Seismic source

- **Green** function, G_{ik}
- **Representation** theorem
- shear **dislocation**

$$u_i(\mathbf{x},t) = \iint_\Sigma [u_n]\nu_j c_{njkq} * \frac{\partial G_{ik}}{\partial \xi_q} d\Sigma$$

- **equivalent** body force

$$u_i(\mathbf{x},t) = \iint_\Sigma m_{kq} * G_{ik,q} d\Sigma$$

- **point** source

$$\boxed{u_i(\mathbf{x},t) = M_{kq} * G_{ik,q}}$$

- **double couple**

$$M_{kq} = \mu A\left(\nu_k[u_q] + \nu_q[u_k]\right)$$

1D problem

- vertically heterogeneous half-space

$$\rho\ddot{\mathbf{u}} = (\lambda+\mu)\nabla(\nabla\cdot\mathbf{u}) + \mu\nabla^2\mathbf{u} +$$

$$+\frac{\partial\lambda}{\partial z}(\hat{\mathbf{z}}\nabla\cdot\mathbf{u}) + \frac{\partial\mu}{\partial z}\left[\hat{\mathbf{z}}\times(\nabla\times\mathbf{u}) + \nabla(\hat{\mathbf{z}}\cdot\mathbf{u})\right]$$

- solution: $\mathbf{u}(\mathbf{x},t) = \mathbf{F}(z)e^{i(\omega t - kx)}$

- **P-SV** Problem **SH** Problem

$$\frac{\partial}{\partial z}\left(\mu\frac{\partial F_y}{\partial z}\right) + \left(\omega^2\rho - k^2\mu\right)F_y = 0$$

$$\left.\left(\mu\frac{\partial F_y}{\partial z}\right)\right|_{z=0} = 0$$

$$\lim_{z\to\infty} F_y = 0$$

•GF for the oscillation **Modes**

$$G_{ik}^{L} = \sum_{m=1}^{\infty} G_{ik}^{mL} \qquad G_{ik}^{R} = \sum_{m=1}^{\infty} G_{ik}^{mR}$$

$$G_{ik}^{mL}(\mathbf{x}_0;\mathbf{x},\omega) = \frac{e^{i\pi/4}}{\sqrt{8\pi}} \frac{e^{-ik_m x}}{\sqrt{k_m x}} \frac{F_y^m(h_s,\omega)}{\sqrt{c_m v_m I_m}} \frac{F_y^m(z,\omega)}{\sqrt{c_m v_m I_m}} \begin{pmatrix} \sin^2\phi & -\sin\phi\cos\phi & 0 \\ -\sin\phi\cos\phi & \cos^2\phi & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$G_{ik}^{mR}(\mathbf{x}_0;\mathbf{x},\omega) = \frac{e^{i\pi/4}}{\sqrt{8\pi}} \frac{e^{-ik_m x}}{\sqrt{k_m x}} \frac{1}{\sqrt{c_m v_m I_m}} \frac{1}{\sqrt{c_m v_m I_m}}$$

$$\begin{pmatrix} F_x^m(h_s)F_x^m(z)\cos^2\phi & F_x^m(h_s)F_x^m(z)\sin\phi\cos\phi & -iF_z^m(h_s)F_x^m(z)\cos\phi \\ F_x^m(h_s)F_x^m(z)\sin\phi\cos\phi & F_x^m(h_s)F_x^m(z)\sin^2\phi & -iF_z^m(h_s)F_x^m(z)\sin\phi \\ iF_x^m(h_s)F_z^m(z)\cos\phi & iF_x^m(h_s)F_z^m(z)\sin\phi & F_z^m(h_s)F_z^m(z) \end{pmatrix}$$

$\mathbf{x}_0=(0,0,h_s)$; $\mathbf{x}=(x,0,z)$;
m is the mode index, $c_m(\omega)$ is the phase velocity, $v_m(\omega)$ is the group velocity, $F_y^m(\omega)$ is the eigenfunction, $I_m(\omega)$ is the energy integral

•Synthetic seismogram

$$u_x^{mR}(\omega) = \frac{e^{-i\pi/4}}{\sqrt{8\pi}} \frac{e^{-ik_m x} e^{-\omega x C_{2R}^m}}{\sqrt{k_m x}} \frac{S(\omega)\chi_R^m(h_s,\varphi,\omega)}{\sqrt{c_m v_m I_m}} \frac{F_x^m(z,\omega)}{\sqrt{c_m v_m I_m}}$$

$$u_y^{mL}(\omega) = \frac{e^{-i\pi/4}}{\sqrt{8\pi}} \frac{e^{-ik_m x} e^{-\omega x C_{2L}^m}}{\sqrt{k_m x}} \frac{S(\omega)\chi_L^m(h_s,\varphi,\omega)}{\sqrt{c_m v_m I_m}} \frac{F_y^m(z,\omega)}{\sqrt{c_m v_m I_m}}$$

$$u_z^{mR}(\omega) = \frac{e^{-i3\pi/4}}{\sqrt{8\pi}} \frac{e^{-ik_m x} e^{-\omega x C_{2R}^m}}{\sqrt{k_m x}} \frac{S(\omega)\chi_R^m(h_s,\varphi,\omega)}{\sqrt{c_m v_m I_m}} \frac{F_z^m(z,\omega)}{\sqrt{c_m v_m I_m}}$$

$$\chi_L^m(h_s,\varphi,\omega) = i(d_{1L}^m \sin\varphi + d_{2L}^m \cos\varphi) + d_{3L}^m \sin 2\varphi + d_{4L}^m \cos 2\varphi$$

$$d_{1L}^m = G(h_s,\omega)\cos\lambda \sin\delta$$

$$d_{2L}^m = -G(h_s,\omega)\sin\lambda \cos 2\delta$$

$$d_{3L}^m = \frac{1}{2} V(h_s,\omega)\sin\lambda \sin 2\delta$$

$$d_{4L}^m = V(h_s,\omega)\cos\lambda \sin\delta$$

$$\chi_R(h_s,\varphi,\omega) = d_0 + i(d_{1R} \sin\varphi + d_{2R} \cos\varphi) + d_{3R} \sin 2\varphi + d_{4R} \cos 2\varphi$$

$$d_0 = \frac{1}{2} B(h_s)\sin\lambda \sin 2\delta$$

$$d_{1R} = -C(h_s)\sin\lambda \cos 2\delta$$

$$d_{2R} = -C(h_s)\cos\lambda \cos\delta$$

$$d_{3R} = A(h_s)\cos\lambda \sin\delta$$

$$d_{4R} = -\frac{1}{2} A(h_s)\sin\lambda \sin 2\delta$$

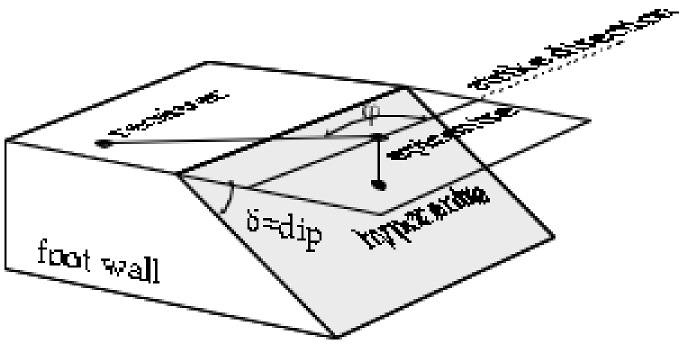
$$A(h_s) = -\frac{F_x{}^*(h_s)}{F_z(0)}$$

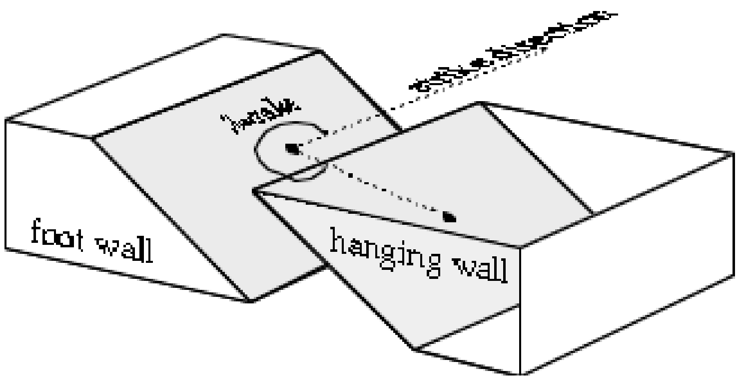
$$B(h_s) = -\left(3 - 4\frac{\beta^2(h_s)}{\alpha^2(h_s)}\right)\frac{F_x{}^*(h_s)}{F_z(0)} - \frac{2}{\rho(h_s)\,\alpha^2(h_s)}\frac{\sigma_{zz}{}^*(h_s)}{\dot{F}_z(0)/c}$$

$$C(h_s) = -\frac{1}{\mu(h_s)}\frac{\sigma_{zx}(h_s)}{\dot{F}_z(0)/c}$$

$$G(h_s) = -\frac{1}{\mu(h_s)}\frac{\sigma_{zy}{}^*(h_s)}{\dot{F}_y(0)/c}$$

$$V(h_s) = \frac{\dot{F}_y(h_s)}{\dot{F}_y(0)} = \frac{F_y(h_s)}{F_y(0)}$$





The quantity I are the *energy integrals* defined as

$$I_L = \int_0^\infty \rho(z)\left(F_y(z)/F_y(0)\right)^2 dz$$

$$I_R = \int_0^\infty \rho(z)\left[y_1^{\,2}(z) + y_3^{\,2}(z)\right] dz$$

$$y_1 = \frac{F_z(z)}{F_z(0)}$$

$$y_3 = -i\frac{F_x(z)}{F_z(0)} = \frac{F_x^*(z)}{F_z(0)}$$

v is the group velocity that can be calculated analytically from the phase velocity

$$v = \frac{c}{1 - \frac{\omega}{c}\frac{\partial c}{\partial \omega}}$$

while C_2 indicates the phase attenuation and expresses the effect due to anelasticity. C_2 can be calculated analytically using variational techniques (e.g., Takeuchi and Saito, 1972; Aki and Richards, 1980).

