SEISMOLOGY I

Laurea Magistralis in Physics of the Earth and of the Environment

Seismic sources 4: earthquake size

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Earthquake size

Earthquake size

- intensity
- magnitude scales
- saturation
- energy concepts
- moment magnitude

How is Earthquake Size Determined?

(1) **Maximum Seismic Intensity**

Outdated method

Does not use seismometers

Many problems

(2) **Magnitudes**

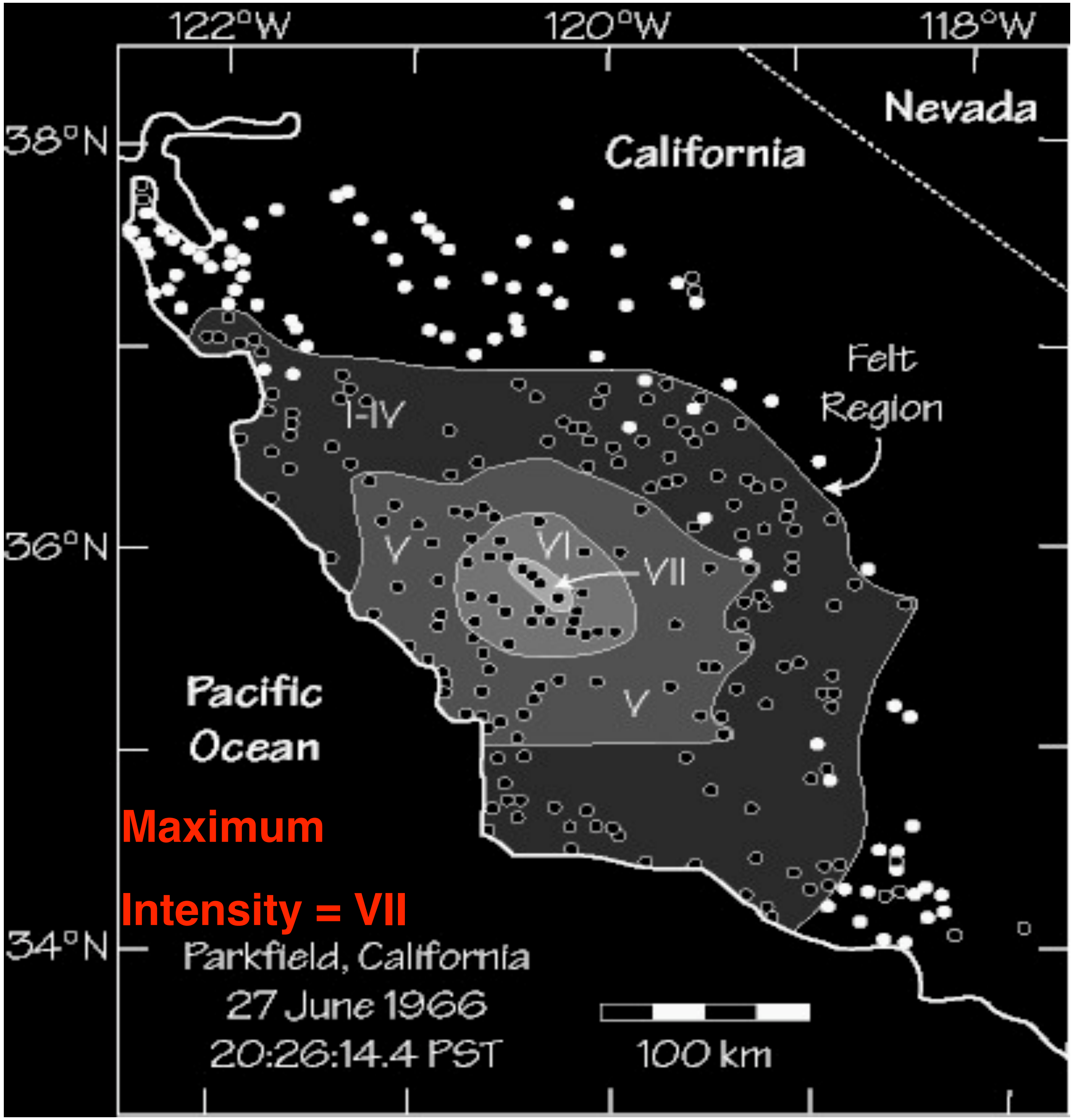
Modern method

Uses seismometers

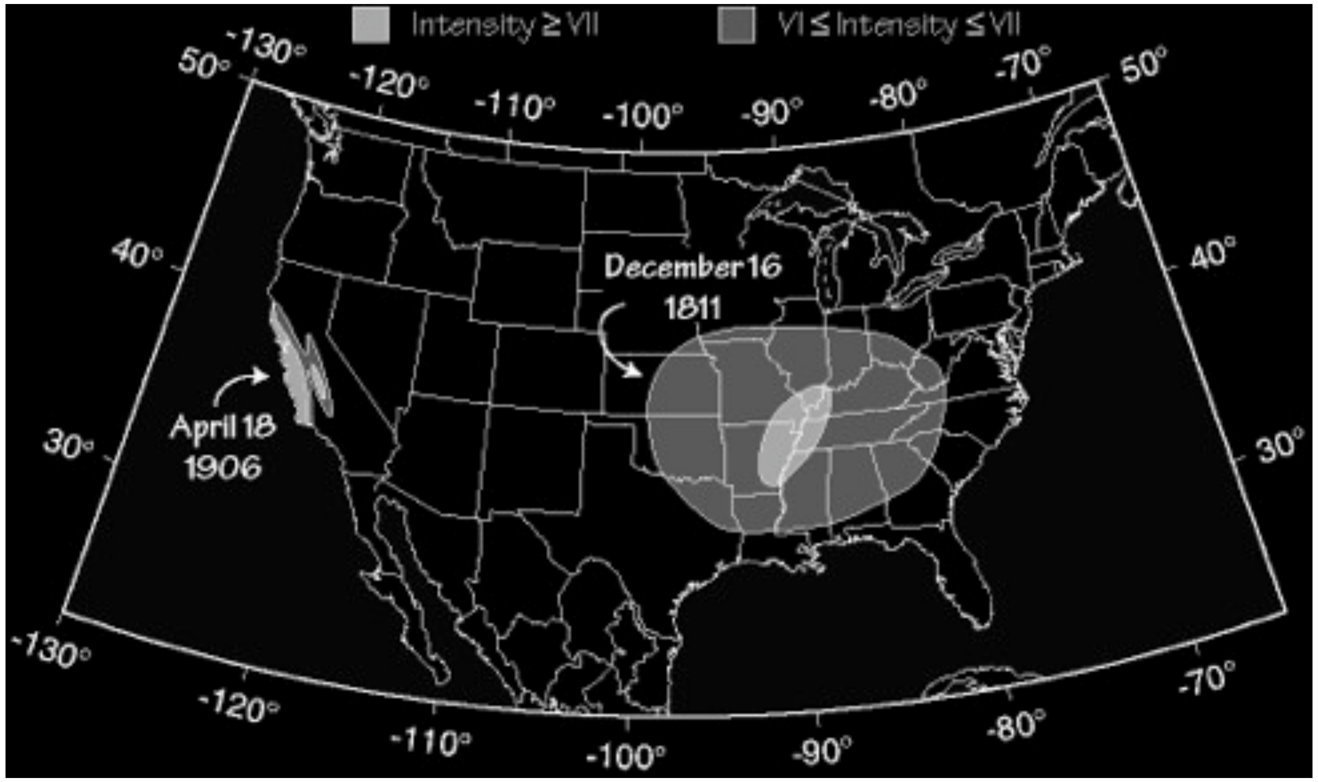
Fewer problems

Maximum Intensity

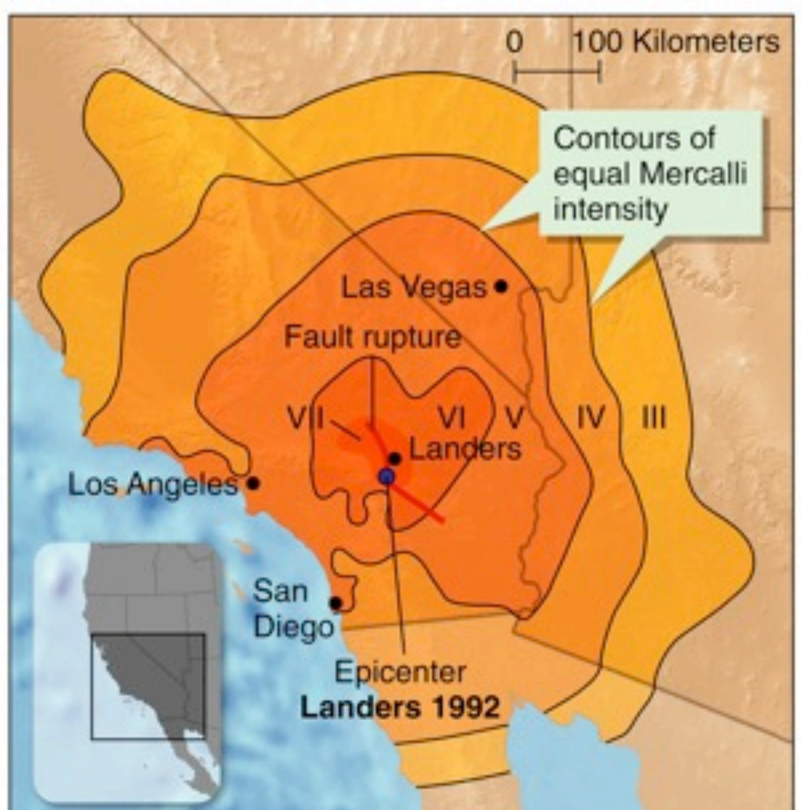
Maximum Intensity is used to estimate the size of historical earthquakes, but suffers from dependence on depth, population, construction practices, site effects, regional geology, etc.

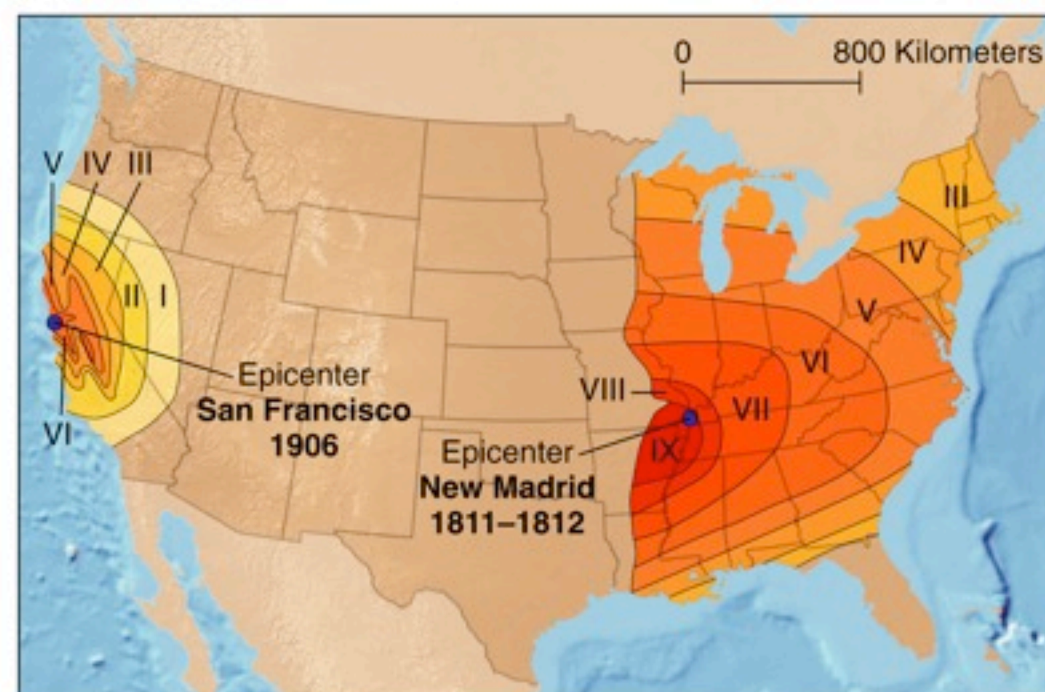


1906 SF and 1811-12 New Madrid



These earthquakes were roughly the same size, but the intensity patterns in the east are broader than in the west (wait for Q...)





