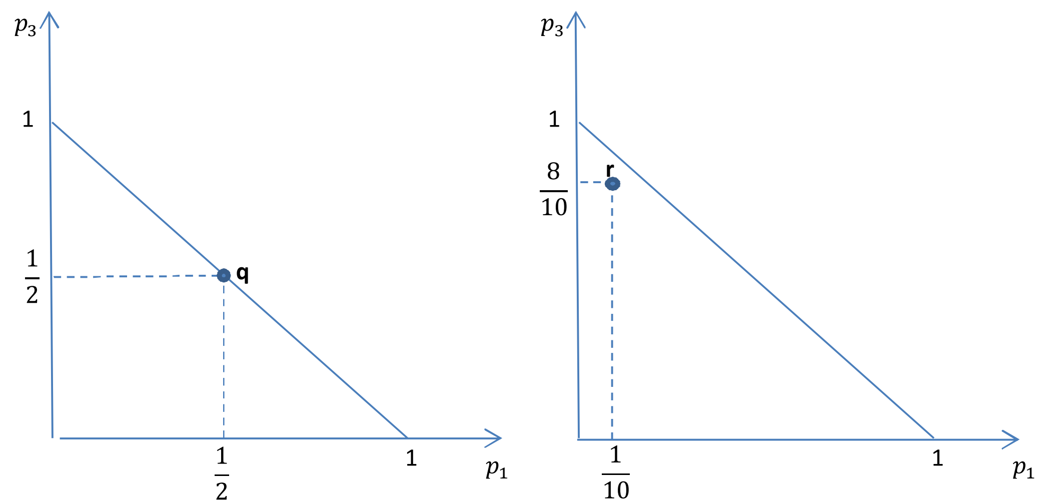
1. Consider the lottery $\boldsymbol{q} = (25, p_3;\ 16,\ p_2; 9, p_1)$, p_3+p_2+p_1=1. Assuming that the utility function is $\mathrm{u(x)} = \sqrt{\mathrm{x}}$ and the weighting function is $\pi(p) = p^2$.
   i. Using the Machina triangle (p on the vertical axis, w on the horizontal one) represent the following prospects: $\boldsymbol{q} = (25, \frac{1}{2};\ 9, \frac{1}{2})$, $\boldsymbol{r} = (25, \frac{8}{10};\ 16,\ \frac{1}{10}; 9, \frac{1}{10})$.
   ii. Check the sign of the slope of the indifference curves passing for $\boldsymbol{q}$
   iii. Check the sign of the slope of the indifference curves passing for $\boldsymbol{r}$

i.



ii. $\boldsymbol{q} = (25, \frac{1}{2};\ 9, \frac{1}{2})$

$U(\boldsymbol{q}) = \sqrt{9}\left(\frac{1}{2}\right)^2 + \sqrt{25}\left(\frac{1}{2}\right)^2 = 2$

$F = \sqrt{9}{p_1}^2 + \sqrt{16}(1 - p_3 - p_1)^2 + \sqrt{25}{p_3}^2 - 2 = 0$

Using the theorem of the implicit function we get

$$\frac{dp_3}{dp_1} = -\frac{\frac{dF}{dp_1}}{\frac{dF}{dp_3}} = -\frac{6p_1 - 8(1 - p_3 - p_1)}{10p_3 - 8(1 - p_3 - p_1)}$$

Evaluated in $\boldsymbol{q}$ we obtain that $\frac{dp_3}{dp_1} = -\frac{6}{10}$

iii. $\boldsymbol{r} = (25, \frac{8}{10};\ 16,\ \frac{1}{10}; 9, \frac{1}{10})$

$U(\boldsymbol{r}) = \sqrt{9}\left(\frac{1}{10}\right)^2 + \sqrt{16}\left(\frac{1}{10}\right)^2 + \sqrt{25}\left(\frac{8}{10}\right)^2 = 3.27$

$F = \sqrt{9}{p_1}^2 + \sqrt{16}(1 - p_3 - p_1)^2 + \sqrt{25}{p_3}^2 - 3.27 = 0$

Using the theorem of the implicit function we get

$$\frac{dp_3}{dp_1} = -\frac{\frac{dF}{dp_1}}{\frac{dF}{dp_3}} = -\frac{6p_1 - 8(1 - p_3 - p_1)}{10p_3 - 8(1 - p_3 - p_1)}$$

Evaluated in $\boldsymbol{r}$ we obtain that

$$\frac{dp_3}{dp_1} = \frac{1}{36}$$

2. An individual prefers high outcomes respect to small ones.
   a. Suppose that lottery r stochastically dominates lottery q. Which is the preferred lottery?
   b. Check for stochastic dominance of the two following lotteries.

$$q = (210, 0.30;\ 500, 0.2; 760, 0.5)$$
$$r = (200, 0.30;\ 500, 0.15; 760, 0.55)$$

a. Lottery r is the preferred one.
b. Let be $F_q(x)$ and $F_r(x)$ the cumulative functions respectively of lotteries q and r.