APPLICATION 1

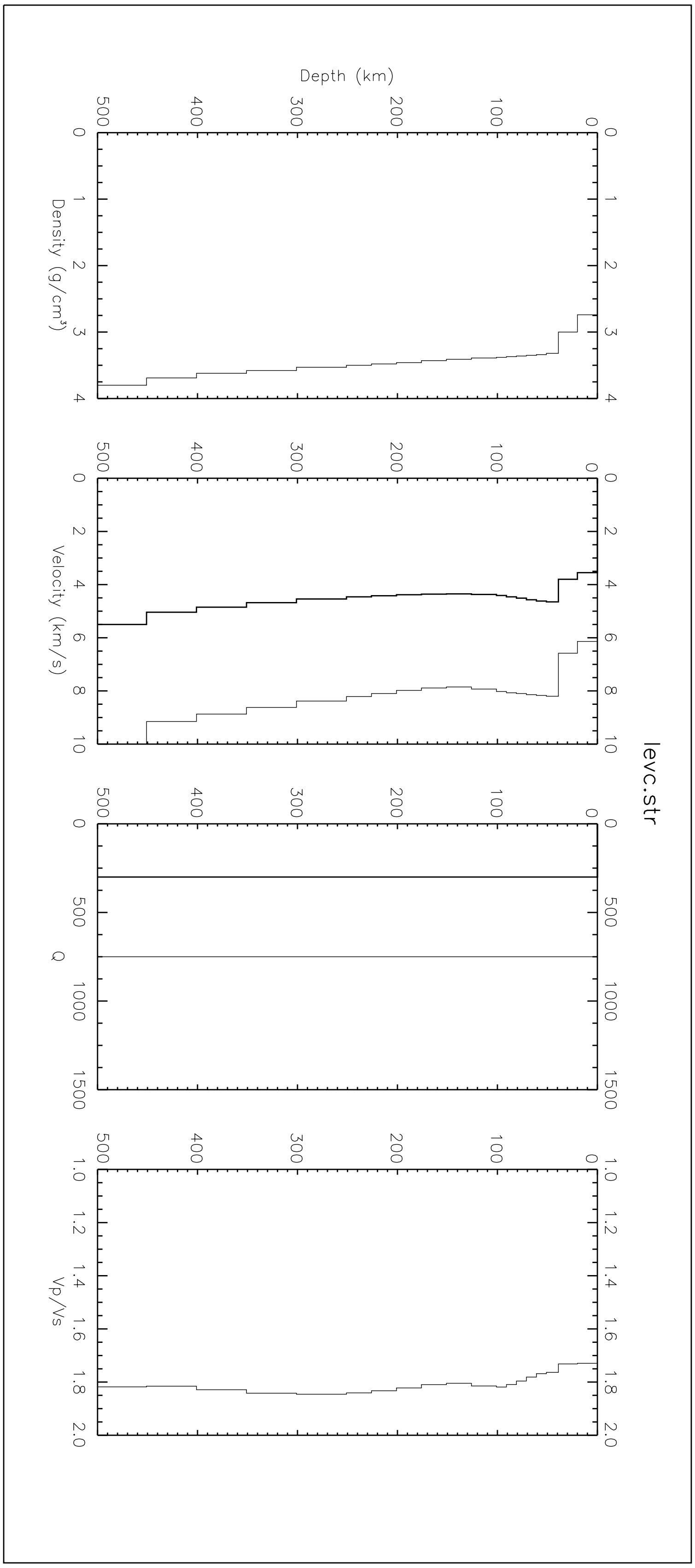
Examples of calculation of:

- Spectral quantities (eigenvalues, eigenfunctions, etc.)

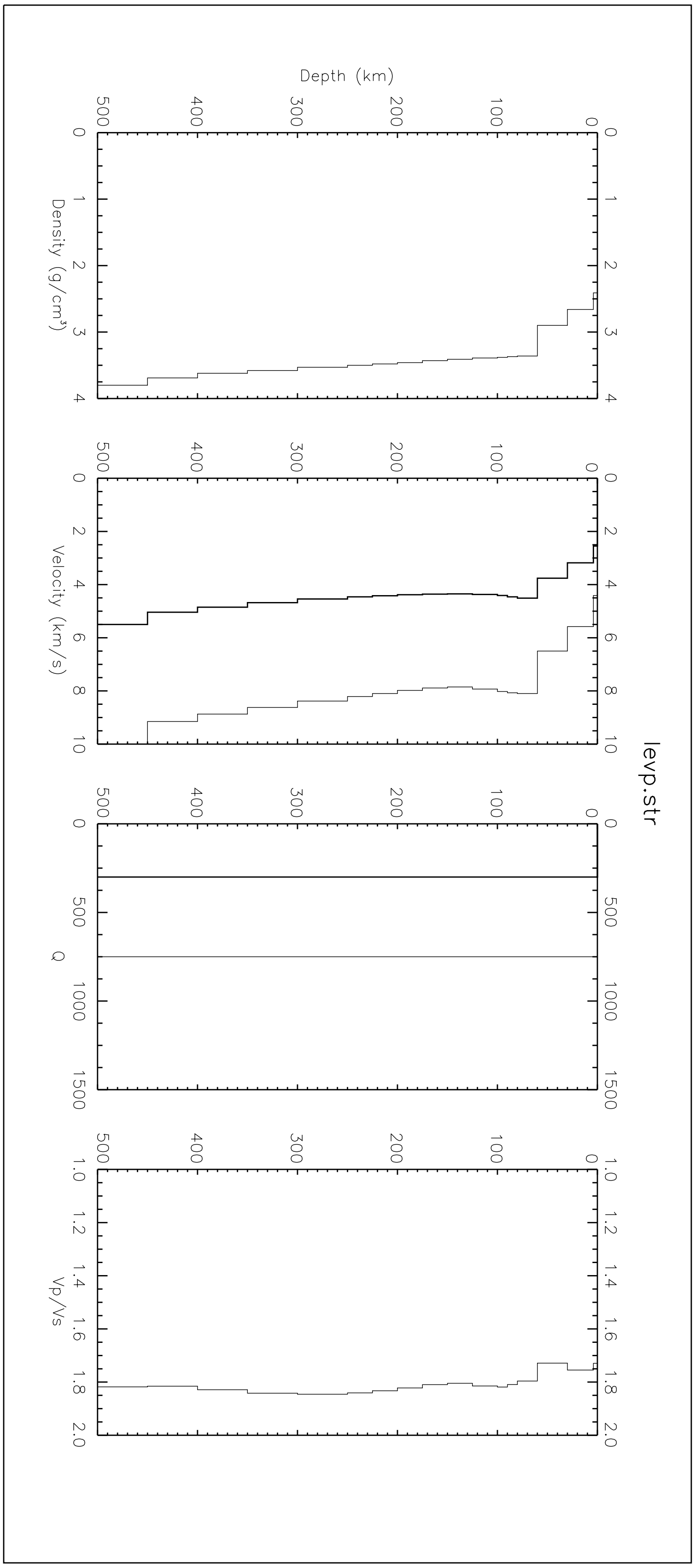
- Synthetic seismograms (modal summation)

- Effect of source parameters (focal mechanism)

