Theoretical Seismology

Astrophysics and Cosmology and Earth and Environmental Physics

FTAN Analysis

Fabio ROMANELLI

Department of Mathematics & Geosciences

University of Trieste

romanel@units.it

FTAN analysis

Frequency-Time representation:

gaussian filters

FTAN maps

Floating filters:

phase equalization

Examples of calculation and filtering:

synthetic signals (Love, Rayleigh, 1D, 2D)

recorded (real) signals

Frequency-Time representation

Surface waves, showing not impulse nor quasi-harmonic behavior, are difficult to be studied in time or spectral domain, since their principal feature, dispersion, is described by a function rather than a single parameter.

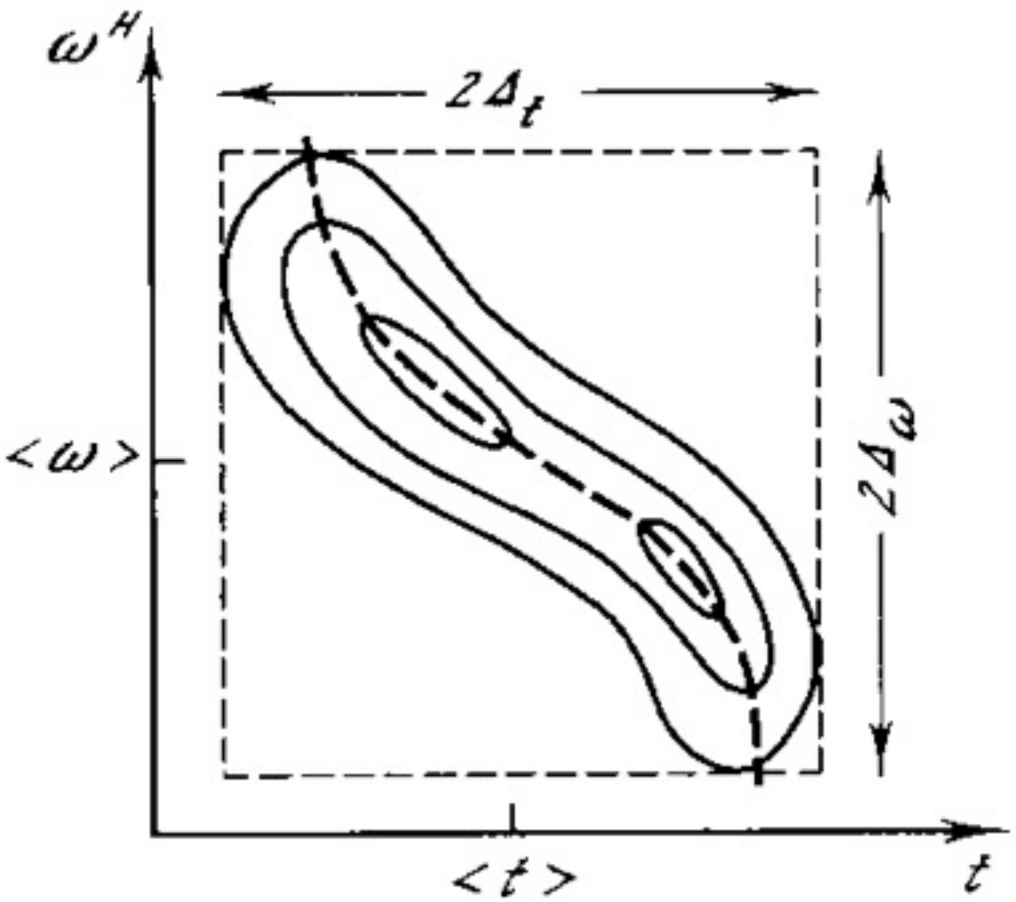
Frequency-time analysis has the property of separating signals in accordance to their dispersion curve, since a visual picture requires a function of two variables.

Let us consider a signal in time $W(t)$ and its Fourier transform, $K(\omega)$, and let it pass through a system of parallel relatively narrow-band filters $H(\omega-\omega^H)$ with varying central frequency ω^H . The combination of all signals at the output of all the filters is a complex function of two variables:

$$\boxed{S\left(\omega^H, t\right) = \int_{-\infty}^{+\infty} H\left(\omega - \omega^H\right) K(\omega) e^{i\omega t} d\omega}$$

FTAN

A contour map of $|S(\omega^H,t)|$ is called **FTAN** (Frequency-Time Analysis) map, and it is used to visualize the dispersion curves, since, for frequency fixed, a "mountain ridge" (increased amplitudes) appears. The frequency-time region of a signal is that part of the (ω^H,t) plane occupied by the relevant ridge, and the statement "the energy of a signal concentrates around its dispersion curve" has a clear meaning.



FTAN map: dashed line is a dispersion curve $t = \tau(\omega^H)$

The function $S(\omega^H,t)$ is not a property of the original signal alone, since it involves also the filter characteristics $H(\omega-\omega^H)$, chosen by the investigator: we have different classes of signal representations that differ in **filter choice**.

When the shape of $H(\omega-\omega^H)$ is known, the function **W(t) or K(ω) can be recovered**:

- $K(\omega)$ from infinitesimally small filters = $\delta(\omega-\omega^H)$
- $W(t)$ from infinitely broad filters = $1/(2\pi)^{1/2}$

with the advantage that the noise can be more easily separated, for surface wave identification.

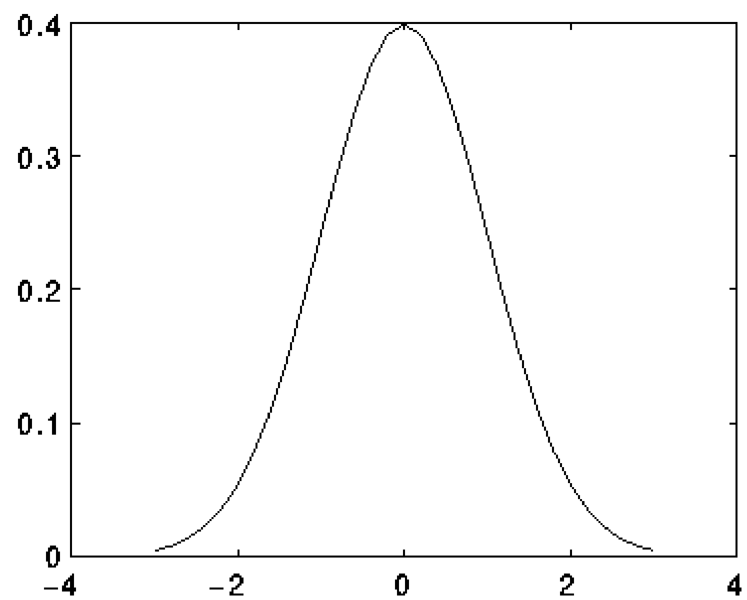
Gaussian filters

The choice of $H(\omega-\omega^H)$ is guided by the typical properties of the signal to be processed. For surface waves two simple rules have to be followed:

- No phase distortion (H has to be real valued)
- Best resolution

and the optimal choice is found to be a **Gaussian** filter, described by two parameters: central frequency, ω^H, and width of the frequency band, σ.

$$G(\omega)=\frac{1}{\sqrt{2\pi\sigma}}e^{-\frac{\omega^2}{2\sigma^2}}$$



And the final FTAN representation is the complex valued function:

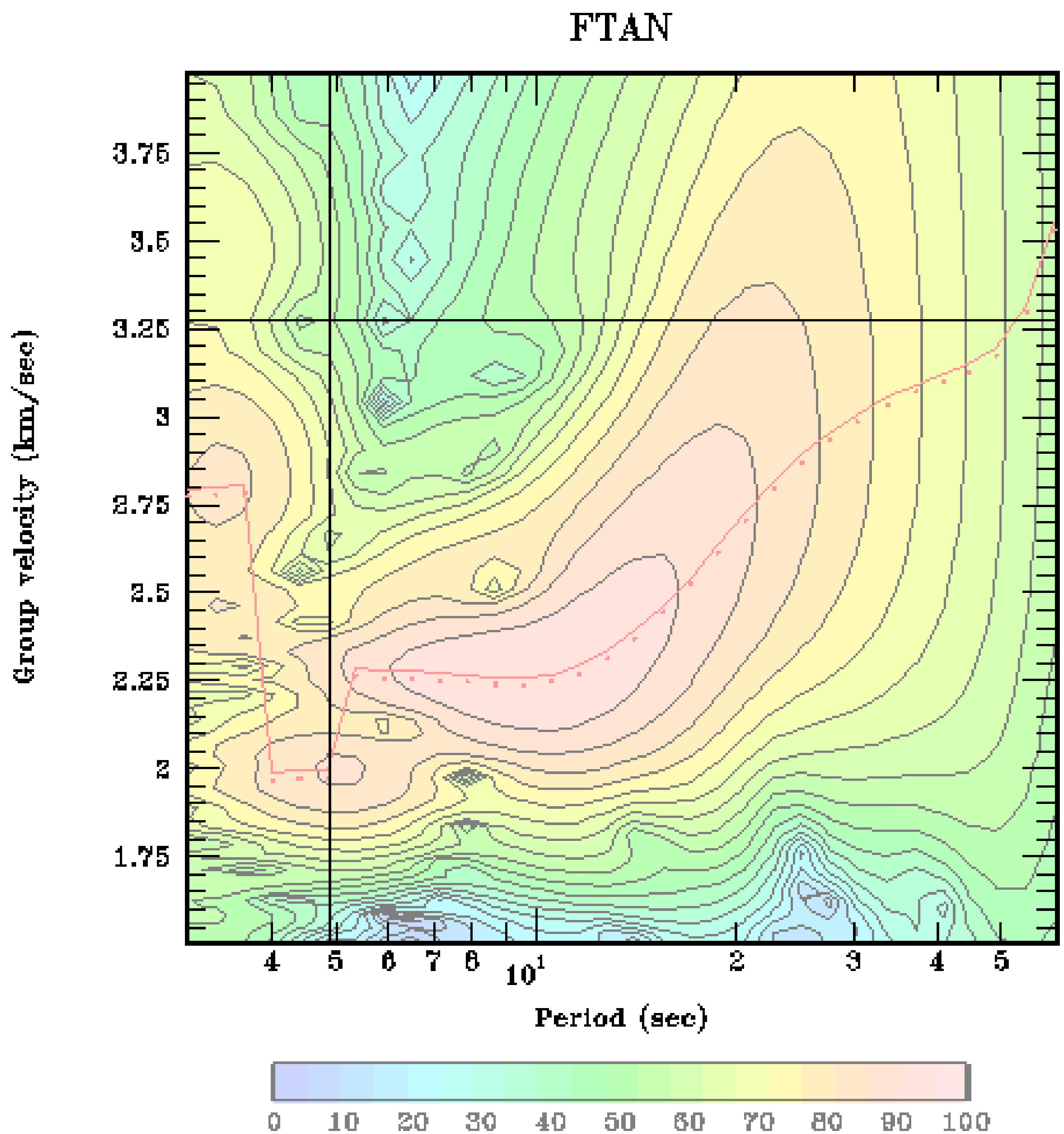
$$\boxed{S\left(\omega^H,t\right)=\frac{1}{\sqrt{2\pi\sigma(\omega^H)}}\int_{-\infty}^{+\infty}e^{-\frac{(\omega-\omega^H)^2}{2\sigma^2(\omega^H)}}K\left(\omega\right)e^{i\omega t}d\omega}$$

FTAN map

The graphical representation consist in the plot of a **matrix** of values, (t_i,ω_i), whose rows are given by:

$$S_\omega(t_i) = 20\log_{10}\left(\frac{\left|S_\omega\right|_i}{\max\left|S_\omega\right|_i}\right)$$

Converting the frequency to period and,
given the **epicentral distance**,
converting arrival times of energy packets to group velocity,
one has the typical **FTAN map**
of a signal:



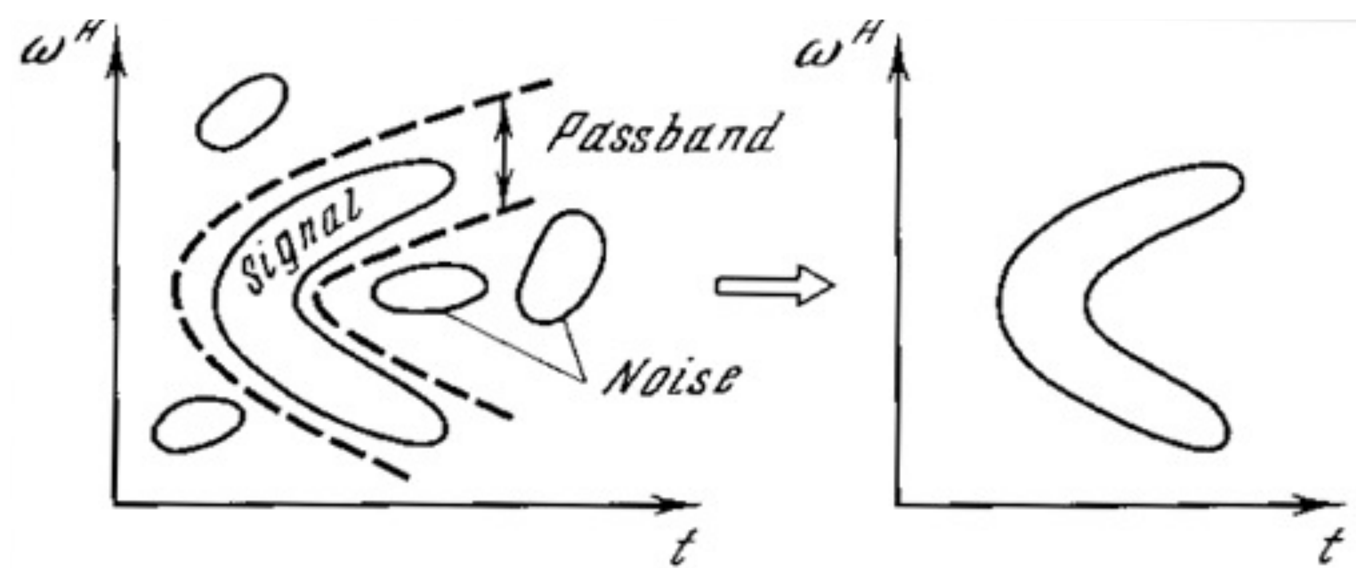
FTAN and linear filters

The term "filter" is usually employed for a transformation whose parameters are invariant under a time shift: a band parallel to time axis, bandpass filtering, or a band parallel to frequency axis, time window.

In a broader sense, the general form of a linear transformation can be written as:

$$K'(\omega) = \int_{-\infty}^{+\infty} F(\omega, \lambda) K(\lambda) d\lambda$$

And the filter should separate, without distortions, the part of the plane where the signal energy is. The filter band has to "**float**" along the dispersion curve:



Floating filters

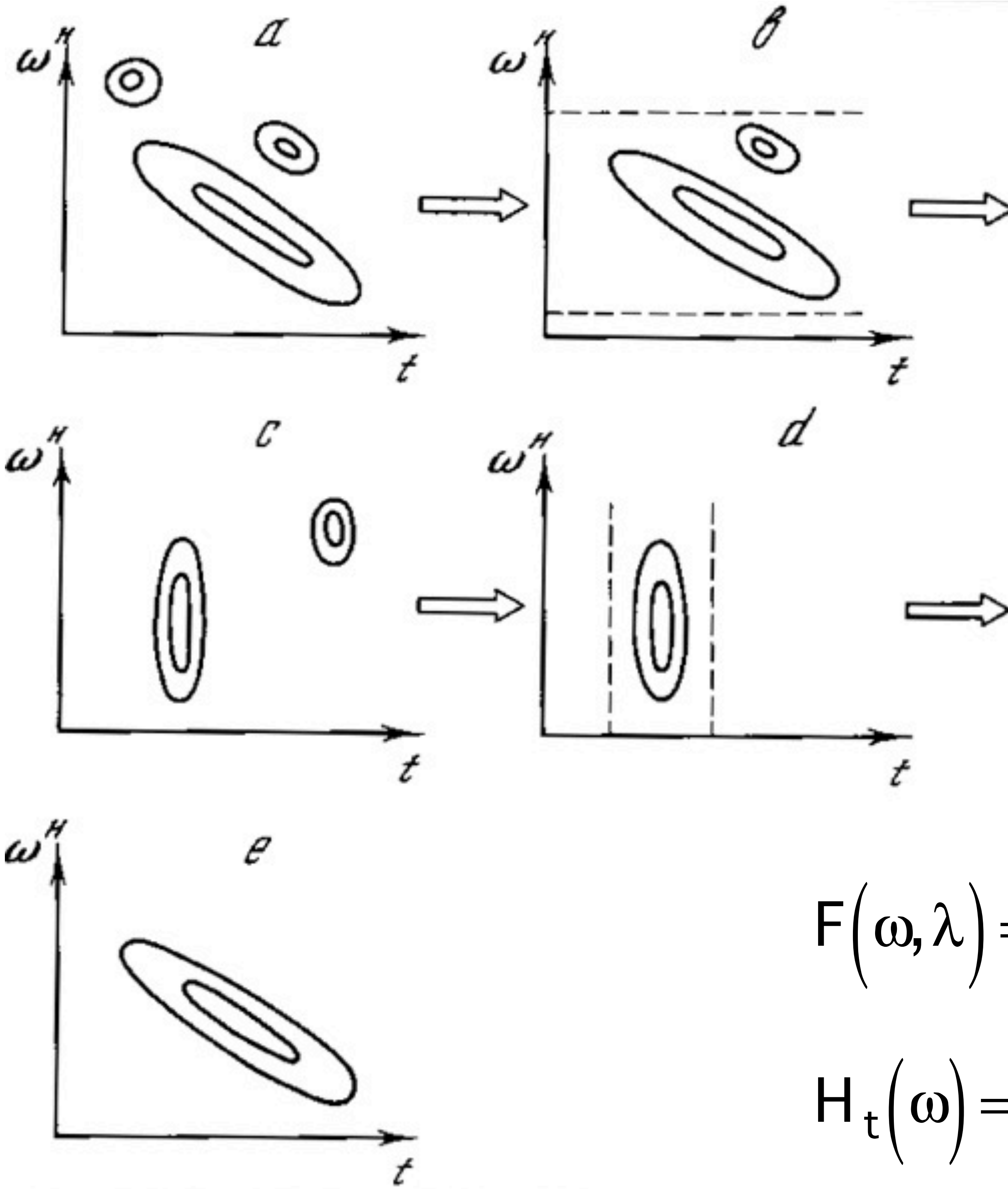
The dispersion curve of a signal, $\tau(\omega)$, is known approximately from FTAN results and the spectral phase of the whole record is transformed according to:

$$K'(\omega) = K(\omega)\, e^{-i\psi(\omega)}$$

$$\psi(\omega) = -\left(\int_0^{\omega} \tau(\eta)\, d\eta + c_1\omega + c_2 \right)$$

to make the signal weakly dispersed, thus transforming into a straight line parallel to the frequency axis. The use of a time window allows to filter out the noise and the original signal shape can be recovered applying the inverse procedure of **phase equalization**.