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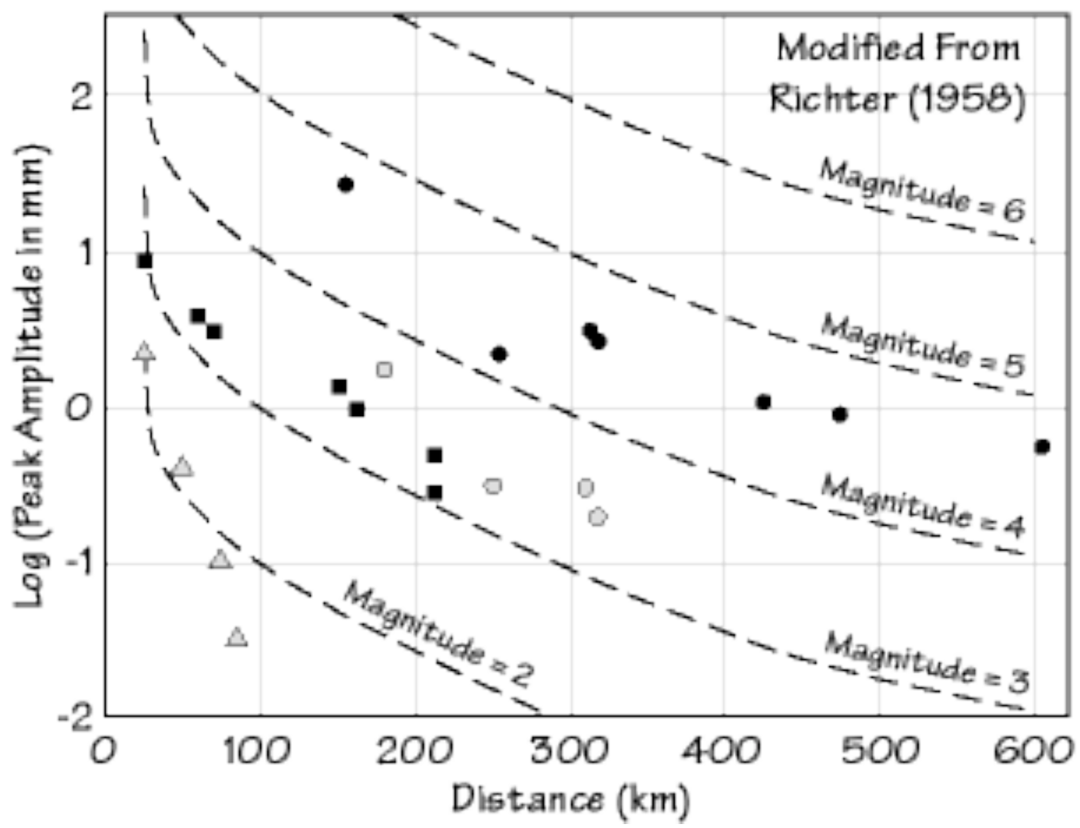
Earthquake Magnitude

Earthquake magnitude scales originated because of

- the desire for an objective measure of earthquake size
- Technological advances -> seismometers

Magnitude Scales - Richter

The concept of magnitude was introduced by Richter (1935) to provide an objective instrumental measure of the size of earthquakes. Contrary to seismic intensity, I, which is based on the assessment and classification of shaking damage and human perceptions of shaking, the magnitude M uses instrumental measurements of earth ground motion adjusted for epicentral distance and source depth.

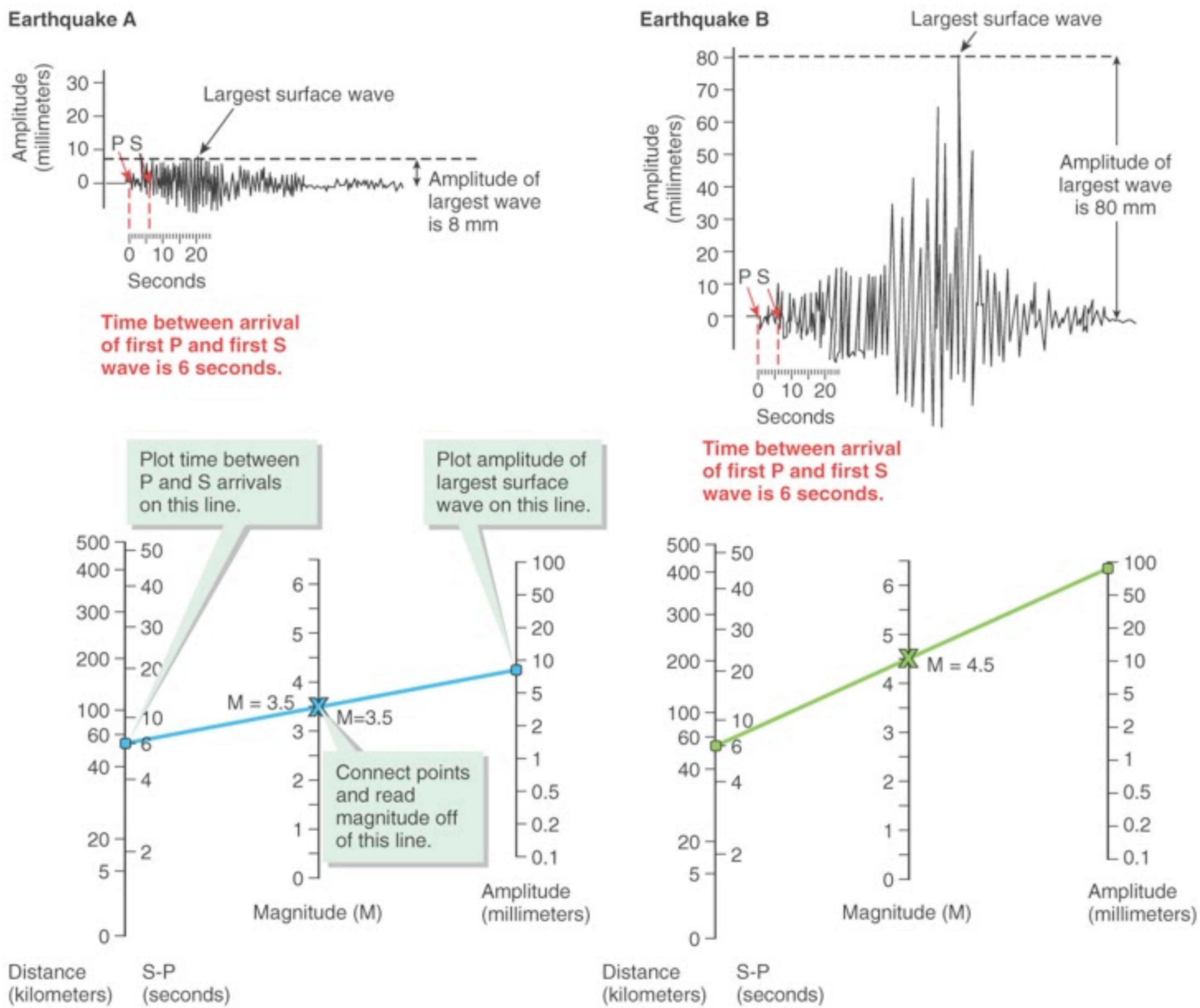


The original Richter scale was based on the observation that the amplitude of seismic waves systematically decreases with epicentral distance.

Data from local earthquakes in California

The relative size of events is calculated by comparison to a reference event, with M_L=0, such that A_0 was 1μm at an epicentral distance of 100 km with a Wood-Anderson instrument:

$$M_L=\log(A/A_0)=\log A-2.48+2.76\Delta.$$



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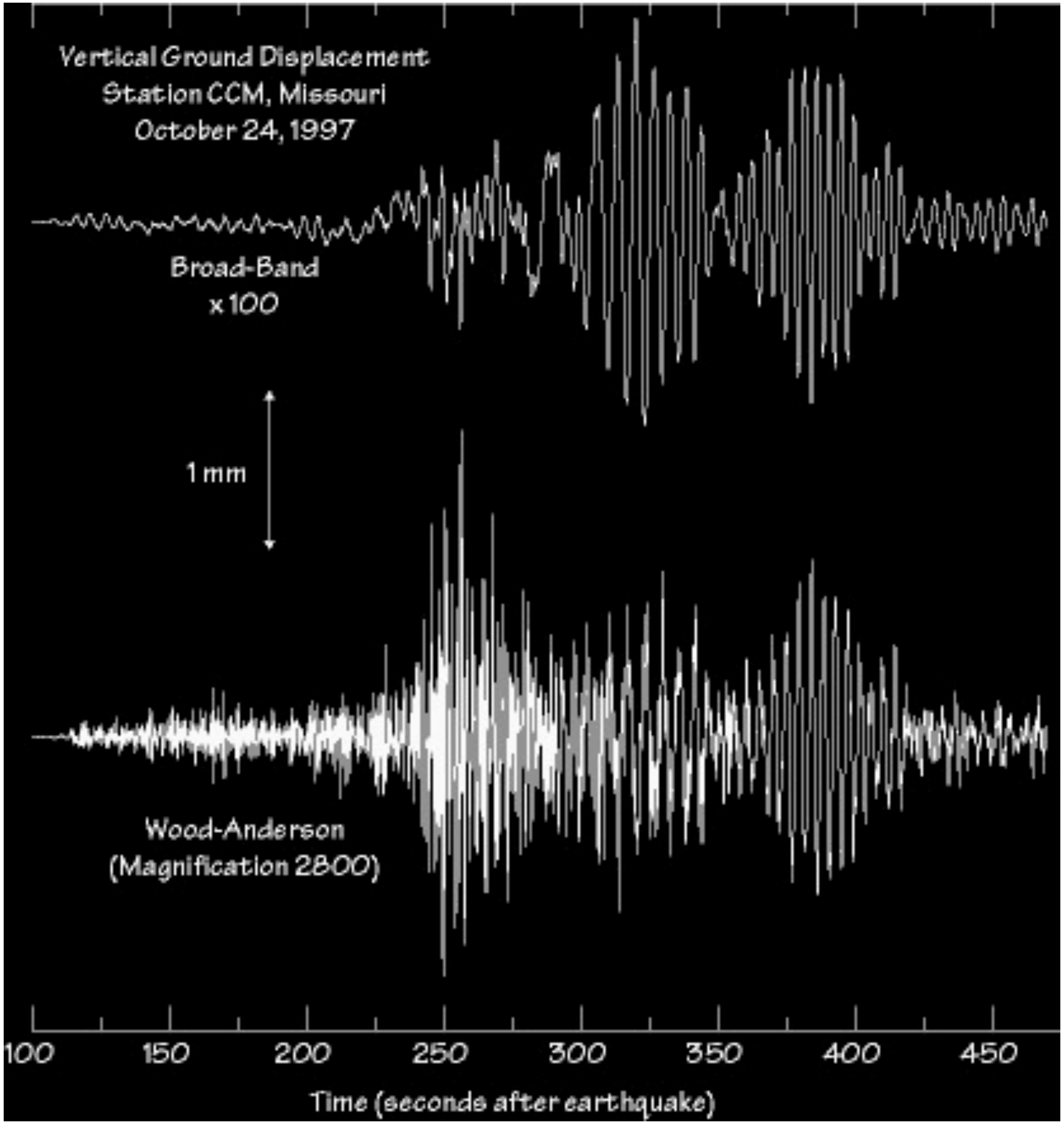
http://siovizcenter.ucsd.edu/library/TLTC/TLTCmag.htm

Wood-Anderson Seismometer

Richter also tied his formula to a specific seismic instrument.



Mercalli Intensity and Richter Magnitude

Magnitude	Intensity	Description
1.0-3.0 **Micro**	**I**	**I.** Not felt except by a very few under especially favorable conditions.
3.0 - 3.9 **Minor**	**II - III**	**II.** Felt only by a few persons at rest, especially on upper floors of buildings. **III.** Felt quite noticeably by persons indoors, especially on upper floors of buildings. Many people do not recognize it as an earthquake. Standing motor cars may rock slightly. Vibrations similar to the passing of a truck. Duration estimated.
4.0 - 4.9 **Light**	**IV - V**	**IV.** Felt indoors by many, outdoors by few during the day. At night, some awakened. Dishes, windows, doors disturbed; walls make cracking sound. Sensation like heavy truck striking building. Standing motor cars rocked noticeably. **V.** Felt by nearly everyone; many awakened. Some dishes, windows broken. Unstable objects overturned. Pendulum clocks may stop.
5.0 - 5.9 **Moderate**	**VI - VII**	**VI.** Felt by all, many frightened. Some heavy furniture moved; a few instances of fallen plaster. Damage slight. **VII.** Damage negligible in buildings of good design and construction; slight to moderate in well-built ordinary structures; considerable damage in poorly built or badly designed structures; some chimneys broken.
6.0 - 6.9 **Strong**	**VII - IX**	**VIII**. Damage slight in specially designed structures; considerable damage in ordinary substantial buildings with partial collapse. Damage great in poorly built structures. Fall of chimneys, factory stacks, columns, monuments, walls. Heavy furniture overturned. **IX.** Damage considerable in specially designed structures; well-designed frame structures thrown out of plumb. Damage great in substantial buildings, with partial collapse. Buildings shifted off foundations.
7.0 and higher **Major great**	**VIII or higher**	**X.** Some well-built wooden structures destroyed; most masonry and frame structures destroyed with foundations. Rails bent. **XI.** Few, if any (masonry) structures remain standing. Bridges destroyed. Rails bent greatly. **XII.** Damage total. Lines of sight and level are distorted. Objects thrown into the air.

Richter's Magnitude Scale

- Only valid for Southern California earthquakes
- Only valid for one specific type of seismometer
- Has not been used by professional seismologists in decades
- Is much abused by the press today

Modern Seismic Magnitudes

- Today seismologists use different seismic waves to compute magnitudes
- These waves have lower frequencies than those used by Richter
- These waves are generally recorded at distances of 1000s of kilometers instead of the 100s of kilometers for the Richter scale

Magnitude Scales

The original M_L is suitable for the classification of local shocks in Southern California only since it used data from the standardized short-period Wood-Anderson seismometer network. The magnitude concept has then been extended so as to be applicable also to ground motion measurements from medium- and long-period seismographic recordings of both surface waves (M_s) and different types of body waves (m_b) in the teleseismic distance range.