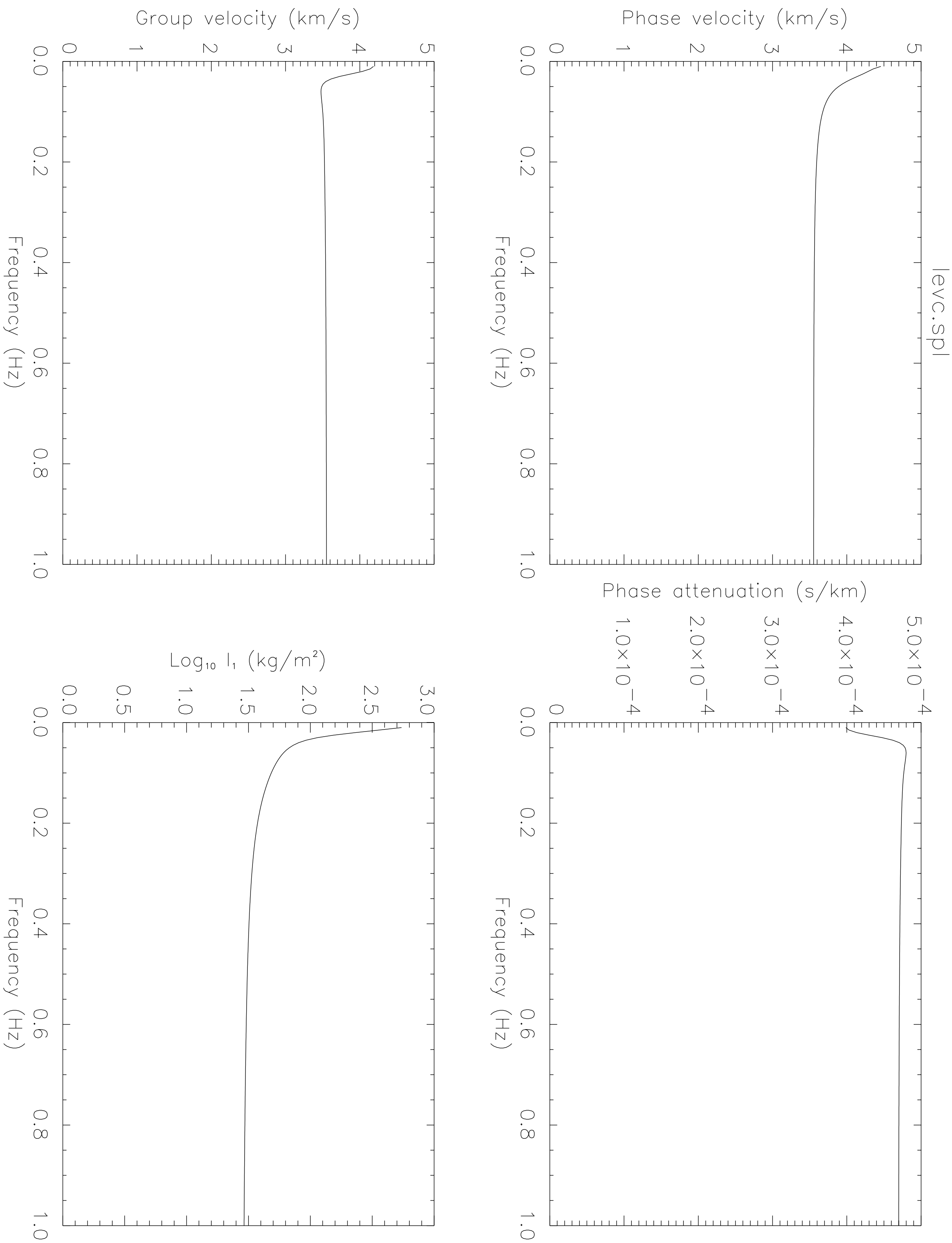
First continental structure - C



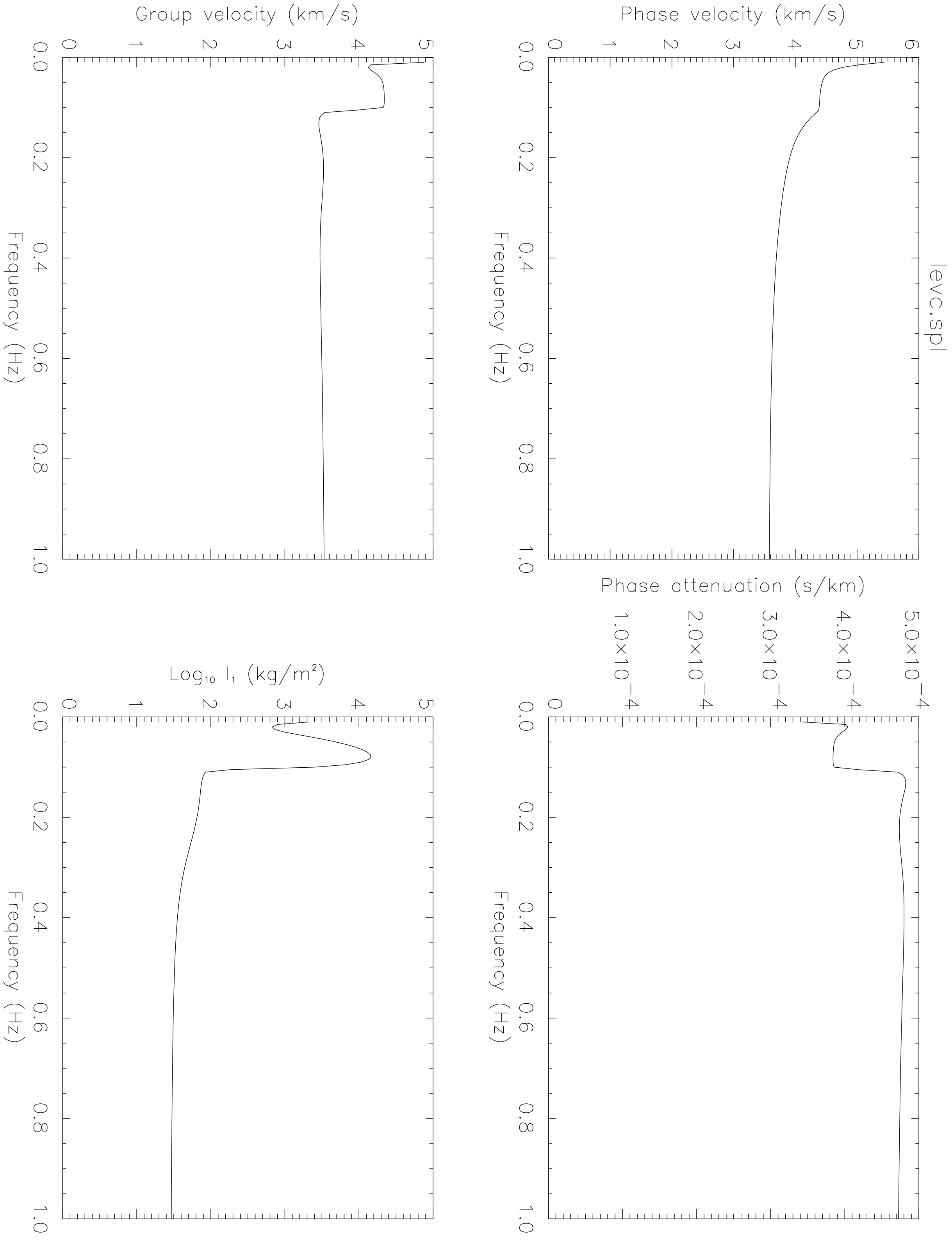
Second continental structure - P

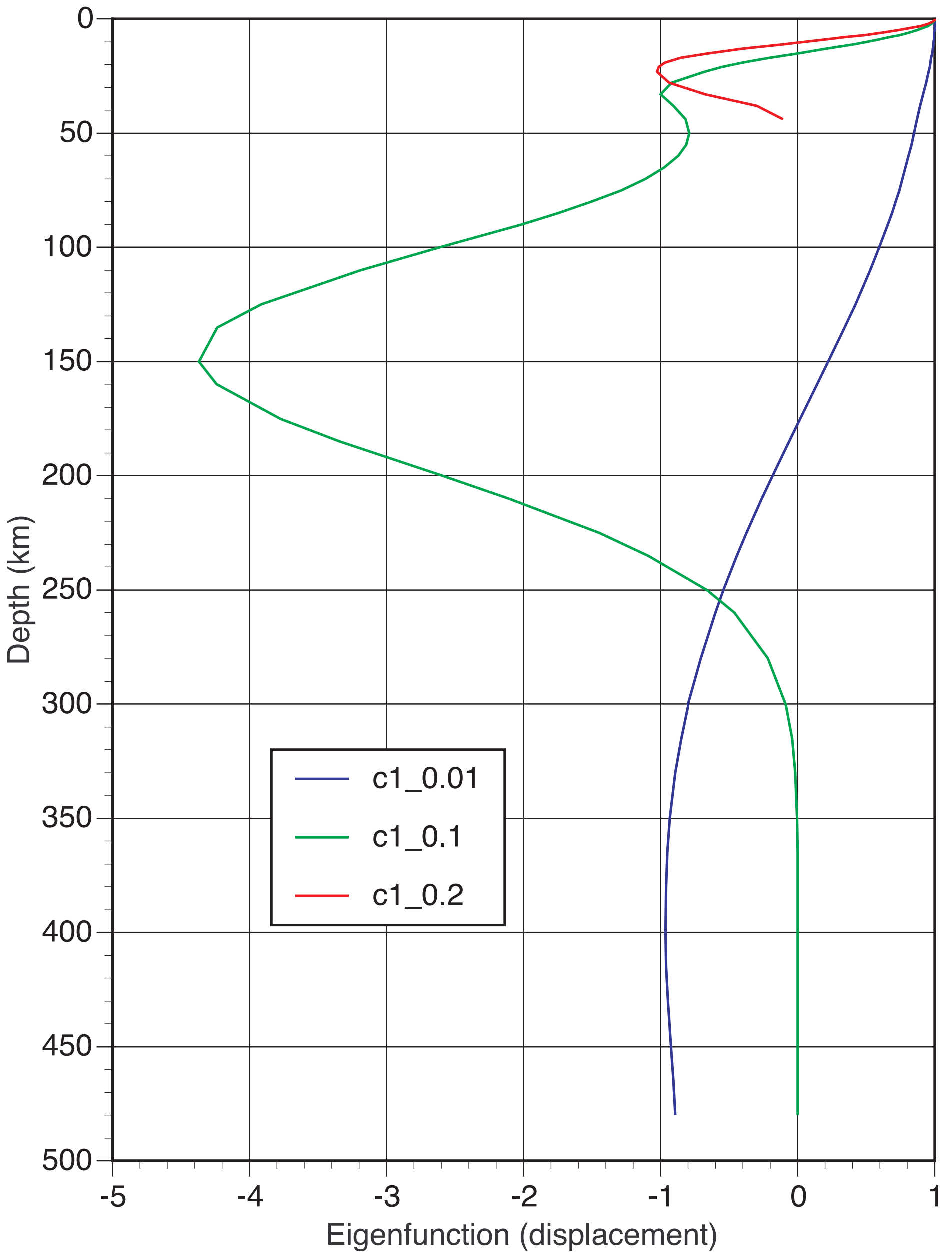


Love spectrum of C: fundamental mode