Floating filters: scheme



b) Bandpass filtering

$$K_1'(\lambda) = H(\lambda)K(\lambda)$$

c) Phase equalization

$$W_2'(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} K_1'(\lambda) e^{i[\lambda t - \psi(\lambda)]} d\lambda$$

d) Time window

$$K_3'(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} G(t) W_2'(t) e^{-i\omega t} d\lambda$$

e) Inverse phase transformation

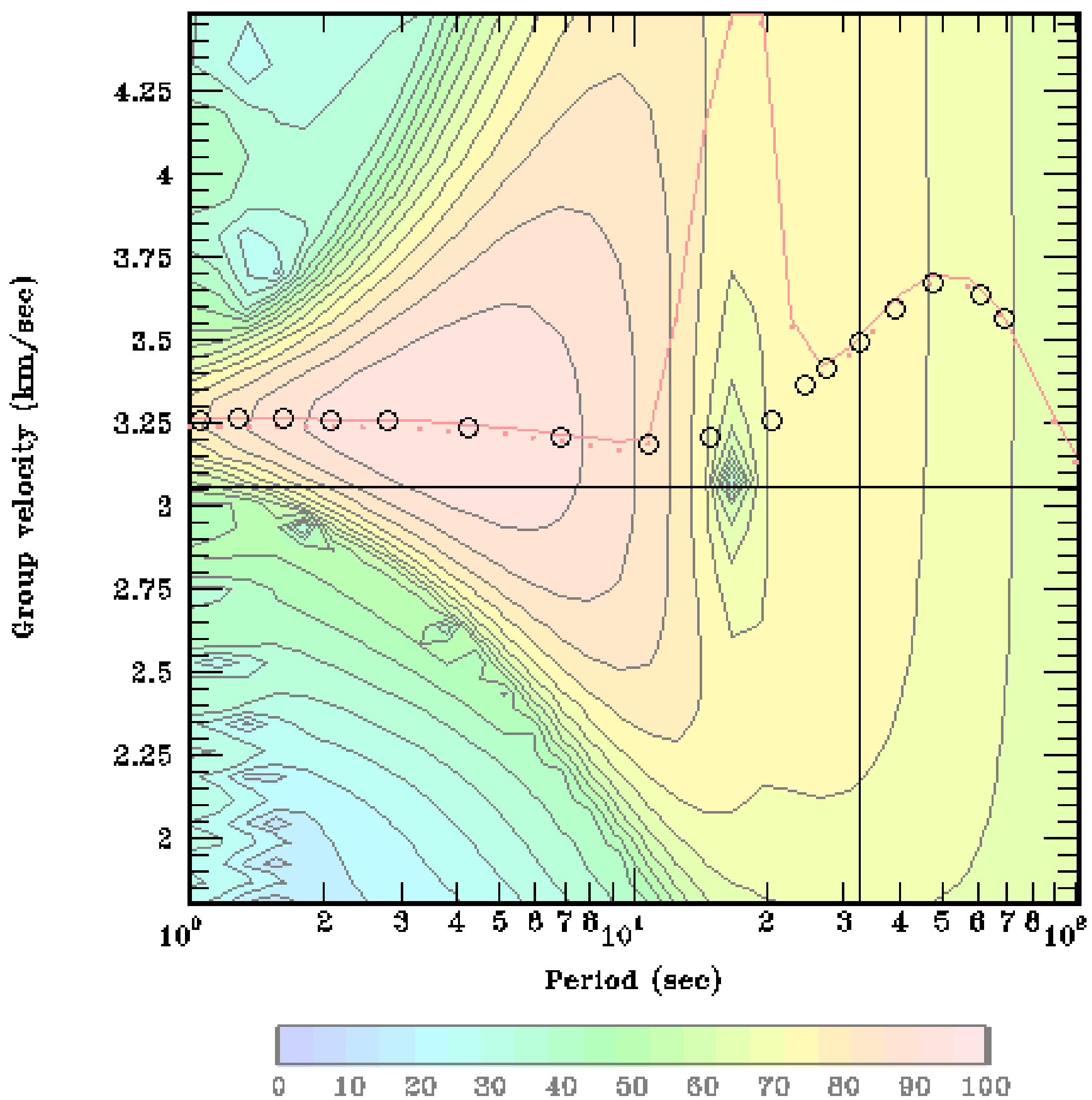
$$K'(\omega) = K_3'(\omega) e^{i\psi(\omega)}$$

$$F(\omega,\lambda) = \frac{1}{\sqrt{2\pi}} H(\lambda) H_t(\omega - \lambda) e^{i[\psi(\omega) - \psi(\lambda)]}$$

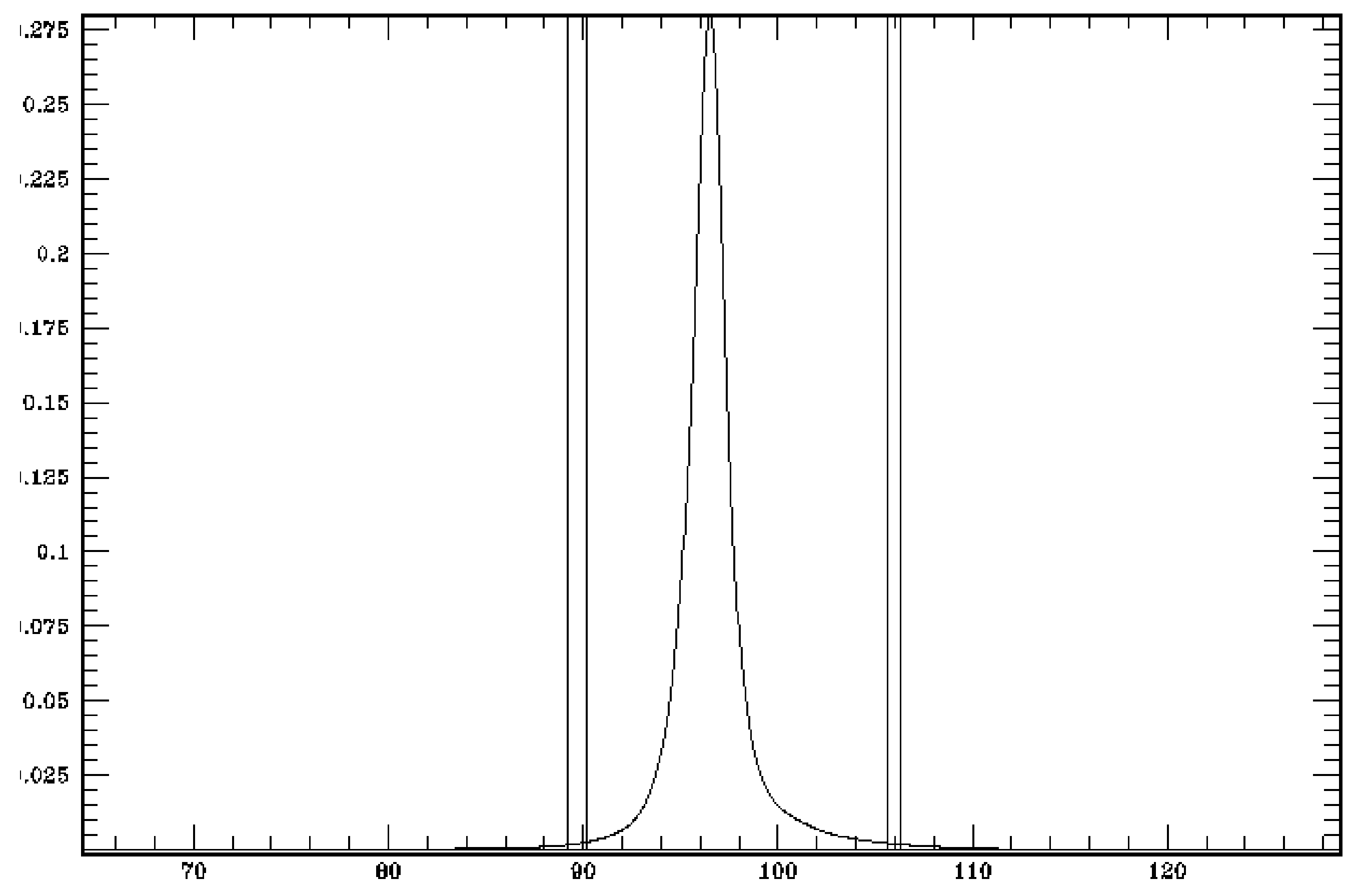
$$H_t(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} G(t) e^{i\omega t} dt$$

Example 1: CFV - FTAN and filters

Analysis of component of synthetic displacement:
Vertical component, **fundamental** mode of structure ***C***



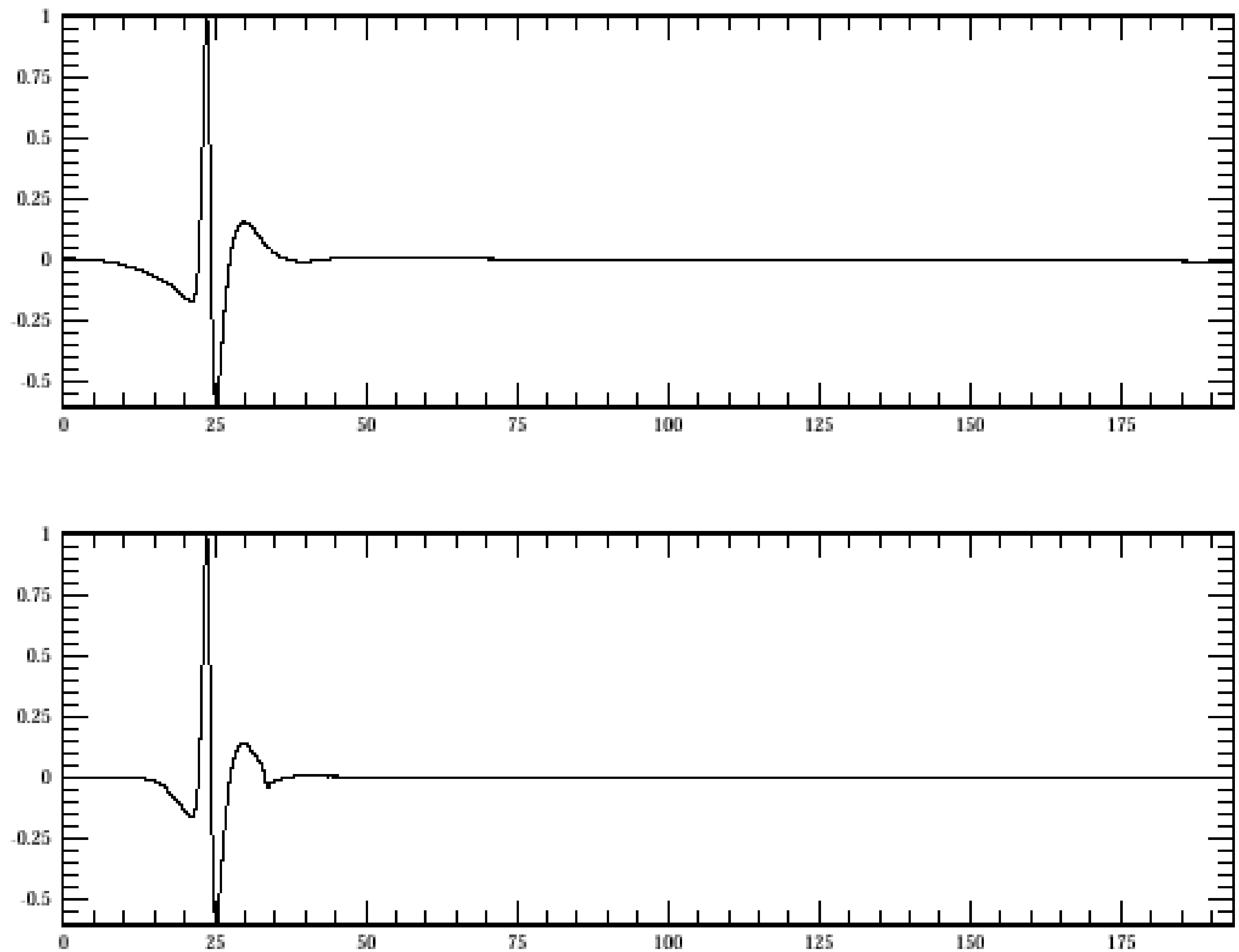
FTAN map and operator-defined dispersion curve, for phase equalization



Time domain: result of phase equalization and operator-defined time window

Example 1: CFV - results

Analysis of component of synthetic displacement:
Vertical component, **fundamental** mode of structure ***C***

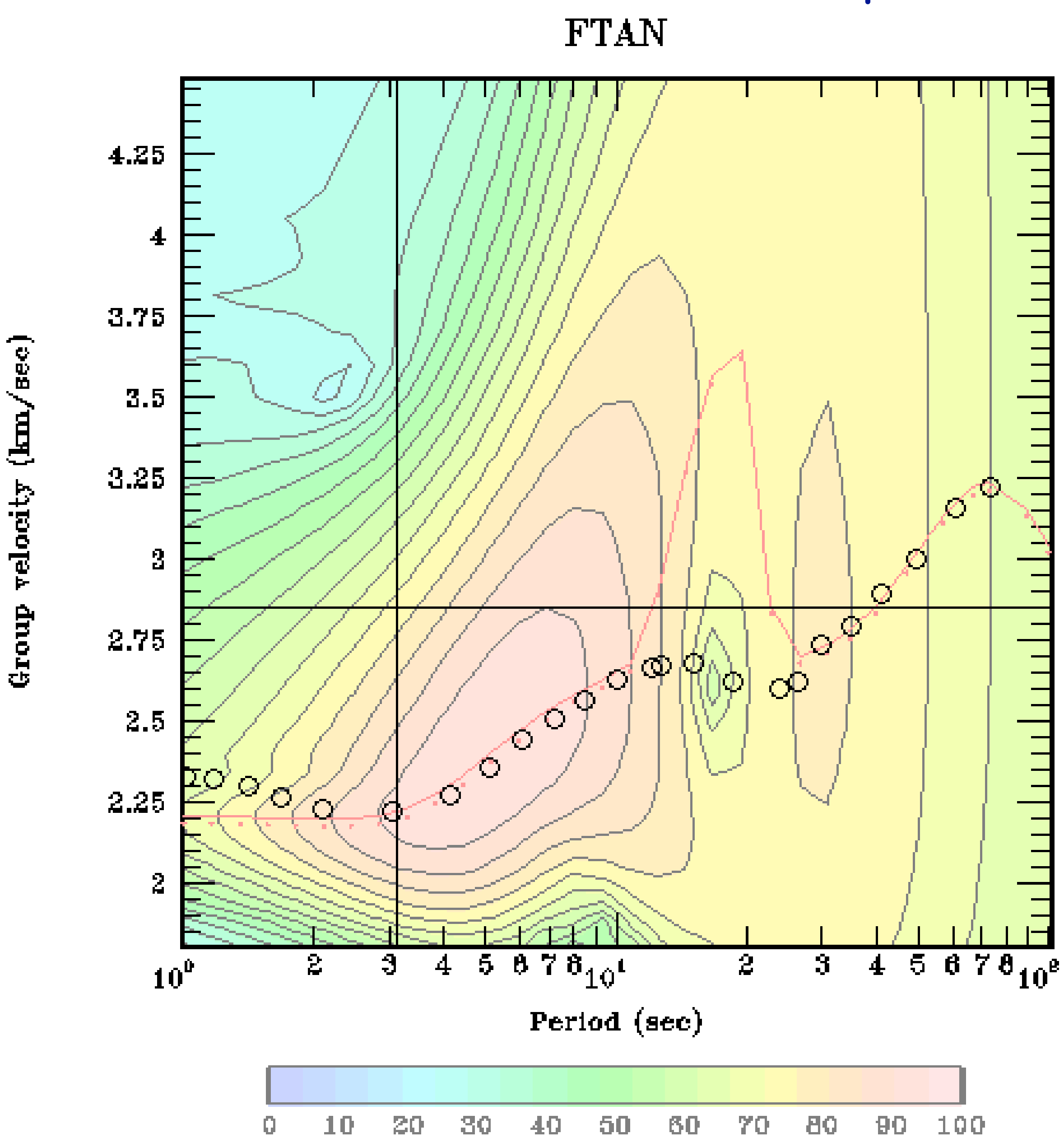


Input and processed signal in time domain

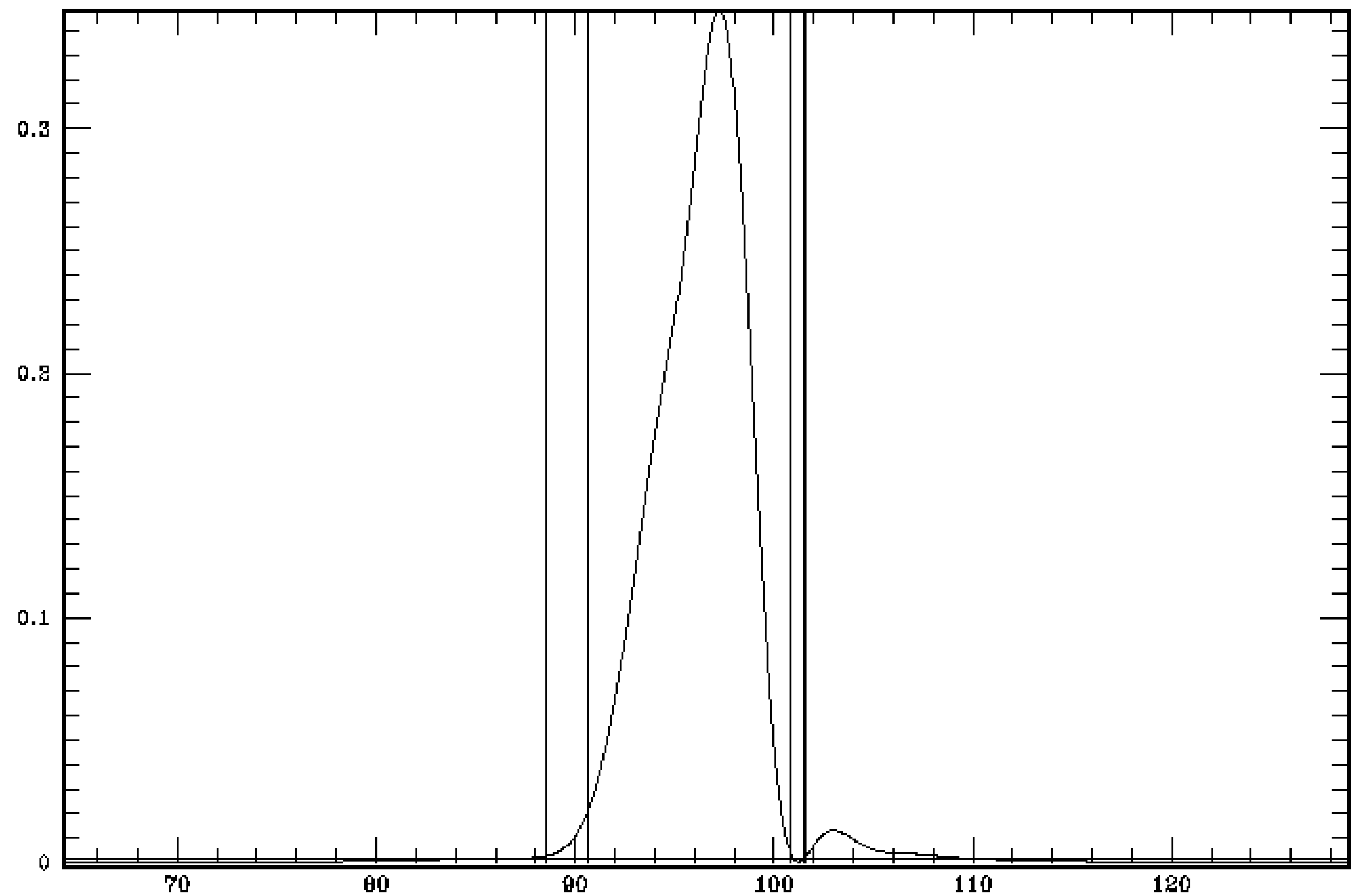
Example 2: PFV - FTAN and filters

Analysis of component of synthetic displacement:

Vertical component, **fundamental** mode of structure **P**



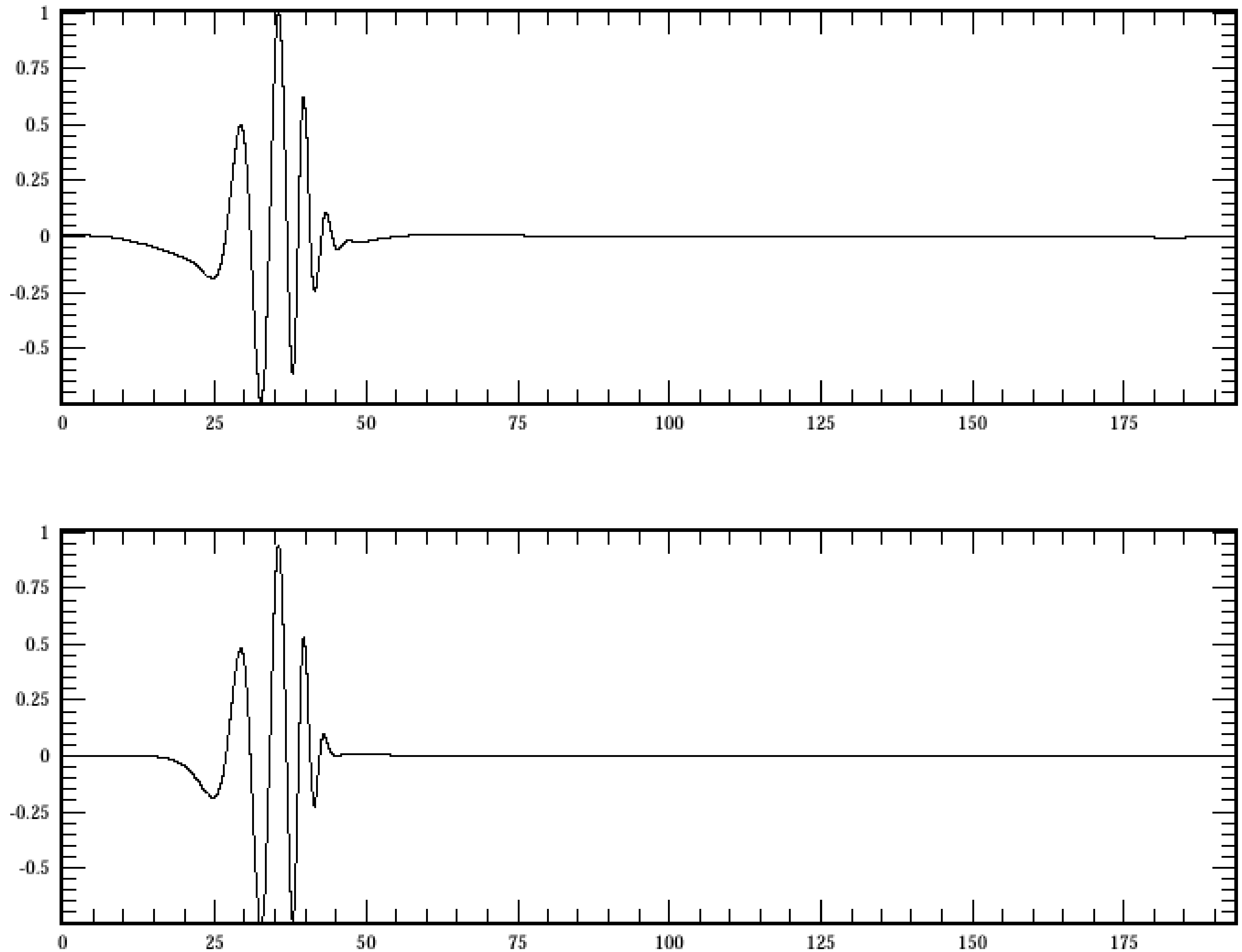
FTAN map and operator-defined dispersion curve, for phase equalization