The general form of all magnitude scales based on measurements of ground displacement amplitudes A and periods T is:

$$M = \log(A/T) + f(\Delta, h) + C_s + C_r$$

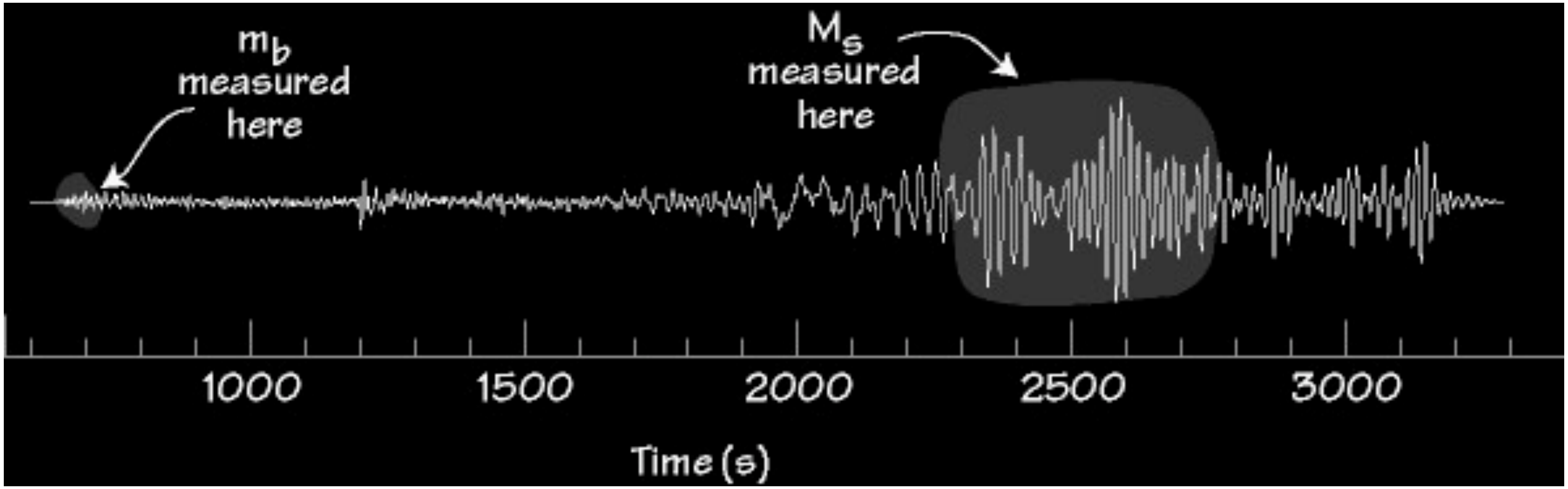
M seismic magnitude
A amplitude
T period
f correction for distance
C_s correction for site
C_r correction for source region

M_L **Local magnitude**
m_b **body-wave magnitude** (1s)
M_s **surface wave magnitude** (20s)

Teleseismic M_S and mb

The two most common modern magnitude scales are:

- M_S, Surface-wave magnitude (Rayleigh Wave, 20s)
- m_b, Body-wave magnitude (P-wave)



Magnitude Discrepancies

Ideally, you want the same value of magnitude for any one earthquake from each scale you develop, i.e.

$M_S = m_b = M_L$

But this does not always happen:

Turkey 8/17/99: $M_S = 7.8$, $m_b = 6.3$

Taiwan 9/20/99: $M_S = 7.7$, $m_b = 6.6$

Why Don't Magnitude Scales Agree?

Simplest Answer:

- Earthquakes are complicated physical phenomena that are not well described by a single number.
- Can a thunderstorm be well described by one number ? (No. It takes wind speed, rainfall, lightning strikes, spatial area, etc.)

Why Don't Magnitude Scales Agree?

More Complicated Answers:

- The distance correction for amplitudes depends on geology.
- Deep earthquakes do not generate large surface waves - M_S is biased low for deep earthquakes.
- Some earthquakes last longer than others, even though the peak amplitude is the same.

Why Don't Magnitude Scales Agree?

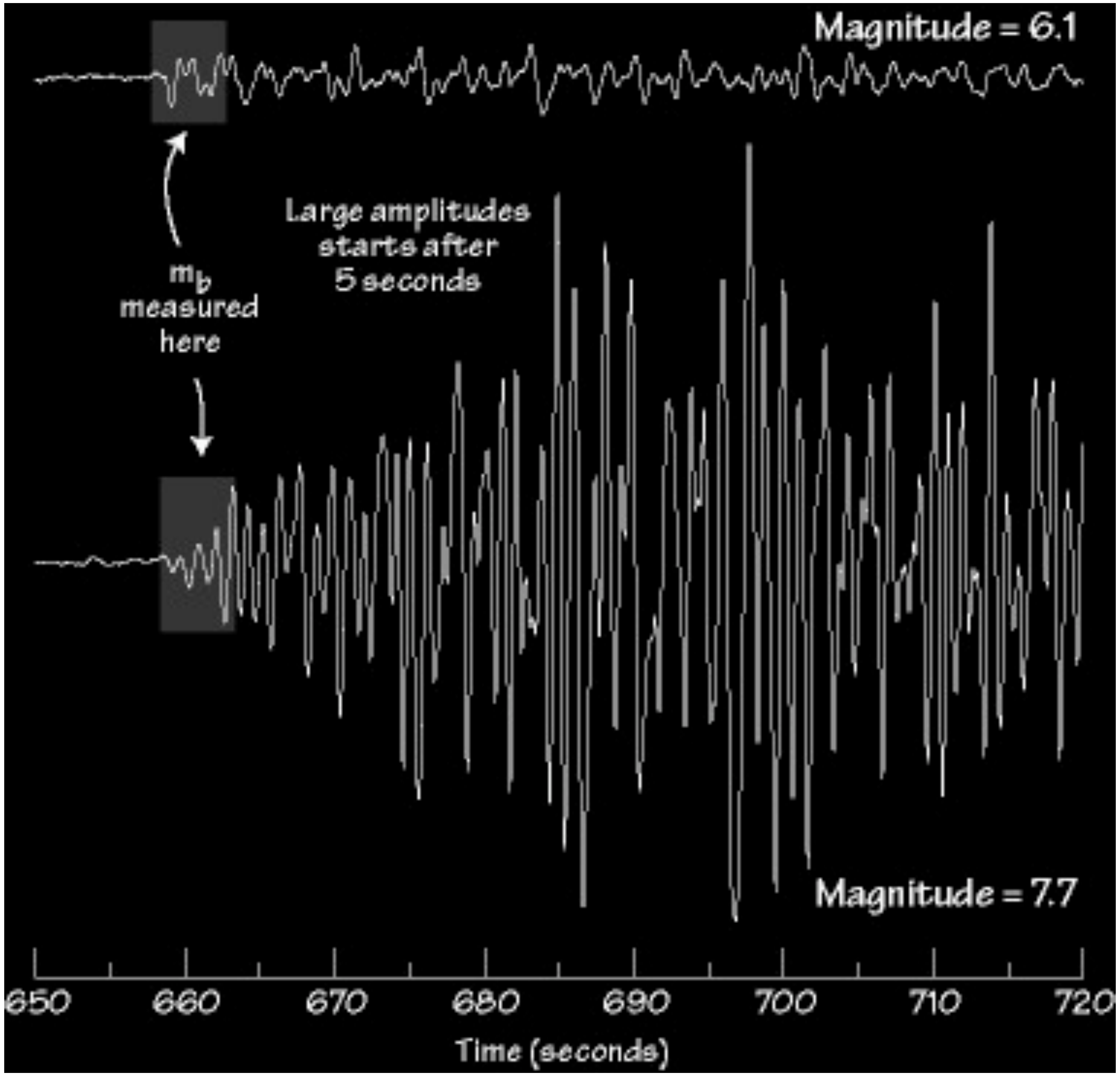
Most complicated reason:

- Magnitude scales saturate
- This means there is an upper limit to magnitude no matter how "large" the earthquake is
- For instance M_s (surface wave magnitude) never gets above 8.2-8.3

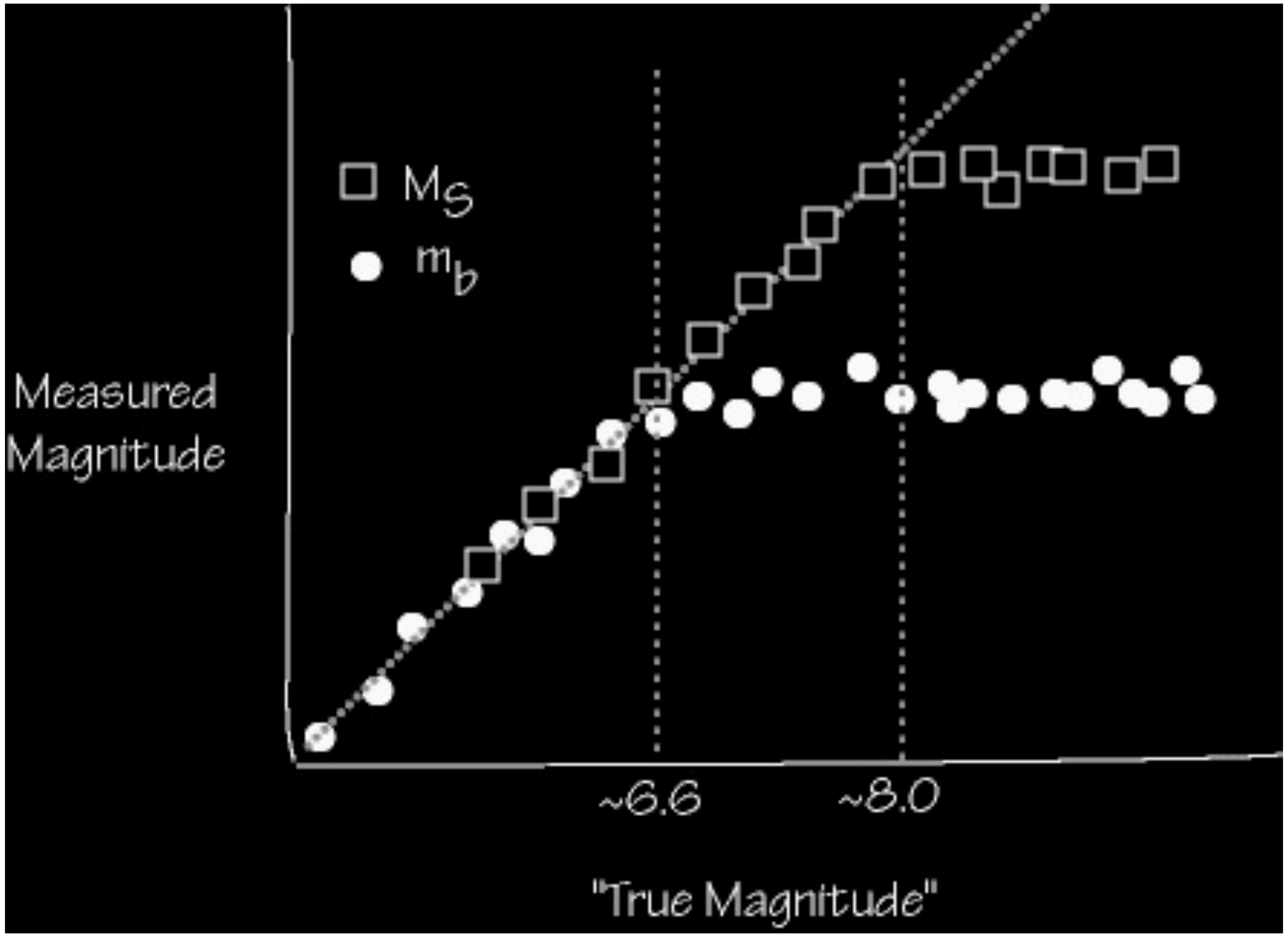
Example: mb "Saturation"

m_b seldom gives values above 6.7 - it "saturates".

m_b must be measured in the first 5 seconds - that's the rule.



