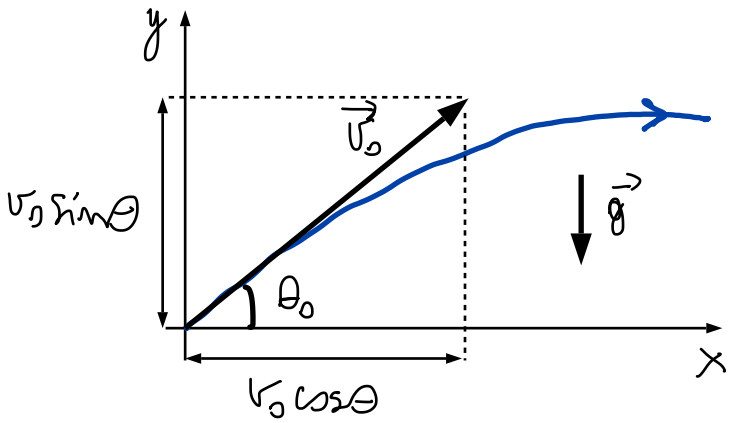
$$= (v_{0x} + a_x t)\,\hat{\imath} + (v_{0y} + a_y t)\,\hat{\jmath}$$

Possiamo quindi scomporre il moto in x e y con due equazioni di moto con $|\vec{a}|$ = costante, indipendenti

$$\vec{r}(t) = \vec{r}_0 + \vec{v}_0 t - \frac{1}{2}\vec{a}t^2 \qquad \begin{cases} x(t) = x_0 + v_{x0}t + \frac{1}{2}a_x t^2 \\ y(t) = y_0 + v_{y0}t + \frac{1}{2}a_y t^2 \end{cases}$$

NB. In 3D aggiungerò eq. per $z(t)$

$\boxed{\text{MOTO DEI PROIETTILI}}$ $\longrightarrow$ oggetto con densità tale da poter non considerare la resistenza dell'aria (ok pallina biliardo, no pallina ping-pong)



$$a_x = 0\,, \quad a_y = -g$$

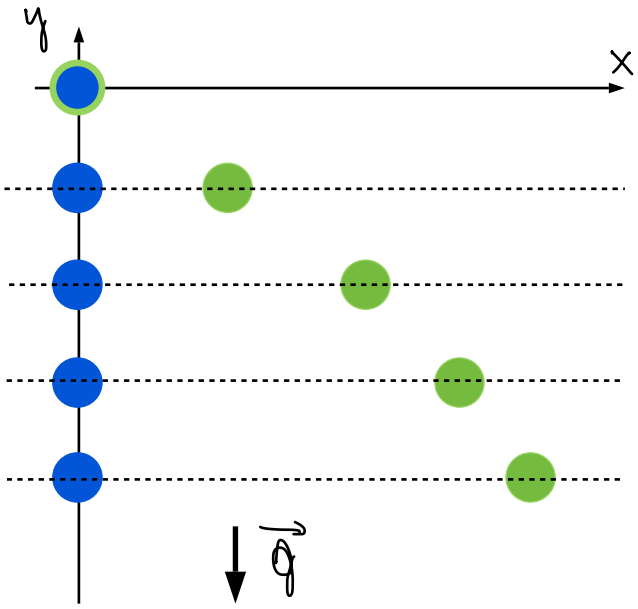
$$\begin{cases} v_x(t) = v_0 \cos\theta_0 \\ v_y(t) = v_0 \sin\theta_0 - gt \end{cases}$$

$$\begin{cases} x(t) = x_0 + v_0 \cos\theta_0\, t \\ y(t) = y_0 + (v_0 \sin\theta_0)t - \frac{1}{2}gt^2 \end{cases}$$

Composizione di moti $\nearrow \hat{\imath}$ MOTO VELOCITÀ COSTANTE

$\searrow \hat{\jmath}$ MOTO ACCELERAZIONE COSTANTE

Vediamo un esempio di due casi estremi per vedere che i moti in $\hat{i}$ e $\hat{j}$ sono disaccoppiati.



