SEISMOLOGY I

Laurea Magistralis in Physics of the Earth and of the Environment

2D modes

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- Wave equation

$$v^2\nabla^2 u = v^2\Delta u = u_{tt}$$

- Laplacian in Cylindrical and Spherical systems

$$\Delta f = \frac{\partial^2 f}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial f}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \varphi^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\Delta f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial \varphi^2}$$

Special Coordinate systems

In these cases, the variable separation approach also facilitates the solution. In the Euclidian case the eigenfunctions were Fourier series. Here, after the substitution:

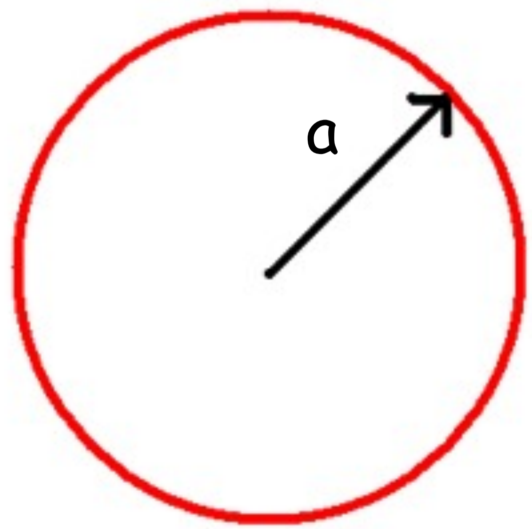
$$f = P(\rho) \cdot \Phi(\varphi) \cdot Z(z)$$

$$f = R(r) \cdot \Phi(\varphi) \cdot \Theta(\theta)$$

The differential equations arise, which solutions are special functions like Legendre polinomials or Bessel functions.

Circular Membrane Problem

A thin circular elastic membrane has a radius a:



and the wave equation, with a circular boundary condition, is:

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = 0$$

and if it has separable solutions:

$$u(r,\theta,t) = R(r)\Theta(\theta)T(t)$$

Variable separation

$$\Theta''(\theta) + m^2\Theta(\theta) = 0$$

$$\Theta(\theta) = C\cos(m\theta) + D\sin(m\theta)$$

m is a positive integer

$$T''(t) + c^2k^2T(t) = 0$$

$$T(t) = A\cos(\omega t) + B\sin(\omega t)$$

$\omega = ck$

$$\frac{1}{R}\frac{\partial^2 R}{\partial r^2} + \frac{1}{rR}\frac{\partial R}{\partial r} - \frac{m^2}{r^2} = \frac{1}{c^2 T}\frac{\partial^2 T}{\partial t^2} = -k^2$$

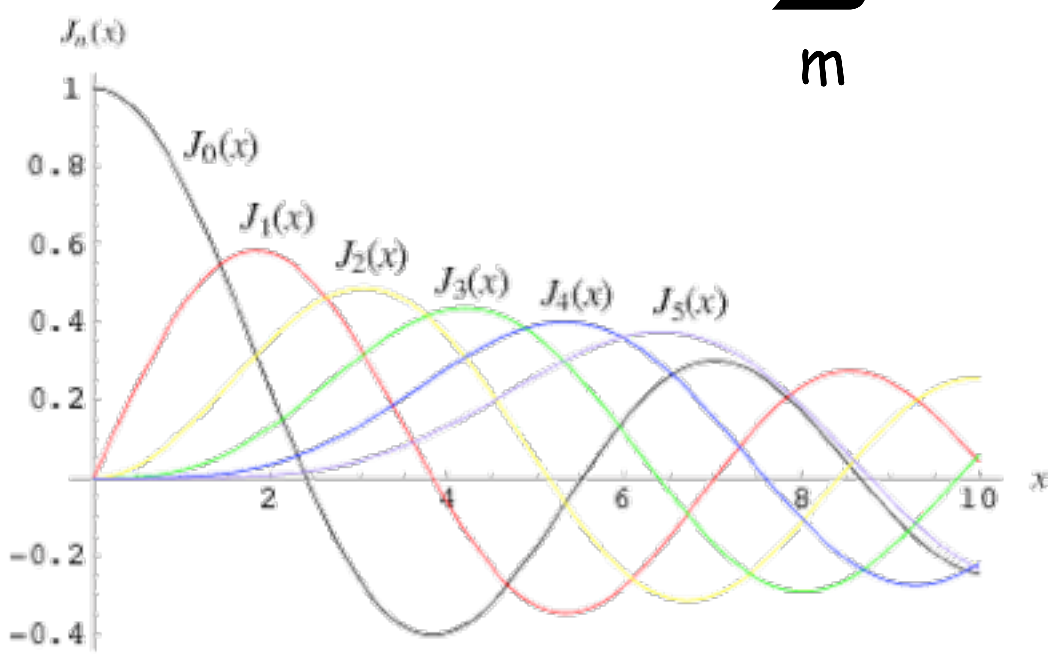
$$s^2\frac{d^2R}{ds^2} + s\frac{dR}{ds} + \left(s^2 - m^2\right)R = 0; \quad s = kr$$