Time domain: result of phase equalization and operator-defined time window

Example 2: PFV - results

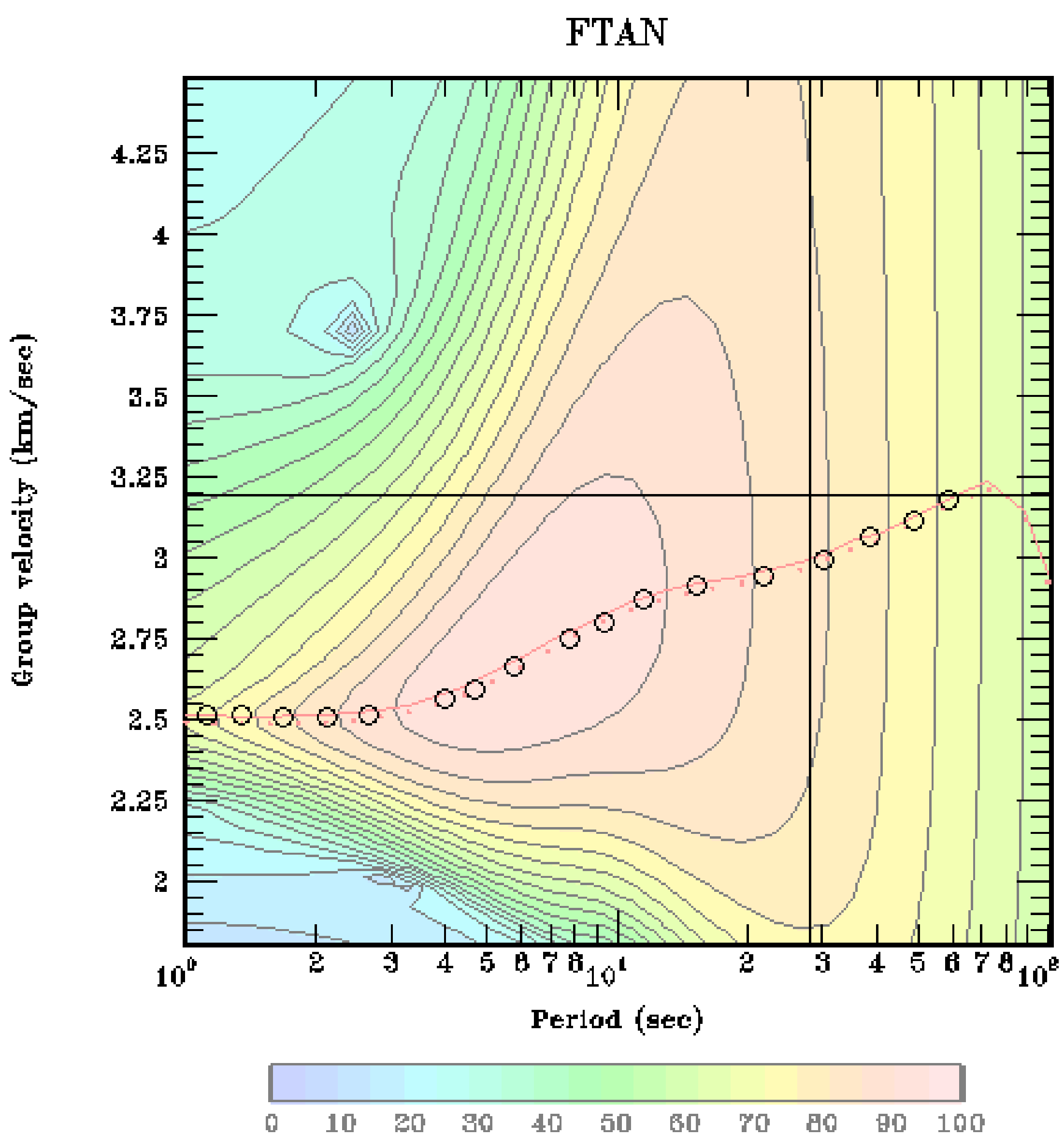
Analysis of component of synthetic displacement:
Vertical component, **fundamental** mode of structure P



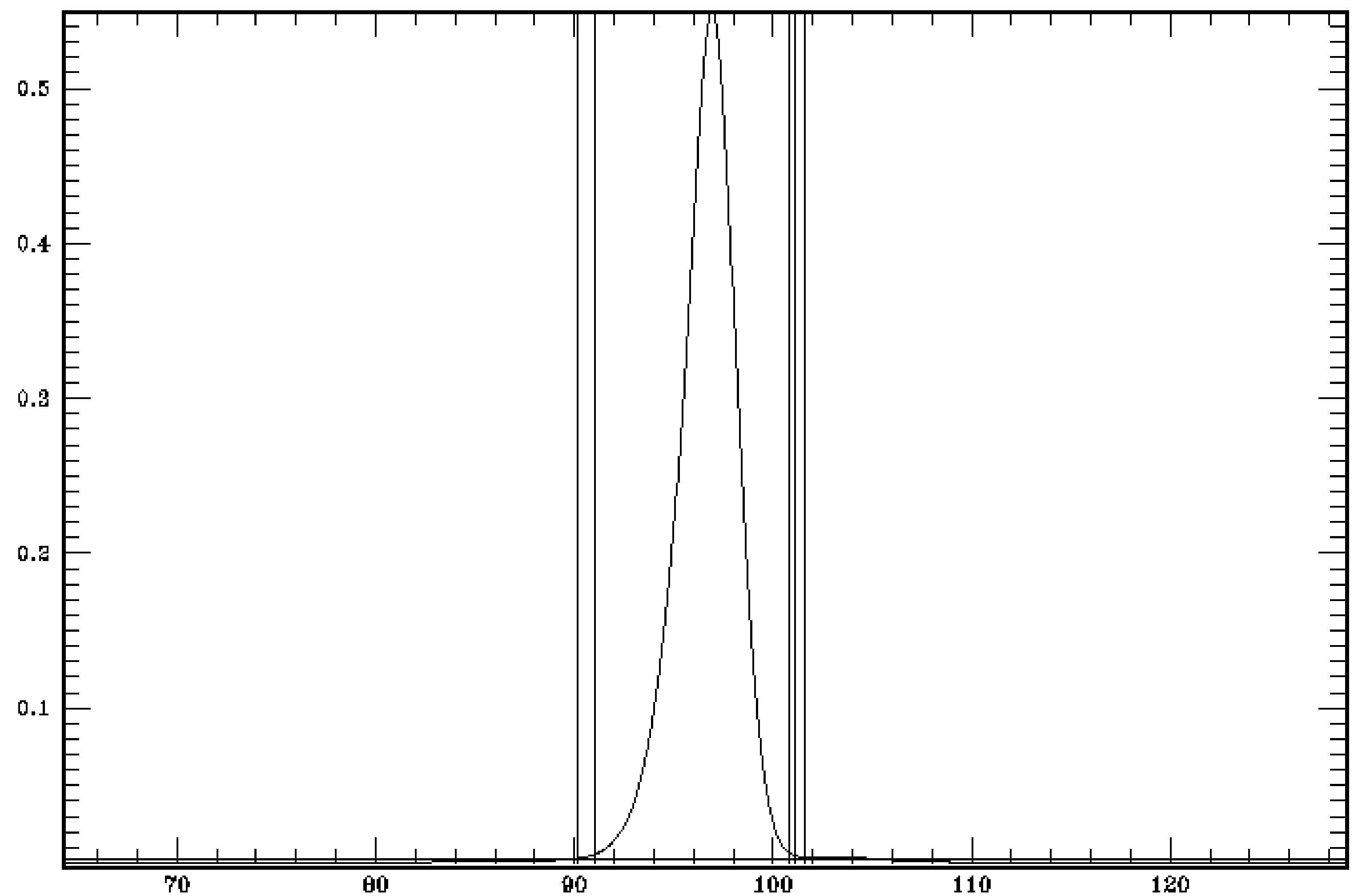
Input and processed signal in time domain

Example 3: PFT - FTAN and filters

Analysis of component of synthetic displacement:
Transverse component, **fundamental** mode of structure P



FTAN map and operator-defined dispersion curve, for phase equalization

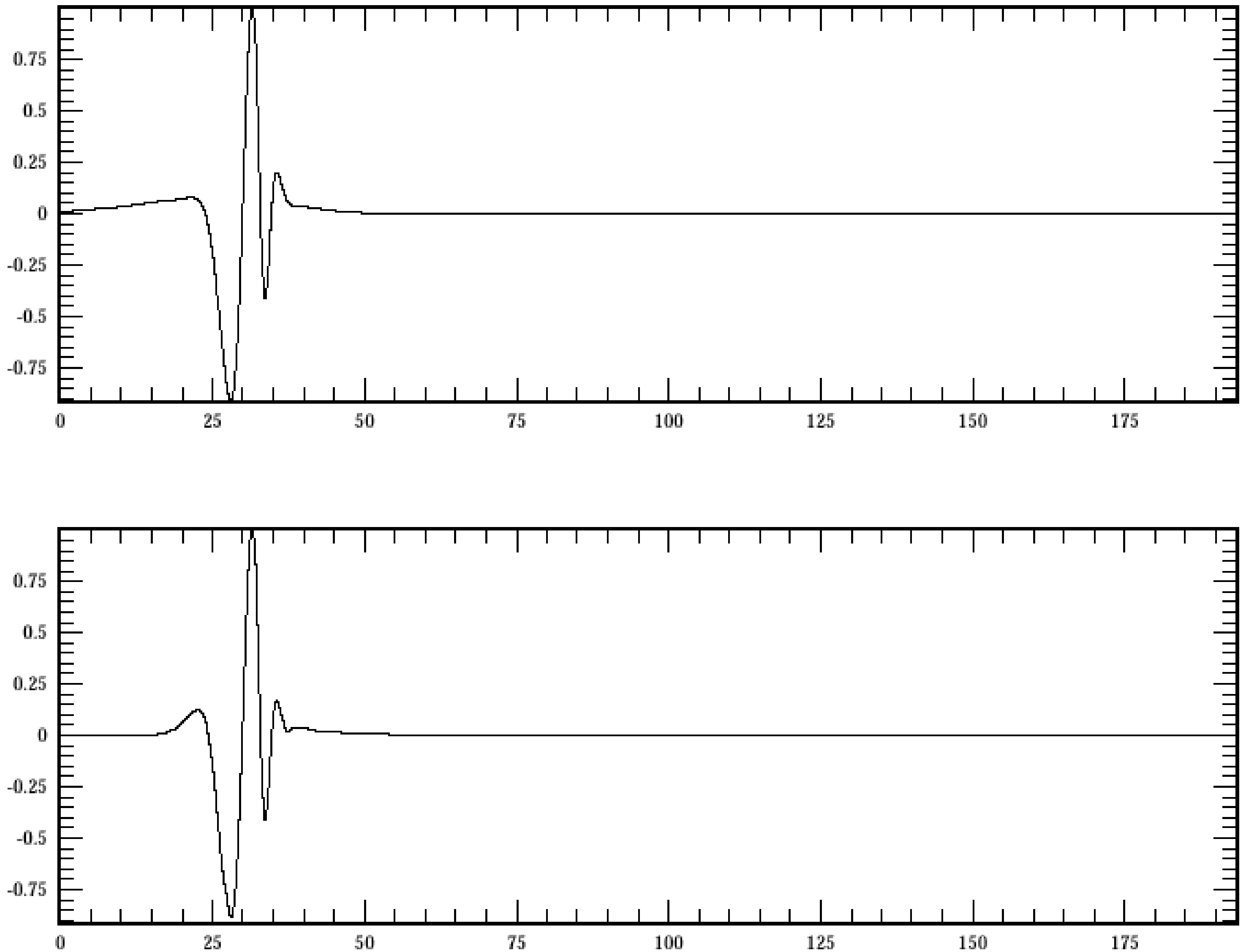


Time domain: result of phase equalization and operator-defined time window

Example 3: PFT - results

Analysis of component of synthetic displacement:

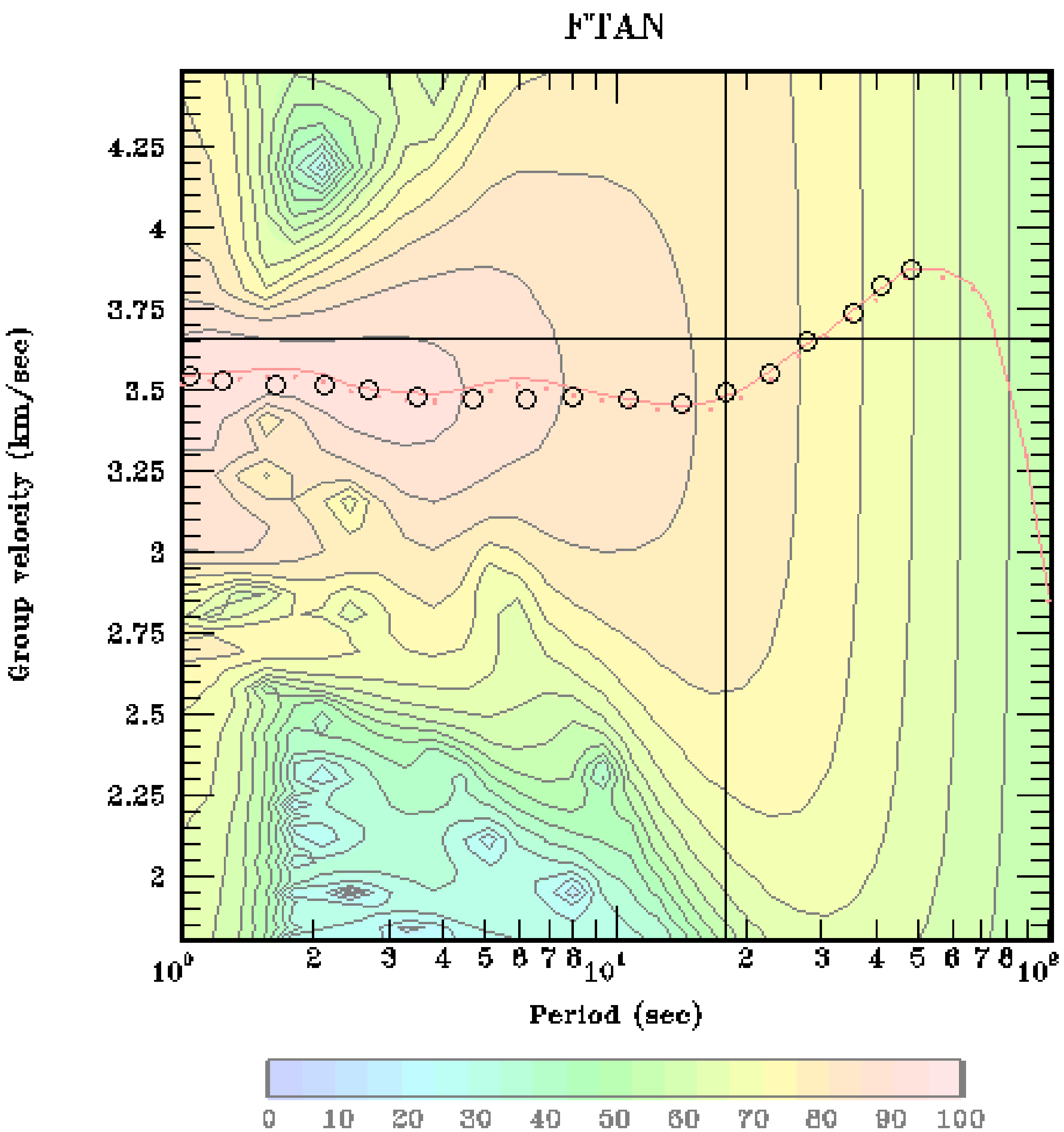
Transverse component, **fundamental** mode of structure **P**



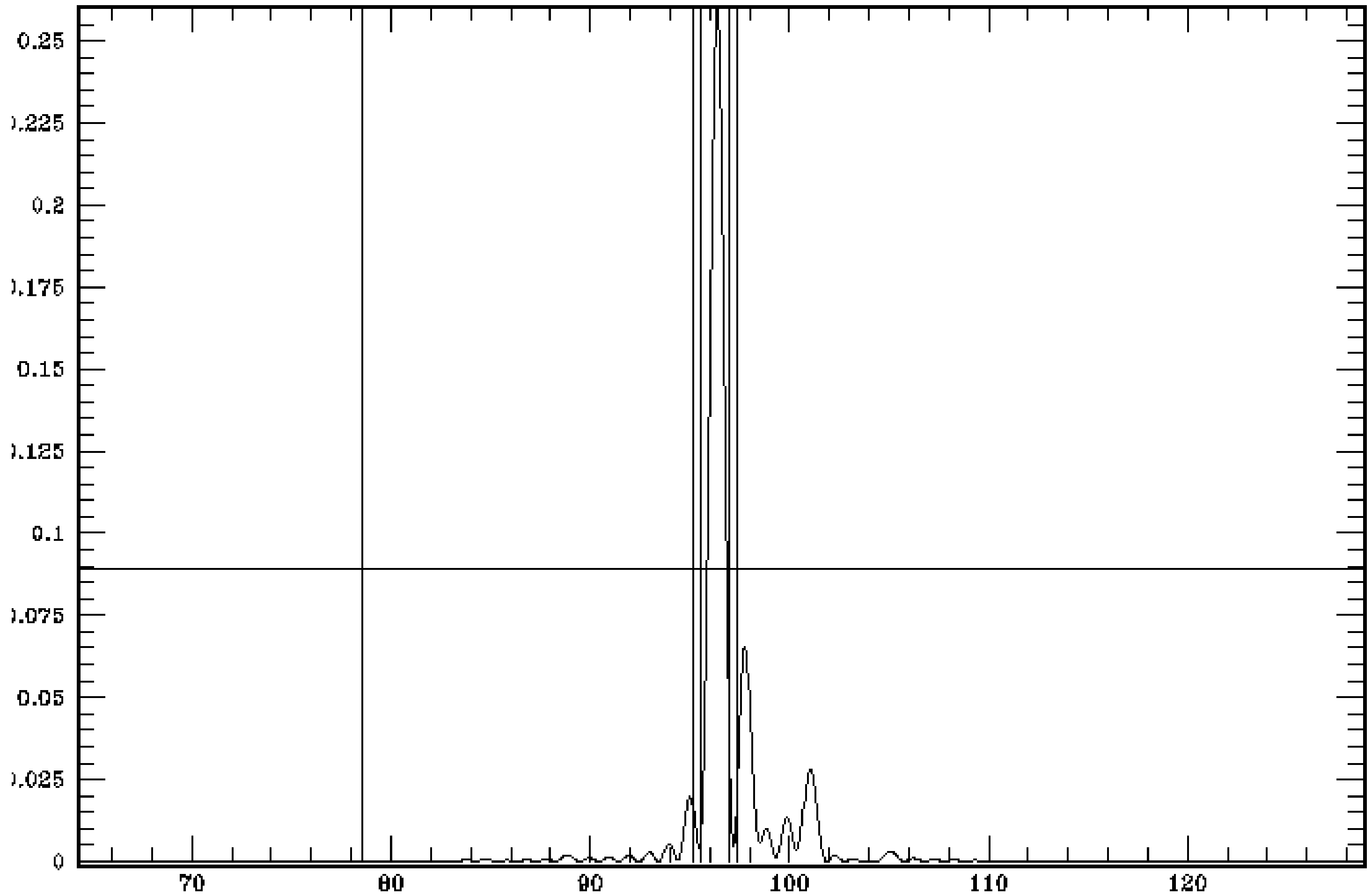
Input and processed signal in time domain

Example 4: CAT - FTAN and filters

Analysis of component of synthetic displacement:
Transverse component, **110** modes of structure ***C***



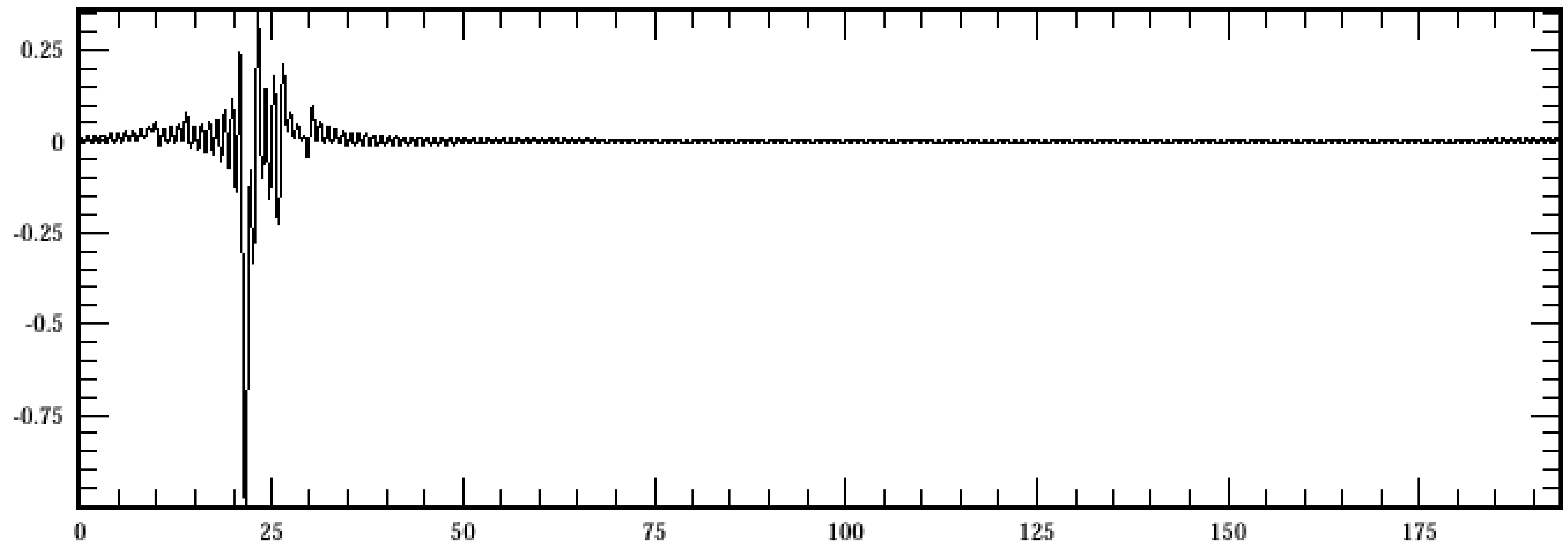
FTAN map and operator-defined dispersion curve, for phase equalization