Saturation

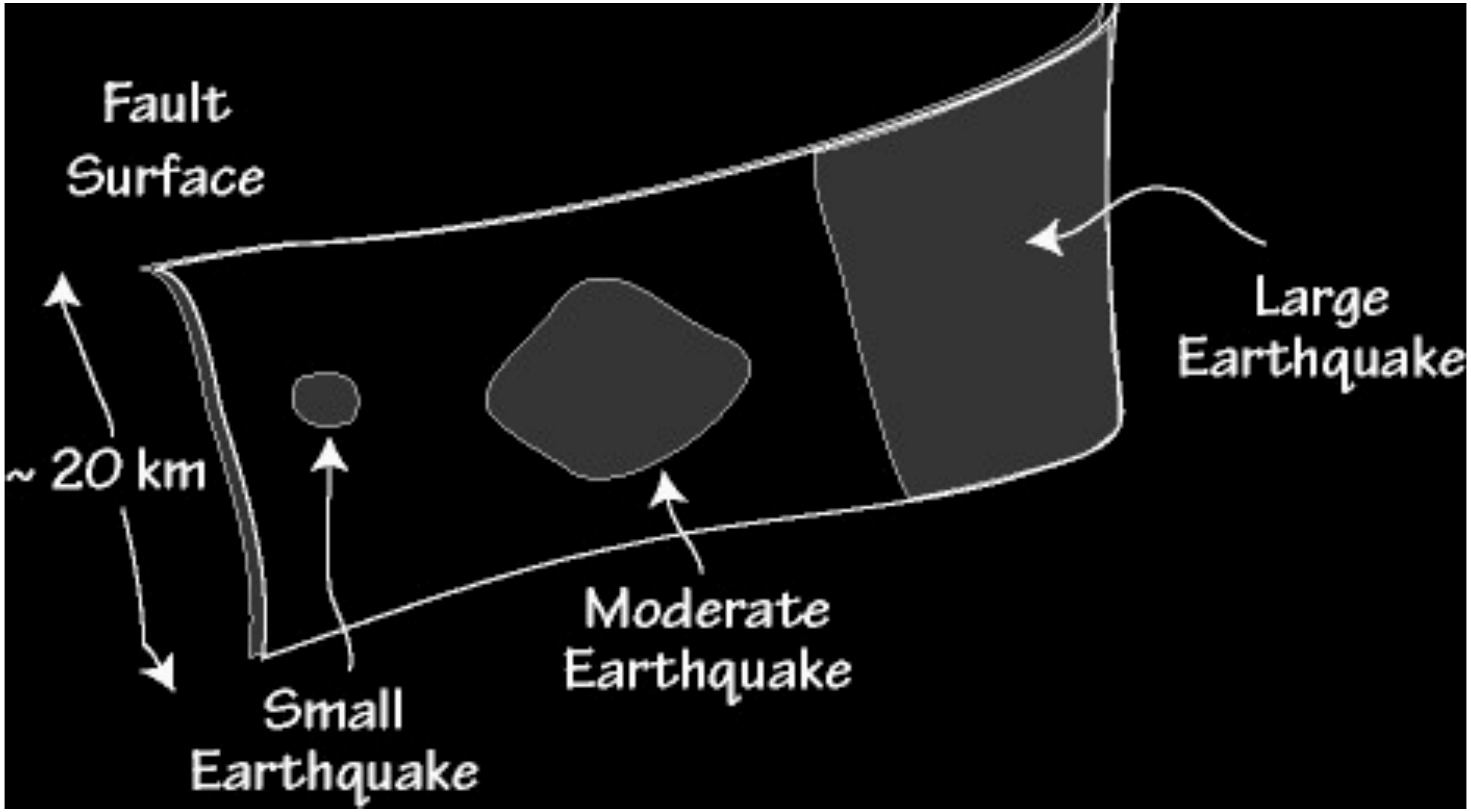


What Causes Saturation?

The rupture process.

Small earthquakes rupture small areas and are relatively enriched in short period signals.

Large earthquakes rupture large areas and are relatively depleted in high frequencies.



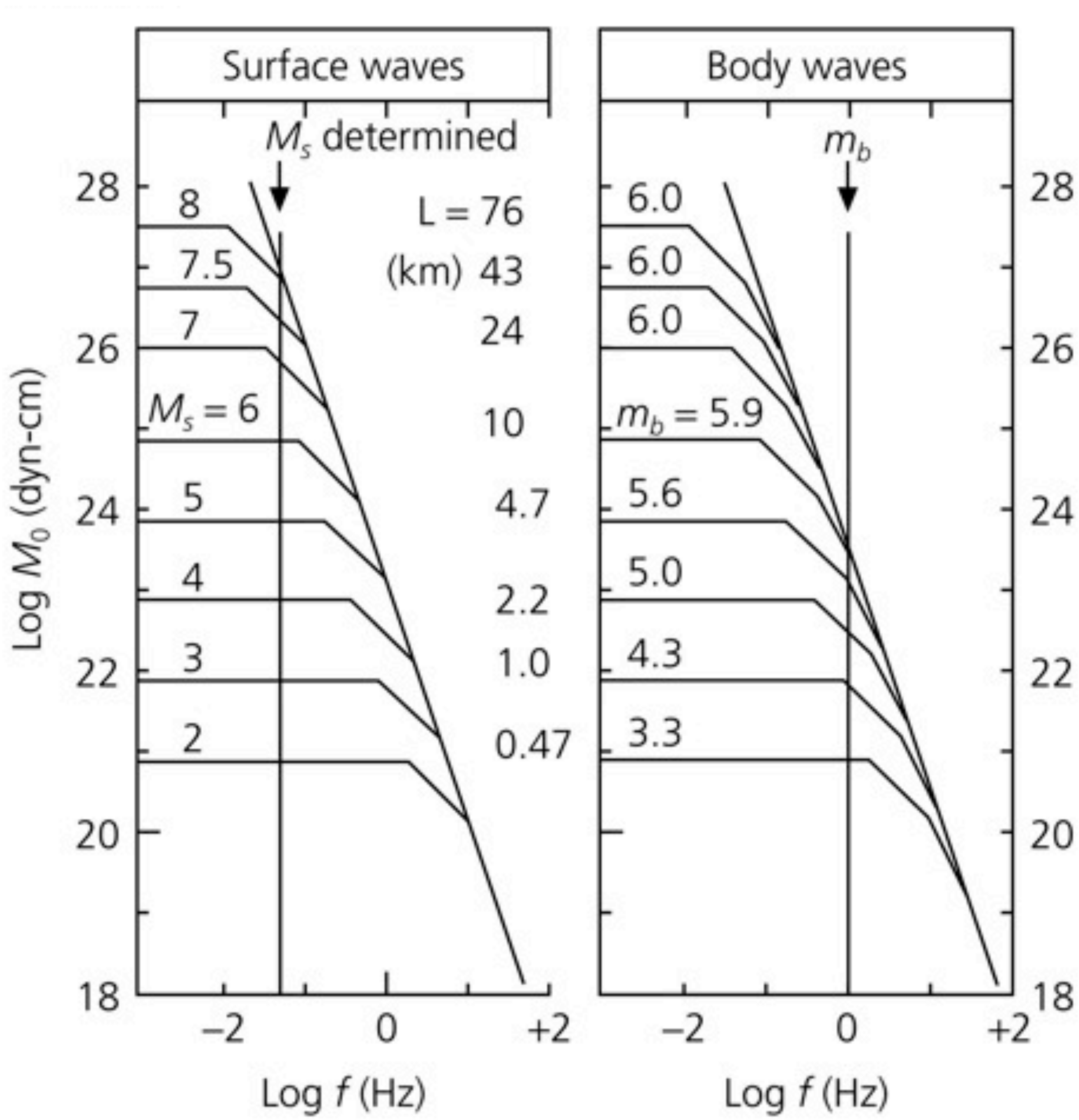
Are m_b and M_s still useful?

YES!

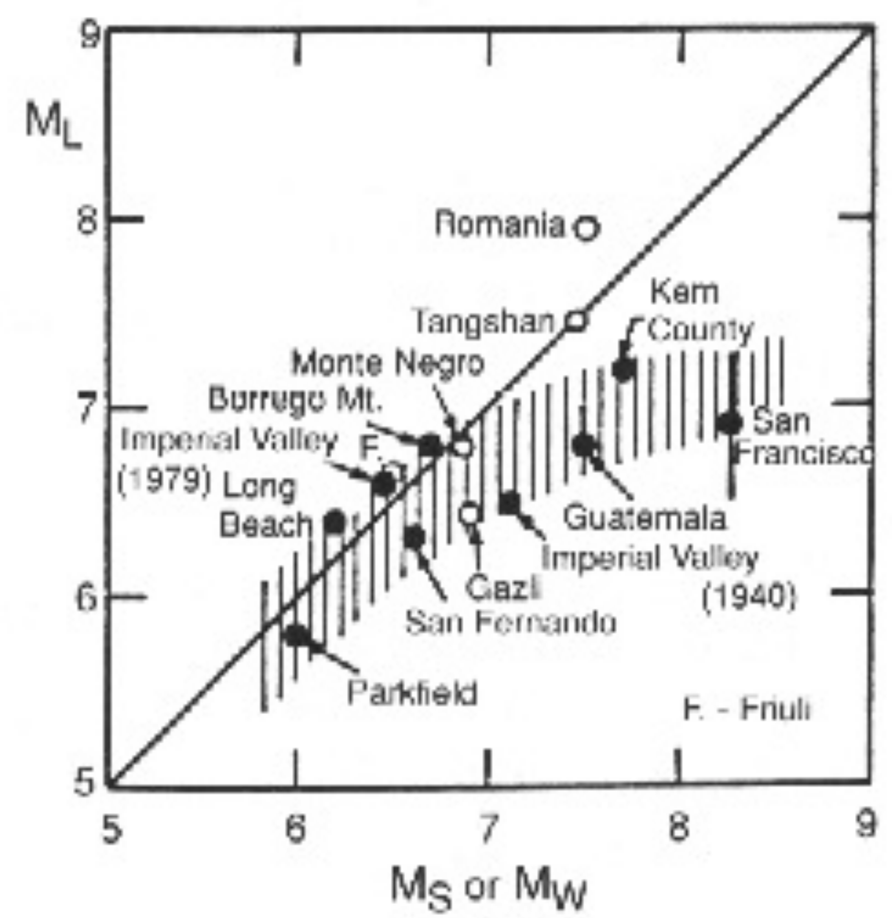
- Many (most) earthquakes are small enough that saturation does not occur
- Empirical relations between energy release and m_b and M_s exist
- The ratio of m_b to M_s can indicate whether a given seismogram is from an earthquake or a nuclear explosion (verification seismology)

Magnitude saturation

There is no a-priory scale limitation or classification of magnitudes as for macroseismic intensities. In fact, nature limits the maximum size of tectonic earthquakes which is controlled by the maximum size of a brittle fracture in the lithosphere. A simple seismic shear source with linear rupture propagation has a typical "source spectrum".



M_s is not linearly scaled with M_0 for $M_s > 6$ due to the beginning of the so-called saturation effect for spectral amplitudes with frequencies $f > f_c$. This saturation occurs already much earlier for m_b which are determined from amplitude measurements around 1 Hz.



Is there anything better?

- Yes, the seismic moment – M_o

- Invented in the 1960s to circumvent magnitude limitations
- Has physical units of energy (Nm, cal, J)
- Is the product of three factors that indicate the size of the earthquake:

M_o = (shear modulus) x (rupture area) x (slip offset)

Stress-drop and similarity conditions

Static stress-drop can be defined as the stress drop integrated over the fault area, A, divided by A; it can be expressed in terms of the **strain drop**, defining a characteristic rupture dimension:

$$\frac{\bar{D}}{\bar{\bar{L}}} \longrightarrow \Delta\sigma = C\mu\left(\frac{\bar{D}}{\bar{\bar{L}}}\right)$$

Under certain assumptions there exist several conditions of static (geometric) and dynamic similarity. With the assumption of a constant stress drop one gets:

$$\frac{W}{L} \approx c_1 \text{ (constant aspect ratio)}$$

$$\frac{\bar{D}}{\bar{\bar{L}}} \approx c_2 \text{ (constant strain)}$$

$$\longrightarrow M_0 = \mu W L \bar{D} = \frac{c_1}{C}\Delta\sigma \bar{L}^3 = \frac{c_1}{C}\Delta\sigma S^{3/2}$$

Result valid for source dimensions smaller than the thickness of the seismogenic layer. Besides this there is a dynamic similarity

$$\frac{\tau v_r}{\tilde{L}} \approx c_3 \text{ (dynamic)}$$

Scaling relations for earthquakes

Write the fault area in terms of a shape factor f and the square of a dimension L:

$$M_0 = \mu \bar{D} S = \mu \bar{D} f L^2$$

For large earthquakes, faults are often approximated as rectangles, so L is the length and f is the ratio of width to length.

For circular faults, L is the radius and $f = \pi$.

The rupture time can be approximated as $T_R = L/V_R = L/(0.7\beta)$

The rise time can be approximated as $T_D = \mu \bar{D} / (\beta \, \Delta\sigma) = 16 L f^{1/2} / (7 \beta \pi^{1.5})$

($\Delta\sigma$ is the stress drop during the earthquake)

Assuming a shear velocity of 4 km/s gives $T_R = 0.35\ L$ $T_D = 0.1\ L f^{1/2}$

Because stress drops are approximately independent of seismic moment, slip is roughly proportional to fault length, allowing for theoretical scaling laws.