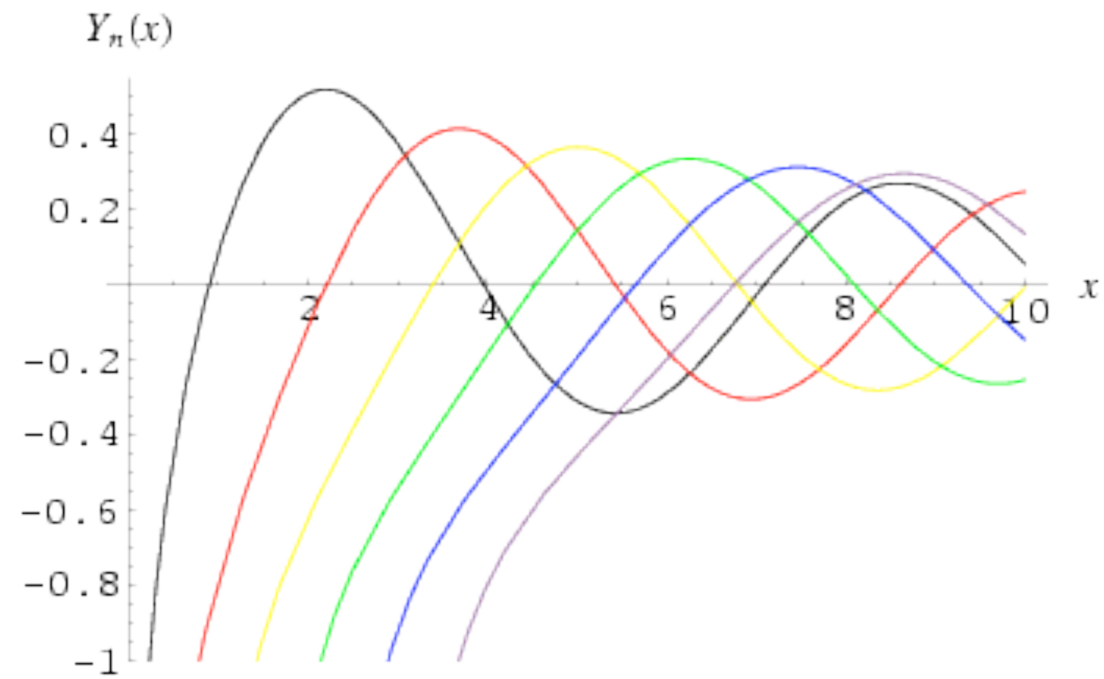
that is a Bessel equation of order m

$$x^2y'' + xy' + \left(x^2 - m^2\right)y = 0$$

and the general solution is:

$$y = \sum_m A_{1m} J_m(x) + A_{2m} Y_m(x)$$





that are to cylindrical waves what cosines/sines are to waves on a straight line.