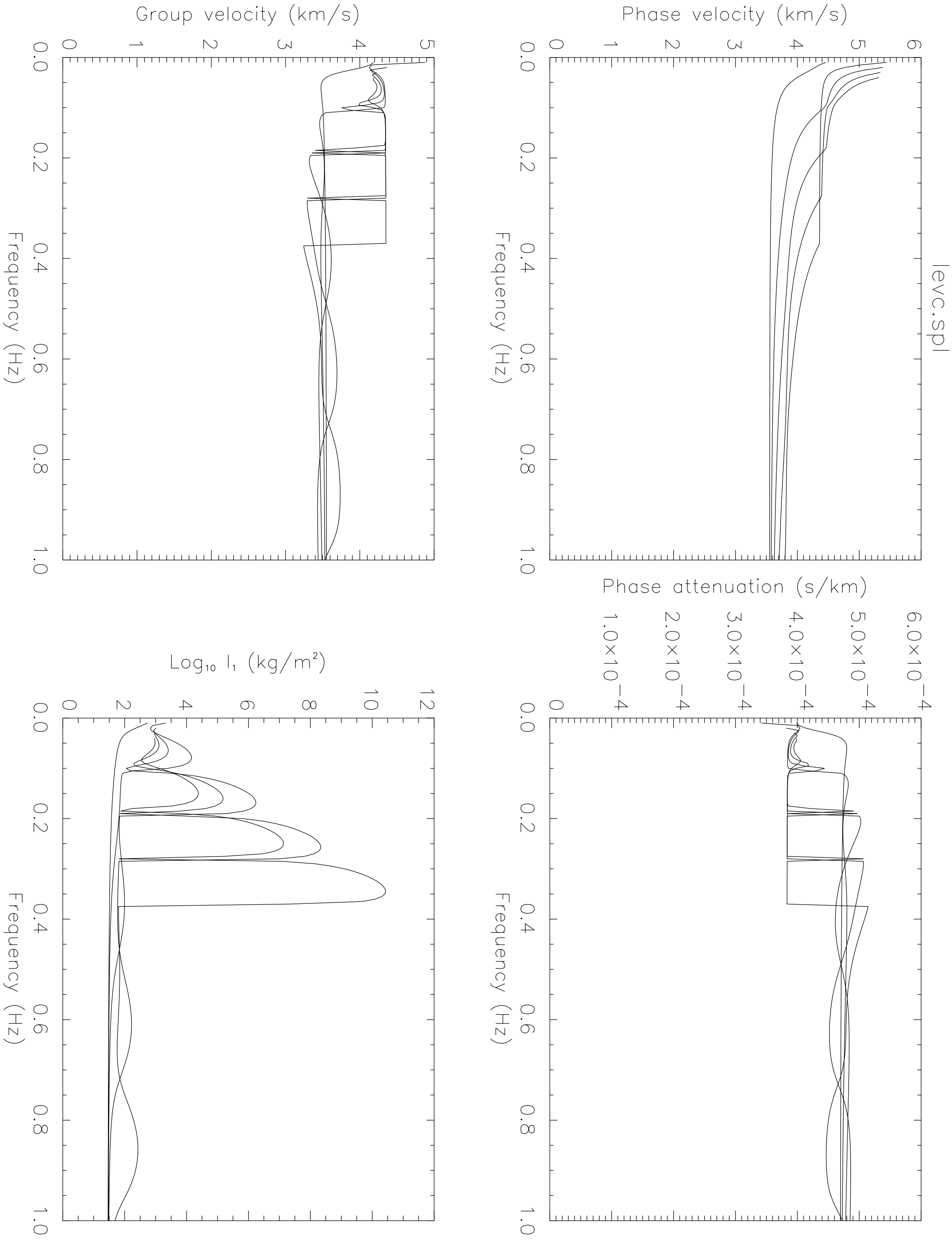
Love spectrum of C: first higher mode
note the effect of the low velocity channel



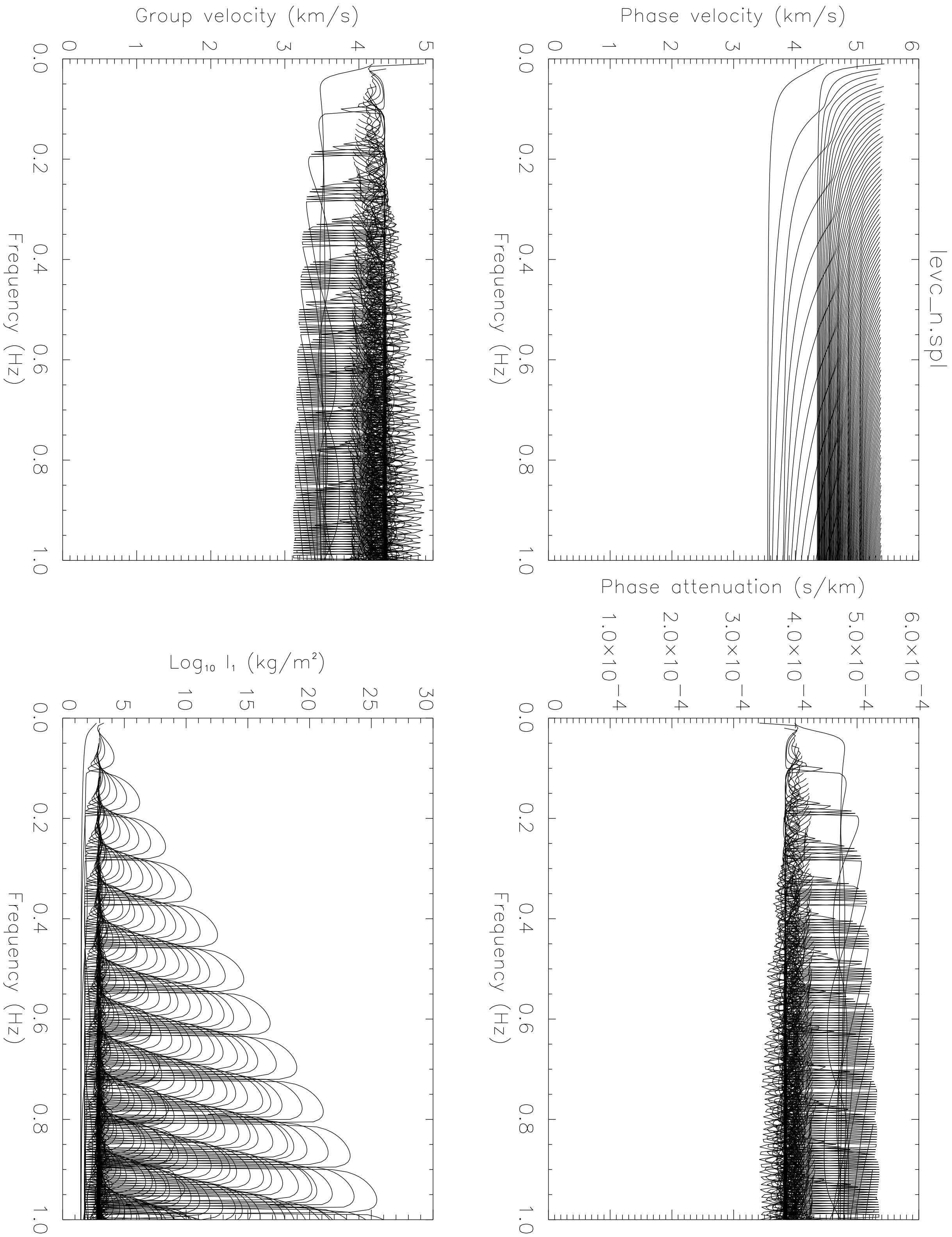


Displacement versus depth for 3 frequencies (0.01, 0.1, 0.2 Hz) of 1st higher mode of structure C