Moment and source area

$$\log M_0 = \frac{3}{2}\log S + \log\left(\frac{c_1}{C}\Delta\sigma\right)$$

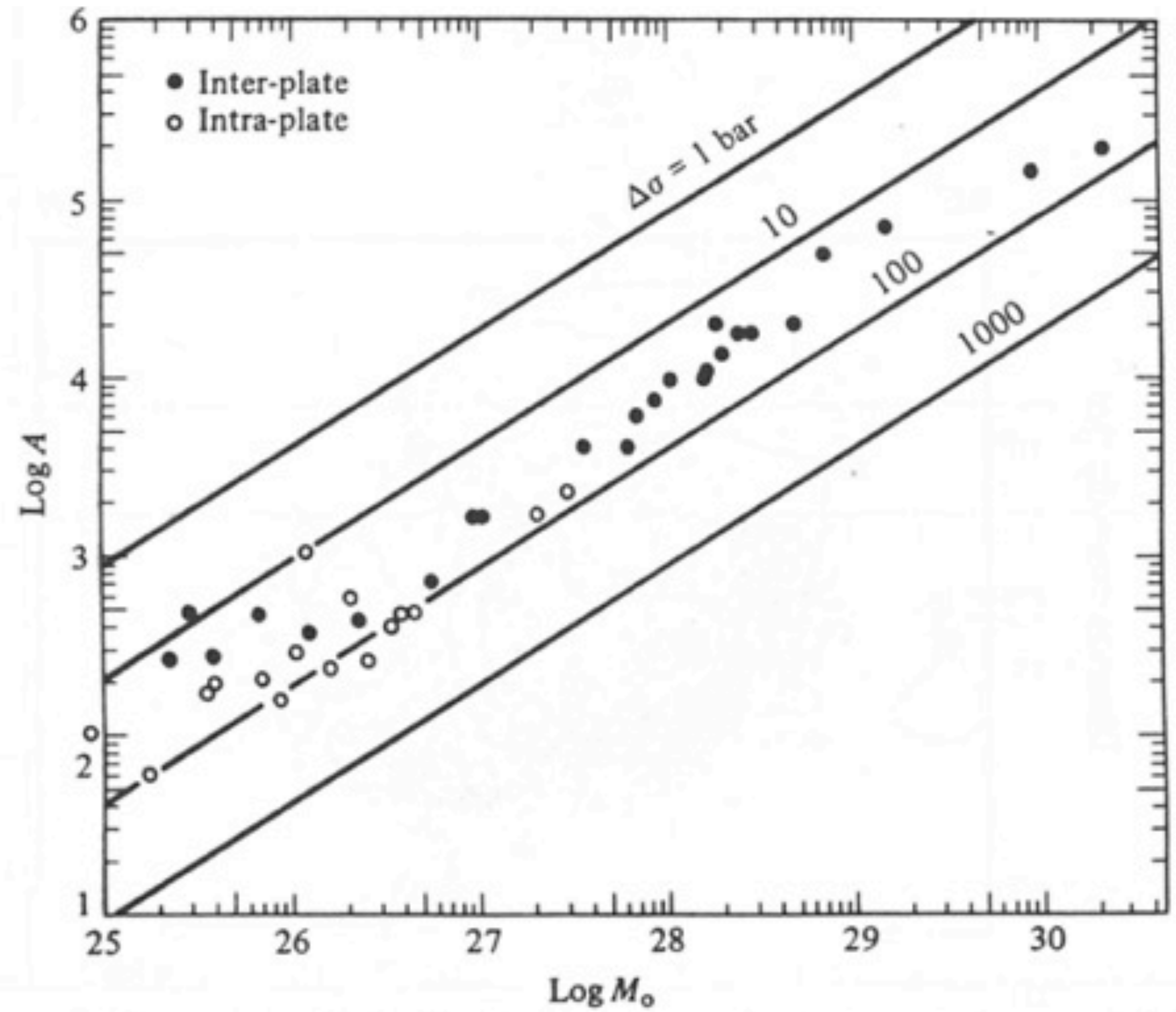


FIGURE 9.25 Area versus moment for inter- and intraplate earthquakes. Note that the interplate earthquakes show little scatter about a stress drop of 30 bars. The intraplate earthquakes have stress drops of ~100 bars. (Modified from Kanamori and Anderson, 1975.)

Magnitude and source spectra

The source spectra can be written as:

$$U(\omega) = M_0 F(\omega) = M_0 \left| \frac{\sin\left(\frac{\omega\tau}{2}\right)}{\left(\frac{\omega\tau}{2}\right)} \right| \left| \frac{\sin\left(\frac{\omega L}{v_r 2}\right)}{\left(\frac{\omega L}{v_r 2}\right)} \right| \approx \begin{cases} M_0 \propto L^3 & \tau < \frac{T}{\pi}, \frac{L}{v_r} < \frac{T}{\pi} \\ M_0 \frac{T}{\pi\tau} \propto L^2 & \tau > \frac{T}{\pi}, \frac{L}{v_r} < \frac{T}{\pi} \\ M_0 \frac{v_r T}{L\pi} \propto L^2 & \tau < \frac{T}{\pi}, \frac{L}{v_r} > \frac{T}{\pi} \\ M_0 \frac{v_r T^2}{\tau L \pi^2} \propto L & \tau > \frac{T}{\pi}, \frac{L}{v_r} > \frac{T}{\pi} \end{cases}$$

And the amplitude of a seismic phase will scale with fault dimension in several ways; if we make our observations at T=20s we can use the relations to scale M_s with L

$$M_s \approx \log\left(\frac{A}{20}\right) \approx \begin{cases} 3\log L \approx \log M_0 \approx \frac{3}{2}\log S & \tau < \frac{T}{\pi}, \frac{L}{v_r} < \frac{T}{\pi} \\ 2\log L \approx \frac{2}{3}\log M_0 \approx \log S & \tau > \frac{T}{\pi}, \frac{L}{v_r} < \frac{T}{\pi} \\ 2\log L \approx \frac{2}{3}\log M_0 \approx \log S & \tau < \frac{T}{\pi}, \frac{L}{v_r} > \frac{T}{\pi} \\ \log L \approx \frac{1}{3}\log M_0 \approx \frac{1}{2}\log S & \tau > \frac{T}{\pi}, \frac{L}{v_r} > \frac{T}{\pi} \end{cases}$$

Moment and times

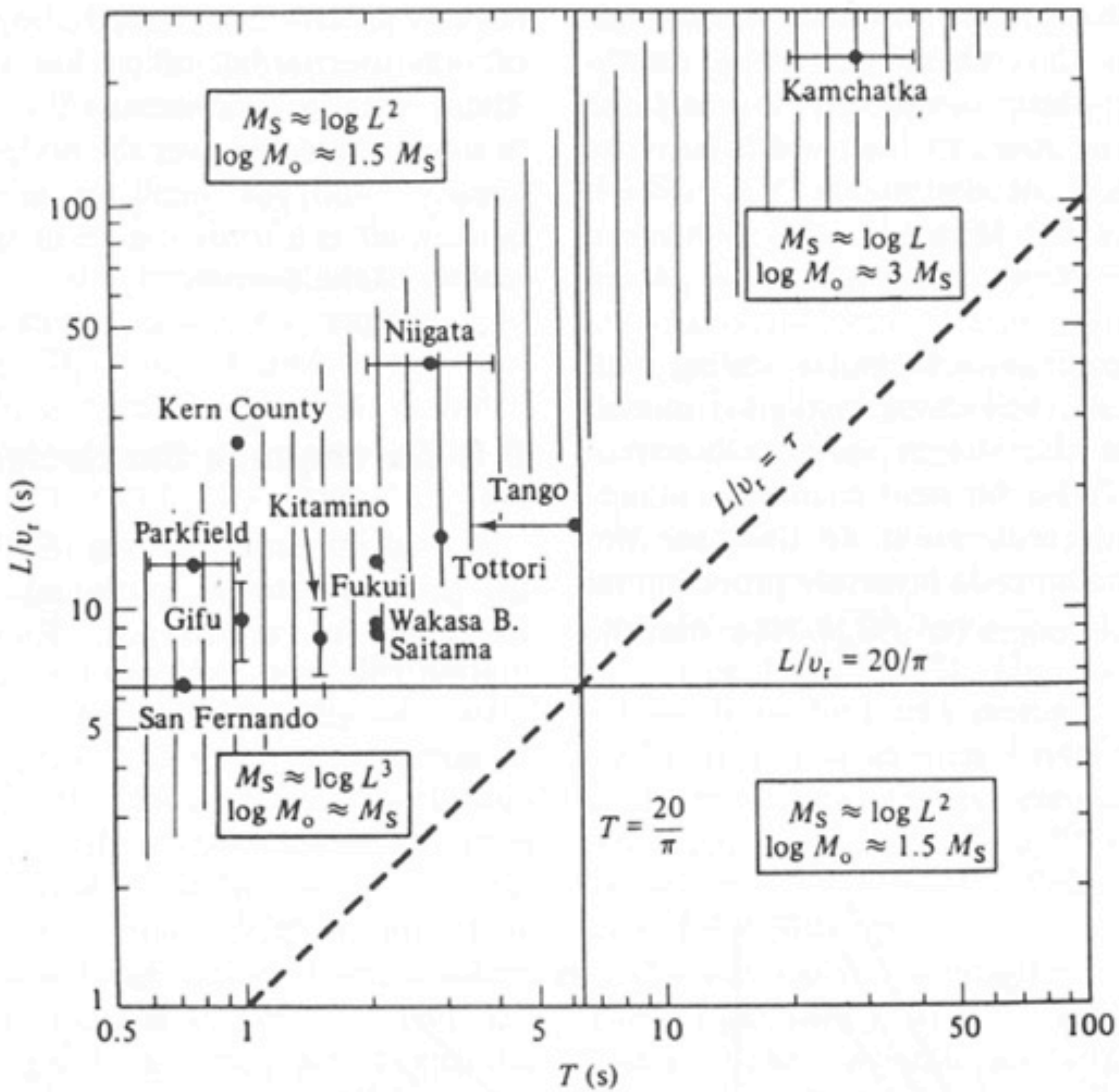


FIGURE 9.27 Scaling for τ_c, τ_r, and the seismic moment. (Modified from Kanamori and Anderson, 1975.)

Moment magnitude

Empirical studies (Gutenberg & Richter, 1956; Kanamori & Anderson, 1975) lead to a formula for the released seismic energy (in Joule), and for moment, with magnitude:

$$\log E = 4.8 + 1.5 M_s \quad \log M_0 = 9.1 + 1.5 M_s$$

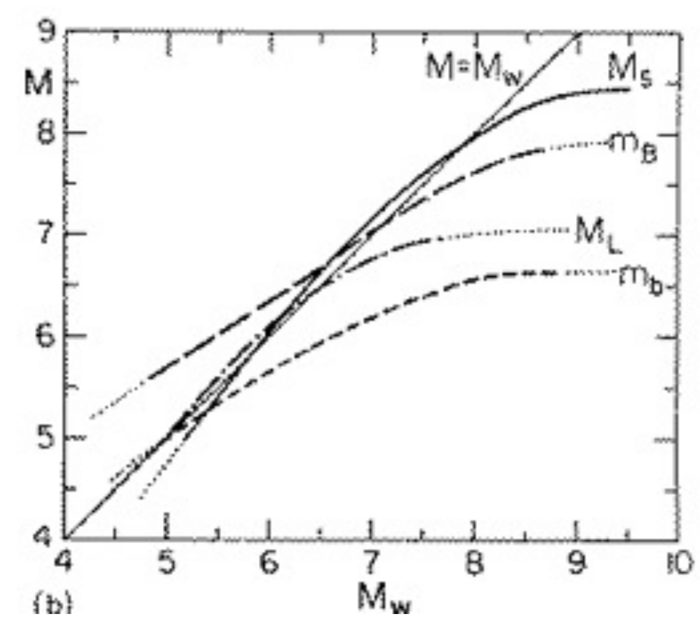
resulting in

$$M_w = 2/3 \log M_0 - 6.07$$

when the Moment is measured in N·m (otherwise the intercept becomes 10.73); it is related to the final static displacement after an earthquake and consequently to the tectonic effects of an earthquake.

$$u(x,t) = A\cos\left(\frac{2\pi t}{T}\right) \Rightarrow v(x,t) \propto \frac{A}{T} u$$

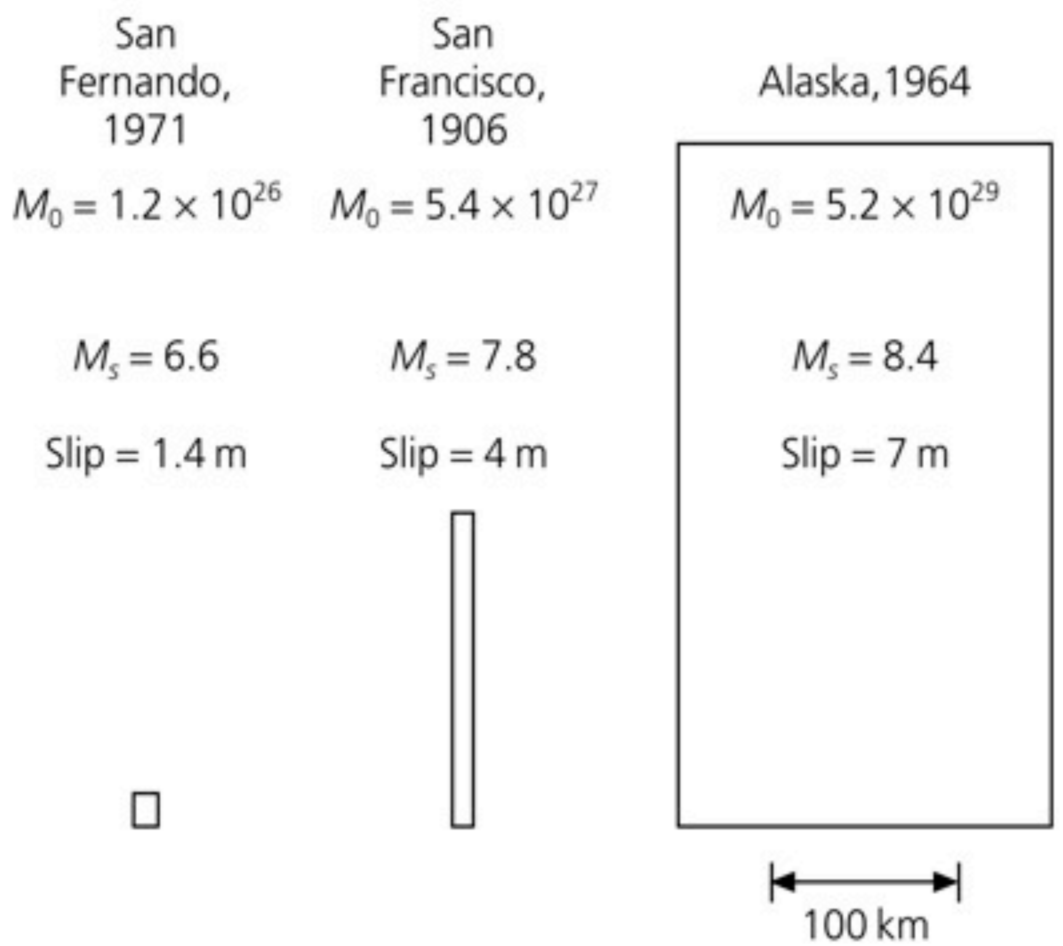
$$\Rightarrow e \propto v^2 \propto \left(\frac{A}{T}\right)^2 \Rightarrow \log E = C + 2\log\left(\frac{A}{T}\right)$$