The BC at the (regular singular) origin point is: R(0) is finite

$$R(s) = R(kr) = \sum_m A_m J_m(kr)$$

The radial factor of the solution is a **Bessel function of the first kind**: **NOT periodic and the distance between zeros is NOT constant.**

The other **BOUNDARY CONDITION** of the circular membrane problem is:

$$u=0 \text{ at } r=a$$

this implies that $$J_m(ka)=0$$

Therefore $$\frac{\omega a}{c} = \gamma_{mn}$$ nth positive zero of J_m

$$\omega_{mn} = \frac{c}{a}\gamma_{mn}$$

$$R(kr) \propto J_m\left(\frac{\gamma_{mn}}{a} r\right)$$

General solution

and the general solution is:

$$u = R(r)\Theta(\theta)T(t) = \sum_{m=0}^{\infty}\sum_{n=1}^{\infty}\left[C_{mn}\cos(m\theta) + D_{mn}\sin(m\theta)\right]\left[A_{mn}\cos(ck_{mn}t) + B_{mn}\sin(ck_{mn}t)\right]J_m(k_{mn}r)$$

but if we assume that the initial conditions are rotationally **symmetric**, i.e. goes like f(r), we have that we need only m=0

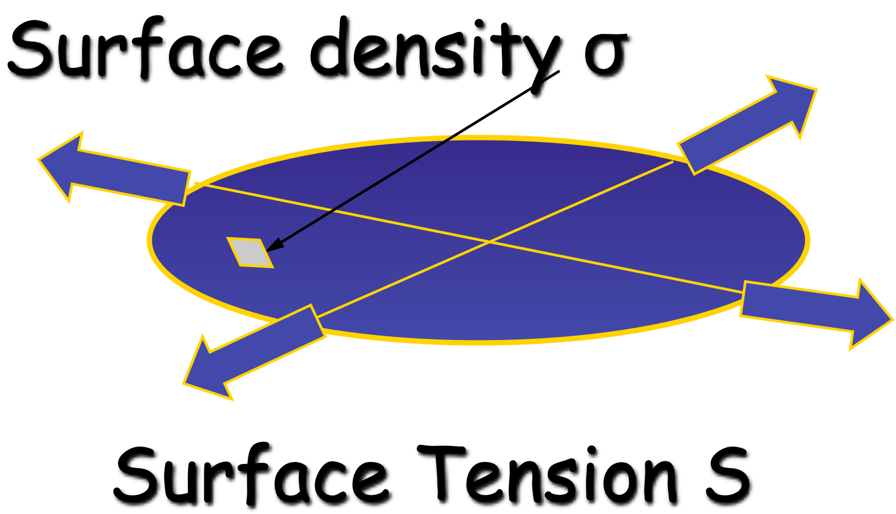
$$u = R(r)\Theta(\theta)T(t) = \sum_{n=1}^{\infty}\left[A_n\cos(ck_n t) + B_{mn}\sin(ck_n t)\right]J_0(k_n r)$$

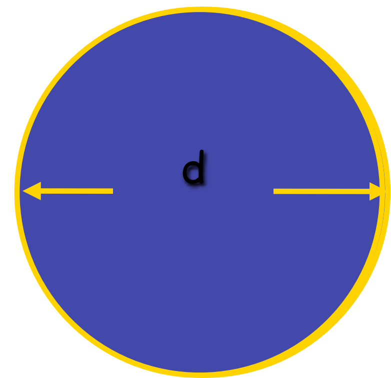
with

$$k_n = \frac{\gamma_n}{a} = \frac{\gamma_{0n}}{a} = \frac{\omega_{0n}}{c}$$

to be determined with the proper **initial conditions!**

Oscillation of a Clamped Membrane





Mode: (0,1)

$f_{0\,1} = v/\lambda; \quad v = \sqrt{(S/\sigma)}$

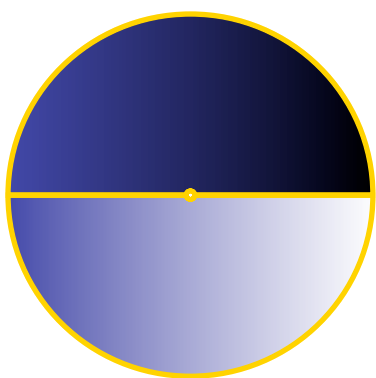
$f_{0\,1} = x_{0\,1}/(\pi\, d) \cdot \sqrt{(S/\sigma)}$