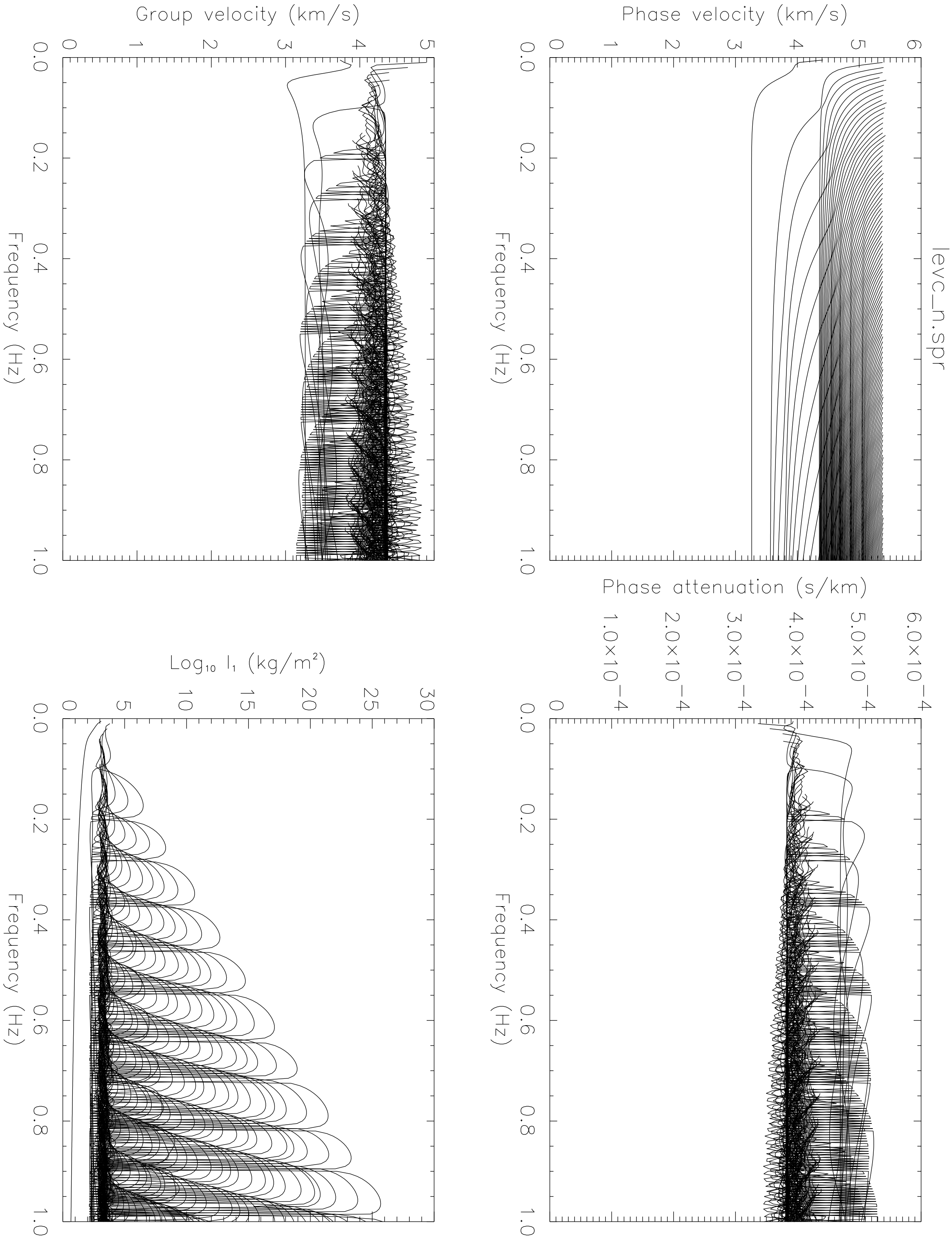
Love spectrum of C: first 5 modes



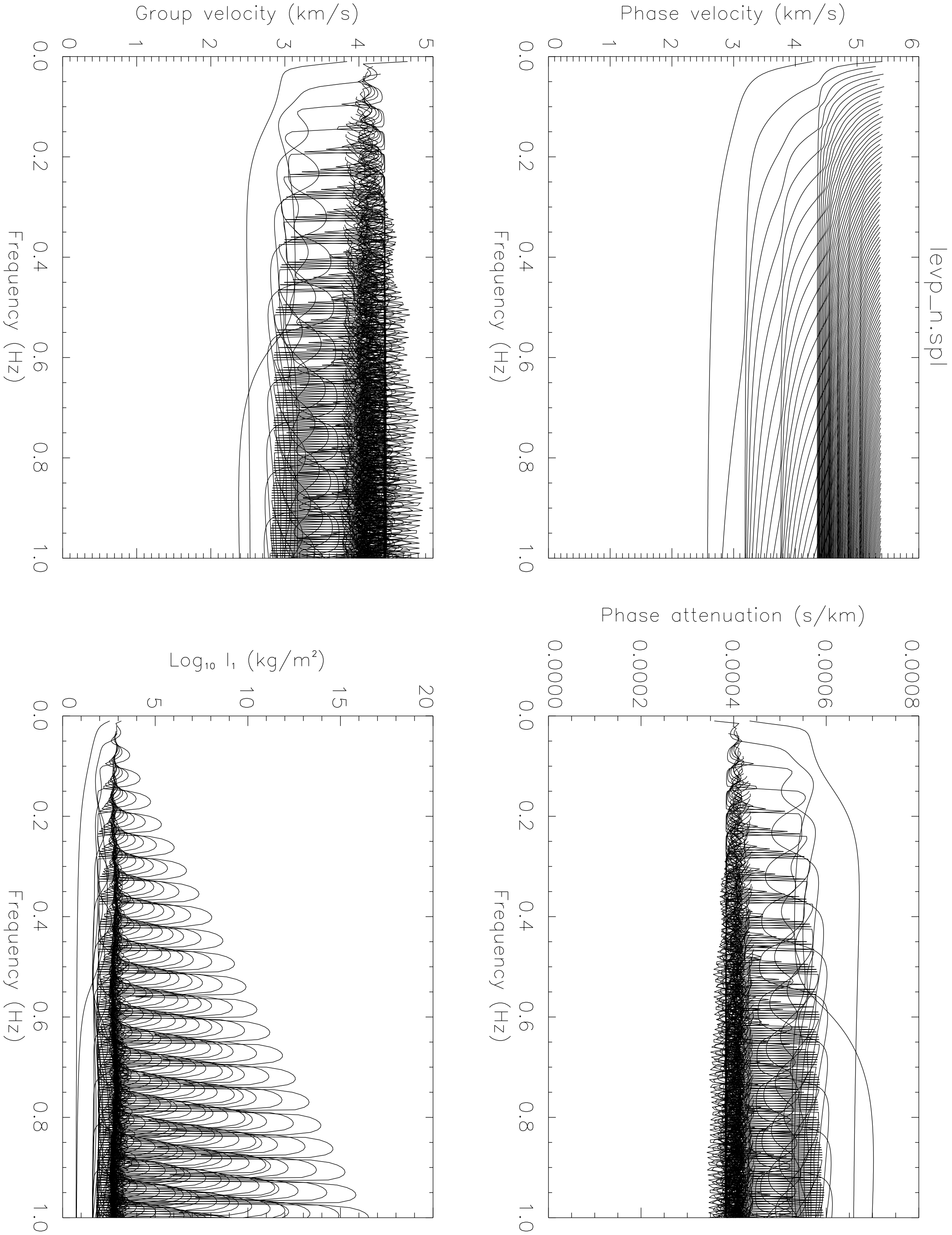
Love spectrum of C: 110 modes



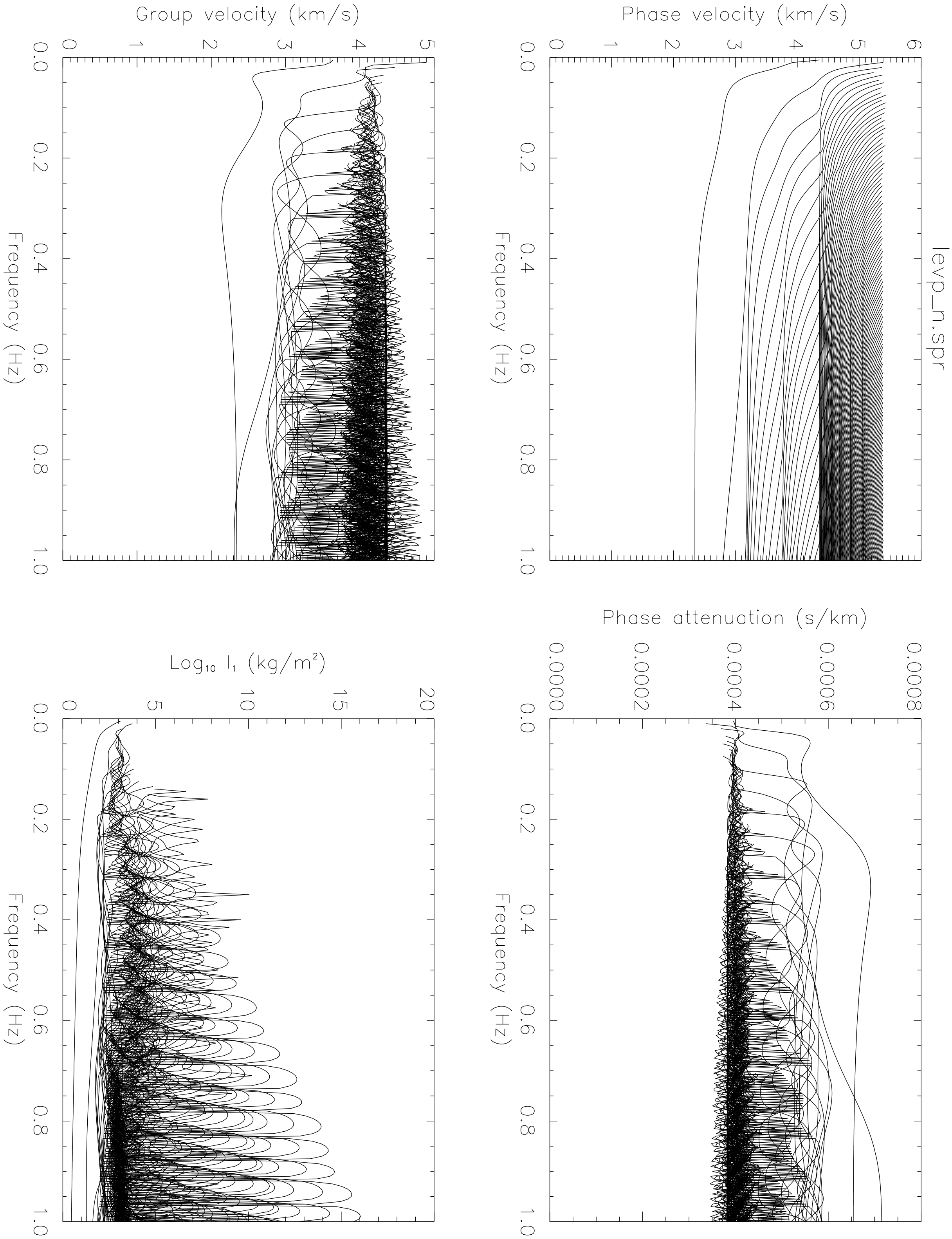
Rayleigh spectrum of C: 110 modes