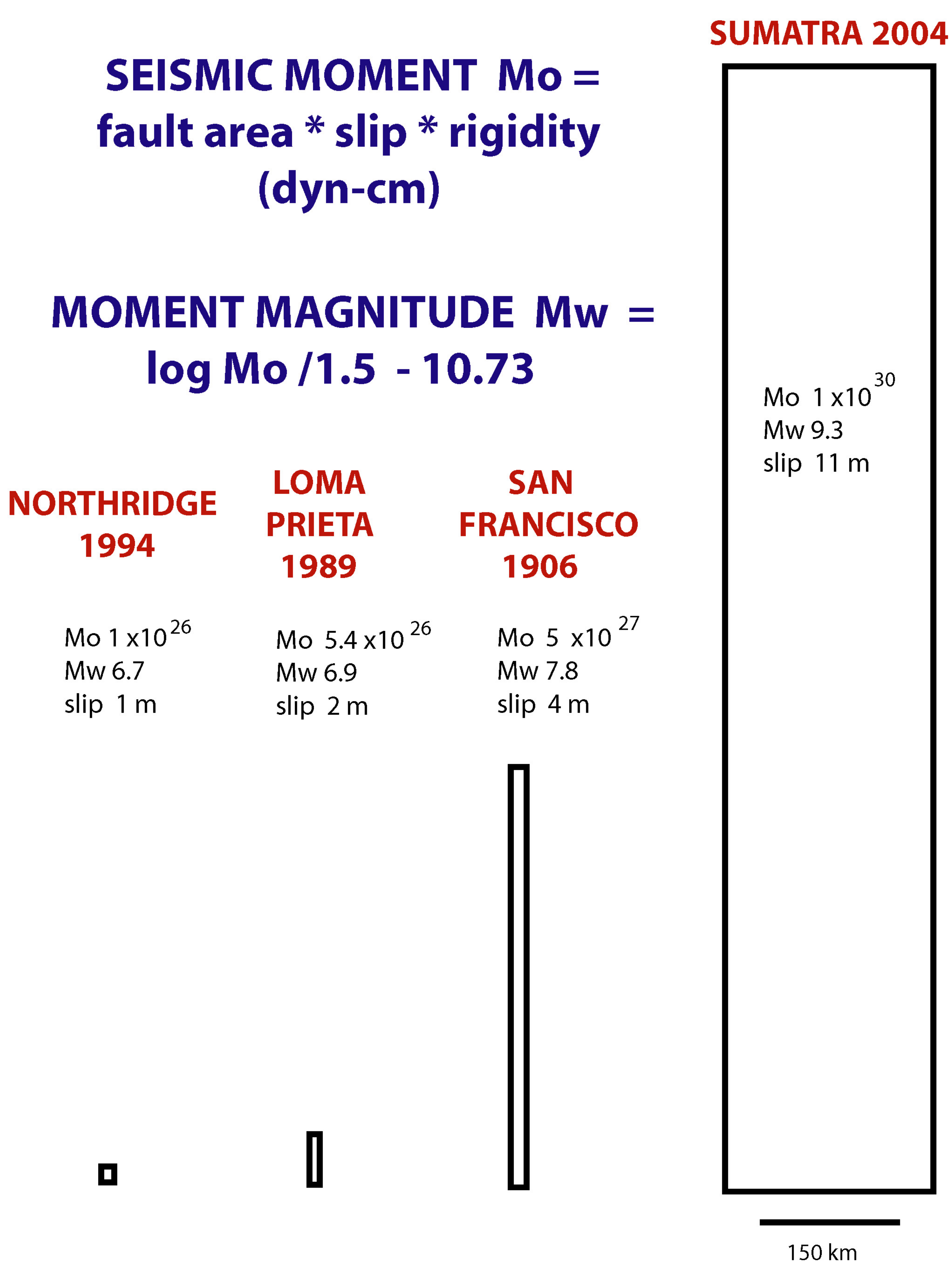


Earthquake	Body wave magnitude m_b	Surface wave magnitude M_s	Fault area (km^2) length × width	Average dislocation (m)	Moment (dyn-cm) M_0	Moment magnitude M_w
Truckee, 1966	5.4	5.9	10 × 10	0.3	8.3×10^{24}	5.8
San Fernando, 1971	6.2	6.6	20 × 14	1.4	1.2×10^{26}	6.7
Loma Prieta, 1989	6.2	7.1	40 × 15	1.7	3.0×10^{26}	6.9
San Francisco, 1906		8.2	320 × 15	4	6.0×10^{27}	7.8
Alaska, 1964	6.2	8.4	500 × 300	7	5.2×10^{29}	9.1
Chile, 1960		8.3	800 × 200	21	2.4×10^{30}	9.5

SEISMIC MOMENT Mo = fault area * slip * rigidity (dyn-cm)

MOMENT MAGNITUDE Mw = log Mo /1.5 - 10.73





The Largest Earthquakes

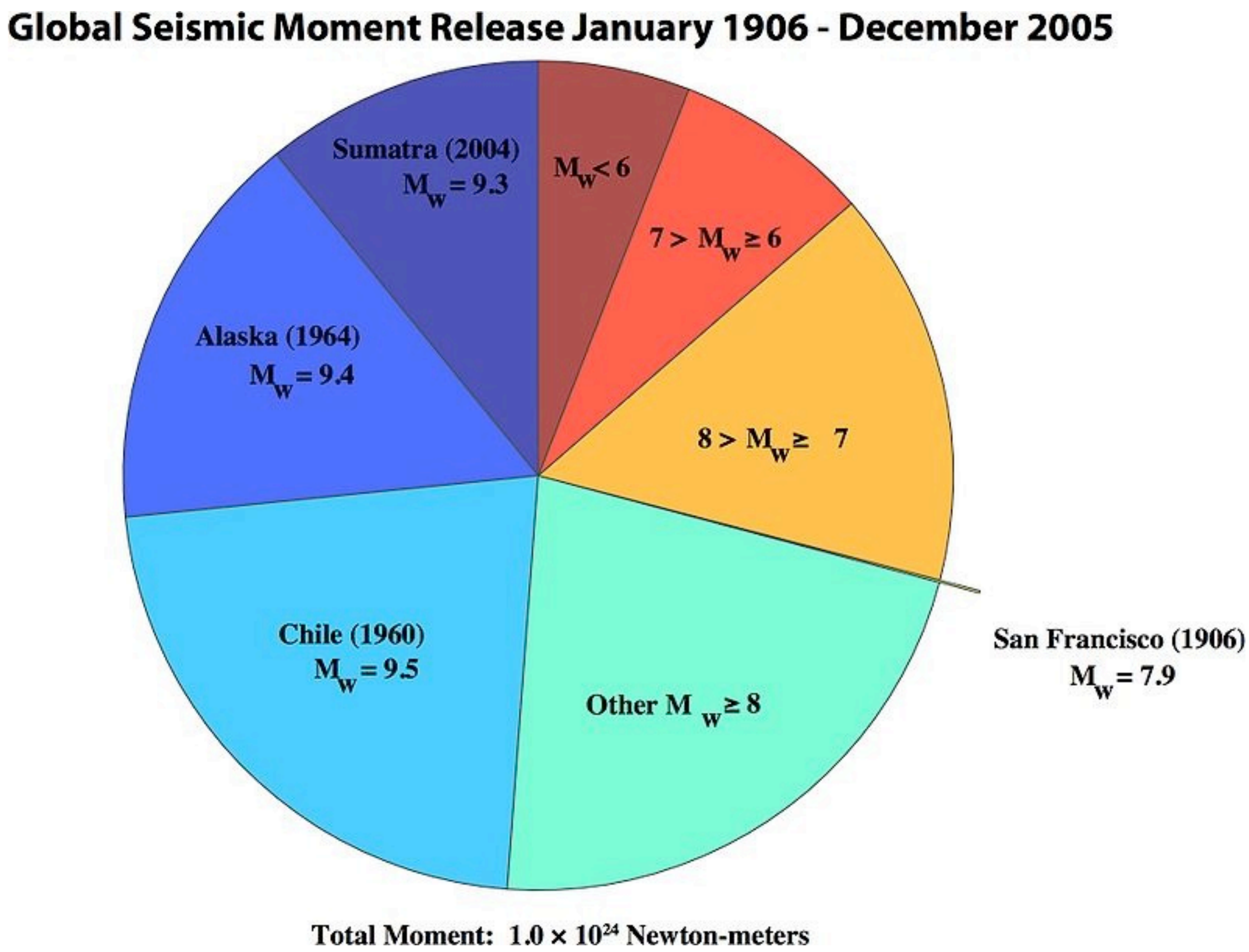
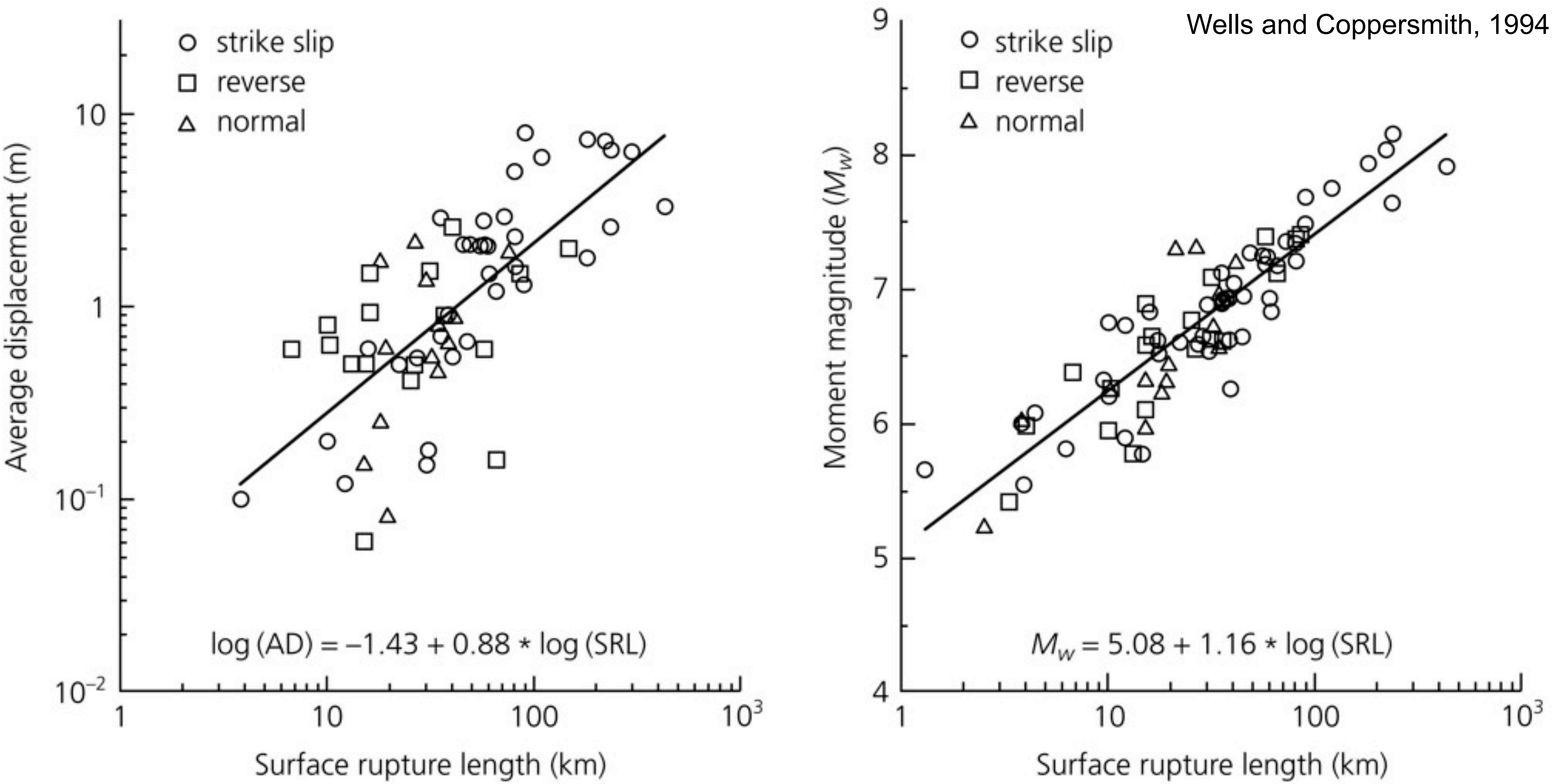


Figure 4.6-7: Empirical relations between slip, fault length, and moment.