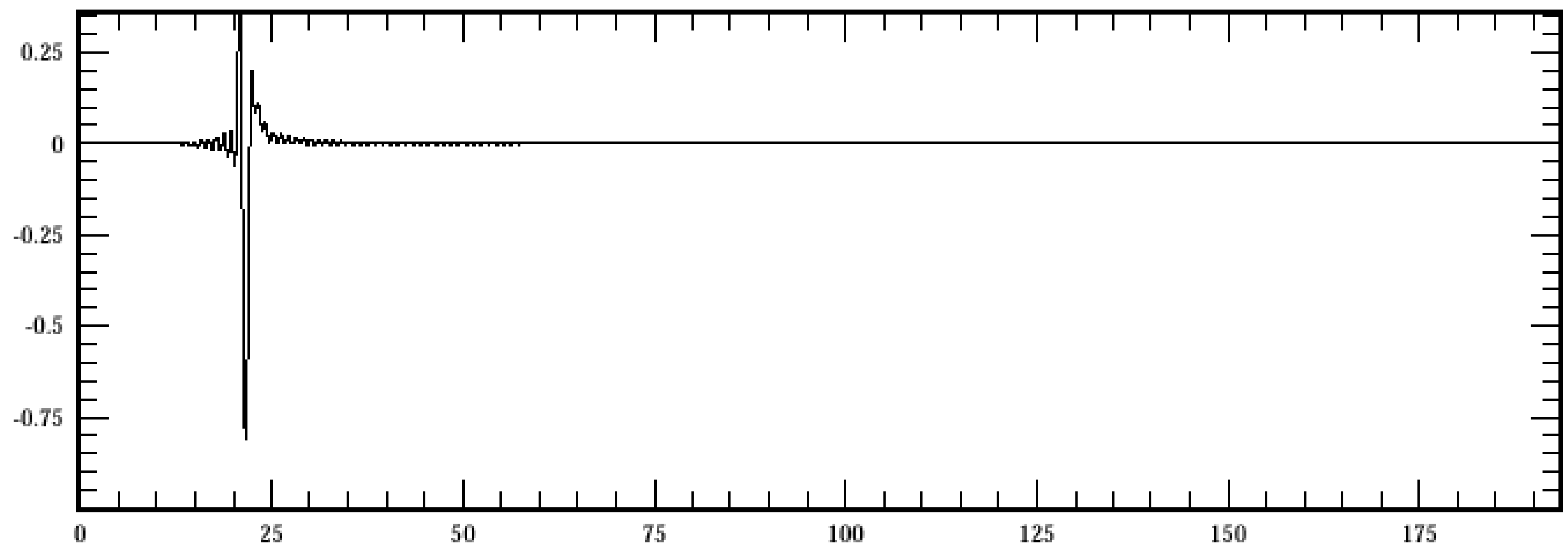


Time domain: result of phase equalization and operator-defined time window

Example 4: CAT - results

Analysis of component of synthetic displacement:
Transverse component, **110** modes of structure ***C***

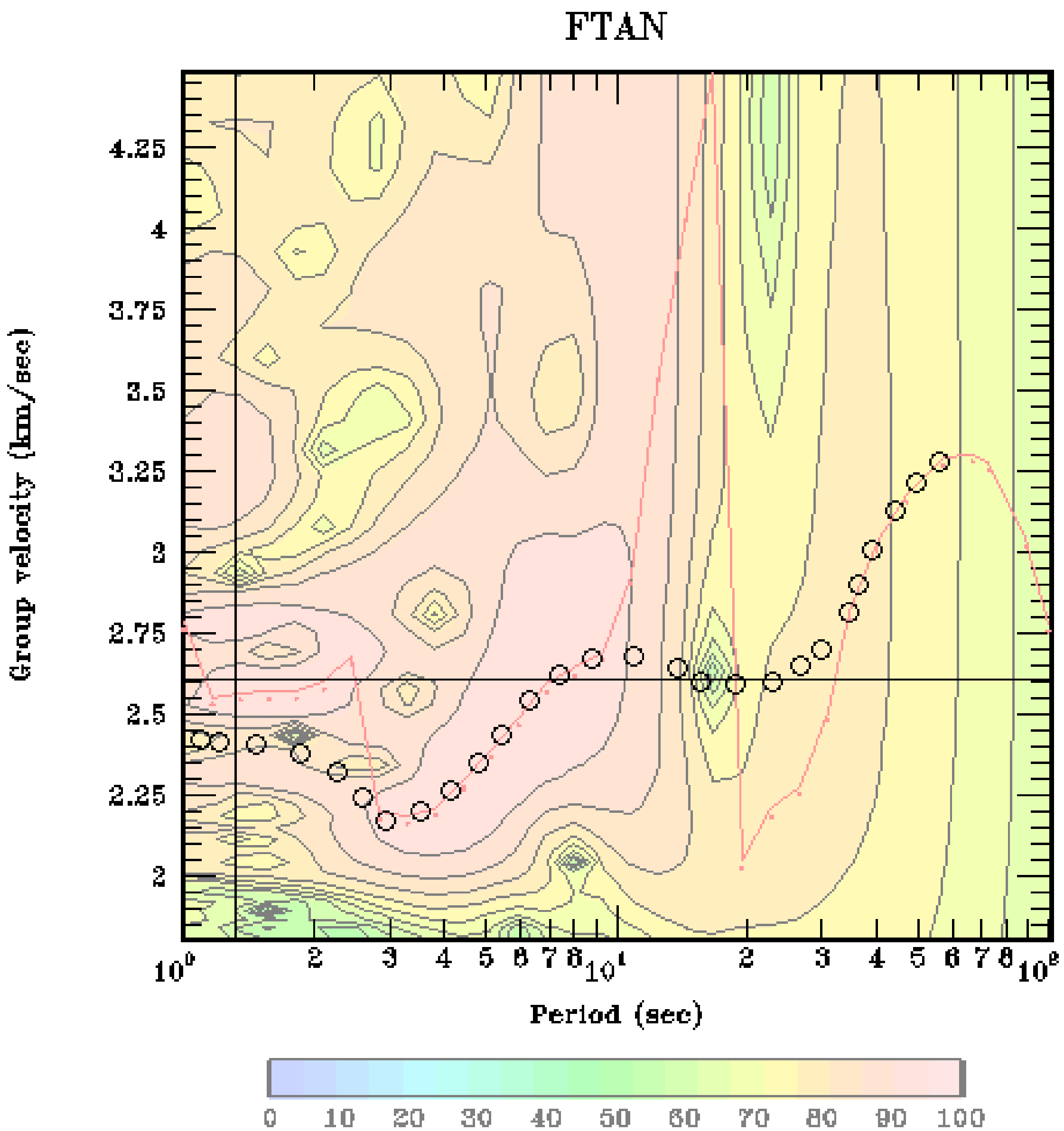




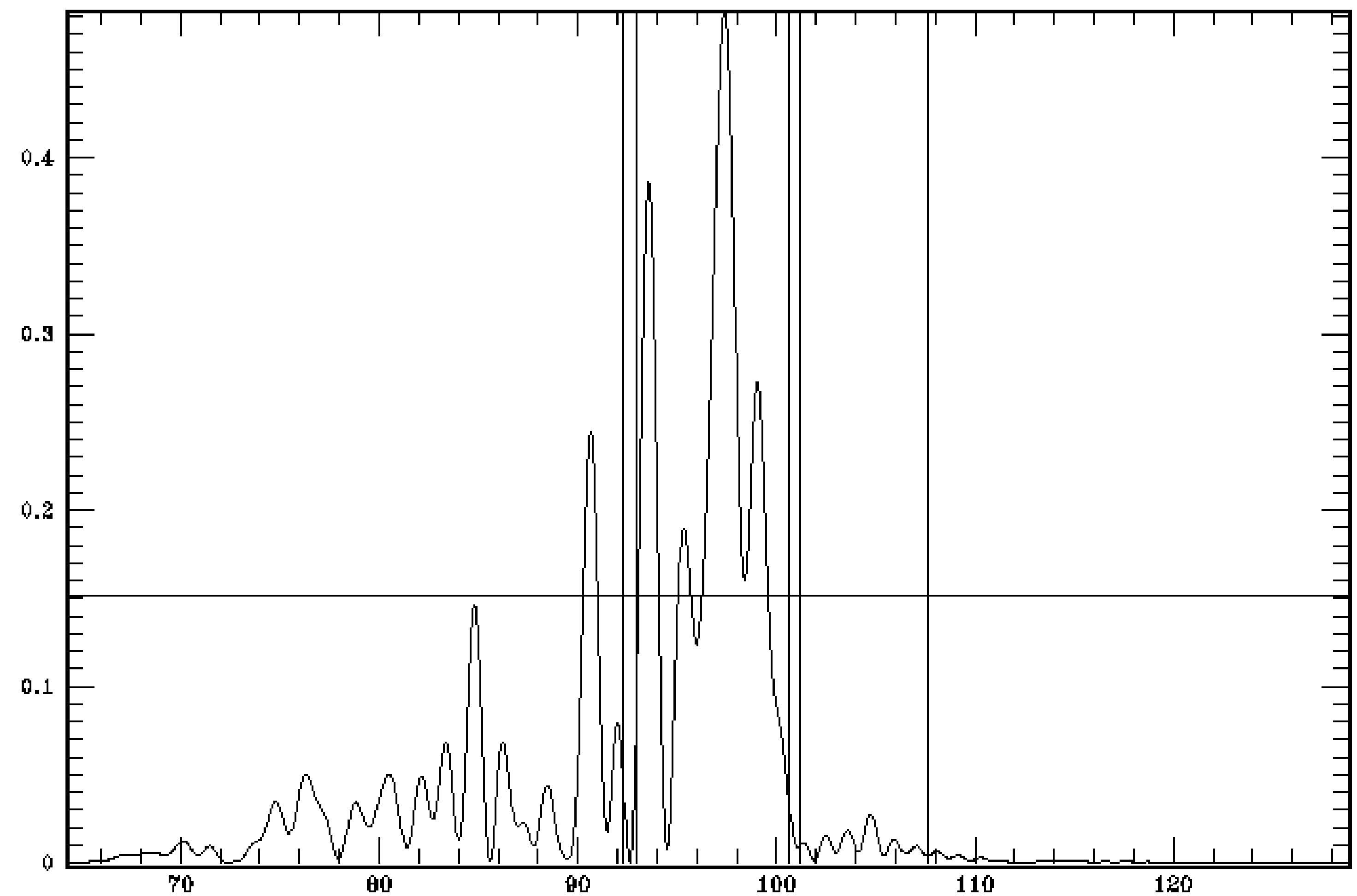
Input and processed signal in time domain

Example 5: PAR - FTAN and filters

Analysis of component of synthetic displacement:
Radial component, **110** modes of structure P



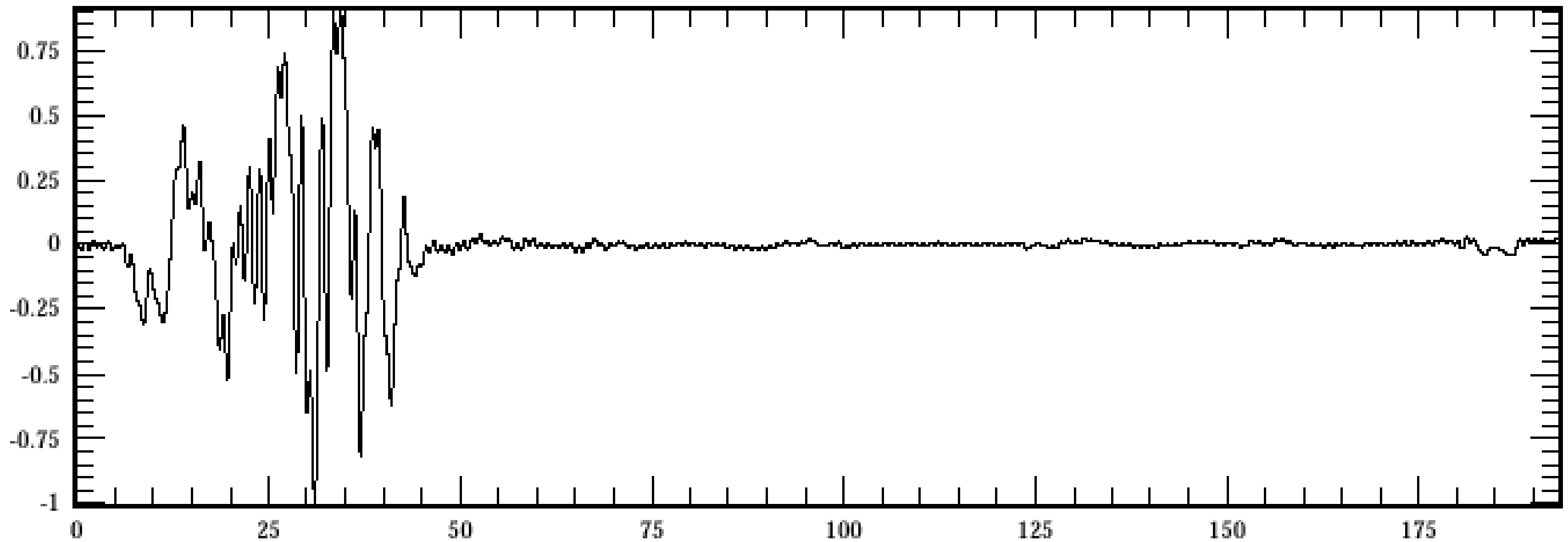
FTAN map and operator-defined dispersion curve, for phase equalization

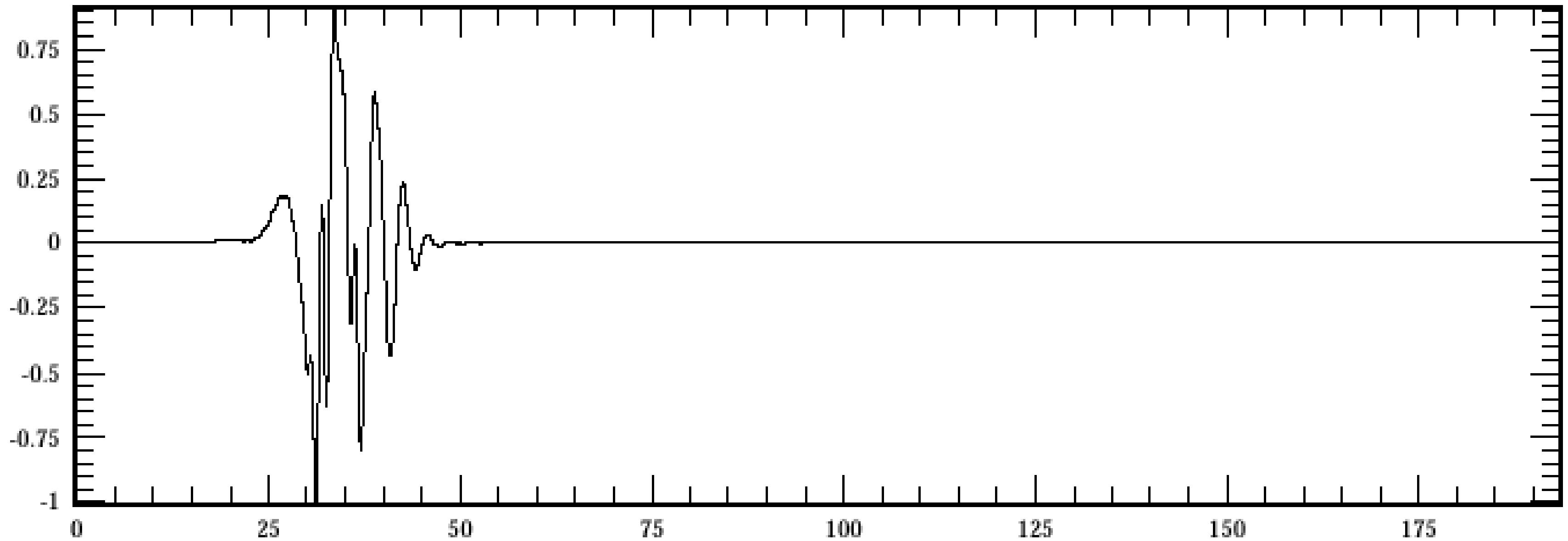


Time domain: result of phase equalization and operator-defined time window

Example 5: PAR - results

Analysis of component of synthetic displacement:
Radial component, **110** modes of structure **P**

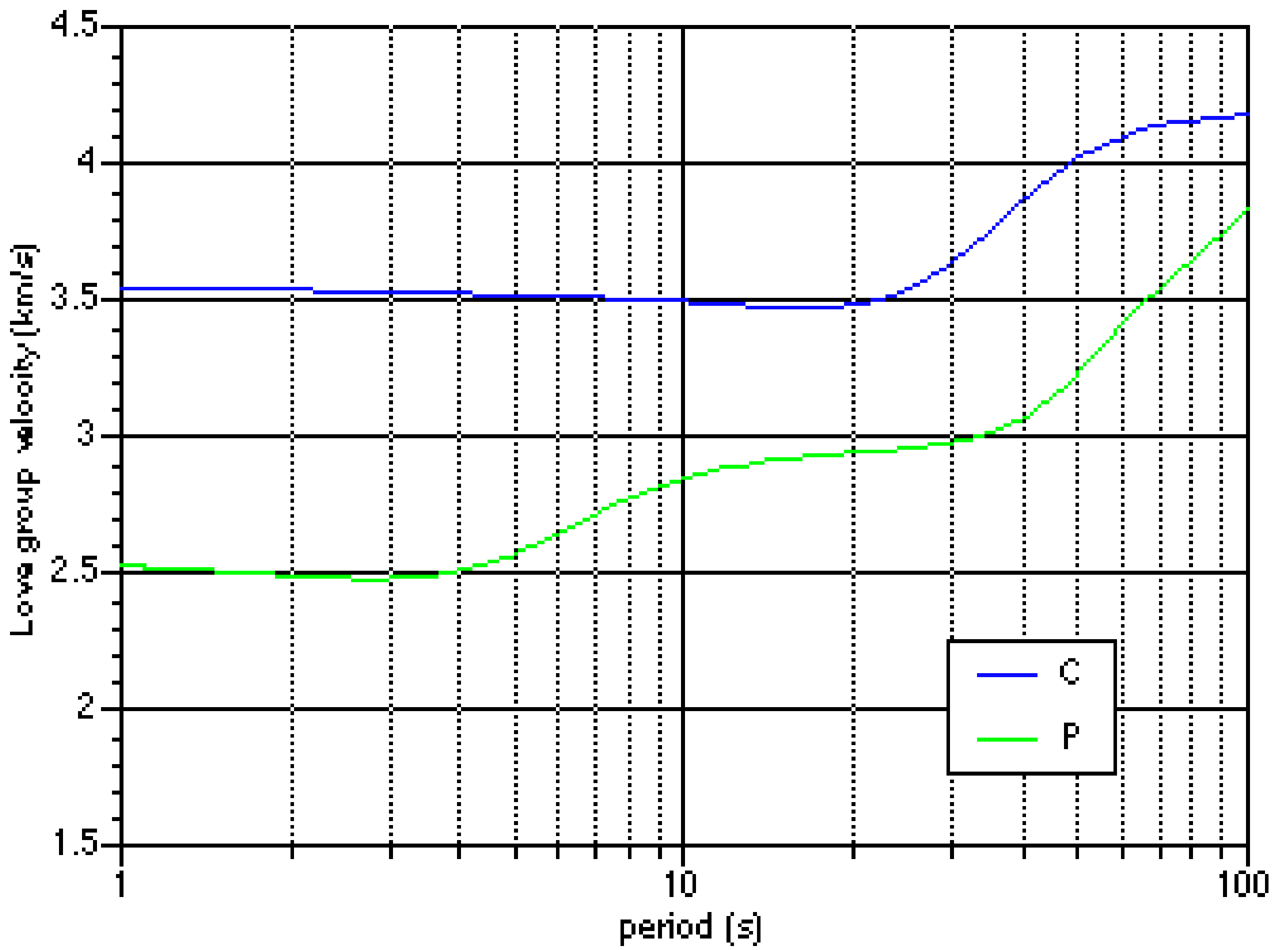


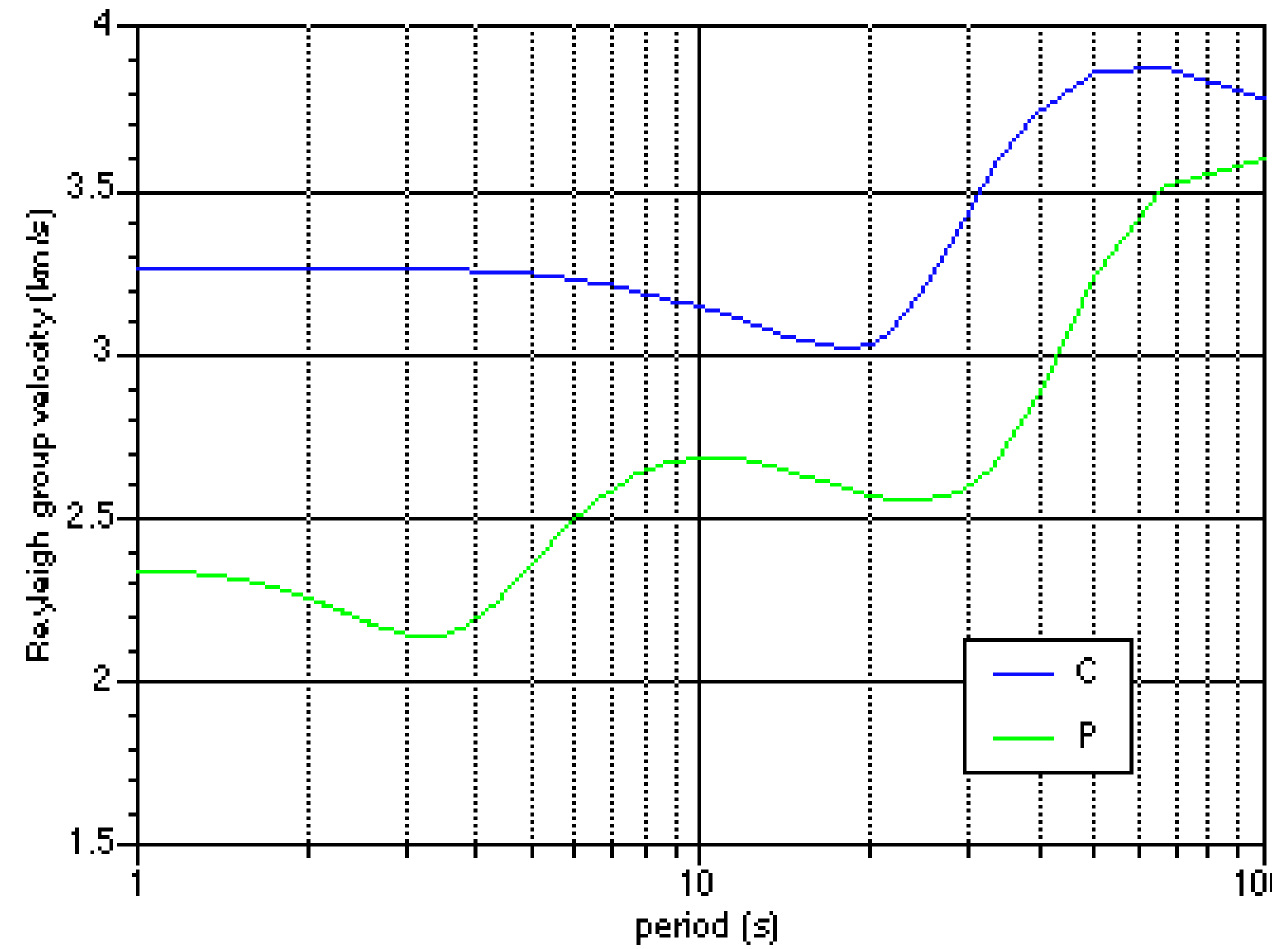


Input and processed signal in time domain

Considerations - 1

The results obtained so far show the power of FTAN and floating filtering approach, but in these theoretical tests we are guided by the "real" dispersion curves of the fundamental modes of structures C and P. Contribution from **higher modes** can be important in FTAN maps (see results from structure P) making the analysis more difficult.