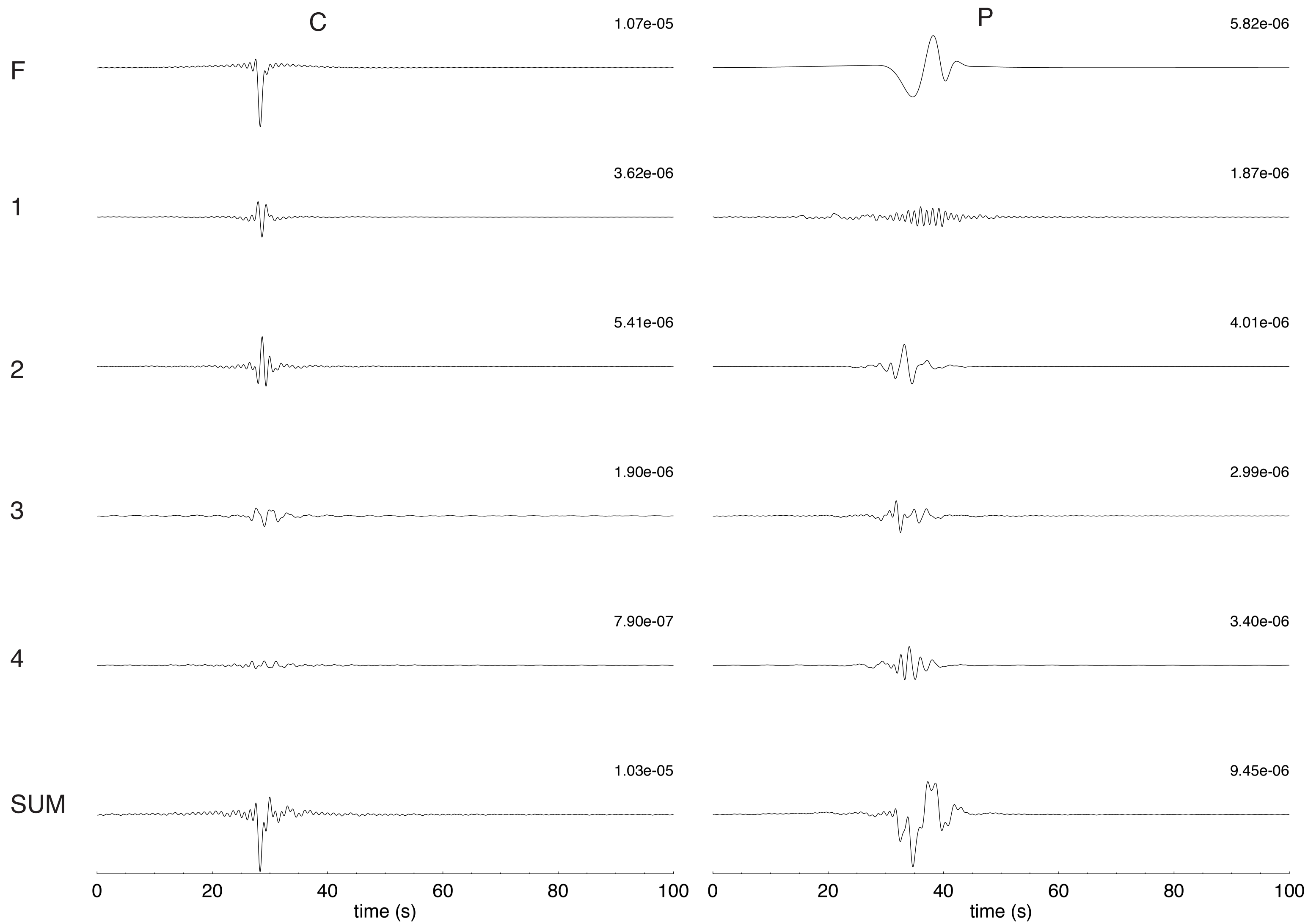
Love spectrum of P: 110 modes



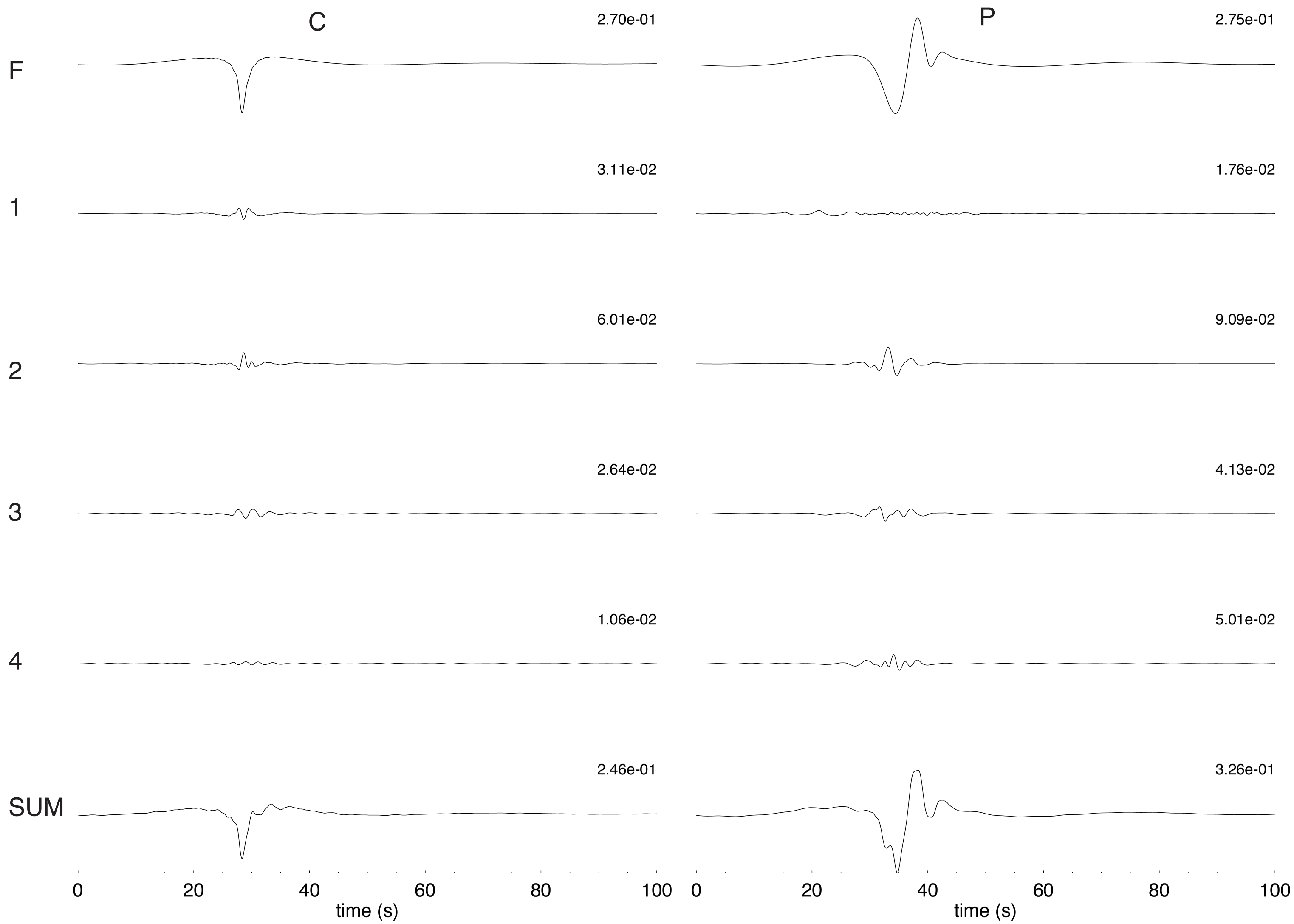
Raqyleigh spectrum of P: 110 modes

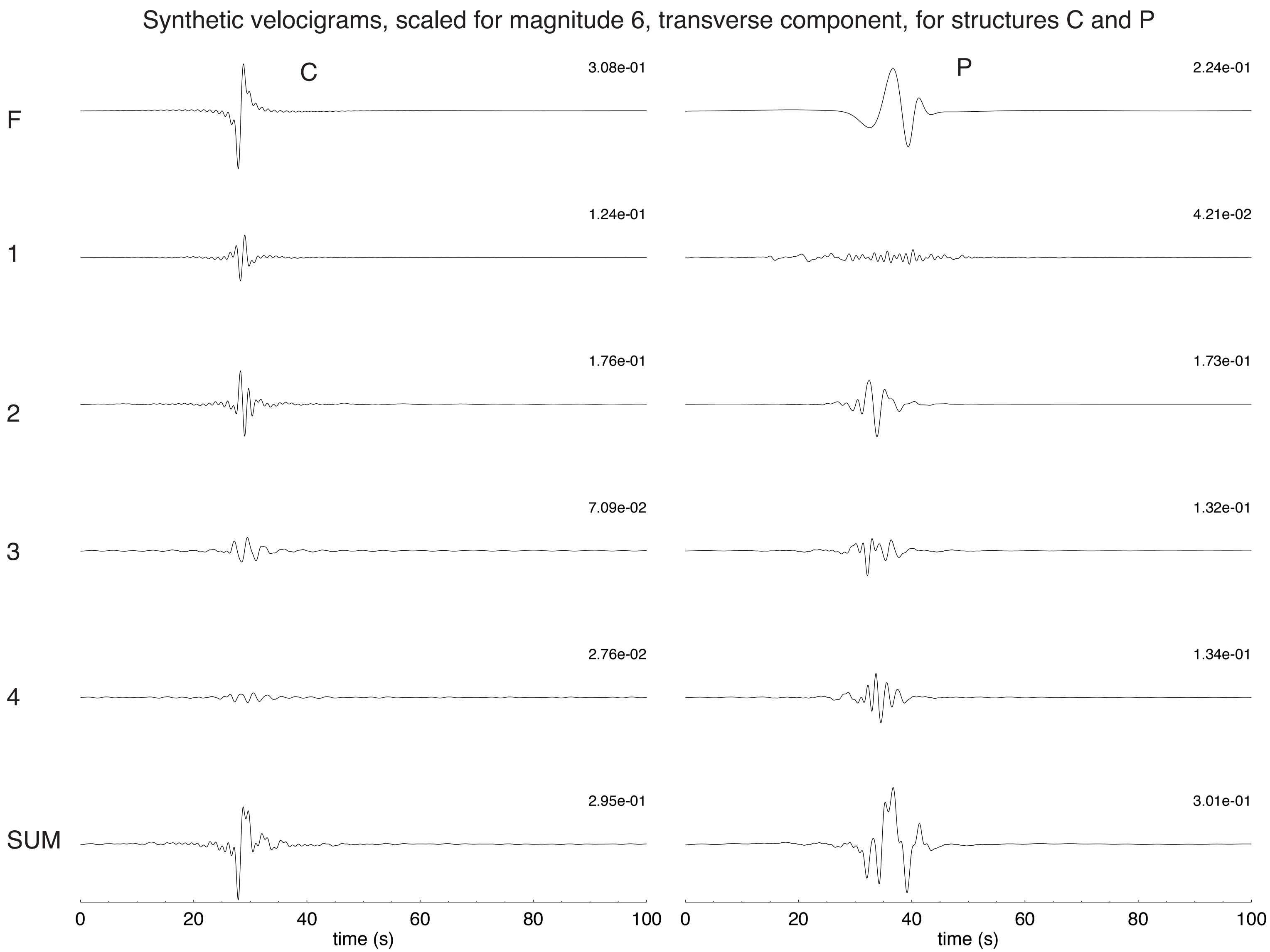


Synthetic seismograms, transverse component, for structures C and P