$F_q(200) = 0, F_q(210) = 0.3, F_q(500) = 0.5, F_q(760) = 1$
$F_r(200) = 0.3, F_q(210) = 0.3, F_q(500) = 0.45, F_q(760) = 1$

No one stochastically dominates the other.

3. All Prospect theory's assumptions are satisfied. Assume that $\mathrm{v(x)} = \begin{cases} \sqrt{x}\ if\ x \geq 0 \\ -\frac{4}{5}\sqrt{-\mathrm{x}}\ if\ x < 0 \end{cases}$ and $\pi(\mathrm{p}) = \mathrm{p}^2$. Evaluate the following prospects and for each one compute the certainty equivalent (reference point 0):
   a. $\mathrm{q} = (625, 0.60;\ -225, 0.40)$
   b. $\mathrm{q} = (154, 0.15; 46, 0.55; 10, 0.30)$

a. $U(q) = \sqrt{625}\ 0.6^2 - \frac{4}{5}\sqrt{225}\ 0.4^2 = 25\ 0.36 - 12\ 0.16 = 7.08$

$$CE = 7.08^2 = 50.13$$

b. $(q) = \sqrt{10} + \left(\sqrt{46} - \sqrt{10}\right) 0.55^2 + \left(\sqrt{154} - \sqrt{10}\right) 0.15^2 = 4.47$

$$CE = 4.47^2 = 19.98$$

4. Consider the following prospects $\boldsymbol{q} = (9,\ 0.12)$ and $\boldsymbol{r} = (27,\ 0.04)$. Assume that $v(x) = x$ and $\pi\left(\frac{1}{3} \cdot 0.12\right) > \frac{1}{3} \cdot \pi(0.12)$. State the preferred prospect.

By contradiction. Assume $\boldsymbol{q}$ is preferred to $\boldsymbol{r}$. In this case $U(r) < U(q)$ i.e. $27\ \pi(0.04) < 9\ \pi(0.12)$ simplifying $\pi(0.04) < \frac{1}{3}\ \pi(0.12)$. A contradiction with the initial assumption.

5. Consider the following prospects $\boldsymbol{r} = (x)$, $\boldsymbol{q} = \left(y,\ 0.10; x,\ 0.89\right)$ $\boldsymbol{r}' = \left(x,\ 0.11\right)$ and $\boldsymbol{q}' = \left(y,\ 0.10\right)$, $x, y > 0$. Assume that $\boldsymbol{q}'$ is indifferent to $\boldsymbol{r}'$ and that $\pi(0.11) + \pi(0.89) < 1$. Which is the preferred lottery between ***r*** and ***q***? Prove your answer assuming a generic subjective value function $v(x)$.

   $\boldsymbol{q}'$ is indifferent to $\boldsymbol{r}'$ means that $\pi(0.10)\ v(y) = \pi(0.11)\ v(x)$
   Compute the expected utilities of ***r*** and ***q***.
   $U(\boldsymbol{r}) = v(x)$ $U(\boldsymbol{q}) = v(y)\pi(0.10) + v(x)\pi(0.89)$
   $U(\boldsymbol{q})$ can be rewritten as $U(\boldsymbol{q}) = v(x)\pi(0.11) + v(x)\pi(0.89) = v(x)(\pi(0.11) + \pi(0.89))$
   Then $U(\boldsymbol{r}) = v(x) > U(\boldsymbol{q}) = v(x)\left(\pi(0.11) + \pi(0.89)\right)$given the assumption $\pi(0.11) + \pi(0.89) < 1$.
   Therefore ***r*** is preferred to ***q***.

6. All Prospect theory's assumptions are satisfied. Consider the following prospects: $\boldsymbol{q} = (x, 2p)$, $\boldsymbol{r} = (2x, p)$ where $x < 0$ and $p < 0.5$. Show that $q \succ r$ implies subadditivity.