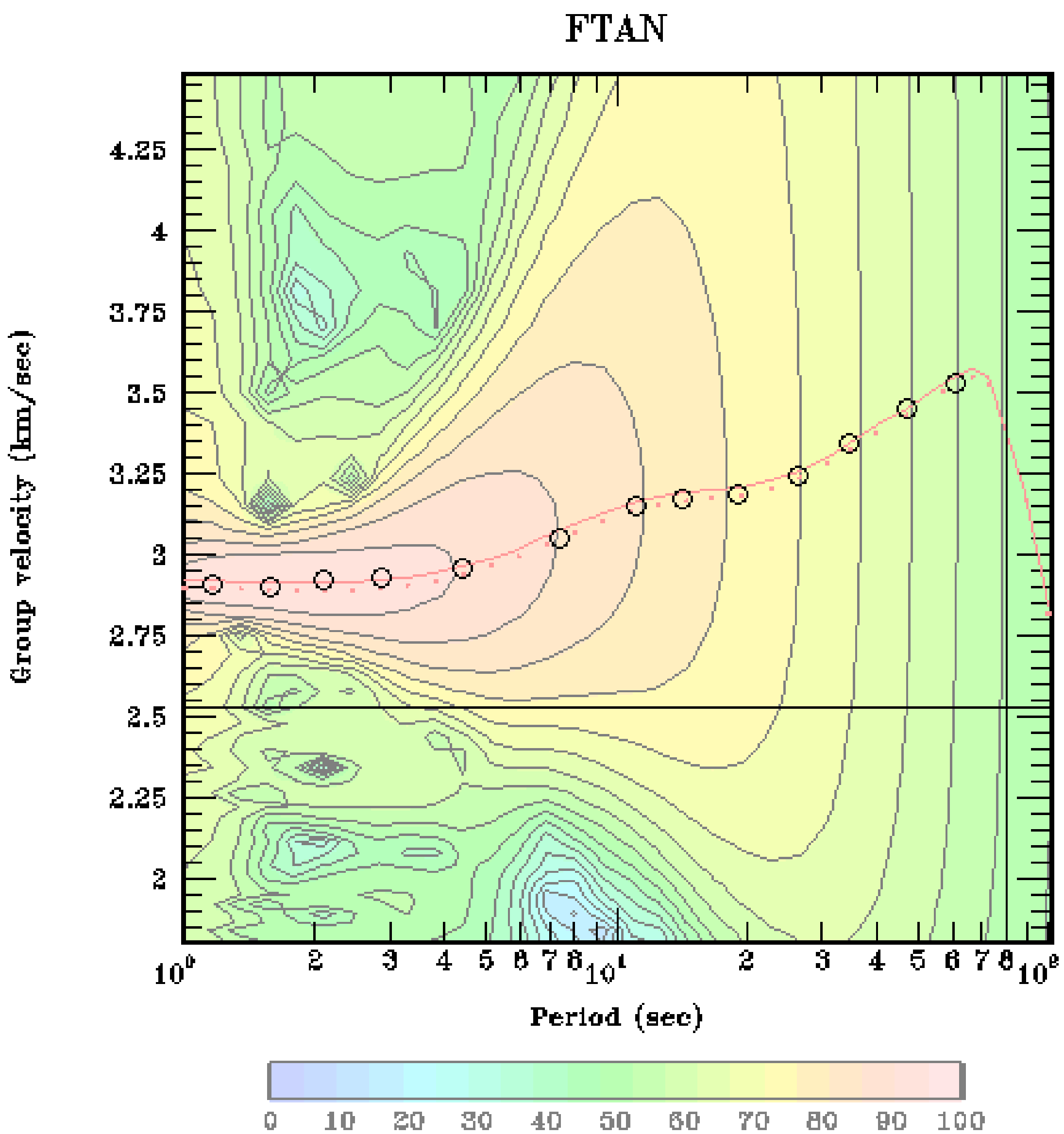




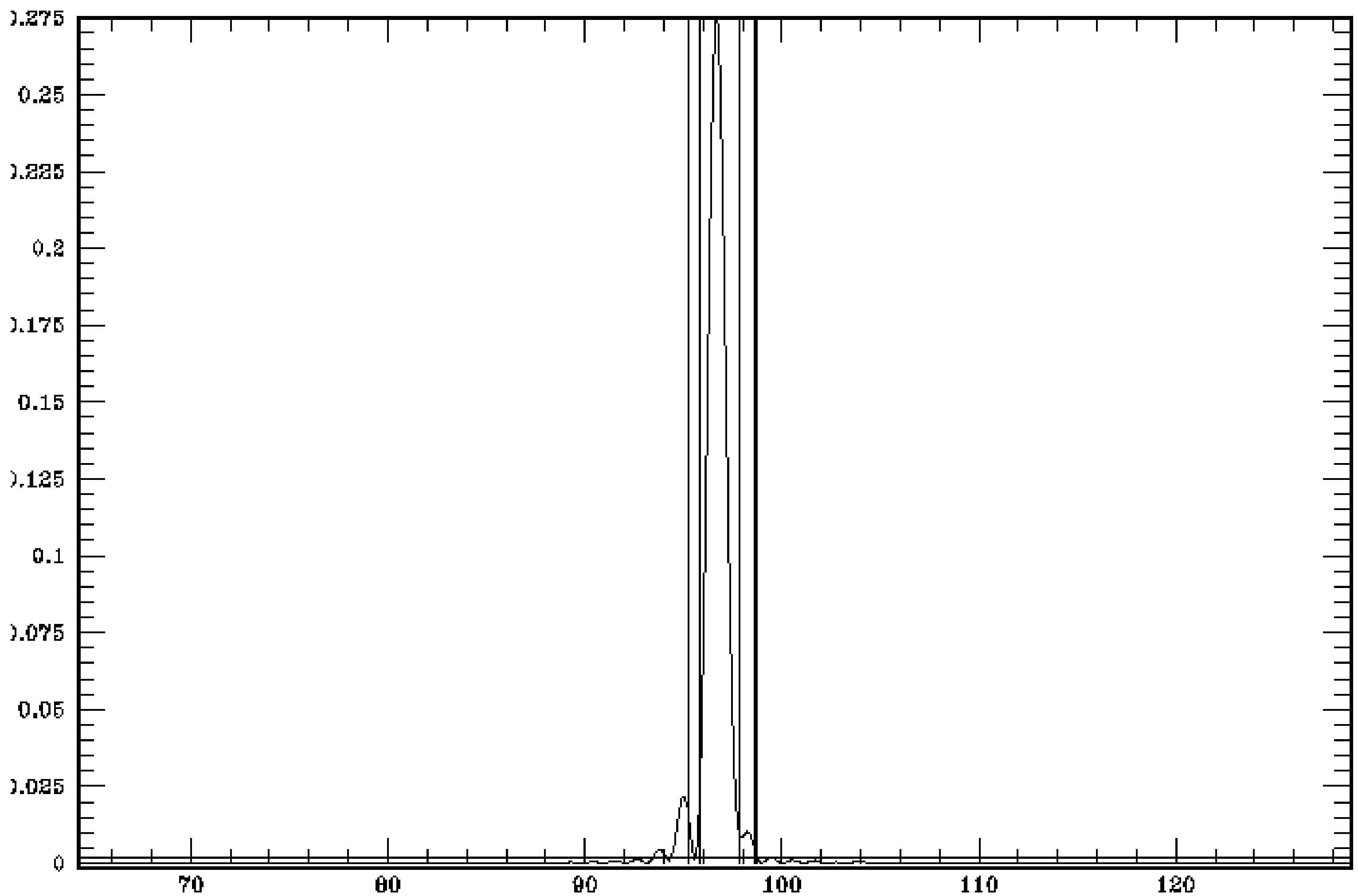
Staying in a synthetic analysis what happens if the signals are propagating in a **2D structure**? FTAN analysis allows to find the dispersion curve of the 1D equivalent model, that represents a sort of "average" of the dispersion properties of the various models.
(The results have to be compared with the signals shown in Application_1.pdf)

Example 6: CPFT - FTAN and filters

Analysis of component of synthetic displacement:
Transverse component, **fundamental** mode of structure **CP**



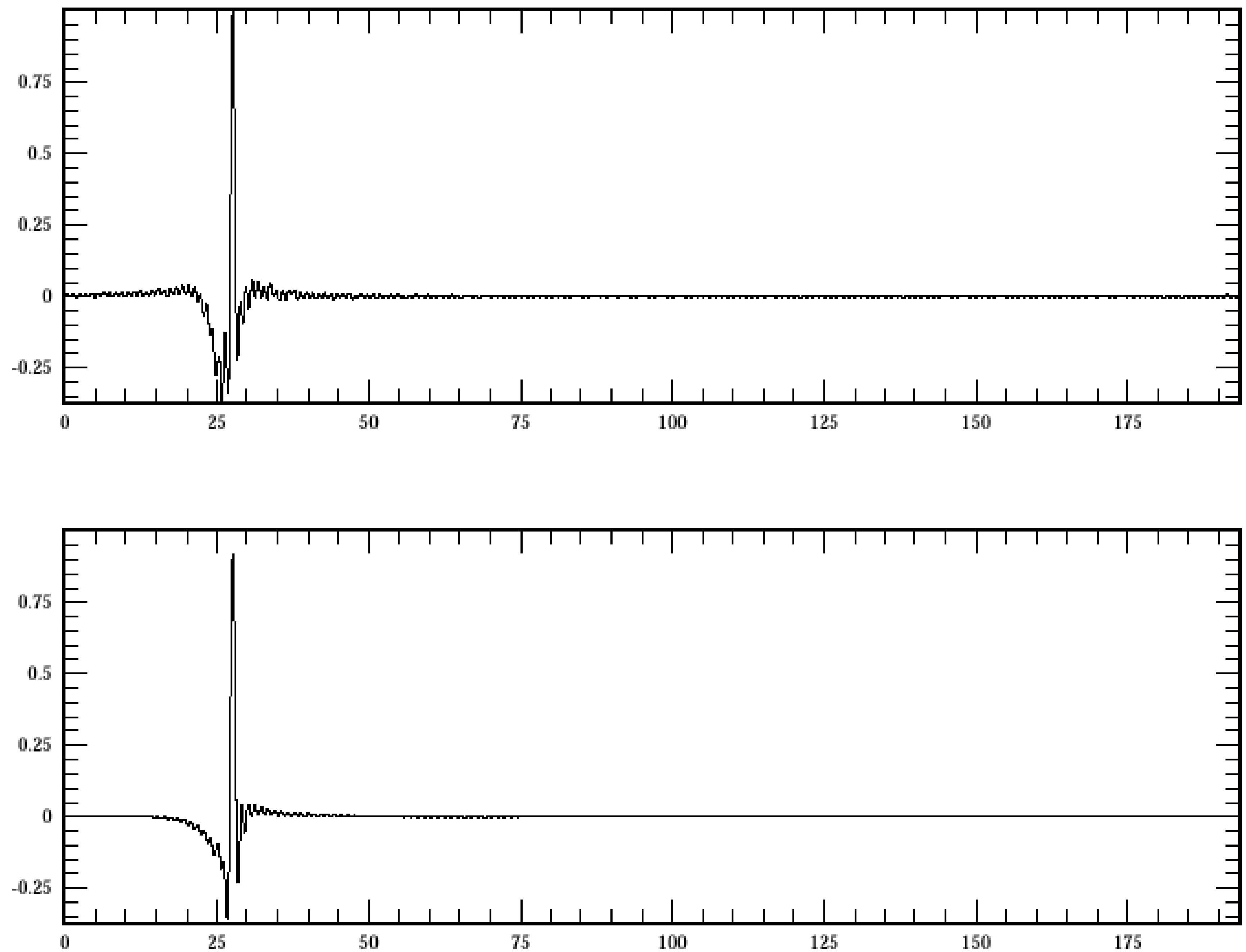
FTAN map and operator-defined dispersion curve, for phase equalization



Time domain: result of phase equalization and operator-defined time window

Example 6: CPFT - results

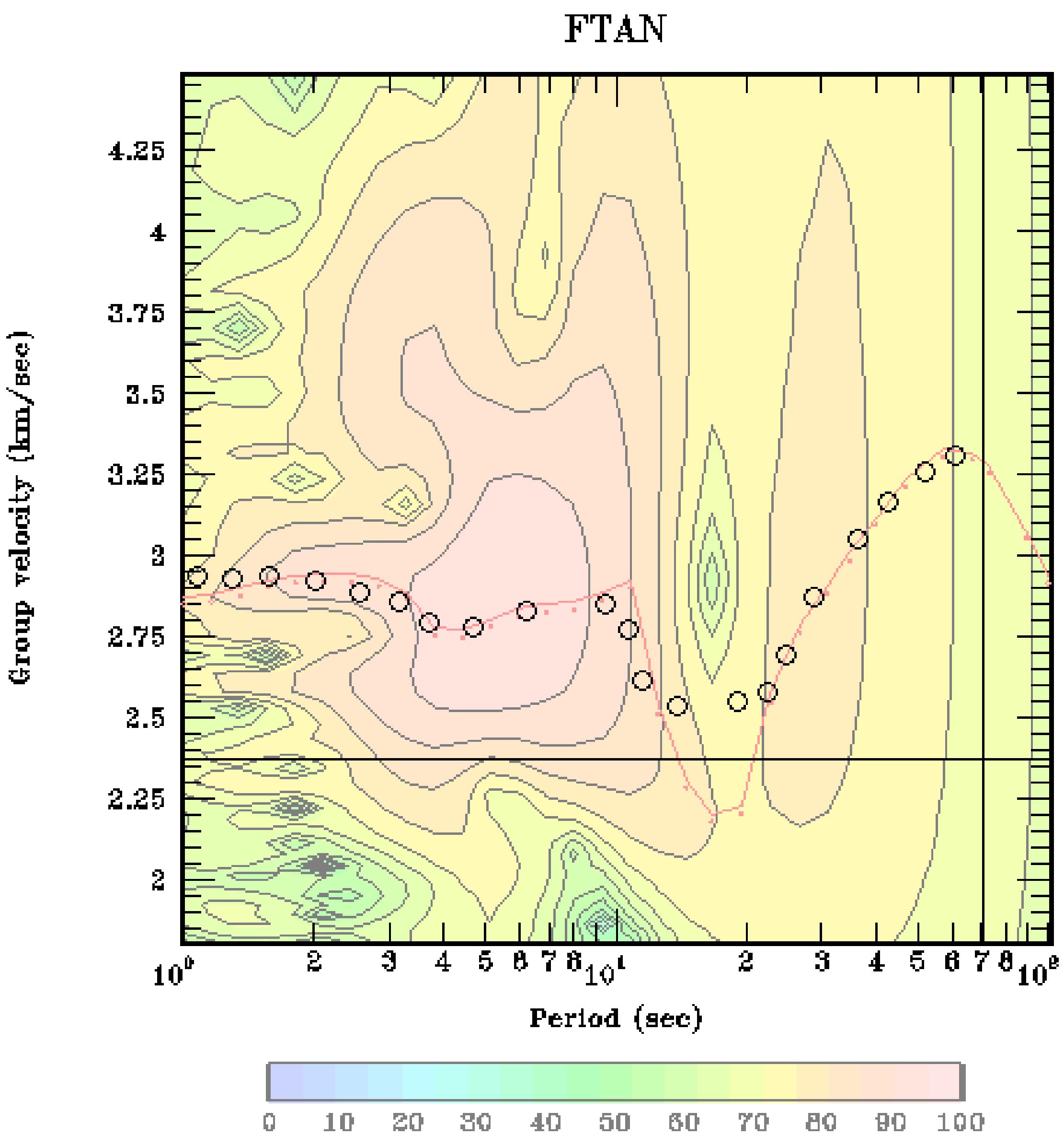
Analysis of component of synthetic displacement:
Transverse component, **fundamental** mode of structure ***CP***



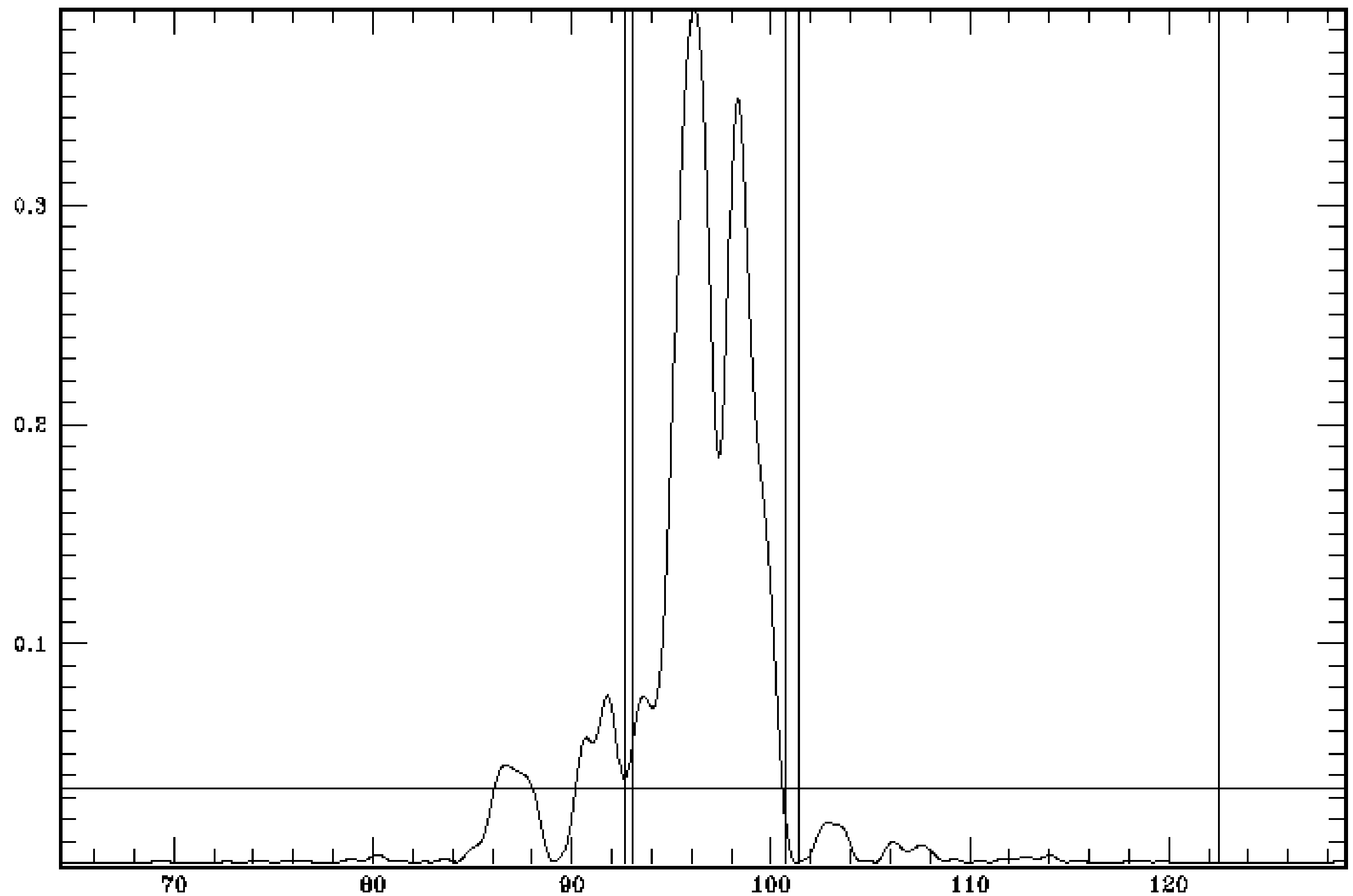
Input and processed signal in time domain

Example 7: PCAV - FTAN and filters

Analysis of component of synthetic displacement:
Vertical component, **110** modes of structure **PC**