$x_{0\,1} = 2.405$

Surface density σ= mass/area σ= density · thickness

Surface Tension S= force/length

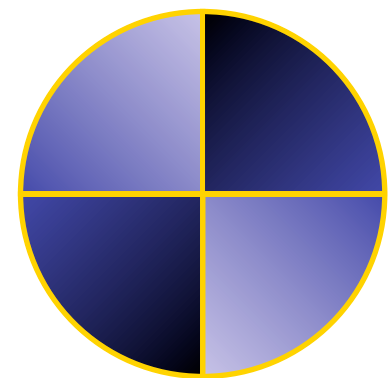
Membrane vs string



Mode: (1,1)

$f_{1\,1} = (x_{1\,1} / x_{0\,1})\, f_{0\,1}$

$x_{1\,1} / x_{0\,1} = 1.594$

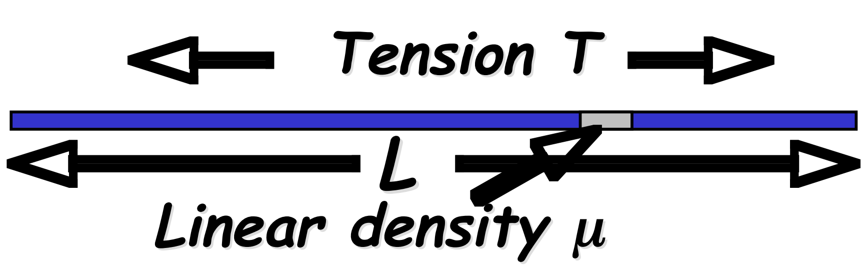


Mode: (2,1)

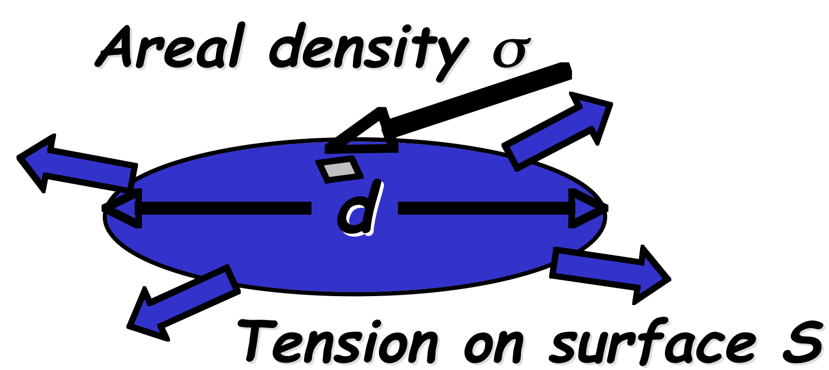
$f_{2\,1} = (x_{2\,1} / x_{0\,1})\, f_{0\,1}$