$\theta_0 = 0$

$v_x(t) = v_0 \ \forall t$

$v_y(t) = -gt$

$x(t) = v_0 t$

$y(t) = -\frac{1}{2} g t^2$

$\vec{v}_0 = \vec{0}$

$v_x(t) = 0 \ \forall t$

$v_y(t) = -gt$

$x(t) = 0 \ \forall t$

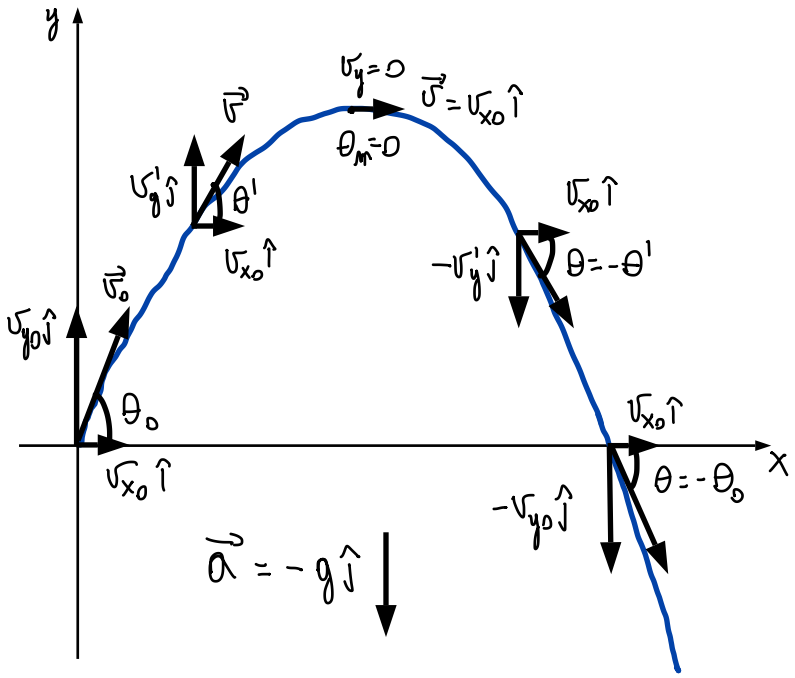
$y(t) = -\frac{1}{2} g t^2$

STESSA $y(t) \ \forall t$

Verifichiamo che è effettivamente un MOTO PARABOLICO

$$\begin{cases} x = (v_0 \cos\theta)\, t \\ y = (v_0 \sin\theta)\, t - \frac{1}{2} g t^2 \end{cases} \longrightarrow t = \frac{x}{v_0 \cos\theta_0}$$

$$y = v_0 \sin\theta_0 \frac{x}{v_0 \cos\theta_0} - \frac{1}{2} g \left(\frac{x}{v_0 \cos\theta_0} \right) = (\tan\theta_0)\, x - \frac{g}{2 v_0^2 \cos^2\theta_0} x^2$$



Possiamo ricavarci un po' di quantità utili

TEMPO (t_m) PER ALTEZZA MASSIMA (h_m) $\longrightarrow$

$$v_y(t_m) = 0 = v_0 \sin\theta_0 - g t_m \rightarrow t_m = \frac{v_0 \sin\theta_0}{g}$$

$$y(t_m) = h_m = v_0 \sin\theta_0 \, t_m - \frac{1}{2} g t_m^2$$

$$= \frac{v_0^2 \sin^2\theta_0}{g} - \frac{1}{2} g \left(\frac{v_0 \sin\theta_0}{g}\right)^2$$

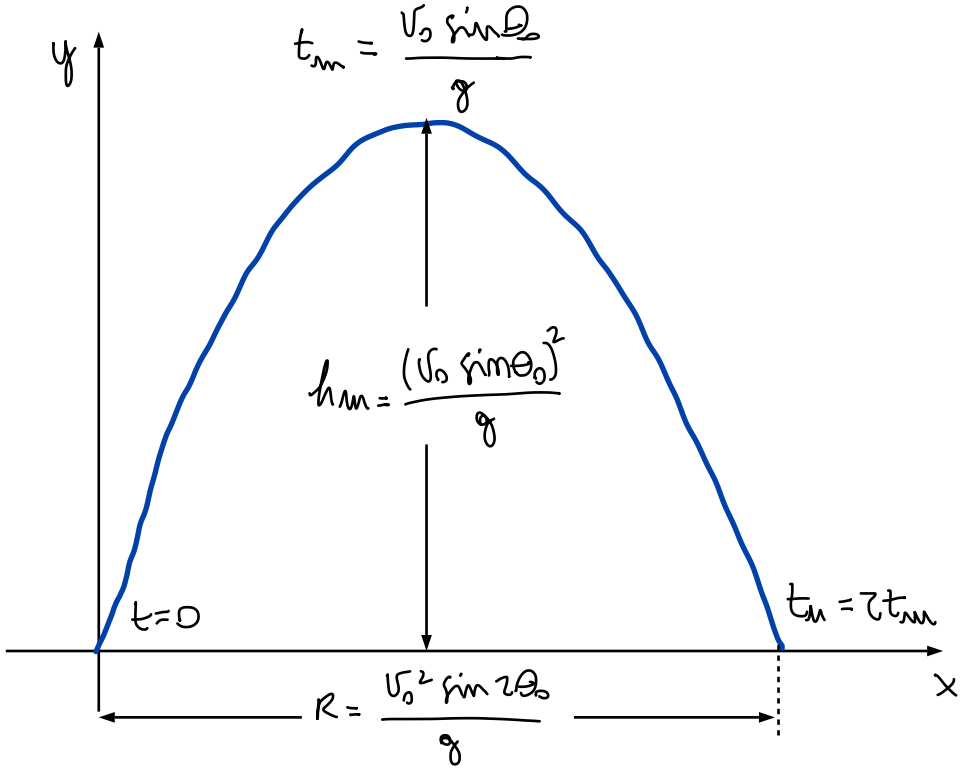
$$= \frac{v_0^2 \sin^2\theta_0}{g}$$

GITATA MASSIMA (R) $\longrightarrow$ distanza massima raggiunta in $\hat{i}$

$\longrightarrow$ per simmetria $t_M = 2t_m$, dove t_m corrisponde a $x(t_M) = R$

$$x(t_M) = R = (v_0 \cos\theta_0) t_M = v_0 \cos\theta_0 \frac{2 v_0 \sin\theta_0}{g} = \frac{v_0^2 \sin 2\theta_0}{g}$$

NB. $\sin 2\theta = 2 \cos\theta \sin\theta$



RIASSUMENDO

$a_x = 0$

$v_x = v_0 \cos\theta_0$

$x = x_0 + (v_0 \cos\theta_0)t$

$a_y = -g$

$v_y = v_0 \sin\theta_0 - gt$

$y = y_0 + (v_0 \sin\theta_0)t - \frac{1}{2}gt^2$

$$y = y_0 + \tan\theta_0 (x - x_0) - \frac{g}{2(v_0 \cos\theta_0)^2}(x - x_0)^2$$