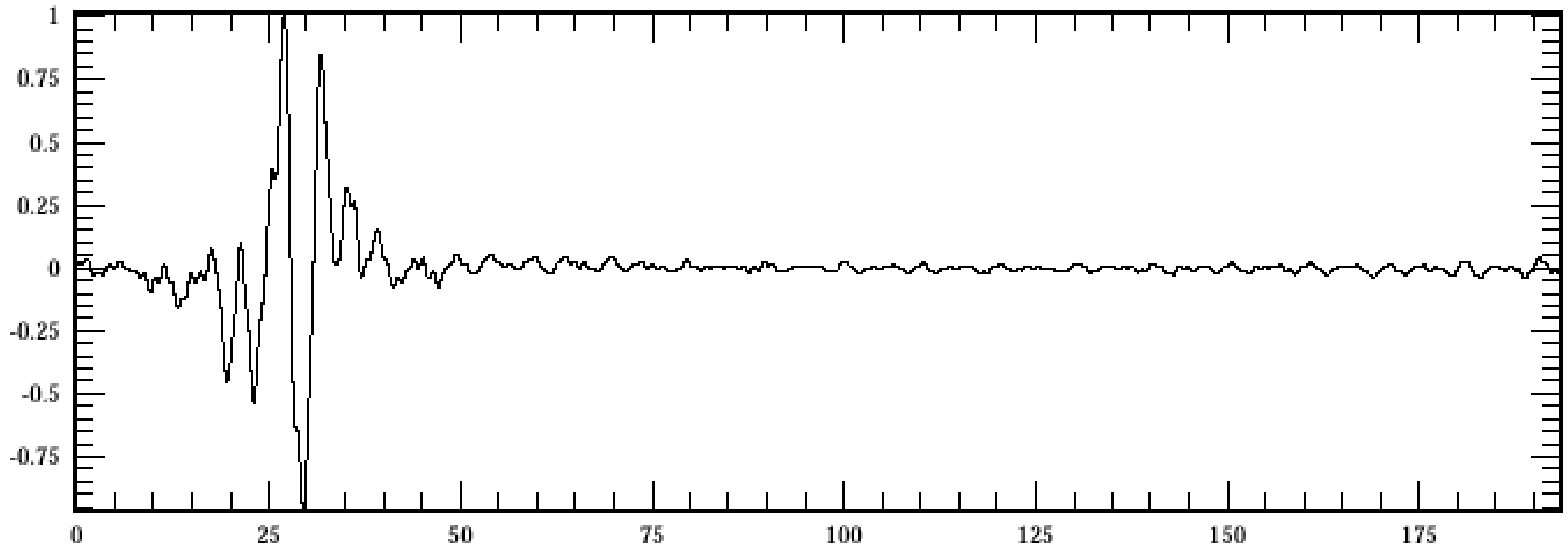
FTAN map and operator-defined dispersion curve, for phase equalization

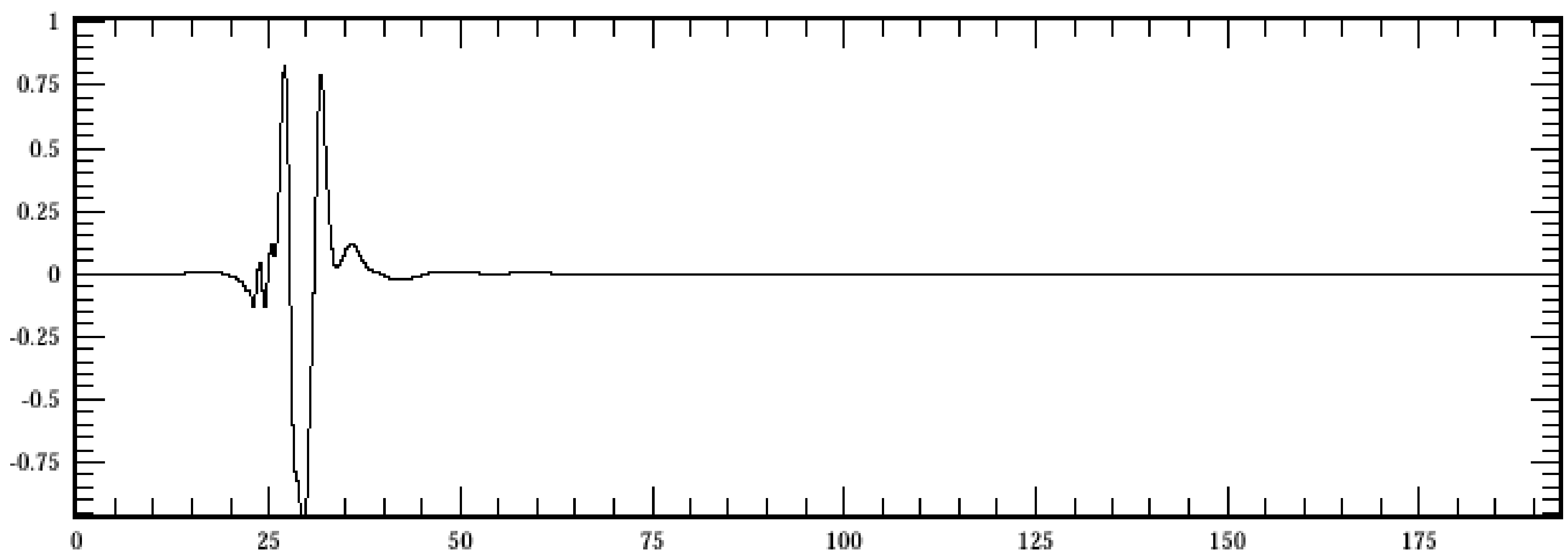


Time domain: result of phase equalization and operator-defined time window

Example 7: PCAV - results

Analysis of component of synthetic displacement:
Vertical component, **110** modes of structure **PC**





Input and processed signal in time domain

Considerations - 2

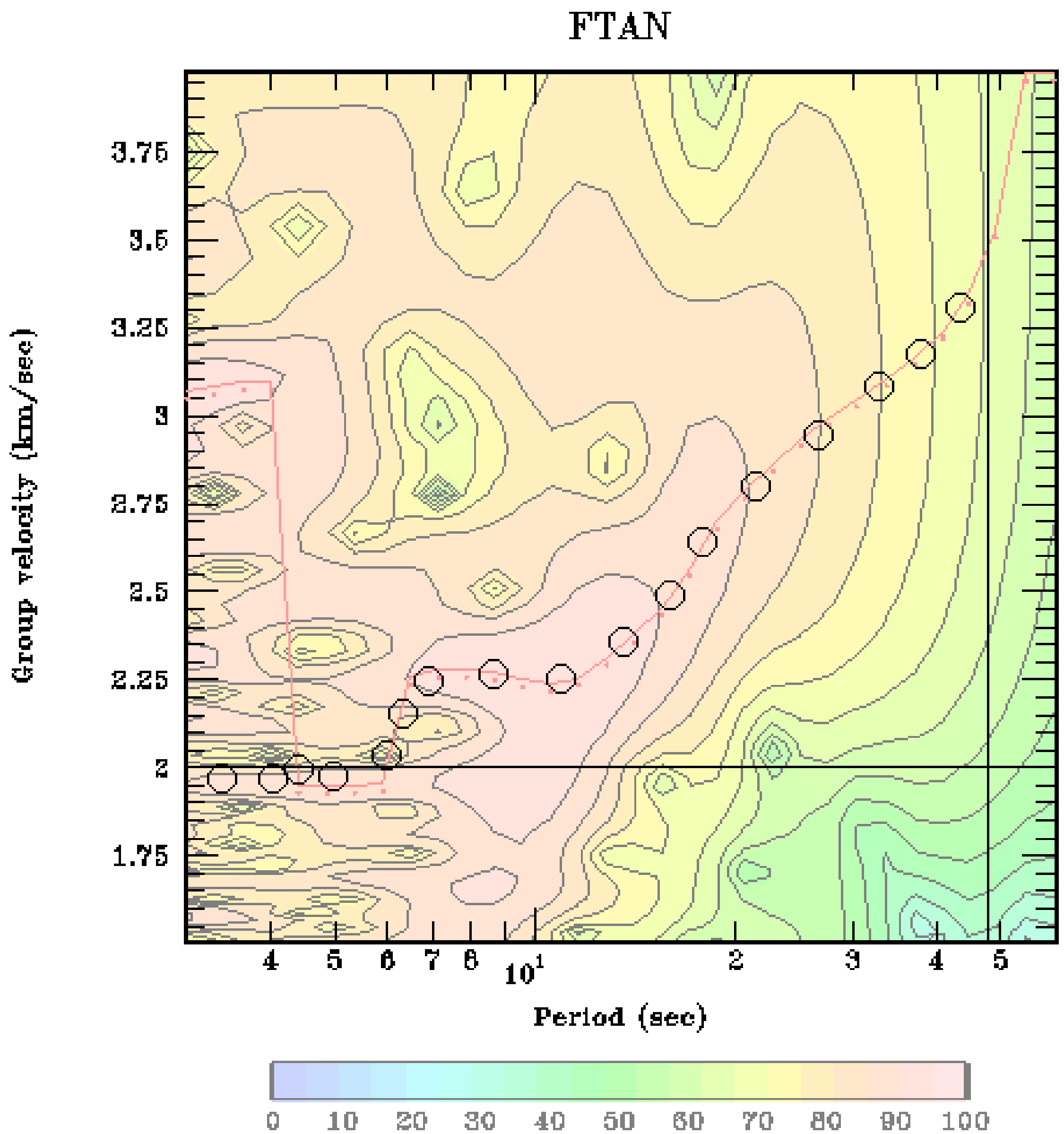
When **real data** has to be examined the situation is much more complicated.

The dispersion properties of a signal may depend on:
the **propagation path**, i.e. the distance traveled and the mechanic characteristics (attenuation and heterogeneity) of the medium,
the **frequency range** to be considered (bandpass filtering may be required) compared to the wavelength of the seismic wavetrain.

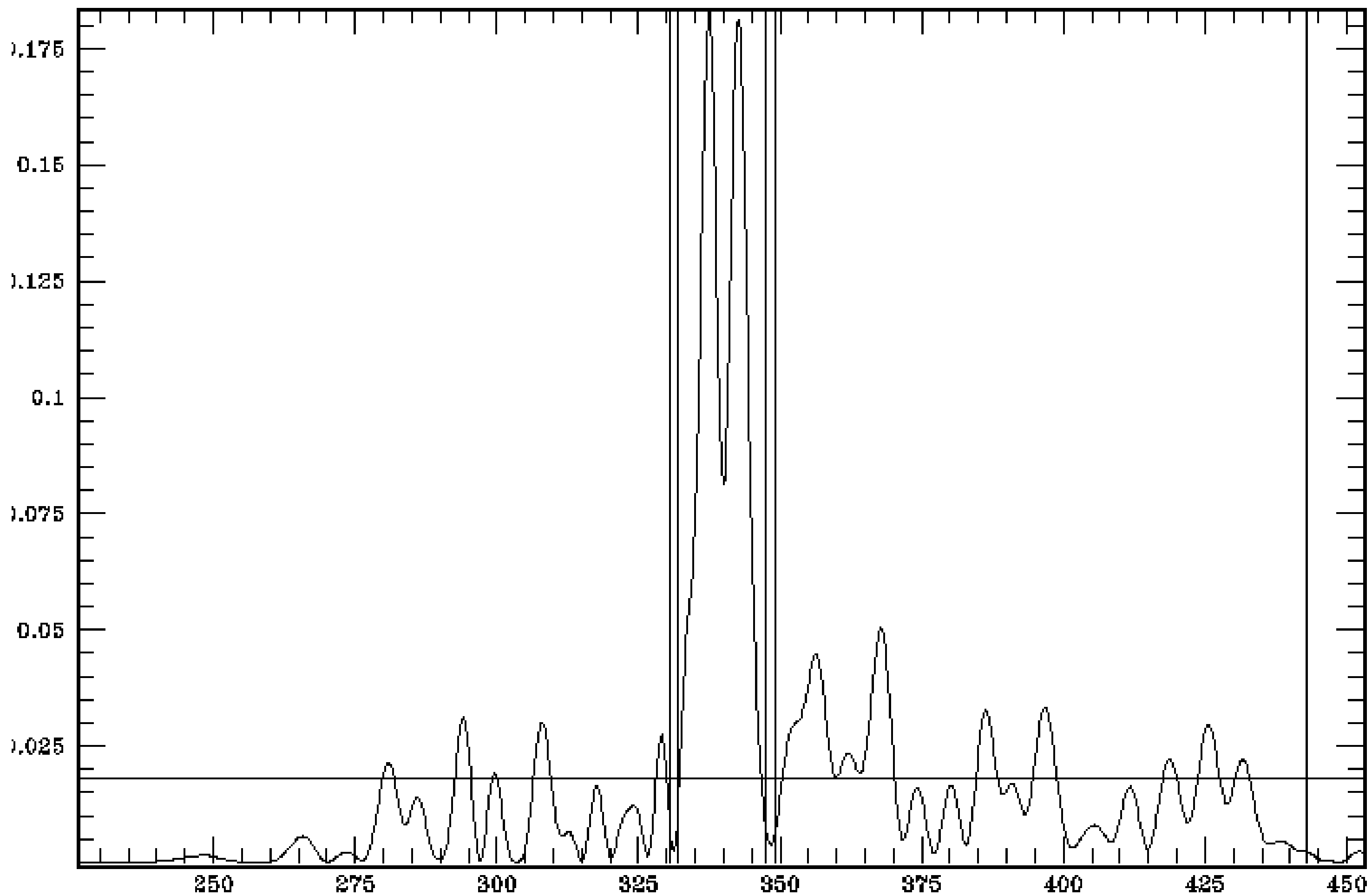
Reprocessing of the signal, i.e. multiple FTAN maps with floating filtering, may improve the Signal/Noise ratio, where the Signal definition depends on the operator choice, i.e. if he is interested in highlighting the contribution of fundamental or higher modes arrivals to energy packets (different velocity ranges).

Example 8: Far event - FTAN and

Analysis of component of recorded displacement:
RADIAL component, event recorded at **311 km**.

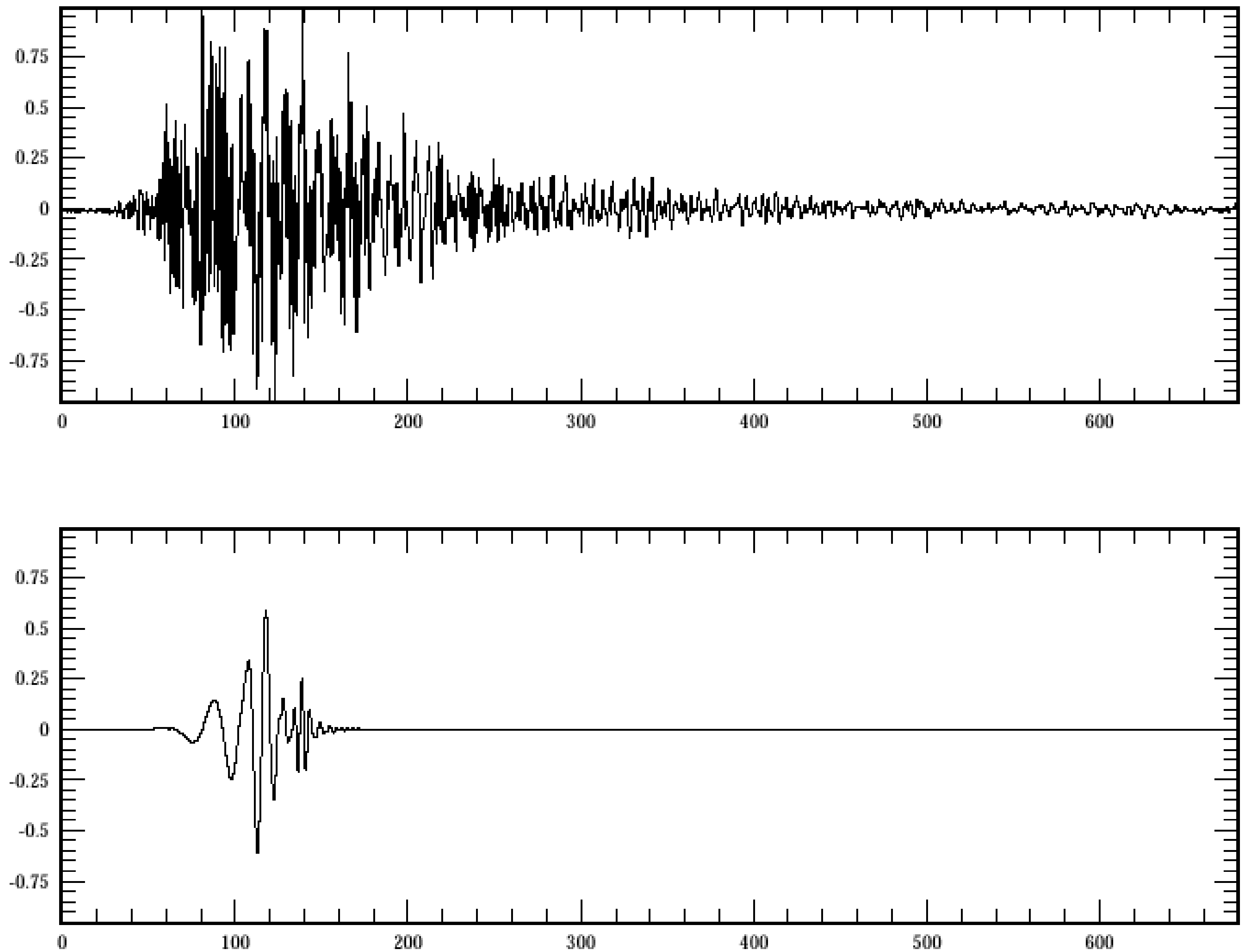


FTAN map and operator-defined dispersion curve, for phase equalization



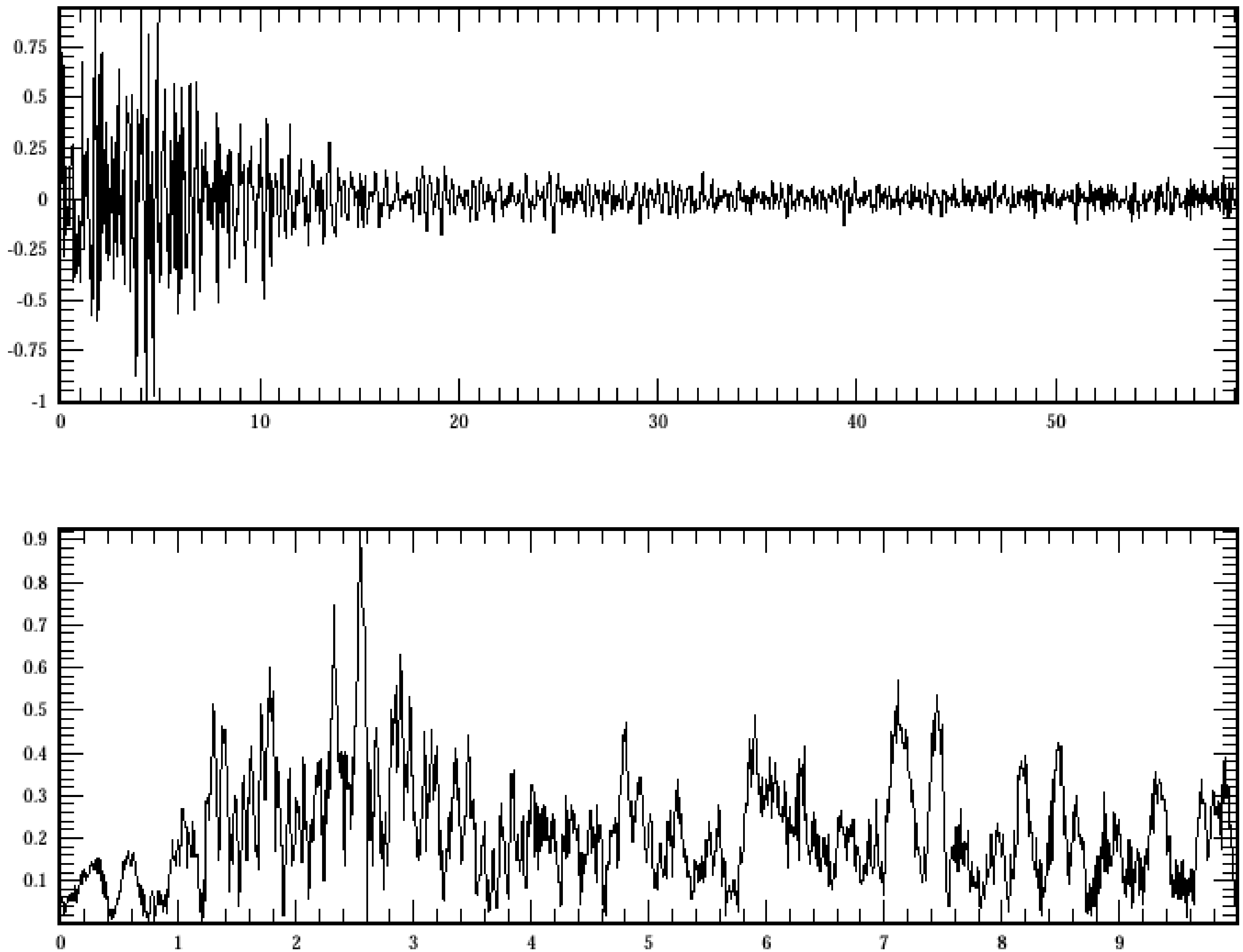
Time domain: result of phase equalization and operator-defined time window

Analysis of component of recorded displacement:
RADIAL component, event recorded at **311 km**.