source depth=10km; receiver at 100km; strike receiver=60º,rake=180º, dip=90º

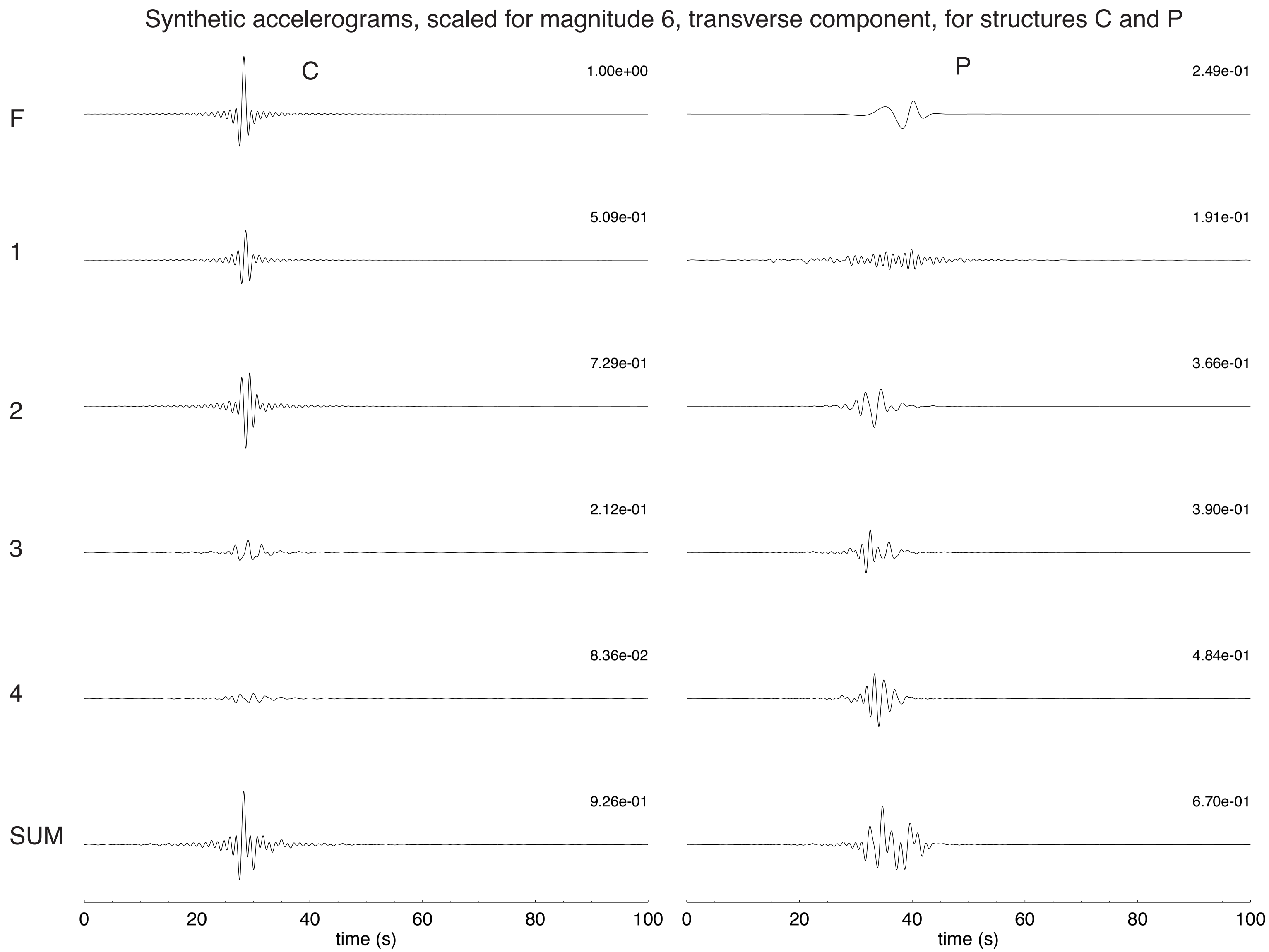


Synthetic seismograms, scaled for magnitude 6, transverse component, for structures C and P