M7, ~ 100 km long, 1 m slip; M6, ~ 10 km long, ~ 20 cm slip

Important for tectonics, earthquake source physics, hazard estimation

Wells D.L. e **Coppersmith** K.J.; **1994**: New empirical relationships among magnitude, rupture length, rupture width, rupture area, and surface displacement. Bull. Seism. Soc. Am., **84**, 974-1002.

Seismic efficiency

This discussion leads naturally to the question of how the seismic wave energy radiated by an earthquake is related to its moment and magnitude. To address this, recall that work equals force times distance, so the strain energy released is the product of the average stress during faulting, $\bar{\sigma}$, the average slip, and the fault area,

$$W = \bar{\sigma}\bar{D}S. \qquad (25)$$

If the stresses before and after faulting are σ_0 and σ_1, then $\Delta\sigma = \sigma_0 - \sigma_1$ and $\bar{\sigma} = \sigma_1 + (\Delta\sigma)/2$. Some of this energy, H, is lost to friction, so the radiated seismic energy is

$$E = W - H = \bar{\sigma}\bar{D}S - \sigma_f\bar{D}S. \qquad (26)$$

where σ_f is the frictional stress, or

$$E = (\Delta\sigma\ /\ 2)\ \bar{D}S + (\sigma_1 - \sigma_f)\bar{D}S = E_0 + (\sigma_1 - \sigma_f)\bar{D}S. \qquad (27)$$

Thus the quantity

$$E_0 = (\Delta\sigma\ /\ 2)\ \bar{D}S = (\Delta\sigma\ /\ 2\mu)\ M_0 \qquad (28)$$

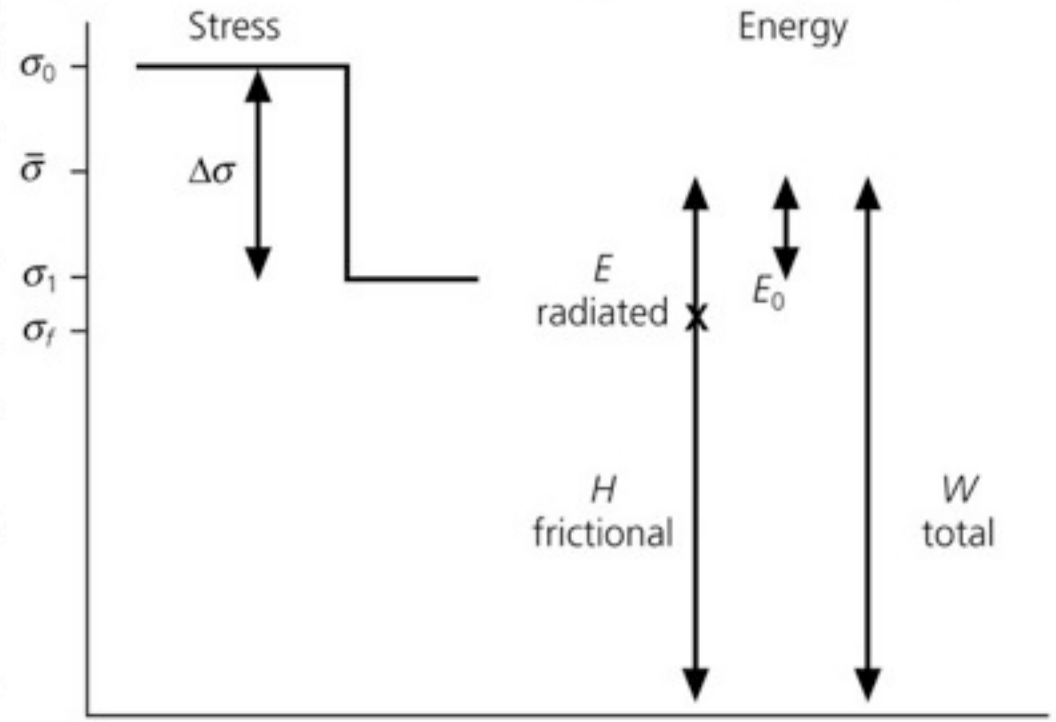
is a lower bound on the radiated seismic energy. If faulting stops once the final stress equals the frictional stress, $\sigma_1 = \sigma_f$, then $E_0 = E$ is the radiated energy. Note that the radiated energy is proportional t o the stress drop.

The ratio of the radiated energy to the total strain energy release is called the *seismic efficiency*

$$\eta = E/W = \Delta\sigma\ /\ (2\bar{\sigma}), \qquad (29)$$

where the last form assumes that $E_0 = E$. The efficiency depends on the final stress or equivalently the ratio of stress drop to the average stress. The case $\Delta\sigma \ll \bar{\sigma}$ is called partial stress drop, whereas $\Delta\sigma \approx \bar{\sigma}$ corresponds to near-total stress drop. It is still unresolved which of these cases is appropriate for earthquakes, because of all the parameters in this model, only the stress drop can be directly estimated from seismological data.

Figure 4.6-14: Cartoon of stress and energy release during an earthquake.



Energy and magnitude

The energy radiated seismically from an earthquake is a small portion of the total energy associated with the rupture:

$$E_0 = (\Delta\sigma / 2)\ \bar{D}S = (\Delta\sigma / 2\mu)\ M_0$$

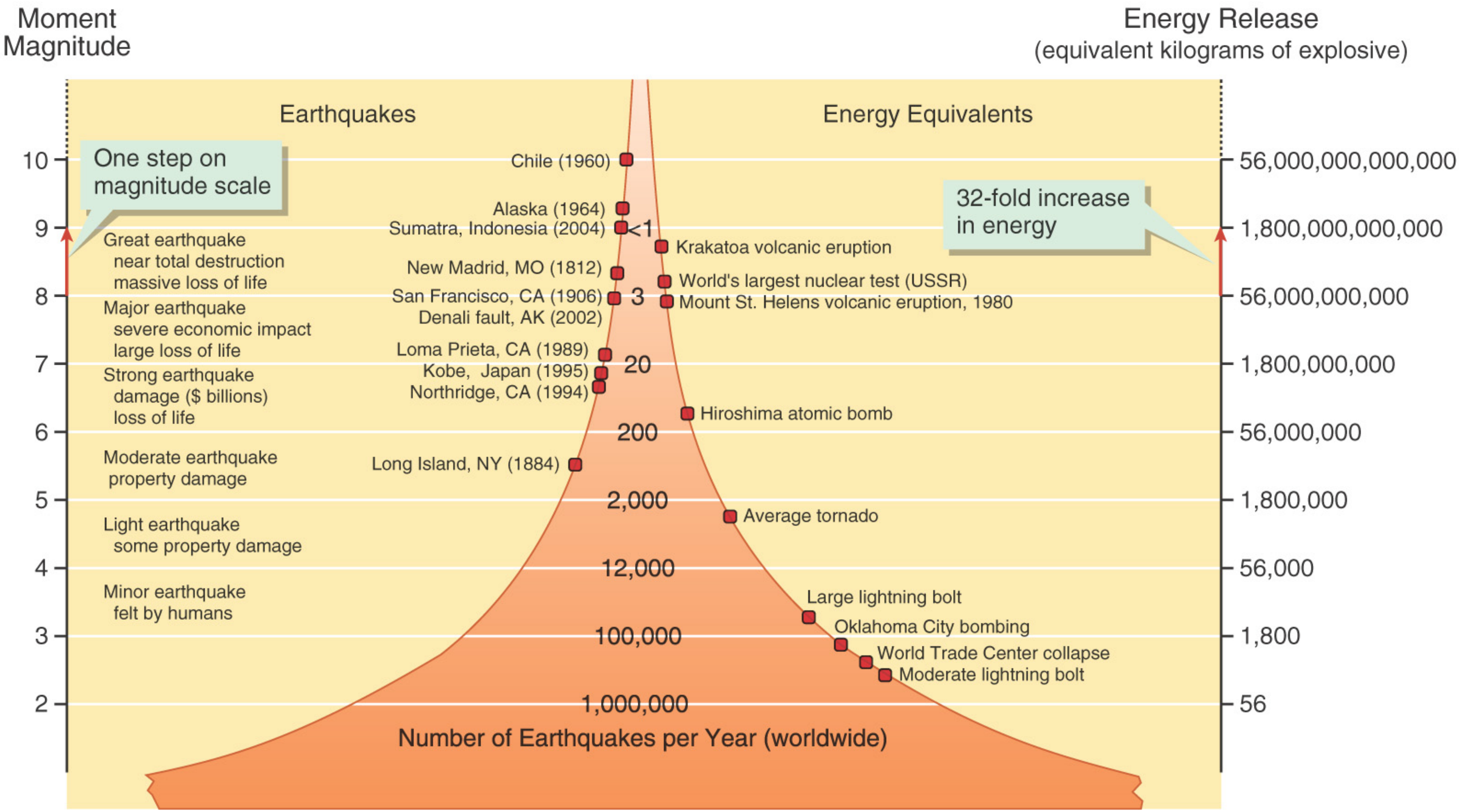
Assuming a stress drop of 50 bars and $\mu = 5 \times 10^{11}$ dyn/cm^2,

$$E_0 = M_0 / 2 \times 10^4 \qquad \log E = \log M_0 - 4.3$$

The radiated energy is only $1 / 2 \times 10^4$ or 0.00005 of the seismic moment released.

$$\log M_0 = 1.5M_w + 16.1 \qquad \log E = 1.5M_w + 11.8$$

An increase in earthquake magnitude of one unit, for example from 5 to 6, increases the radiated energy by a factor of $10^{1.5}$ or about 32. So a magnitude 7 earthquake releases 10^3, or, 1000 times more energy than a magnitude 5 event. This ratio is strictly only valid for earthquakes with the same stress drop, but is a good general approximation.



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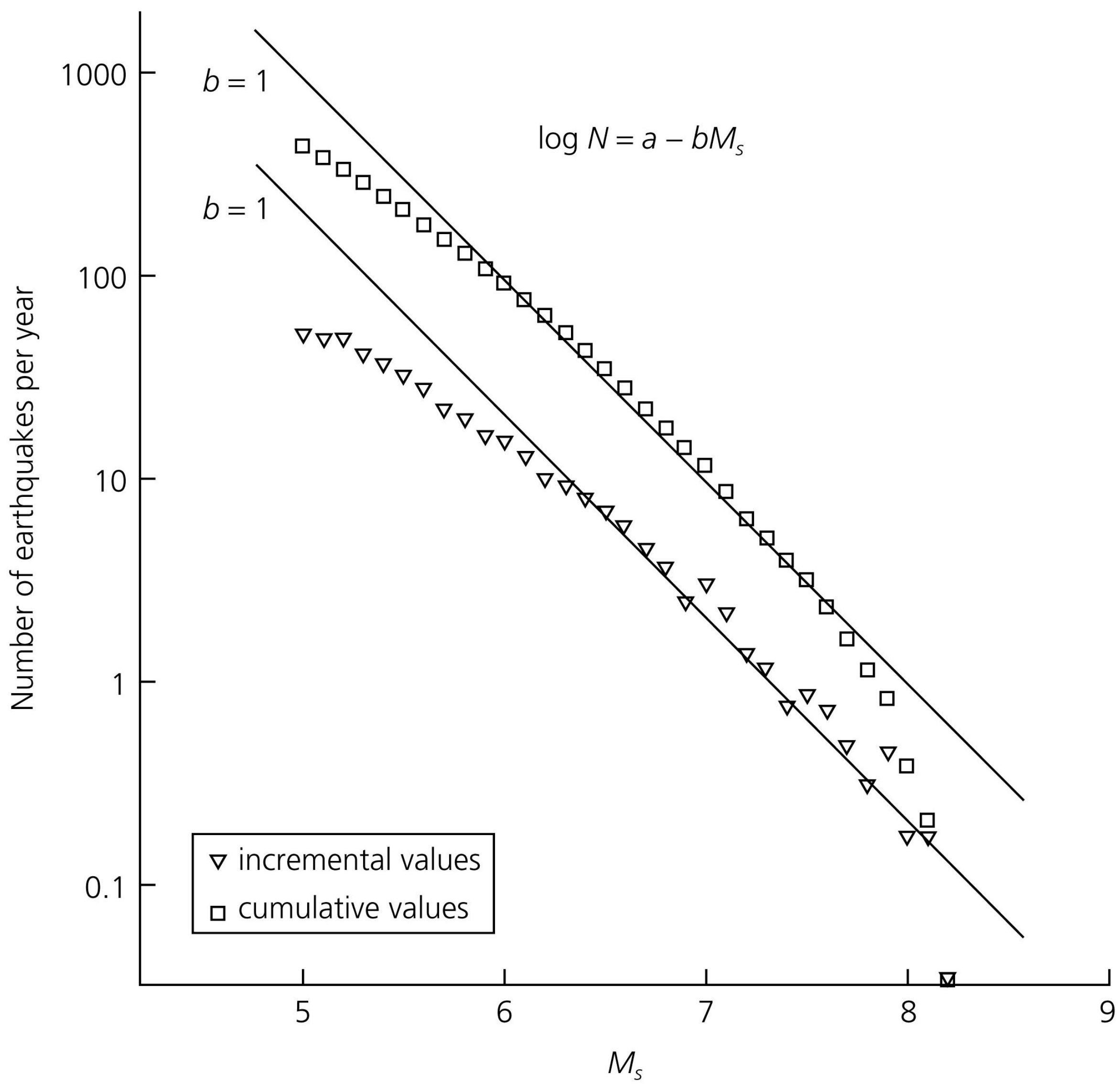
Gutenberg-Richter law