Input and processed signal in time domain

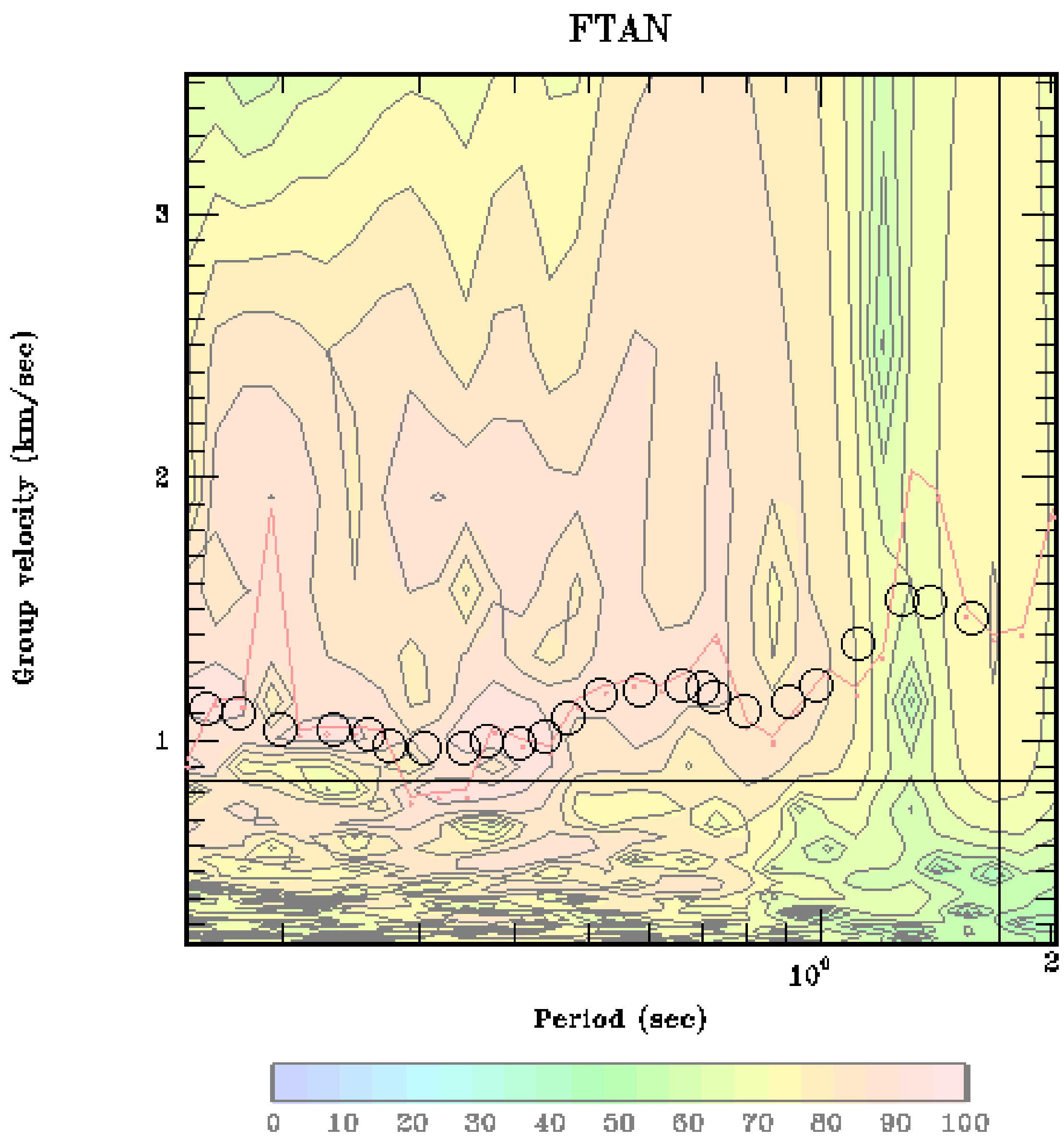
Analysis of component of recorded displacement:
TRANSVERSE component, event recorded at **8 km**.



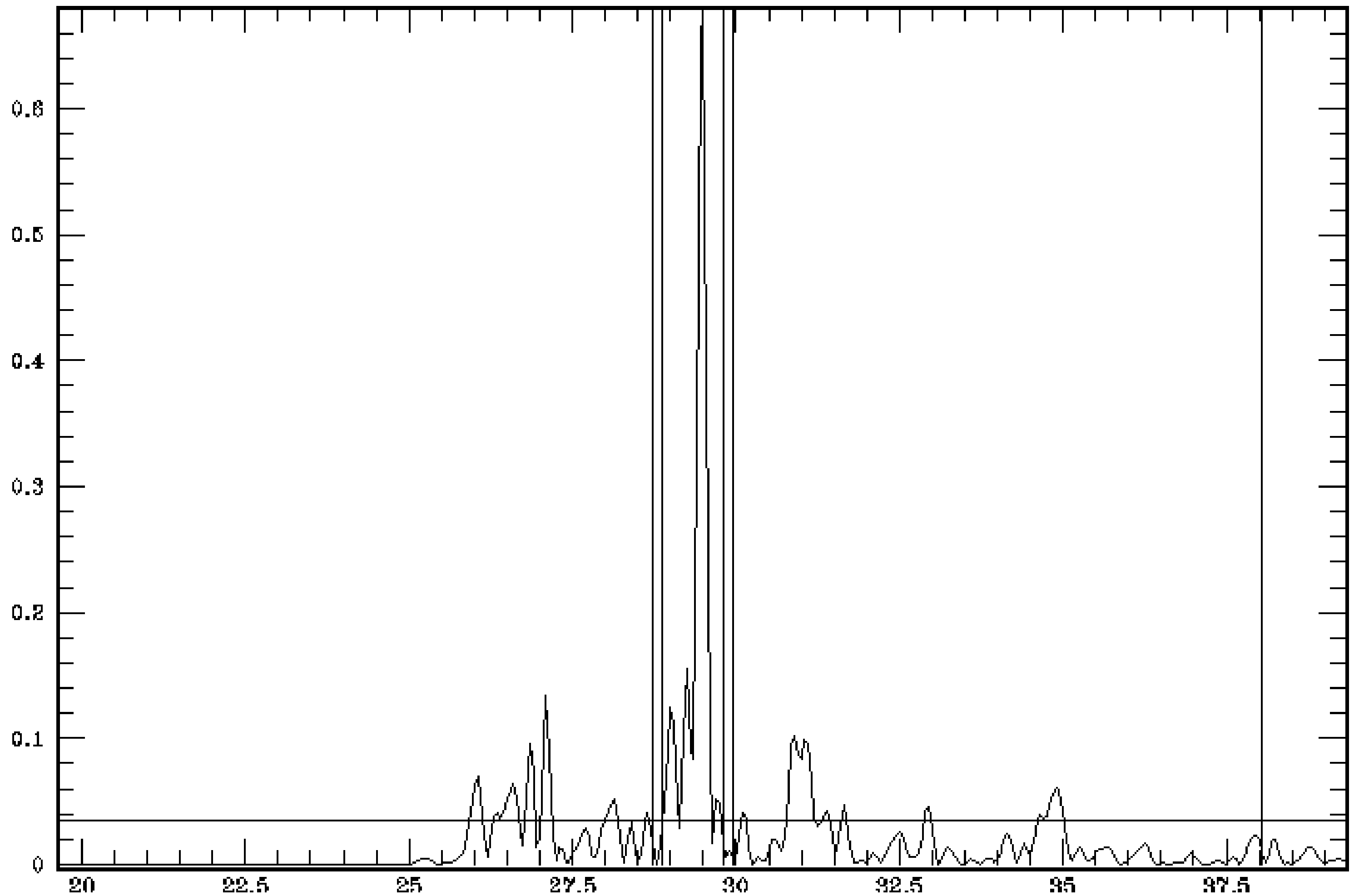
Input signal and its Fourier amplitude spectrum

Example 9: Near event - FTAN and

Analysis of component of recorded displacement:
TRANSVERSE component, event recorded at **8 km**.

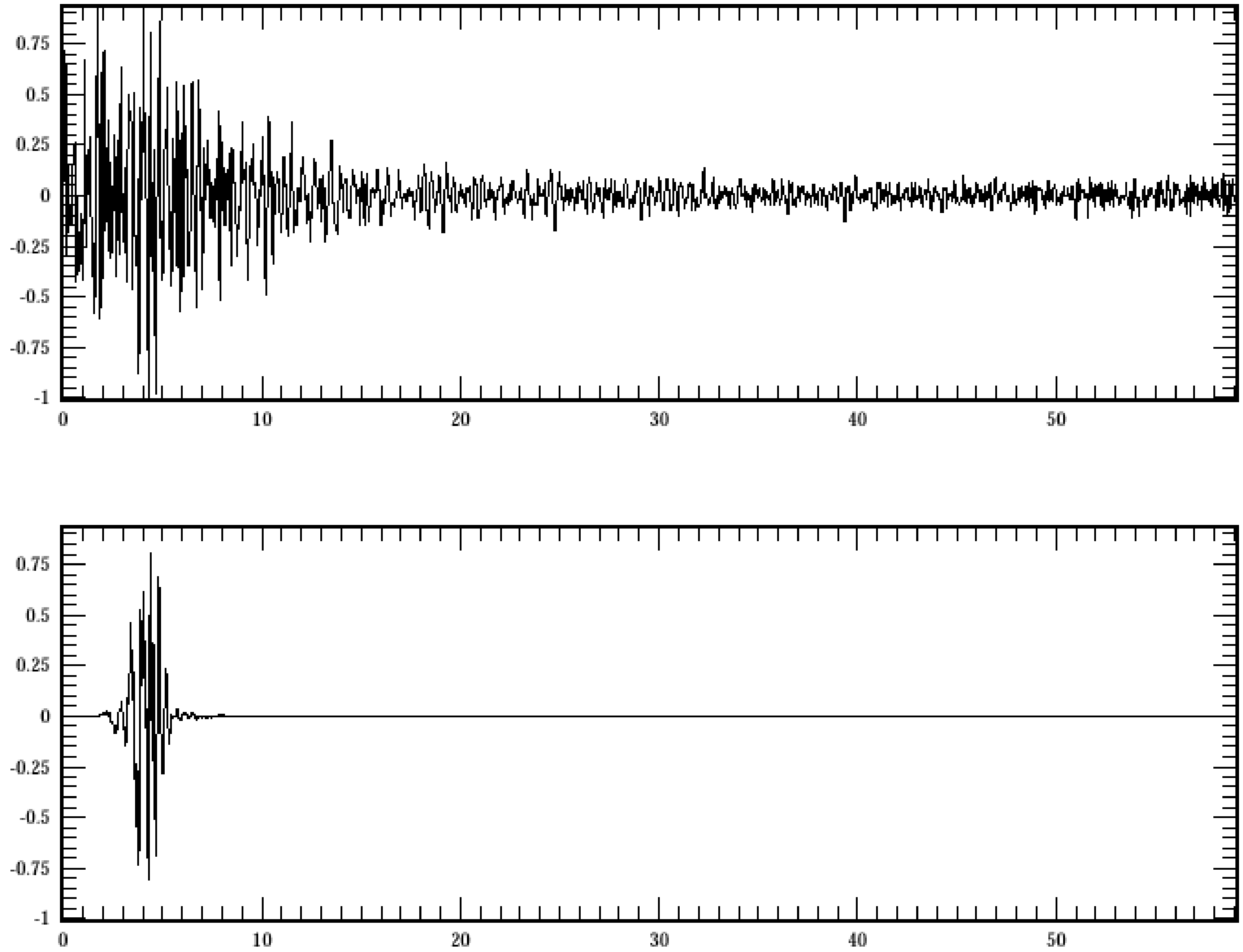


FTAN map and operator-defined dispersion curve, for phase equalization



Time domain: result of phase equalization and operator-defined time window

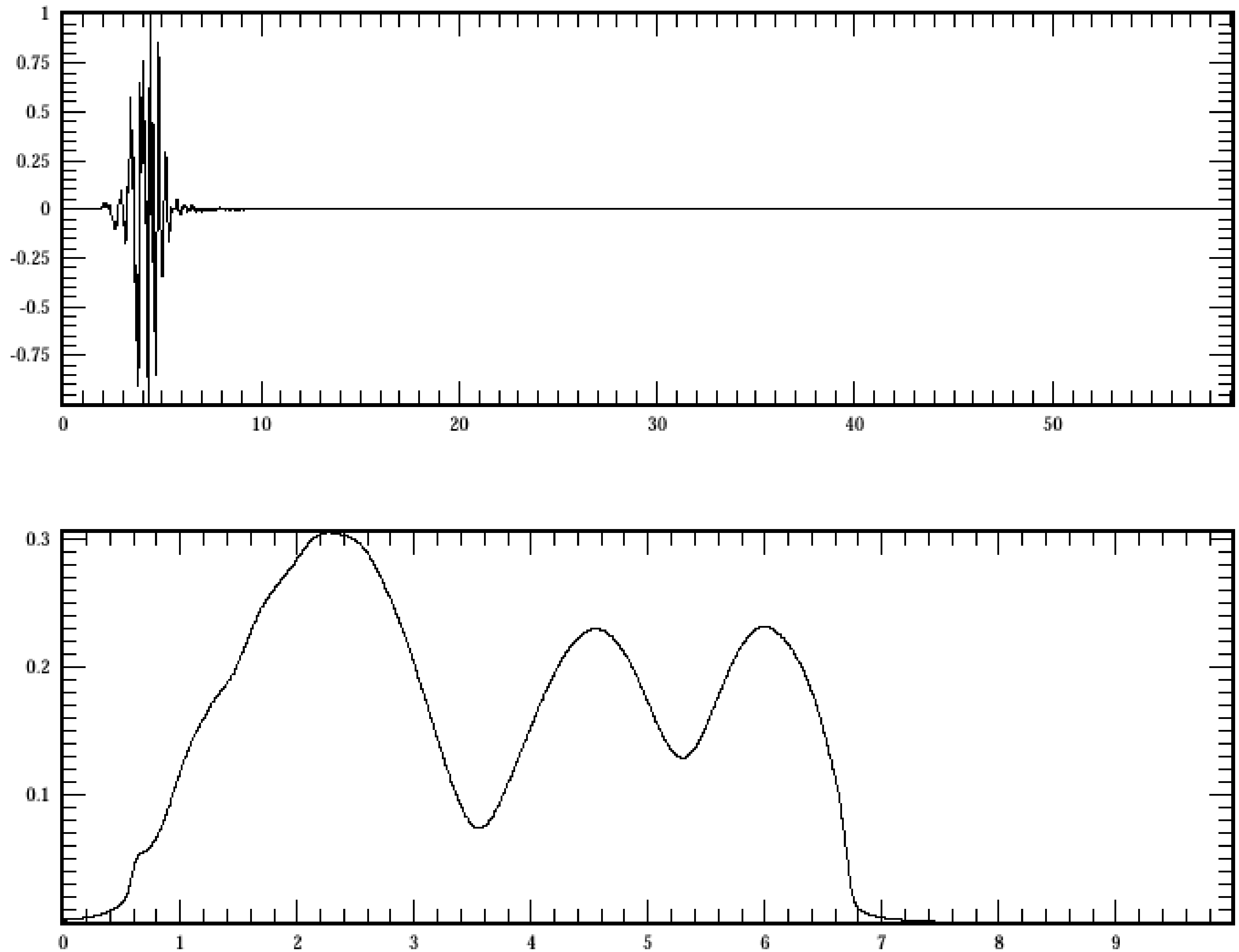
Analysis of component of recorded displacement:
TRANSVERSE component, event recorded at **8 km**.



Input and processed signal in time domain

Example 10: Near event - input

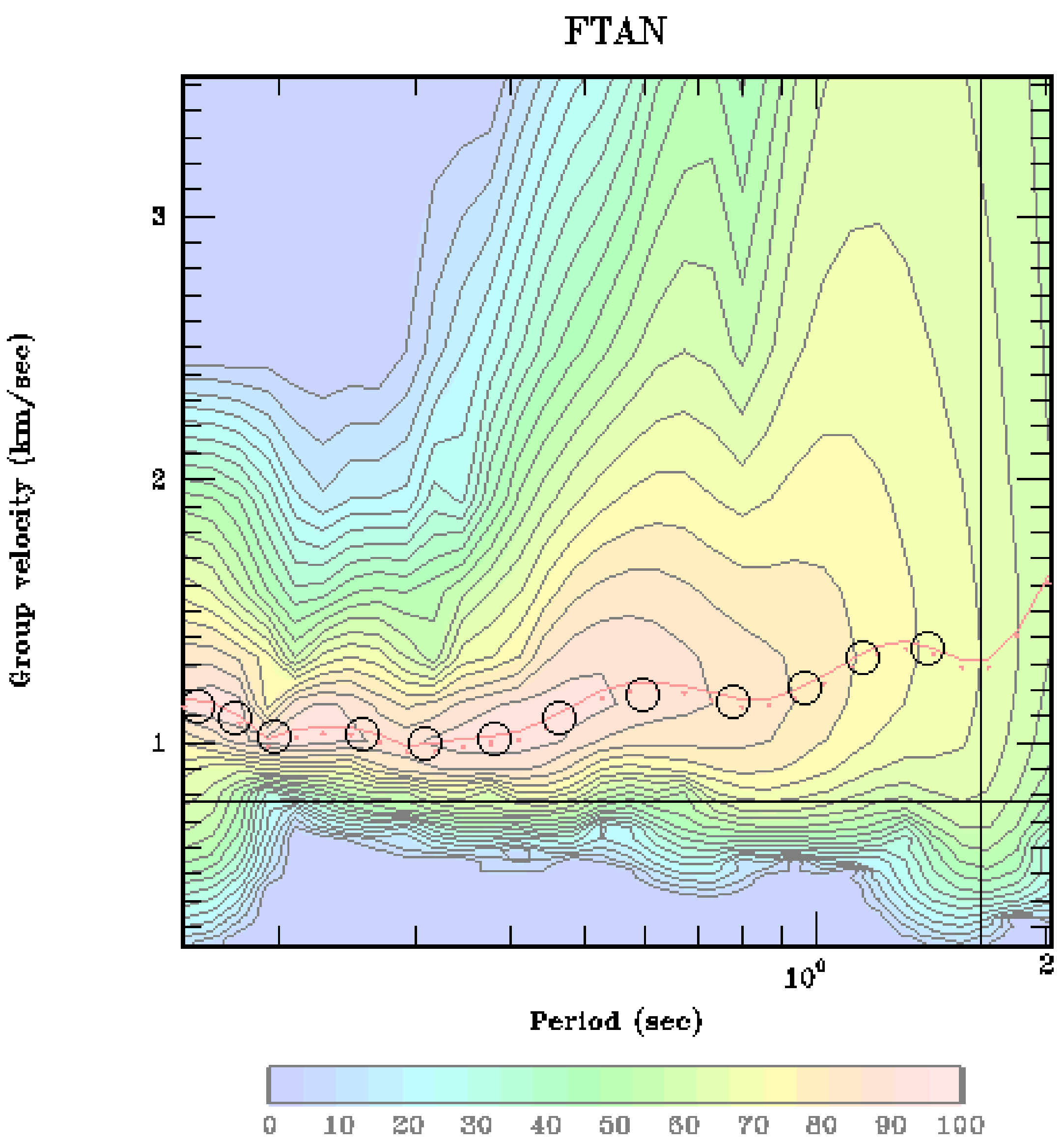
Analysis of component of **FTAN already processed** displacement:
TRANSVERSE component, event recorded at **8 km** .



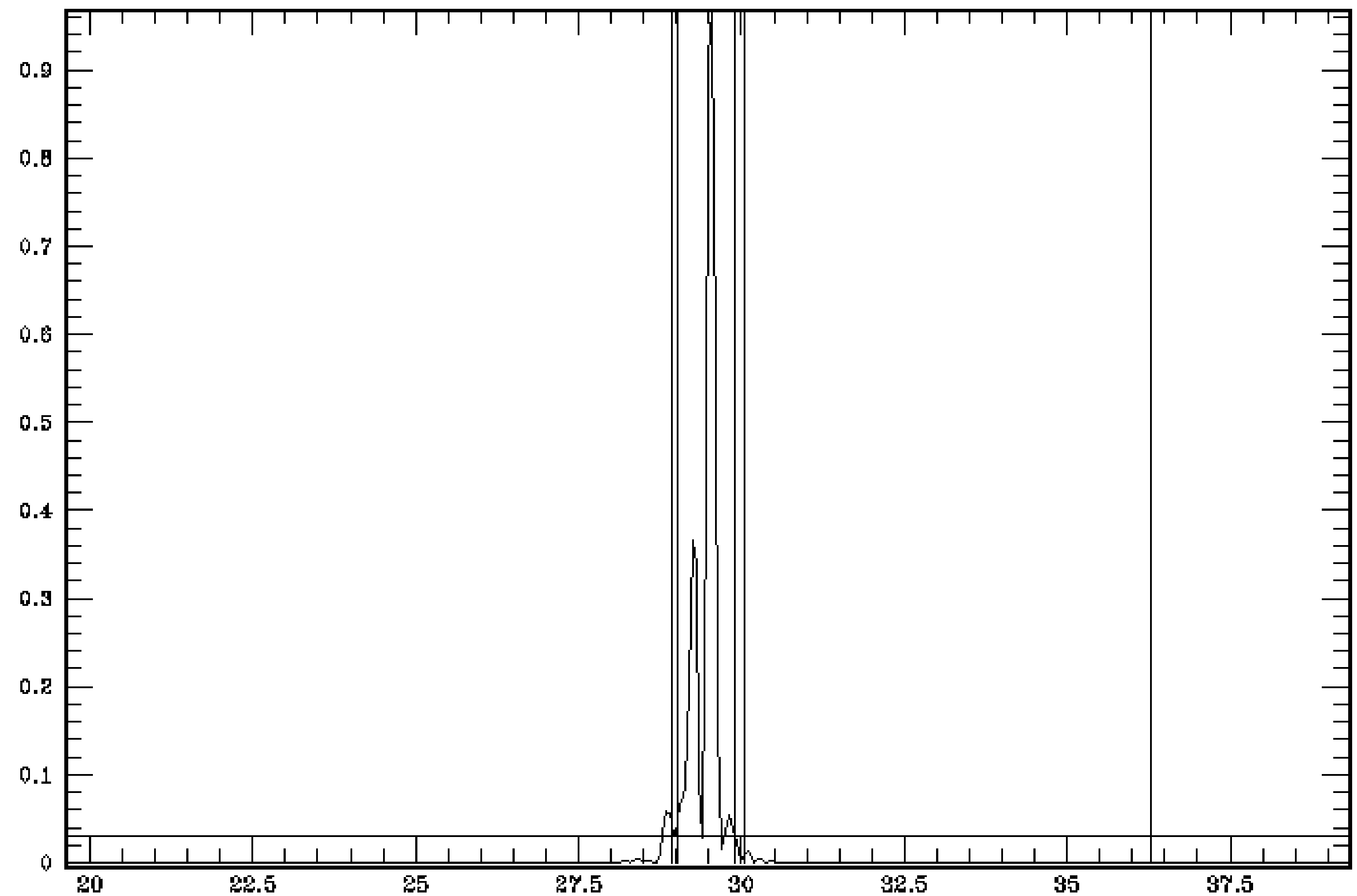
Input signal and its Fourier amplitude spectrum

Example 10: Near event - FTAN and

Analysis of component of **FTAN already processed** displacement: **TRANSVERSE** component, event recorded at **8 km** .