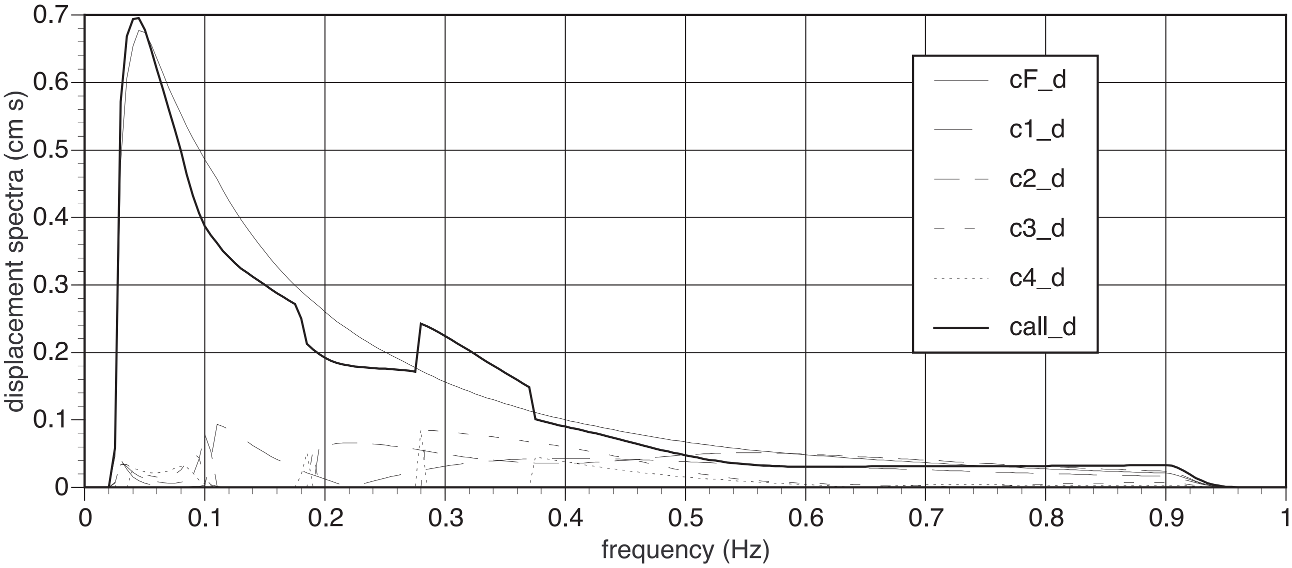




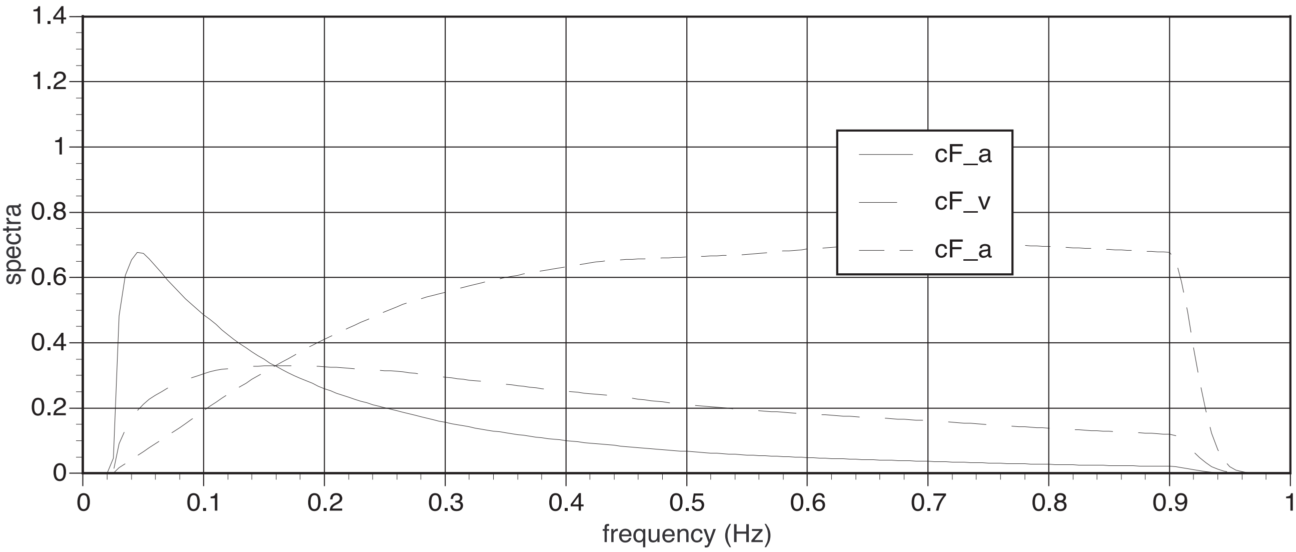
Synthetic velocigrams, scaled for magnitude 6, transverse component, for structures C and P
C
P
F
1
2
3
4
SUM
3.08e-01
1.24e-01
1.76e-01
7.09e-02
2.76e-02
2.95e-01
2.24e-01
4.21e-02
1.73e-01
1.32e-01
1.34e-01
3.01e-01
0
20
40
60
80
100
time (s)
time (s)



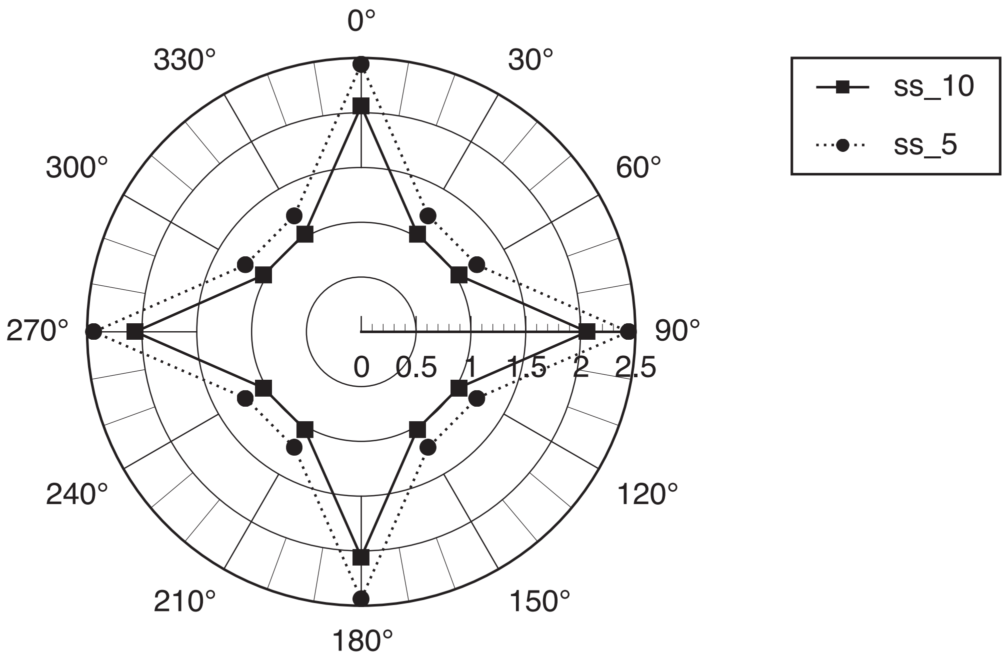
Synthetic accelerograms, scaled for magnitude 6, transverse component, for structures C and P
C
P
F
1
2
3
4
SUM
1.00e+00
5.09e-01
7.29e-01
2.12e-01
8.36e-02
9.26e-01
2.49e-01
1.91e-01
3.66e-01
3.90e-01
4.84e-01
6.70e-01
0
20
40
60
80
100
time (s)



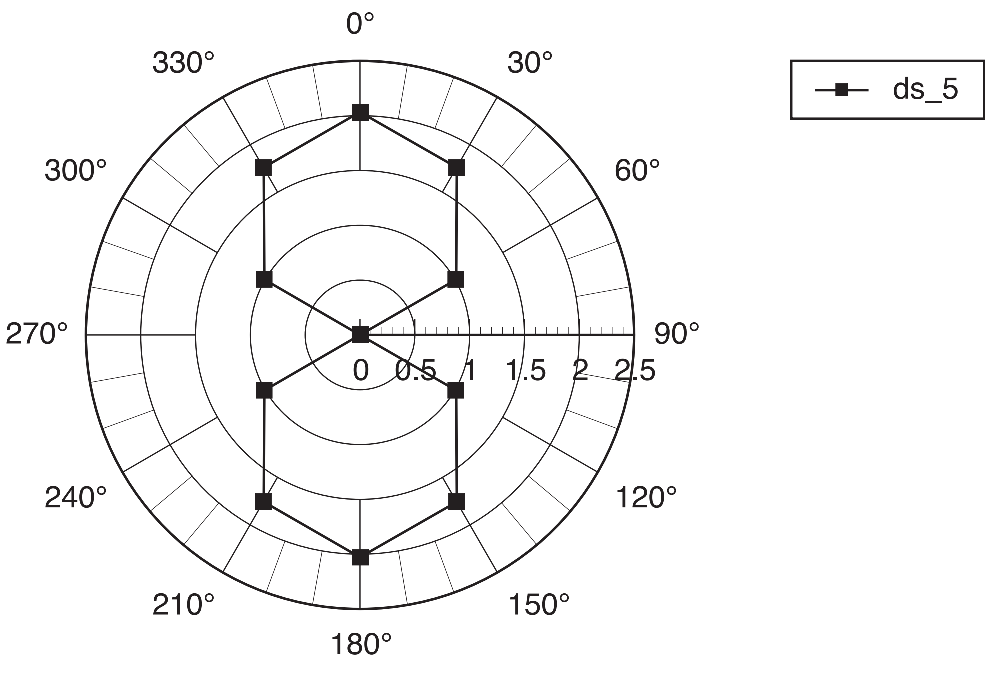
Fourier spectrum of displacement for first 5 modes of structure C



Fourier spectrum of displacement, velocity and acceleration
for fundamental mode of structure C



Maximum amplitude of displacement for two source depths (10 and 5 km) versus the strike-receiver angle: STRIKE SLIP MECHANISM (rake=180º)



Maximum amplitude of displacement forsource depths of 5 km versus the strike-receiver angle: DIP SLIP MECHANISM (rake=90º)