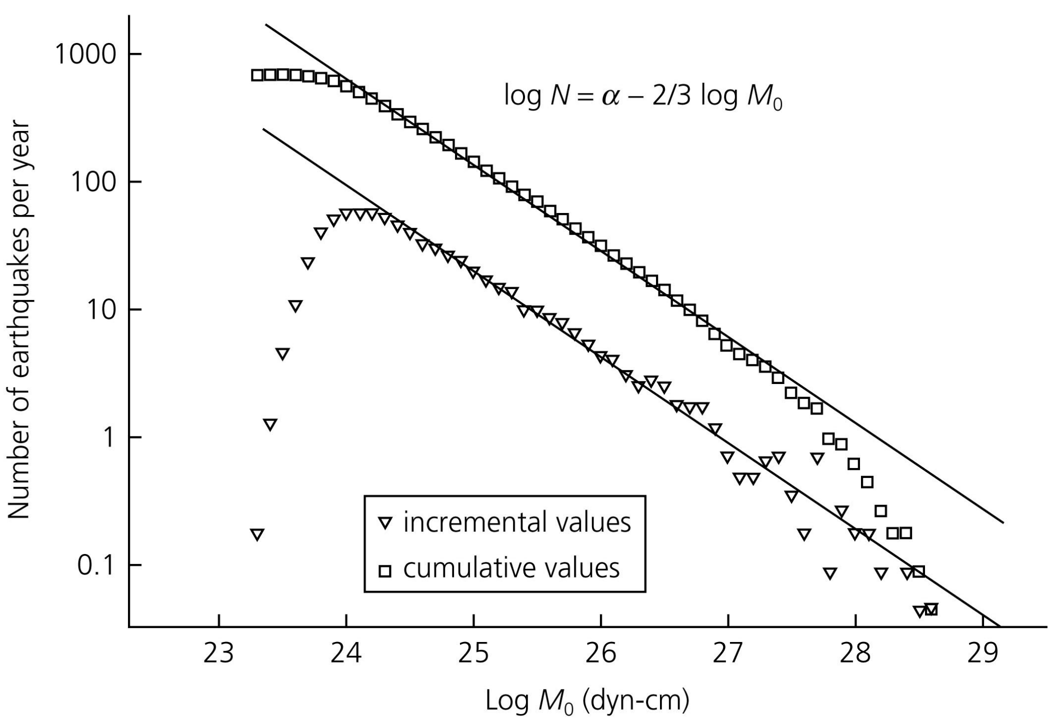
They proposed that in a given region and for a given period of time, the frequency of occurrence can be represented by: **$\log N = A - bM_s$**, where N is the number of earthquakes with magnitudes in a fixed range around M_s. It can be written also as a power-law for moment, distribution that arises from the self-similarity of earthquakes. While the a-value is a measure of earthquake productivity, the b-value is indicative of the ratio between large and small quakes. Both a and b are, therefore, important parameters in hazard analysis. Usually b is close to a unity.

Figure 4.7-1: Frequency-magnitude plot for earthquakes during 1968-1997.



$$N = \frac{10^{A}}{\left(10^{M_s}\right)^{b}} = \frac{C}{\left(M_0\right)^{2b/3}} = CM_0^{-2b/3} \approx CM_0^{-2/3}$$

Figure 4.7-2: Frequency-moment plot for earthquakes during 1976-1998.

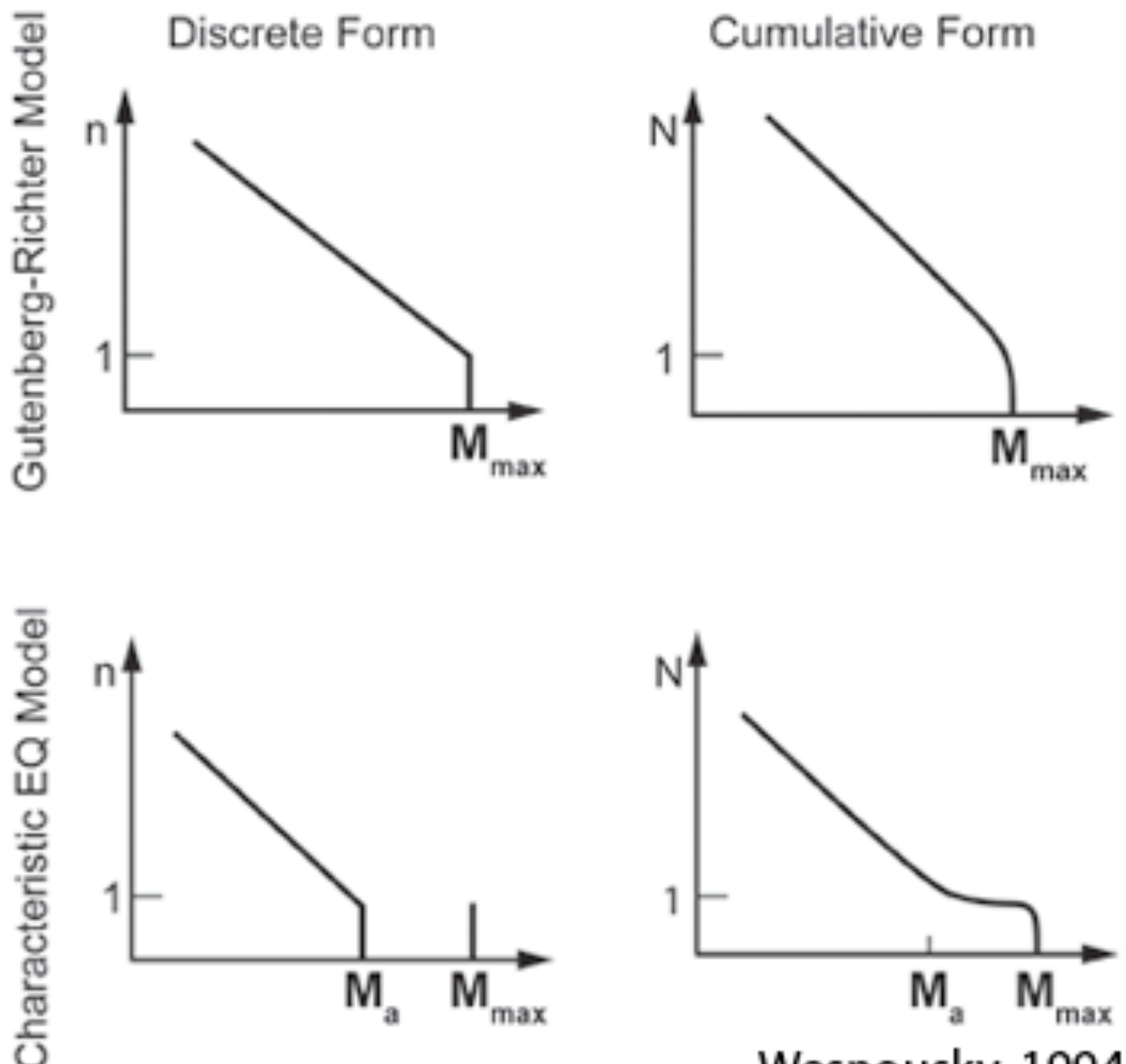


GR & Characteristic

Two end-member models can explain the G-R statistics:

• Each fault exhibits its own G-R distribution of earthquakes.

• There is a power-law distribution of fault lengths, with each fault exhibiting a characteristic distribution.



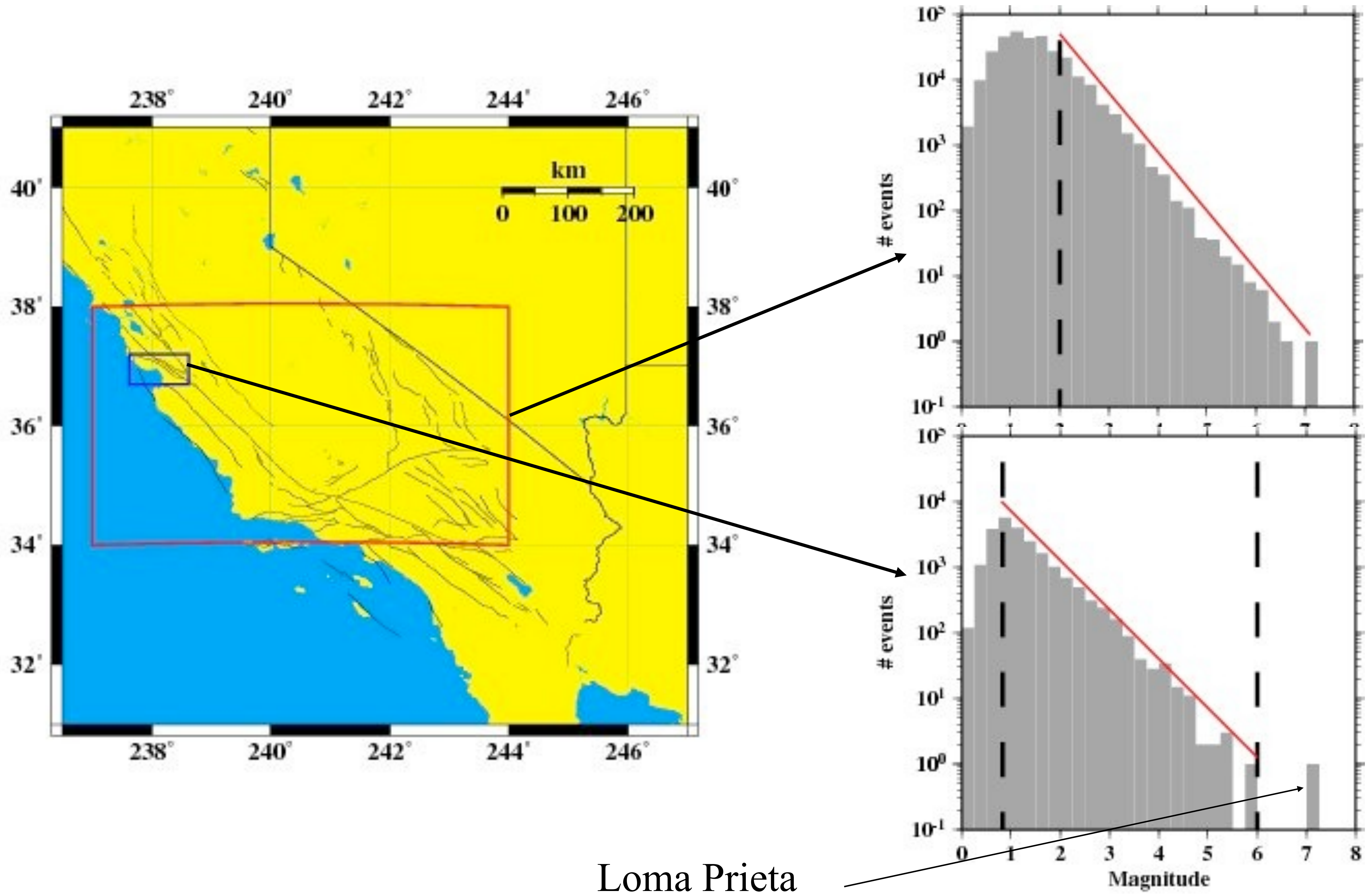
Wesnousky, 1994

• For a statistically meaningful population of faults, the distribution is often consistent with the G-R relation.

• For a single fault, on the other hand, the size distribution is often characteristic.

• Note that the extrapolation of the b-value inferred for small earthquakes may result in under-estimation of the actua hazard, if earthquake size-distribution is characteristic rather than power-law.

GR and scales



Loma Prieta

Magnitude summary

- Magnitude is a measure of ground shaking amplitude.
- More than one magnitude scales are used to study earthquakes.
- All magnitude scales have the same logarithmic form.
- Since different scales use different waves and different period vibrations, they do not always give the same value.

Magnitude	Symbol	Wave	Period
Local (Richter)	M_L	S or Surface Wave*	0.8 s
Body-Wave	m_b	P	1 s
Surface-Wave	M_s	Rayleigh	20 s
Moment	M_w	Rupture Area, Slip	100's-1000's