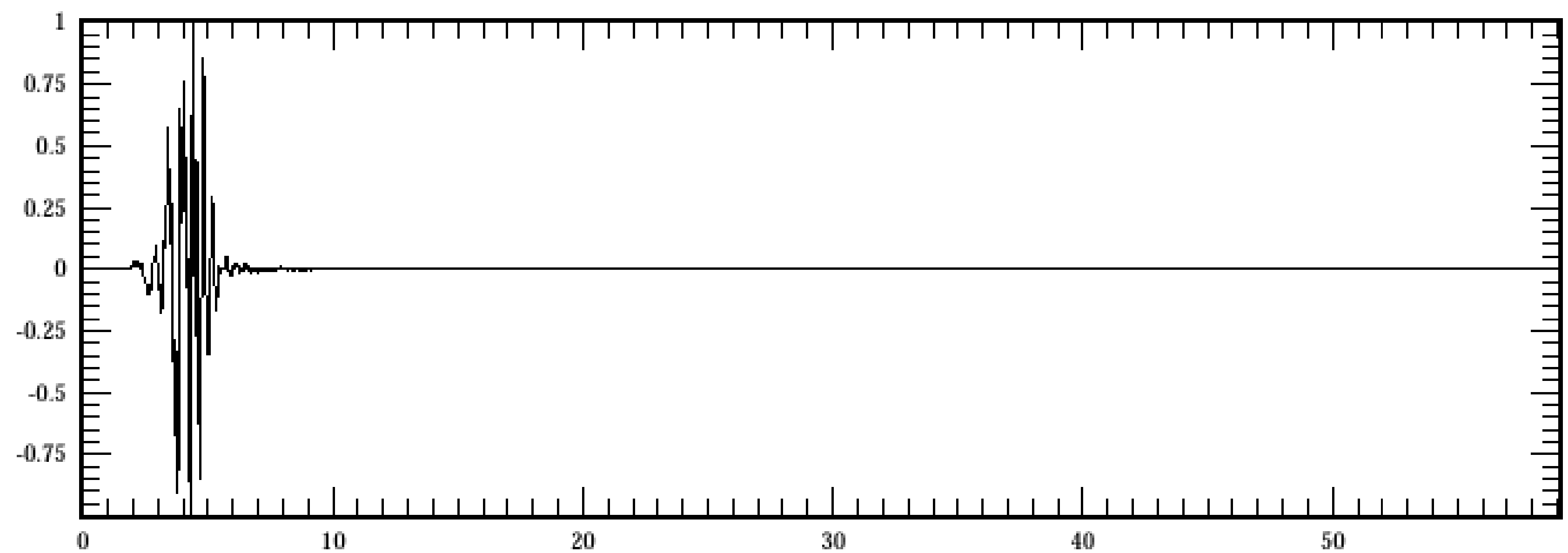
FTAN map and operator-defined dispersion curve, for phase equalization

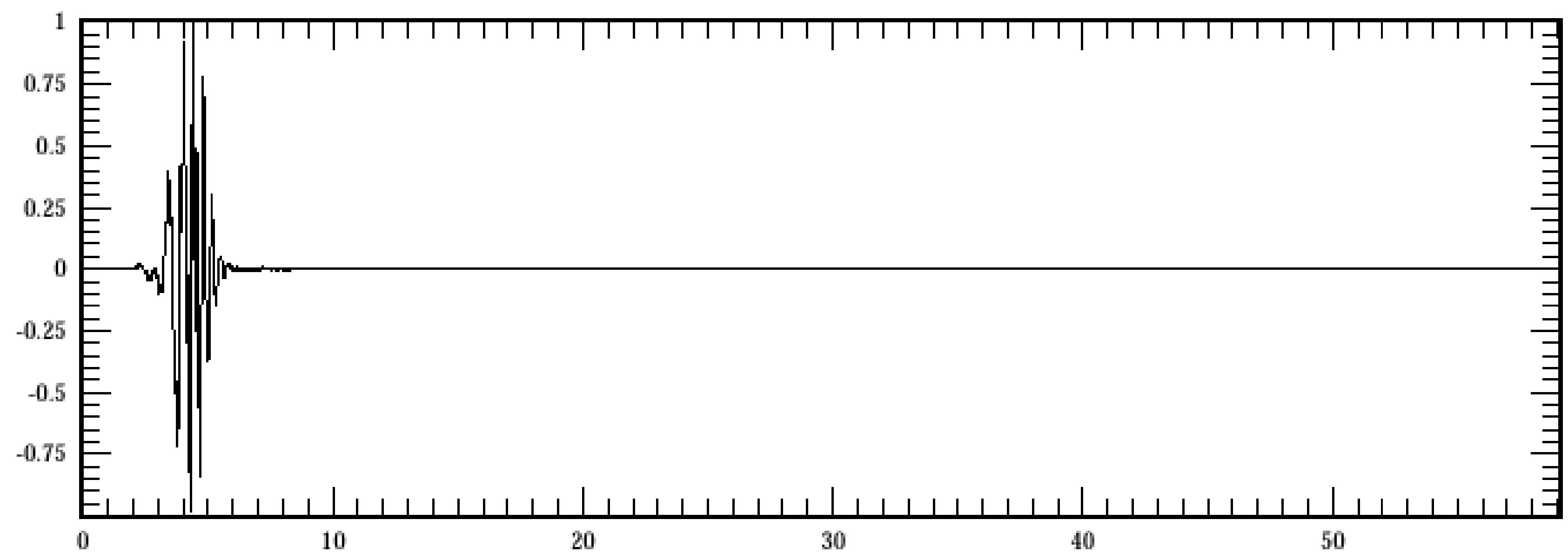


Time domain: result of phase equalization and operator-defined time window

Example 10: Near event - results

Analysis of component of **FTAN already processed** displacement: **TRANSVERSE** component, event recorded at **8 km** .



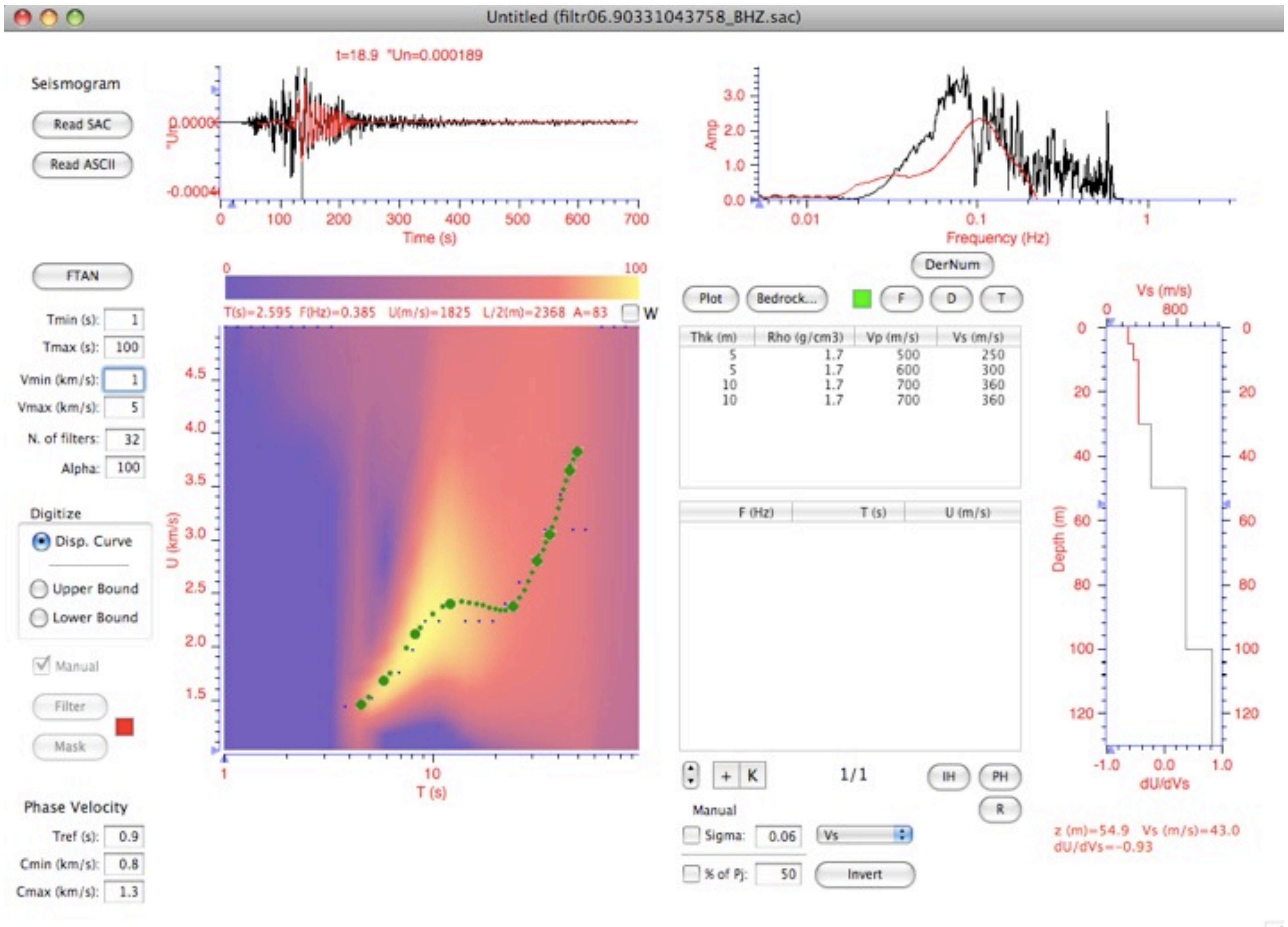


Input and processed signal in time domain

FTAN

- Frequency-Time representation:
  - Gaussian filters; FTAN maps
  - Floating filters: Phase equalization

e.g. Levshin et al., 1972

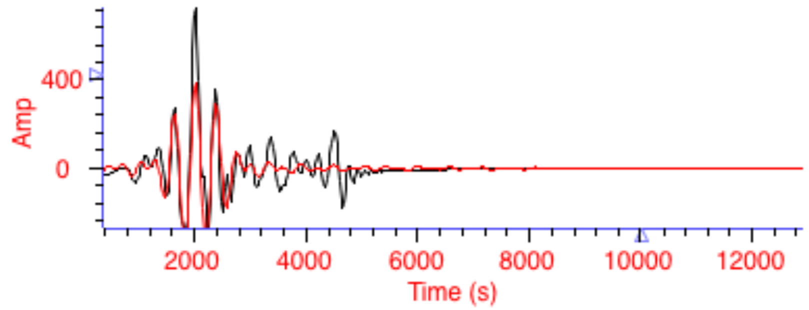


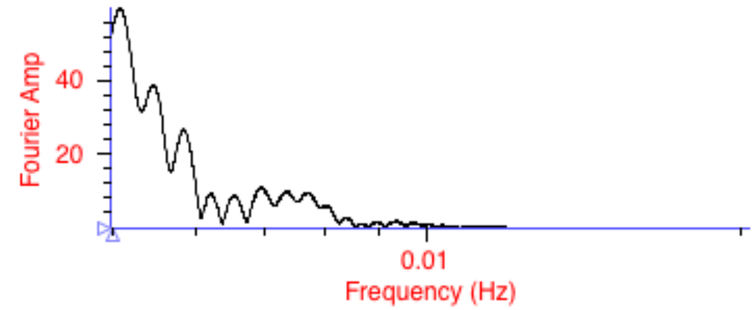
from XFTAN2012 (F. Vaccari)

FTAN - Tsunami signal

Untitled (japan10012.tsz.00001.ft)

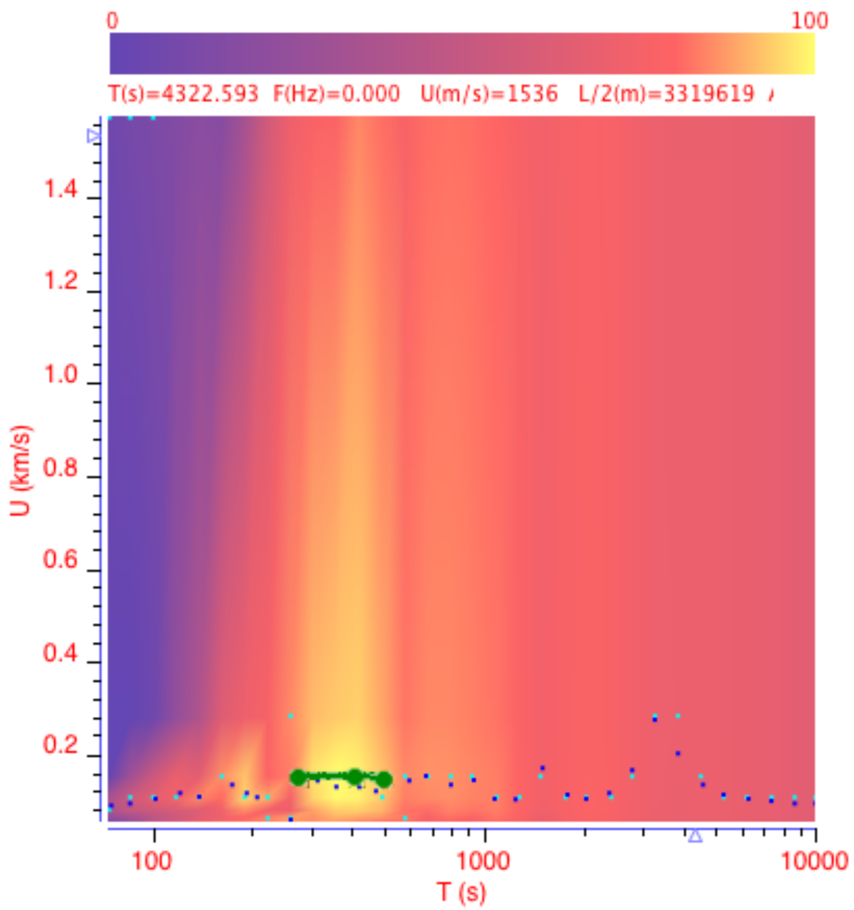
Dist. (km):	269.39
Samples:	4096
dt (s):	24.414





FTAN analysis

Tmin (s):	73.233
Tmax (s):	10000.
Umin (km/s):	0.055
Umax (km/s):	1.576
N. Filters:	32
Alpha:	100

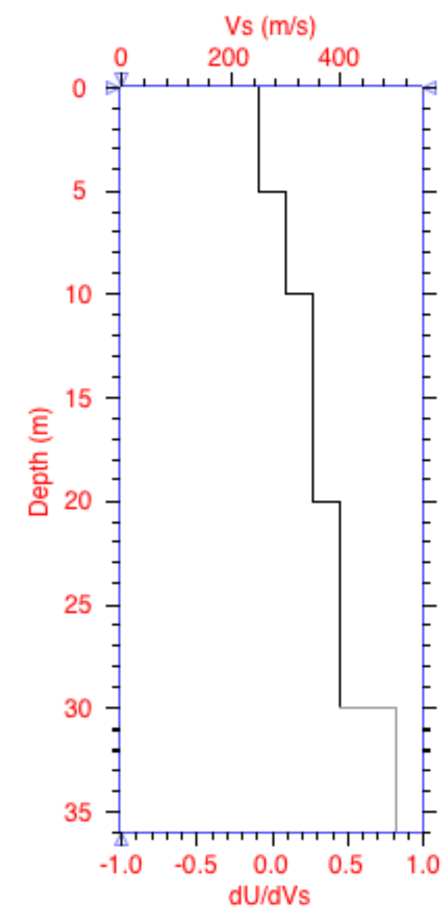


Structural Model

Depth (m)	Thk (m)	D (g/cm3)	Vp (m/s)	Vs (m/s)	Qp	Qs
5	5	1.7	500	250	220	100
10	5	1.7	600	300	220	100
20	10	1.7	700	350	220	100
30	10	1.7	800	400	220	100
50	20	1.8	900	500	220	100
100	50	1.9	1600	900	220	100
200	100	2	2000	1200	220	100
500	300	2.3	2250	1300	220	100
1000	500	2.3	2600	1500	220	100
1500	500	2.3	3200	1800	220	100
2000	500	2.3	3600	2100	220	100
2500	500	2.3	4000	2300	220	100
3000	500	2.3	4400	2500	220	100
3500	500	2.45	4800	2700	220	100
4000	500	2.5	5400	3100	220	100

Fundamental Mode

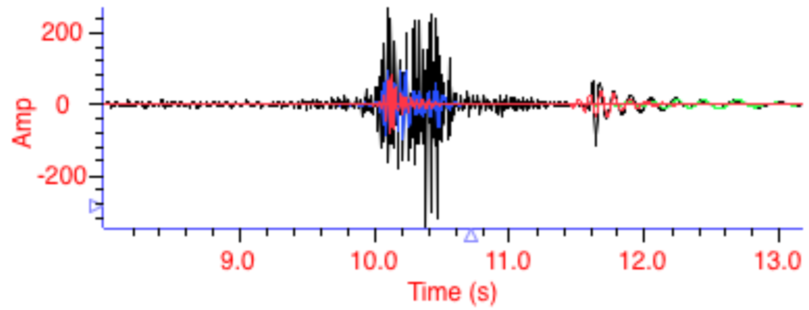
F (Hz)	T (s)	U (m/s)	C (m/s)

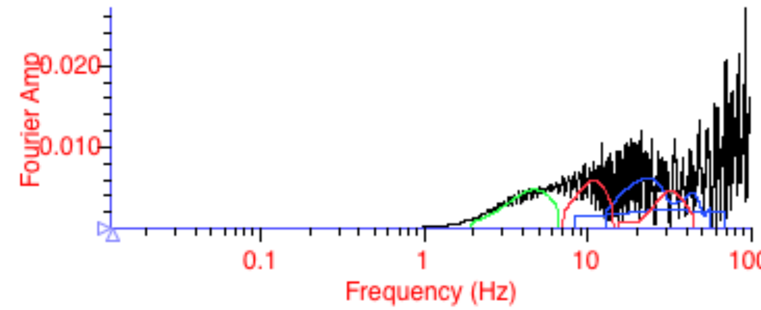


FTAN - Acoustic signal

nosed1 (edi.syz.00001.ft)

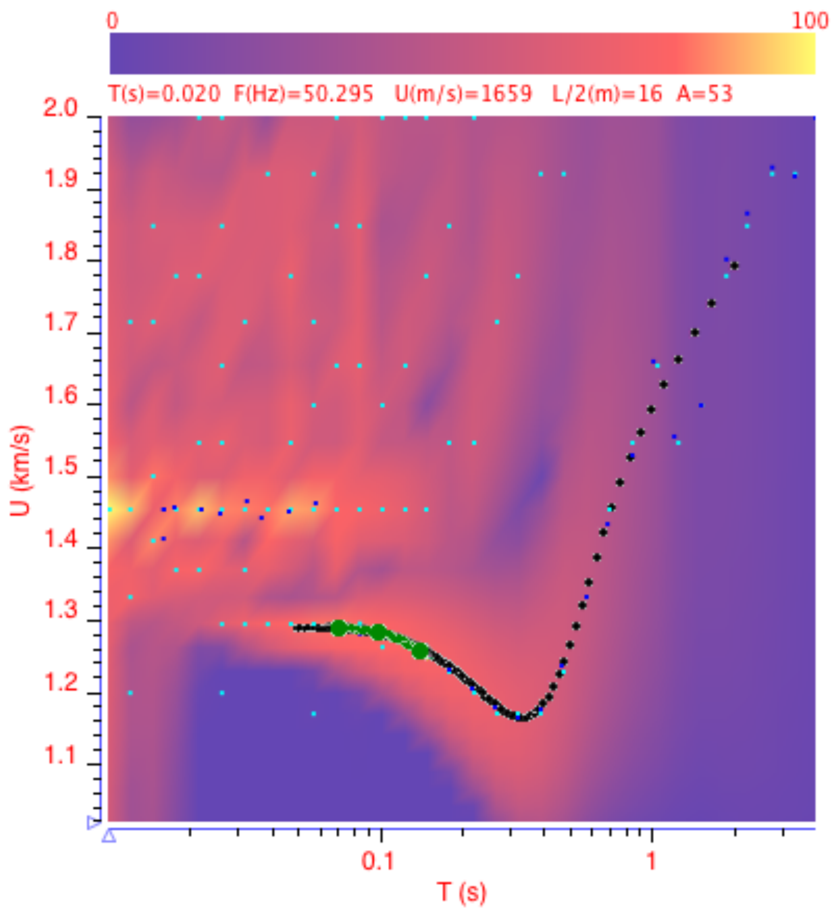
Dist. (km):	14.942
Samples:	8192
dt (s):	0.005





FTAN analysis

Tmin (s):	0.01
Tmax (s):	4.
Umin (km/s):	1.019
Umax (km/s):	2.
N. Filters:	32
Alpha:	100



Structural Model

Depth (m)	Thk (m)	D (g/cm3)	Vp (m/s)	Vs (m/s)	Qp	Qs
130	130	1	1500	0	10000	10000
230	100	2.45	2800	1618	1000	500
330	100	2.45	3000	1734	1000	500
430	100	2.45	3400	1965	1000	500
530	100	2.45	3500	2023	1000	500
780	250	2.45	3600	2081	1000	500
1030	250	2.45	3700	2139	1000	500
1530	500	2.45	3800	2196	1000	500
2030	500	2.45	3900	2254	1000	500
3030	1000	2.45	4000	2312	1000	500

Fundamental Mode

F (Hz)	T (s)	U (m/s)	C (m/s)
0.5000	2.00000	1791	1949
0.6000	1.66667	1741	1916
0.7000	1.42857	1700	1885
0.8000	1.25000	1663	1857
0.9000	1.11111	1628	1831
1.0000	1.00000	1594	1806
1.1000	0.90909	1559	1782
1.2000	0.83333	1525	1759
1.3000	0.76923	1490	1737