   $q \succ r$ implies $v(x)\pi(2p) > v(2x)\pi(p)$
   By convexity 2 $v(x) < v(2x)$ then $v(x)\pi(2p) > v(2x)\pi(p) > 2\ v(x)\pi(p)$
   Then

$$v(x)\pi(2p) > 2\ v(x)\pi(p)$$
$$\frac{1}{2}\pi(2p) < \pi(p)$$

7. Consider the following prospects $\boldsymbol{r} = (20, p)$, $\boldsymbol{q} = \left(24,\ \frac{2}{3}p\right)$ $\boldsymbol{r}' = \left(\boldsymbol{r},\ 0.5\right)$ and $\boldsymbol{q}' = \left(\boldsymbol{q},\ 0.5\right)$. Assume subcertainty and that $\boldsymbol{q}'$ is indifferent to $\boldsymbol{r}'$. Which is the preferred lottery between ***r*** and ***q***? Prove your answer assuming a generic subjective value function $v(x)$.

   TEXT WAS WRONG, THE ASSUMPTION IS SUBPROPORTIONALITY and NOT SUBCERTAINTY

   $\boldsymbol{q}'$ is indifferent to $\boldsymbol{r}'$ implies $v(20)\ \pi\left(\frac{p}{2}\right) = v(24)\pi\left(\frac{p}{3}\right) \rightarrow \frac{v(20)}{v(24)} = \frac{\pi\left(\frac{p}{3}\right)}{\pi\left(\frac{p}{2}\right)}$ by subproportionality we have that: $\frac{v(20)}{v(24)} = \frac{\pi\left(\frac{p}{3}\right)}{\pi\left(\frac{p}{2}\right)} > \frac{\pi\left(\frac{2p}{3}\right)}{\pi(p)} \rightarrow \frac{v(20)}{v(24)} > \frac{\pi\left(\frac{2p}{3}\right)}{\pi(p)} \rightarrow v(20)\ \pi(p) > v(24)\ \pi\left(\frac{2}{3}p\right)$ where the LHS is the expected utility of r and the RHS is the expected utility of q. Then r is the preferred one.

8. Assume $\pi(x) = x$ and that all other Prospect theory's assumptions are satisfied. Suppose the following two prospects: $\boldsymbol{q} = (-9,\ 0.12)$ and $\boldsymbol{r} = (-27,\ 0.04)$. Which is the preferred prospect when the reference point is 0? Which is the preferred prospect when the reference point is -27?

Reference point 0:
the preferred prospect is ***r***. Indeed by convexity of v we have that $v(-27)0.04 > v(-9)0.12.$ → $\frac{v(-27)}{v(-9)} < 3.$

Reference point -27.

$\boldsymbol{q} = (27,\ 0.88;\ 18, 0.12)$ and $\boldsymbol{r} = (27,\ 0.96)$

$U(q) = v(18) + 0.88\ (v(27) - v(18))\ U(r) = v(27)0.96$

Assume **q** is preferred
I$U(q) >\ U(r)$ → $v(18) + 0.88\ (v(27) - v(18)) > v(27)0.96$→

$0.12\ v(18) + 0.88\ v(27) > v(27)0.96$
$0.12\ v(18) > v(27)0.08$
$1.5 > \frac{v(27)}{v(18)}$

That is true given the concavity of $v()$ for positive values.

9. Check if $\pi(x) = x^2$ satisfies subadditivity, subproportionality and subcertainty.

Subadditivity is $\pi(r \cdot p) > r \cdot \pi(p)\ \forall p, r \in (0,\ 1)$.
Assuming $\pi(p) = p^2$ we have $(r\ p)^2 > r\ p^2\ \forall p, r \in (0,\ 1)$. Simplifying we get $r\ p^2 > p^2$ a contradiction because $r \in (0,\ 1)$.
Then subadditivity is not satisfied.

Subproportionality principle is: $\frac{\pi(p \cdot q \cdot r)}{\pi(p \cdot r)} \geq \frac{\pi(p \cdot q)}{\pi(p)}\quad 0 \leq p,\ q,\ r \leq 1.$
Assuming $\pi(p) = p^2$ we have : $\frac{(p\ q\ r)^2}{(p\ r)^2} \geq \frac{(p\ q\ )^2}{(p)^2}\quad 0 \leq p,\ q,\ r \leq 1.$
Simplifying we get that is verified by equality.

Subcertainty principle is:
$\pi(p) + \pi(1 - p) < 1\ \forall p \in (0,\ 1)$
Assuming $\pi(p) = p^2$ we have $(p)^2 + (1 - p)^2 < 1$, simplifying we have $2\ p^2 + 1 - 2\ p < 1$ →
$2\ p^2 - 2\ p < 0$ that is true for $0 \leq p \leq 1$
Then it is verified.