$x_{2\,1} / x_{0\,1} = 2.136$

http://www.kettering.edu/~drussell/Demos/MembraneCircle/Circle.html

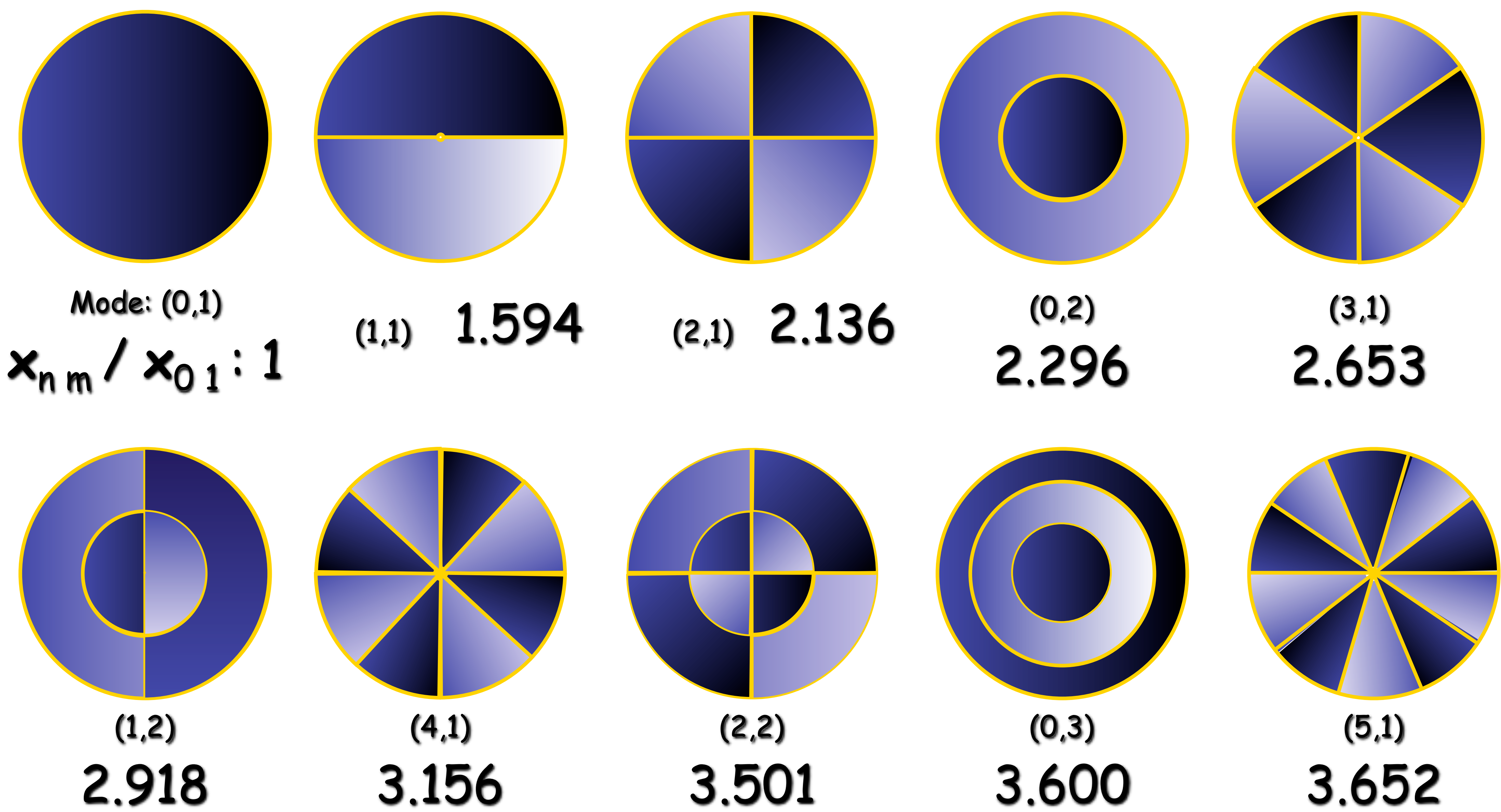


$f_n = n/(2L)\,(T/\mu)^{1/2}$
$n = 1, 2, 3, 4, 5, 6, \ldots$

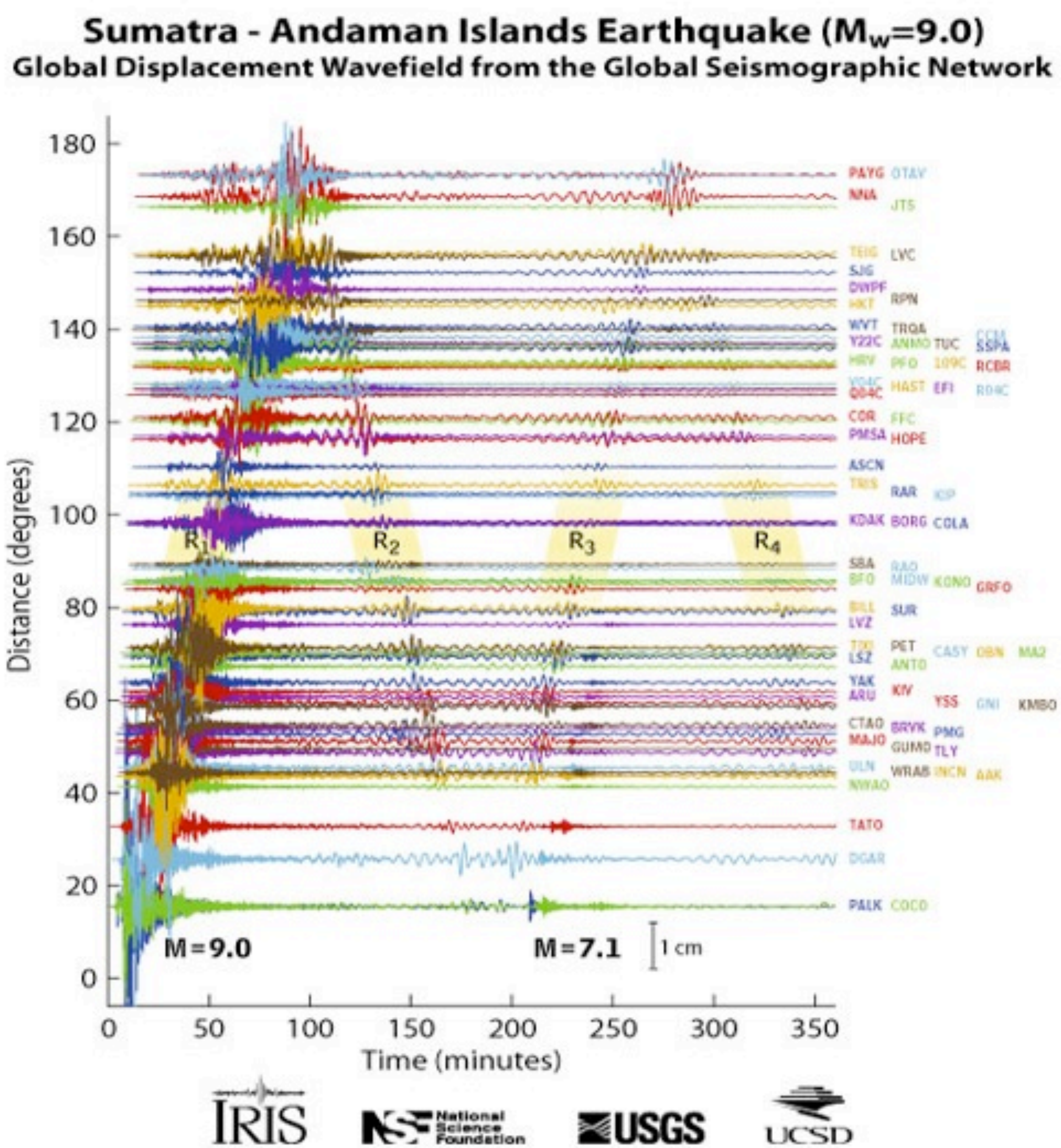


$f_{nm} = x_{nm}/(\pi d)(S/\sigma)^{1/2}$
$x_{0\,1} = 2.405$

Membrane modes



Traveling surface waves



Vertical displacements of the Earth's surface recorded by seismometers.
The traces are arranged by distance from the epicenter in degrees. The earliest, lower amplitude, signal is that of the compressional (P) wave, which takes about 22 minutes to reach the other side of the planet (the antipode).
The largest amplitude signals are seismic surface waves that reach the antipode after about 100 minutes. The surface waves can be clearly seen to reinforce near the antipode (with the closest seismic stations in Ecuador), and to subsequently circle the planet to return to the epicentral region after about 200 minutes.
A major aftershock (magnitude 7.1) can be seen at the closest stations starting just after the 200 minute mark (note the relative size of this aftershock, which would be considered a major earthquake under ordinary circumstances, compared to the mainshock).